Part IV

NONLOCAL ELECTRODYNAMICS

B. Mashhoon

NONLOCAL ELECTRODYNAMICS*

Bahram Mashhoon
Department of Physics and Astronomy
University of Missouri-Columbia
Columbia, Missouri 65211, USA

Abstract

The physical structure of the relativistic theory of gravitation is discussed. The significant role that the hypothesis of locality plays in the framework of general relativity is elucidated, and the limitations of this hypothesis are pointed out. Nonlocal electrodynamics of a uniformly rotating system is presented; the theory is based on the idea that the propagation of electromagnetic radiation is independent of observers. The general nonlocal theory of accelerated observers in Minkowski spacetime goes beyond the hypothesis of locality and appears to be in agreement with available experimental data. The nonlocal theory excludes the possibility of existence of a fundamental scalar field in nature.

*This paper is based on lectures delivered at the VII Brazilian School of Cosmology and Gravitation (August 1-14, 1993).

Chapter 15

Physical Structure of General Relativity

Of all the basic theories of physics, the only one that does not have a basis in microphysics is gravitation. It is not clear at present how to bring the macrophysical description of gravitation into harmony with the quantum theory. It would therefore be interesting to reduce general relativity to its basic elements in order to clarify the physical structure of the macrophysical theory of gravitation. The standard relativistic theory of gravitation, i.e., Einstein's general relativity, is the most successful theory of gravitation at present, since it is in excellent agreement with all available observational data [1]. This theory is based on two main ideas: the first is the notion that the result of any physical measurement should be independent of the choice of coordinates that are assigned to events in spacetime, so that observables are scalar invariants. The second idea is the Einstein principle of equivalence, which provides a heuristic connection between a gravitational field and an accelerated frame. These ideas provide a framework in which Newtonian gravity is generalized in such a way that it becomes consistent with Lorentz invariance.

To reduce the theory to its essential elements, it is necessary to begin with the Lorentz invariance of Maxwell's equations. Imagine an observer at rest in Minkowski spacetime. The absolute spacetime of Minkowski corresponds to an ensemble of inertial reference frames each moving uniformly with respect to the others. Let us choose a member of this ensemble; for instance, this preferred inertial frame could correspond to the rest frame of the cosmic microwave background radiation. Observers at rest in this frame can be thought of as carrying an orthonormal tetrad frame $\tilde{\lambda}^{\mu}_{(\alpha)}$, where $\tilde{\lambda}^{\mu}_{(0)}$ is the vector tangent to the worldline of the observer and thus corresponds to the time axis and the spatial axes are then determined by $\tilde{\lambda}^{\mu}_{(i)}$, $i = 1, 2, 3$. All measurements are performed with respect to the observers' space-time axes, which for the preferred set of observers are $\tilde{\lambda}^{\mu}_{(\alpha)} = \delta^{\mu}_{\alpha}$; therefore, the electric and magnetic fields that appear in Maxwell's equations in the preferred frame are the fields as measured by the static observers in that frame. The Lorentz invariance of Maxwell's equations implies that the electromagnetic field in any inertial frame is in fact the field as measured by inertial observers at rest in that frame. Alternatively, one could consider all such observers as moving uniformly in a preferred spacetime frame; then, each such observer would carry an orthonormal tetrad $\lambda^{\mu}_{(\alpha)}$ that is related to the tetrad of the preferred static observers by a member of the Lorentz group. The electromagnetic field measured by such an observer is simply the projection of the Faraday tensor on the tetrad frame of the observer,

$$F'_{\alpha\beta} = F_{\mu\nu}\lambda^{\mu}_{(\alpha)}\lambda^{\nu}_{(\beta)} \quad , \tag{1.1}$$

where $F_{\mu\nu}$ is the Faraday tensor measured by the preferred static observers.

The description of phenomena according to inertial observers via the theory of Lorentz invariance is limited in two significant respects. The first restriction is that inertial observers use only Cartesian coordinates $x^{\alpha} = (t, \mathbf{x})$ such that the metric is always of the form $ds^2 = \eta_{\alpha\beta}dx^{\alpha}dx^{\beta}$, where $\eta_{\alpha\beta}$ is the Minkowski metric tensor with signature +2 and the speed of light c is unity unless specified otherwise. Consider assigning coordinates $(t', \mathbf{x}')$ to the event characterized by inertial coordinates $(t, \mathbf{x})$ such that $t' = t'(t, \mathbf{x})$ and $\mathbf{x}' = \mathbf{x}'(t, \mathbf{x})$. *Inertial* observers can refer physical phenomena to arbitrary systems of smooth coordinates. To show this, let us assume that the spacetime interval is invariant under such passive coordinate transformations; this assumption is a simple generalization of the fact that the Euclidean distance between two points in space is independent of whether one uses Cartesian coordinates or, say, spherical coordinates. It is then a simple matter to express the equation of motion of a particle with respect to the new coordinate system. In particular, the Lorentz force law

$$m\frac{d^2x^\mu}{d\tau^2} = qF^\mu{}_\nu \frac{dx^\nu}{d\tau} \tag{1.2}$$

for a particle of mass m and charge q in Minkowski spacetime takes the form

$$m\left[\frac{d^2x'^\mu}{d\tau^2} + \Gamma'^\mu_{\rho\sigma}(x')\frac{dx'^\rho}{d\tau}\frac{dx'^\sigma}{d\tau}\right] = qF'^\mu{}_\nu(x')\frac{dx'^\nu}{d\tau} \quad , \tag{1.3}$$

where

$$F'^{\rho\sigma}(x') = \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu}F^{\mu\nu}(x) \quad , \tag{1.4}$$

and in the new coordiante system indices are raised and lowered via $g'_{\mu\nu}(x')$ given by

$$g'_{\mu\nu} = \eta_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} \quad , \tag{1.5}$$

so that $ds^2 = g'_{\mu\nu}(x')dx'^\mu dx'^\nu$ remains invariant. The Christoffel connection $\Gamma'^\mu_{\rho\sigma}$ is given in Minkowski spacetime by

$$\Gamma'^\mu_{\rho\sigma} = \left(\frac{\partial x'^\mu}{\partial x^\alpha}\right)\frac{\partial^2 x^\alpha}{\partial x'^\rho \partial x'^\sigma} \quad , \tag{1.6}$$

which vanishes if the coordinate transformation is linear. The Christoffel connection is therefore absent in all inertial frames. It is important to recognize that the proper time interval τ along the path of the charged particle is in fact determined by static inertial observers in Minkowski spacetime; these observers determine the speed of the particle and hence the particle's Lorentz γ-factor. The proper time interval is then calculated from the integration of $d\tau = dt/\gamma$. The auxiliary field variables satisfy the covariant form of Maxwell's equations; that is, equation (1.4) can be inverted and the field in the inertial frame can be expressed in terms of the field with respect to arbitrary coordinates (i.e., the auxiliary field). Maxwell's equations for the field in the inertial frame then imply that the auxiliary field satisfies the covariant form of Maxwell's equation with the covariant derivative defined through the connection (1.6). In a given situation, the boundary conditions and the symmetries of the problem may be such that the solution of the covariant equations could be simpler; the actual field is then determined via the inverse of equation (1.4), i.e.,

$$F'_{\rho\sigma}\,\frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}} = F_{\mu\nu} \quad . \tag{1.7}$$

This is entirely analogous to the standard treatment of electrodynamics where curvilinear coordinates are routinely employed for the sake of simplicity. The auxiliary field $F'_{\mu\nu}$ has no direct physical significance, it can only facilitate the determination of the Faraday tensor $F_{\mu\nu}$; however, $F'_{\mu\nu}(x')$ can be considered to be the components of the electromagnetic field in the new coordinates since the form $F_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu}$ is a geometric invariant. That is, the components of the tetrad frame of the preferred inertial observers with respect to the new coordinates are

$$\tilde{\lambda}'^{\mu}_{(\alpha)} = \frac{\partial x'^{\mu}}{\partial x^{\sigma}}\tilde{\lambda}^{\sigma}_{(\alpha)} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \quad . \tag{1.8}$$

Therefore, the tensor transformation rule (1.7) becomes $F'_{\mu\nu}\tilde{\lambda}'^{\mu}_{(\alpha)}\tilde{\lambda}'^{\nu}_{(\beta)} = F_{\alpha\beta}$, so that the field as measured in the inertial frame can be equally well determined in the new coordinate system by the projection of the field on the tetrad frame of the observer. The resulting framework is consistent as well as elegant. Thus the Maxwell-Lorentz theory has been extended to arbitrary coordinates by inertial observers without introducing any new basic physical assumption into the theory. The structure of the argument is important: The renaming of events by new coordinates is pointwise; therefore, the equation of motion of a point charge with respect to the new coordinates can naturally lead to the identification of the auxiliary electromagnetic field and the transformation rule for the field under arbitrary coordinate transformations. The approach outlined here can be extended to other fields as well through their interactions with the electromagnetic field. The mathematical procedure that emerges is tensor calculus on the flat spacetime manifold. It can be simply extended to the Riemannian manifolds of general relativity.

The first limitation of the theory of Lorentz invariance has thus turned out to be of a purely mathematical nature; that is, inertial observers can employ any system of admissible coordinates. The second restriction of the theory is that only inertial observers are permitted to make physical measurements. This is a fundamental physical restriction, since all actual observers are accelerated. Inertial observers are fictitious: any attempt to verify that an observer is indeed inertial will necessarily leave the (presumed inertial) observer accelerated. It is important to note that the quantum theory of measurement is thus far exclusively concerned with inertial observers. Most laboratory experiments, however, take place on the Earth which—among other motions—rotates about its axis. In fact, the whole observational basis of Lorentz invariance as well as the quantum theory rests upon experiments in accelerated reference frames. The success of these theories must therefore be accounted for by any reasonable theory of accelerated systems. It is natural to suppose that a connection needs to be established between actual accelerated observers and ideal inertial observers.

The second restriction is removed in the standard theory by a hypothesis of locality: at any instant, an accelerated observer is presumed to be locally equivalent to a comoving inertial observer. This pointwise equivalence of an accelerated observer with a class containing a continuous infinity of hypothetical ideal instantaneously comoving inertial observers is the physical basis in the standard theory for extending the theory of Lorentz invariance to accelerated observers. It follows from the hypothesis of locality that a tetrad frame can be associated with the accelerated observer along its path. This frame coincides at each instant with the tetrad frame of the comoving inertial observer up to a spatial rotation. Thus all pointwise measurements of the accelerated observer can be expressed in terms of the corresponding measurements of comoving inertial observers. For instance, the time measured by the accelerated observer using a comoving clock is in fact the proper time τ in accordance with the hypothesis of locality. This is usually referred to as the "clock hypothesis"; therefore, the hypothesis of locality replaces various more specialized hypotheses that exist in the literature of standard relativity theory regarding the measurements of accelerated systems.

The removal of both restrictions from the theory of Lorentz invariance leads to a theory that satisfies the so-called principle of general covariance [2]. The physical content of general covariance is a prescription for what observers measure. A generally covariant theory accommodates observers with arbitrary acceleration employing any system of coordinates that is suitable for the problem under consideration. It should be clear from the foregoing discussion that once a consistent prescription is given for the measurements of an accelerated observer, a generally covariant theory can be developed. The measured quantities are scalar invariants since this must be the case in the Lorentz invariant theory and this property is preserved by tensor calculus once the theory is extended to arbitrary coordinate systems. In the case under consideration, for instance, the electromagnetic field measured by an accelerated observer with respect to inertial coordinates is $F'_{(\alpha)(\beta)} = F_{\mu\nu}\lambda^{\mu}_{(\alpha)}\lambda^{\nu}_{(\beta)}$, where

$\lambda^{\mu}_{(\alpha)}(\tau)$ is the tetrad frame of the accelerated observer. The measured field component is determined via the hypothesis of locality by equation (1.1) for a momentarily comoving inertial observer; however, the result is expressed here as $F'_{(\alpha)(\beta)}$ for notational convenience. The accelerated observer can employ coordinates in which it is at rest ("comoving coordinates"), in that case $F'_{(\alpha)(\beta)} = F'_{\mu\nu}\lambda'^{\mu}_{(\alpha)}\lambda'^{\nu}_{(\beta)}$, where $F'_{\mu\nu}$ is the Faraday tensor with respect to the accelerated coordinates and satisfies the covariant form of Maxwell's equations. This tensor, $F'_{\mu\nu}$, still has no direct physical significance; only its projection onto the frame of the observer expressed with respect to the accelerated coordinates is physically significant, since it is the measured electromagnetic field.

Gravitation is introduced into this scheme via Einstein's principle of equivalence. According to this heuristic principle, an observer—i.e., a classical measuring device—moving freely in a gravitational field is locally equivalent with an observer that is accelerated with respect to Minkowski spacetime; the uniqueness of this acceleration is of central importance for the development of the geometric theory of gravitation. The Newtonian version of this idea is a direct consequence of the universality of the gravitational interaction, i.e., the equality of gravitational and inertial masses for any classical point particle.

In Newtonian mechanics, any force acting on a test particle of unit inertial mass is at each instant equivalent to the acceleration of the particle according to Newton's second law of motion. Furthermore, the test particle is instantaneously inertial. That is, the state of a particle is determined by its absolute position and velocity at any instant of absolute time; therefore, the hypothesis of locality holds in Newtonian mechanics since the accelerated particle and the instantaneously comoving inertial particle have the same classical state. It follows from these considerations that every test particle under the influence of Newtonian forces is locally inertial. It proves useful to state this result in a different way: The action of a force on a free test particle is locally equivalent to the action of a certain inertial force. In this way, one can obtan a simple generalization of Larmor's theorem that would be applicable to a particle in an arbitrary classical force field [3]. Thus at each instant of time and every point in space, the forces may be replaced by local accelerated frames. It turns out that in the case of gravitation such accelerated frames are *unique* in the sense that they are independent of the nature of the test particles involved. This follows from the principle of equivalence of inertial and gravitational masses. The universality of the gravitational interaction thus implies that the corresponding local accelerated frames are only functions of position and time. The pointwise unique gravitational acceleration thus serves as the connection between the local inertial frames. These underlying ideas can be simply generalized to the relativistic domain [4]. They made it possible for Einstein and Grossmann to propose a geometric theory of gravitation in 1913. The theory took its final form in Einstein's general relativity of 1916.

In brief, Einstein's principle of equivalence together with the hypothesis of locality implies that any observer in a gravitational field is locally inertial; therefore, at all (nonsingular) events in a gravitational field observers can define local inertial frames which are then connected through the structure of spacetime. The gravitational field must therefore reside in the spacetime structure. The flat Minkowski spacetime, for instance, has no structure capable of accommodating a gravitational field. The simplest possibility is to identify the gravitational field with the curvature of spacetime manifold. This is postulated in the standard geometric interpretation of general relativity; furthermore, the gravitational field equations are generalizations of Poisson's equation, $\nabla^2\Phi = 4\pi G\rho$, for the Newtonian gravitational potential Φ in a way that is consistent with the Riemannian structure of spacetime. The simplest possibility, i.e.,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \quad , \tag{1.9}$$

involves the energy-momentum tensor which replaces the matter density ρ. In general relativity, the Newtonian acceleration of gravity is transformed into the Christoffel connection of spacetime. Only nongravitational forces produce true translational accelerations which are then absolute; that is, a translational acceleration can be represented by a vector a^μ such that the magnitude of acceleration $g(x)$, $g^2(x) = a_\mu a^\mu$, is totally independent of the coordinate system employed. The equation of motion of a test particle is thus given by

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = a^\mu(x) \quad , \tag{1.10}$$

where $a^\mu(x)$ is the sum total of nongravitational forces acting on the particle per unit of its inertial mass. In general, a system can have both translational and rotational accelerations, however. It follows from Einstein's principle of equivalence that locally—i.e., to the extent that spacetime curvature can be neglected—gravitational effects are the same as inertial effects; therefore, gravitation can be approximately described in terms of gravitoelectric and gravitomagnetic fields corresponding to translational and rotational inertia, respectively. This is the gravitational Larmor theorem [3], which is very useful in the post-Newtonian approximation to general relativity. The gravitomagnetic field of a massive rotating body is a measure of its absolute rotation. If rotation were relative, the gravitomagnetic field would have to be determined by the rotation of the rest of the universe with respect to the body. In contrast to Coriolis and centripetal accelerations that are only proportional to kinematic quantities relevant to motion (e.g., angular frequency), the dragging frequency of the local inertial frames—which is essentially the gravitomagnetic field—is proportional to the *moment of inertia* of the rotating body in addition to purely kinematic variables. Two adjacent bodies rotating at the same frequency could have different moments of inertia; therefore, it is impossible that the motion of the rest of the universe could produce an effect proportional to the moment of inertia [5,6].

Thus Lorentz invariance, the hypothesis of locality, Einstein's principle of equivalence, and the field equations constitute the basic physical components of general relativity. This paper is mainly concerned with the question of validity of the hypothesis of locality.

Chapter 16

Hypothesis of Locality

The presumed equivalence of an accelerated observer with a momentarily comoving inertial observer is exactly true for point particles in Newtonian mechanics since the state of a particle is determined by its position and velocity alone. At each instant, the accelerated particle has the same state as the comoving inertial particle; therefore, the hypothesis of locality must be valid for pointlike coincidences. In case of realistic measurements, however, the acceleration of the measuring device is locally immaterial only when the influence of inertial effects can be neglected over the length and time scales characteristic of elementary local measurements [7]. Such a classical measuring device—for which the hypothesis of locality is approximately valid under the experimental conditions—will be referred to as a *standard* measuring device.

The hypothesis of locality implies that every standard observer carries an orthonormal tetrad $\lambda^{\mu}_{(\alpha)}$ such that

$$\frac{D\lambda^{\mu}_{(\alpha)}}{D\tau} = \phi_{\alpha}{}^{\beta}\, \lambda^{\mu}_{(\beta)} \quad , \tag{2.1}$$

where the scalars $\phi_{\alpha\beta}$ indicate the deviation of the tetrad frame from parallel transport and form an antisymmetric tensor under local Lorentz transformations. Any length or time scale formed via $\phi_{\alpha\beta}$, $d\phi_{\alpha\beta}/d\tau$, etc., is an acceleration scale characteristic of the motion of the standard device. In most situations, however, the *proper* acceleration scales formed from the invariants of $\phi_{\alpha\beta}$, i.e.,

$$\frac{1}{2}\phi_{\alpha\beta}\phi^{\alpha\beta} = -g^2 + f^2 \quad , \quad \frac{1}{2}\phi^*_{\alpha\beta}\phi^{\alpha\beta} = \mathbf{g}\cdot\mathbf{f} \quad , \tag{2.2}$$

bring out the main acceleration aspects (as opposed to the velocity aspects) of the motion. Here $\phi^*_{\alpha\beta}$ is the dual acceleration tensor, and $\mathbf{g}$ and $\mathbf{f}$ are the translational ("electric") and rotational ("magnetic") parts of the acceleration tensor $\phi_{\alpha\beta}$, respectively. It follows that in general there exists a translational acceleration length and a rotational acceleration length. Let $\mathcal{L}$ be a characteristic acceleration length and λ be the intrinsic scale of the phenomenon under observation; then, $\lambda/\mathcal{L}$ is expected to indicate the degree of divergence from the assumption of locality. This deviation is expected to be far below the measurement accuracy for a standard device. For instance, for optical phenomena with $\lambda \sim 10^3$ Å in a system with linear acceleration $g \sim 10^3$ cm/sec^2, $\mathcal{L} = c^2/g \sim 1$ ℓyr and $\lambda/\mathcal{L} \sim 10^{-23}$, so that experiments thus far do not show any deviations from the hypothesis of locality (cf. Section 5).

It should be clear from the foregoing discussion that the hypothesis of locality is an important and useful approximation; however, it is not a general law of nature. Imagine, for instance, a particle of mass m and charge q moving uniformly with speed v in Minkowski spacetime; if the particle is accelerated, it will radiate and the characteristic wavelength of the radiation is comparable to the acceleration length, i.e., $\lambda \sim \mathcal{L}$. The hypothesis of locality should then be violated for a hypothetical observer comoving with the particle on the basis of intuitive considerations presented above. It turns out that this is indeed the case since the work of Abraham, Lorentz and Dirac has shown that a radiating charged particle must satisfy an equation of the form

$$m\frac{d^2\mathbf{x}}{dt^2} - \frac{2q^2}{3c^3}\frac{d^3\mathbf{x}}{dt^3} + \cdots = \mathbf{F} \quad , \tag{2.3}$$

where the radiation reaction term violates the hypothesis of locality: the position and velocity of the radiating particle are not sufficient to determine its state and hence the particle is not instantaneously equivalent to a hypothetical comoving inertial particle.

The hypothesis of locality is in general violated for wave phenomena that are intrinsically nonlocal [8-10]. In the eikonal limit ($\lambda \to 0$), locality is recovered for the ray picture. It is necessary, however, to have access to standard classical measuring devices—for which the hypothesis of locality is valid—in order to be able to establish local reference frames for accelerated observers. The hypothesis of locality imposes limitations on standard devices and it is interesting to describe the basis for these classical limitations [11].

Consider an accelerated observer in Minkowski spacetime. Let $\bar{x}^\mu(\tau)$ be the worldline of the observer; at proper time τ, the observer is locally equivalent to an ideal inertial observer with tetrad frame $\lambda^\mu_{(\alpha)}(\tau)$. The instantaneous inertial observer would naturally assign coordinates $X^0 = \tau$ and $X^i = \sigma\xi_\mu\lambda^\mu_{(i)}$ to spacetime events. Here ξ_μ, $\xi_\mu\lambda^\mu_{(0)} = 0$, is the unit vector at $\bar{x}^\mu(\tau)$ along a spacelike line that connects $\bar{x}^\mu(\tau)$ to the event with Minkowski coordinates x^μ, and σ is the proper length of this spacelike segment. Thus X^μ constitute the accelerated coordinate system by the hypothesis of locality, and the two coordinate systems are related by

$$x^\mu = \bar{x}^\mu(X^0) + X^i\lambda^\mu_{(i)} \quad . \tag{2.4}$$

It follows from differentiating both sides of this equation that

$$dx^\mu = \left[(1 - \mathbf{g}\cdot\mathbf{X})\,\lambda^\mu_{(0)} + (\mathbf{f}\times\mathbf{X})^j\,\lambda^\mu_{(j)}\right] dX^0 + \lambda^\mu_{(i)}dX^i \quad , \tag{2.5}$$

where equation (2.1) and the definitions of $\mathbf{g}$ and $\mathbf{f}$ (i.e., $\phi_{0i} = -g_i$ and $\phi_{ij} = \epsilon_{ijk} f^k$) have been used. Writing the metric as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = g_{\mu\nu} dX^\mu dX^\nu$, we find the metric in the accelerated observer's geodesic system of coordinates

$$g_{00} = -(1 - \mathbf{g} \cdot \mathbf{X})^2 + (\mathbf{f} \times \mathbf{X})^2 \quad , \tag{2.6}$$

$$g_{0i} = (\mathbf{f} \times \mathbf{X})_i \quad , \quad g_{ij} = \delta_{ij} \quad . \tag{2.7}$$

These equations reflect the fact that space is Euclidean at each instant by the hypothesis of locality. The observer's acceleration vector a^μ is always spacelike with $g_i = -a_\mu \lambda^\mu_{(i)}$ and $a^\mu a_\mu = g^2$, while the observer's spatial frame rotates with frequency $\mathbf{f}$ about a nonrotating (i.e., Fermi-Walker transported) frame.

The spatial and temporal extent of validity of an accelerated coordinate system is generally limited on the basis of physical considerations. To illustrate this point, consider the coordinate transformation (2.4) for the simple case of uniform linear acceleration of magnitude g along the z-axis in Minkowski spacetime. The worldline of the observer is given by $\bar{t} = c^2 \beta\gamma/g$, $\bar{x} = \bar{y} = 0$ and $\bar{z} = c^2(\gamma - 1)/g$, where the speed of the observer is $c\beta = c \tanh(g\tau/c)$ and the corresponding Lorentz γ-factor is $\gamma = \cosh(g\tau/c)$. The only nonzero components of the observer's nonrotating tetrad frame are $\lambda^0_{(0)} = \lambda^3_{(3)} = \gamma$, $\lambda^3_{(0)} = \lambda^0_{(3)} = \beta\gamma$, and $\lambda^1_{(1)} = \lambda^2_{(2)} = 1$. The inertial coordinates (t, x, y, z) are related to the accelerated coordinates (T, X, Y, Z) via

$$t = \left(Z + \frac{c^2}{g}\right) \sinh(gT/c) \quad , \tag{2.8}$$

$$x = X \quad , \quad y = Y \quad , \tag{2.9}$$

$$z = \left(Z + \frac{c^2}{g}\right) \cosh(gT/c) - \frac{c^2}{g} \quad , \tag{2.10}$$

where the observer is at the origin of spatial coordinates in the accelerated system. At any given instant τ, the transformation between the inertial coordinates and the coordinates of the momentarily comoving inertial frame is given by

$$t - \bar{t} = \gamma(t' + \beta z') \quad , \quad x = x' \quad , \quad y = y' \quad , \quad \text{and} \quad z - \bar{z} = \gamma(z' + \beta t') \quad . \tag{2.11}$$

A comparison of these transformations with equations (2.8)-(2.10) makes it clear that the former simply represent a consistent and continuous form of the latter equations. To study the limitations inherent in the hypothesis of locality, imagine two observers $\bar{\text{P}}$ and P at rest along the z-axis and a distance ℓ apart with $\bar{\text{P}}$ at the origin of coordinates. At $t = 0$, they are accelerated from rest in exactly the same way with uniform acceleration g. Their distance according to inertial observers is always ℓ, but according to observer $\bar{\text{P}}$, P is at a distance L given by [11]

$$1 + gL/c^2 = \left(1 + 2\epsilon\gamma + \epsilon^2\right)^{\frac{1}{2}} \quad , \tag{2.12}$$

where $\epsilon = g\ell/c^2$ and the Fermi system (2.8)-(2.10) along the path of $\bar{\text{P}}$ has been employed. Alternatively, in the instantaneous rest frame of $\bar{\text{P}}$ and P, their distance is $\ell' = \gamma\ell$ by Lorentz contraction—if at each instant the momentarily comoving inertial system given by equation (2.11) is employed. As $\epsilon \to 0$,

$$L/\ell' \sim 1 - \frac{1}{2}\beta^2\gamma\epsilon \quad ; \tag{2.13}$$

therefore, consistency is achieved only when ℓ is negligible compared to the proper acceleration length $\mathcal{L} = c^2/g$. On the other hand, the difference between L and ℓ' could become appreciable for a given $\epsilon < 1$ once τ exceeds c/g.

Consider now a standard device that is accelerated, and let $\bar{\text{P}}$ and P be two points of this device. It follows from the general argument outlined above that the dimension of a standard measuring device must be very small compared to $\mathcal{L}$ and the duration of the measurement must be very short compared to $\mathcal{L}/c$. On the other hand, since a standard device of mass m is classical in nature, the intrinsic scales of length and time that characterize the classical device must greatly exceed its wave characteristics given by its Compton wavelength $\hbar/mc$ and period $\hbar/mc^2$. The classical and quantum limitations together imply that $\mathcal{L} >> \hbar/mc$. For translational acceleration this inequality implies that $g << mc^3/\hbar$; therefore, for a device of mass m there is a maximal translational acceleration [12] given by $mc^3/\hbar$. Similar limitations apply for other accelerations including tidal accelerations suffered by a standard device in a gravitational field; a singularity develops when the relevant acceleration length goes to zero [11,13].

It should be clear from these considerations that the measurements of an accelerated observer are linked to those of the hypothetical comoving inertial observers; in fact, an accelerated observer passes through an infinite sequence of such ideal observers. Let $\mathcal{F}_{\alpha\beta}(\tau)$ represent the electromagnetic field that is actually measured by the accelerated observer and $F'_{\alpha\beta}(\tau) = F_{\mu\nu}\lambda^{\mu}_{(\alpha)}\lambda^{\nu}_{(\beta)}$ represent the field that is measured by the ideal momentarily comoving inertial observers. It is interesting to consider the most general relationship between $\mathcal{F}_{\alpha\beta}$ and $F'_{\alpha\beta}$ that would be consistent with causality and would preserve the superposition principle. A general linear relation is

$$\mathcal{F}_{\alpha\beta}(\tau) = F'_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')F'_{\gamma\delta}(\tau')d\tau' \quad , \tag{2.14}$$

where τ_0 represents the instant at which the observer's acceleration begins. The kernel $K_{\alpha\beta\gamma\delta}$ is antisymmetric in its first and second pairs of indices, but is otherwise totally undetermined at this point; a new assumption is required in order to specify the kernel. The nonlocal part of equation (2.14) is expected to be of order $\lambda/\mathcal{L}$ and can be determined from the notion that the propagation of electromagnetic radiation is observer-independent. This idea is developed in the following section.

Chapter 17

Duality of Absolute and Relative Motion

The general theory of relativity, which agrees with all observational data available at present, constitutes the culmination of efforts to extend the principle of relativity (Lorentz invariance) to the relativity of motion in general. It turns out that even in this theory accelerated motion is absolute, i.e., local inertial effects cannot be described in terms of the relative motion of moving distant masses [5,6]. This circumstance provides the motivation to look at the problem of motion from the viewpoint of complementarity. That is, the theory of relativity involves absolute motion, while the theory of absolute motion (including classical mechanics, electrodynamics, and quantum mechanics) involves the principle of relativity (Lorentz invariance).

The description of motion in classical physics is based upon two distinct pictures. The particle picture involves Newtonian point particles while the wave picture involves classical electromagnetic radiation. Mach pointed out that the intrinsic and extrinsic states of a Newtonian point particle are not directly connected

[14]. That is, the mass of a classical particle has no direct connection with the state of the particle characterized by its position and velocity in Newton's absolute space and time. Particles are, however, directly connected with each other via interactions such as gravity. Therefore, classical particle motion is *relative*, since the particle can be referred directly to other particles while it cannot be referred directly to absolute space and time.

The same general argument for the motion of classical electromagnetic waves would imply that classical electromagnetic wave motion is *absolute*, i.e., *nonrelative*, since relative and absolute movements are mutually exclusive: The intrinsic state of a wave—characterized by its frequency, wavelength, intensity, and polarization—is directly connected to its extrinsic state characterized by its wave function. Thus a classical wave can be directly referred to absolute space and time while a classical particle can only be directly referred to other particles. Hence classical wave motion is absolute, since an electromagnetic wave propagates independently of any observer.

The idea that the propagation of electromagnetic waves is independent of observers is clearly valid for inertial observers as a consequence of Lorentz invariance. It is assumed here that this notion is valid for any observer; this hypothesis can be tested experimentally (cf. Section 5). It goes beyond the hypothesis of locality and hence the standard theory of accelerated observers.

It has not been possible to establish a physical theory solely on the basis of relativity of motion; similarly, absolute motion of electromagnetic waves incorporates the principle of relativity (Lorentz invariance). Thus Newtonian mechanics and the Maxwell-Lorentz electrodynamics incorporate the duality of relative and absolute motion; for instance, a charged particle emits electromagnetic waves only when it undergoes acceleration.

It is interesting to extend wave-particle duality to include the duality of absolute and relative motion. Thus neither the picture of particle motion as relative nor the picture of wave motion as absolute is complete; motion has complementary classical aspects in relative and absolute movements. To illustrate this idea, let us consider the nonrelativistic motion of a particle in a potential V according to the Heisenberg picture. In this "particle" representation, the Hamiltonian is

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{\mathbf{x}}) \tag{3.1}$$

and the momentum is $\hat{\mathbf{p}} = m\, d\hat{\mathbf{x}}/dt$, so that the fundamental quantum condition, $[\hat{x}^j, \hat{p}^k] = i\hbar\delta_{jk}$, implies

$$\left[\hat{x}^j, \frac{d\hat{x}^k}{dt}\right] = i\frac{\hbar}{m}\delta_{jk} \quad . \tag{3.2}$$

The observables corresponding to the position and velocity of the particle are related to its intrinsic property—i.e., mass m—via $\hbar$ as a consequence of the particle's nonlocality. These observables commute, however, and Newtonian mechanics is recovered when $m \to \infty$; that is, a massive system behaves in accordance with Newtonian mechanics since the influence on the system of any disturbance accompanying an act of observation is expected to be negligibly small. In terms of the Schrödinger picture, the state of the particle is characterized by a wave function $\Psi(t, \mathbf{x})$ that satisfies the Schrödinger equation. This equation depends explicitly upon the particle mass and thus connects the intrinsic and extrinsic states of the particle in the "wave" picture. More generally, the intrinsic inertial properties of the particle are determined by its mass as well as spin; mass and spin describe the irreducible unitary representations of the Poincaré group [15].

It follows from the fundamental quantum condition that it is impossible to prepare a system in a simultaneous eigenstate of position and momentum; hence, a matter wave can never stand completely still with respect to an inertial observer.

The validity of this assertion for the appearance of any fundamental wave with respect to *arbitrary* observers is the basic hypothesis employed in this paper. It is an immediate consequence of this hypothesis that it would be impossible to describe the relative motion of a physical system with respect to a fundamental wave since no observer can ever be comoving with the wave.

Chapter 18

Nonlocality

It is a general property of the Volterra system (2.14) that in the space of continuous functions the relationship between $F_{\alpha\beta}$ and $\mathcal{F}_{\alpha\beta}$ is unique. Moreover, it is reasonable to assume that the continuous function $K_{\alpha\beta\gamma\delta}(\tau,\tau')$ depends only upon $\tau-\tau'$ so that this function is a convolution-type kernel. It proves useful to introduce instead of $F^{\mu\nu}$ a six-vector F^A, where the index A ranges over the set $(01,02,03,23,31,12)$. Thus in matrix notation we have

$$\mathcal{F} = F' + \int_{\tau_0}^{\tau} K(\tau-\tau')F'(\tau')d\tau' \quad , \tag{4.1}$$

where $K = (K^A{}_B)$, the matrix K_{AB} is related to the kernel in equation (2.14) through

$2K_{\alpha\beta\gamma\delta}(\tau,\tau') \to K_{AB}(\tau-\tau')$, and $F' = \Lambda F$. The basic assumption that if $\mathcal{F}$ is constant, then F must be constant as well implies that

$$\Lambda(\tau) + \int_{\tau_0}^{\tau} K(\tau-\tau')\Lambda(\tau')d\tau' = \Lambda(\tau_0) \quad , \tag{4.2}$$

since $\mathcal{F}(\tau_0) = \Lambda(\tau_0)F(\tau_0)$ in any case. The integral equation (4.2) can be solved by means of the ***resolvent kernel*** $\mathcal{R}$ such that

$$\Lambda(\tau_0) + \int_{\tau_0}^{\tau} \mathcal{R}(\tau - \tau')\Lambda(\tau_0)d\tau' = \Lambda(\tau) \quad . \tag{4.3}$$

It follows immediately upon differentiating equation (4.3) that

$$\mathcal{R}(\tau - \tau_0) = \frac{d\Lambda(\tau)}{d\tau}\Lambda^{-1}(\tau_0) \quad , \tag{4.4}$$

so that the resolvent kernel and hence the kernel are proportional to the acceleration. Once the resolvent kernel is calculated via equation (4.4), K can be determined using standard procedures of the theory of Volterra integral equations [16]. In particular, Laplace transformations provide a useful method due to the assumption of convolution-type kernels [17]. If the observer is inertial, $\mathcal{R} = 0$ and hence $K = 0$ and the standard result of the Lorentz-invariant theory is thus recovered. On the other hand, when the observer is accelerated the nonlocal part is generally of order $\lambda/\mathcal{L}$, which tends to zero when $\lambda \to 0$; hence, the hypothesis of locality is recovered in the eikonal limit.

Let us suppose that the acceleration of the observer is turned off at τ_1, and for $\tau > \tau_1$ the observer is inertial again; then, equation (4.1) implies that $\mathcal{F}(\tau) = F'(\tau) + C$, where

$$C = \int_{\tau_0}^{\tau_1} K(\tau_1 - \tau')F'(\tau')d\tau' \tag{4.5}$$

is a constant field that is the residue of past acceleration. Maxwell's equations are *differential* equations for the electromagnetic field; therefore, the field is determined in any given situation up to a constant electromagnetic field. Boundary conditions are needed in general to specify the field uniquely. It follows that in any measuring device the influence of past accelerations would be canceled when the device is reset [17].

Let us now consider a concrete example. Imagine an inertial observer moving uniformly with speed v parallel to the y-axis of an inertial frame; at $t = 0$, $x = r > 0$ and $y = z = 0$, the observer is accelerated such that for $t \geq 0$ it moves on a circle of radius r around the origin in the (x, y)-plane with constant frequency Ω. The azimuthal angle indicating the position of the uniformly rotating observer is given by $\varphi = \Omega t = \gamma\Omega\tau$, where γ is the Lorentz factor corresponding to $v = r\Omega$. The natural tetrad frame of the uniformly rotating observer is given by

$$\lambda^{\mu}_{(0)} = \gamma(1, -\beta\sin\varphi, \beta\cos\varphi, 0) \quad , \tag{4.6}$$

$$\lambda^{\mu}_{(1)} = (0, \cos\varphi, \sin\varphi, 0) \quad , \tag{4.7}$$

$$\lambda^{\mu}_{(2)} = \gamma(\beta, -\sin\varphi, \cos\varphi, 0) \quad , \tag{4.8}$$

$$\lambda^{\mu}_{(3)} = (0, 0, 0, 1) \quad , \tag{4.9}$$

with respect to inertial coordinates. Here $\beta = v/c$, and the spatial axes $\lambda^{\mu}_{(i)}$, $i = 1, 2, 3$, correspond, respectively, to the radial, tangential and normal directions for the circular motion of the observer. Representing F as a column vector

$$F = \begin{bmatrix} \mathbf{E} \\ \mathbf{B} \end{bmatrix} \quad , \tag{4.10}$$

we find $F' = \Lambda F$, where Λ is of the form

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ -\Lambda_2 & \Lambda_1 \end{bmatrix} \quad , \tag{4.11}$$

with

$$\Lambda_1 = \begin{bmatrix} \gamma\cos\varphi & \gamma\sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & \gamma \end{bmatrix} \quad , \tag{4.12}$$

and

$$\Lambda_2 = -\beta\gamma \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ \cos\varphi & \sin\varphi & 0 \end{bmatrix} \quad . \tag{4.13}$$

It is now possible to compute the resolvent kernel using equation (4.4), and the kernel by employing Laplace transforms. It turns out that $K = (K^A{}_B)$ is a constant matrix of the form

$$K = \begin{bmatrix} K_1 & K_2 \\ -K_2 & K_1 \end{bmatrix} \quad , \tag{4.14}$$

where K_1 and K_2 are antisymmetric matrices

$$K_1 = \gamma^2\Omega \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \tag{4.15}$$

and

$$K_2 = \beta\gamma^2\Omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad . \tag{4.16}$$

Note that K_1 is proportional to I_3, the generator of rotations about the z-axis, while K_2 is proportional to I_1, the generator of rotations about the x-axis; in general, $(I_i)_{jk} = -\epsilon_{ijk}$. The physical interpretation of this result becomes clear upon computing the acceleration tensor $\phi_{\alpha\beta}$ using equations (4.6)-(4.9). The result is

$$\phi = \begin{bmatrix} 0 & -g_1 & 0 & 0 \\ g_1 & 0 & f_3 & 0 \\ 0 & -f_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \tag{4.17}$$

where $g_1 = \beta\gamma^2\Omega$ and $f_3 = \gamma^2\Omega$ are scalar invariants. It follows that $K_1 = \mathbf{f}\cdot\mathbf{I}$ and $K_2 = \mathbf{g}\cdot\mathbf{I}$. Moreover, it is interesting to note that the kernel is antisymmetric, i.e., $K_{AB} = -K_{BA}$.

The relationship between $\mathcal{F}$ and F, given by equation (4.1) for a radiation field, can be explicitly written for the uniformly rotating observer as

$$\mathcal{E}_1 = \gamma(\cos\varphi E_1 + \sin\varphi E_2) + \beta\gamma B_3 + \gamma^2\Omega\int_0^\tau (\sin\varphi' E_1 - \cos\varphi' E_2)d\tau' \quad , \tag{4.18}$$

$$\mathcal{E}_2 = -\sin\varphi E_1 + \cos\varphi E_2 + \gamma\Omega\int_0^\tau (\cos\varphi' E_1 + \sin\varphi' E_2)d\tau' \quad , \tag{4.19}$$

$$\mathcal{E}_3 = \gamma E_3 - \beta\gamma(\cos\varphi B_1 + \sin\varphi B_2) + \beta\gamma^2\Omega\int_0^\tau (-\sin\varphi' B_1 + \cos\varphi' B_2)d\tau' \quad , \tag{4.20}$$

$$\mathcal{B}_1 = \gamma(\cos\varphi B_1 + \sin\varphi B_2) - \beta\gamma E_2 + \gamma^2\Omega\int_0^\tau (\sin\varphi' B_1 - \cos\varphi' B_2)d\tau' \quad , \tag{4.21}$$

$$\mathcal{B}_2 = -\sin\varphi B_1 + \cos\varphi B_2 + \gamma\Omega\int_0^\tau (\cos\varphi' B_1 + \sin\varphi' B_2)d\tau' \quad , \tag{4.22}$$

$$\mathcal{B}_3 = \gamma B_3 + \beta\gamma(\cos\varphi E_1 + \sin\varphi E_2) + \beta\gamma^2\Omega\int_0^\tau (\sin\varphi' E_1 - \cos\varphi' E_2)d\tau' \quad . \tag{4.23}$$

It should be clear that if F does not represent a radiation field, then the nonlocal part does not exist and $\mathcal{F} = F'$.

Imagine now an incident monochromatic plane electromagnetic wave of frequency ω propagating along the z-axis. The incident field is given by

$$F = i\omega a \begin{bmatrix} \mathbf{e}_\pm \\ \mathbf{b}_\pm \end{bmatrix} e^{-i\omega(\gamma\tau - z)} \quad , \tag{4.24}$$

where a is a complex amplitude, $\mathbf{e}_\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$, $\mathbf{b}_\pm = \mp i\mathbf{e}_\pm$, and the upper (lower) sign represents positive (negative) helicity radiation; we adopt the convention that only the real part of equation (4.24) indicates the (classical) field. Here $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors along the x and y axes, respectively. The nonlocal theory of accelerated observers is linear; therefore, it is sufficient to explore the consequences of the theory for the wave under consideration, since a general pulse may be expressed as a linear superposition of monochromatic plane waves. It follows from $F' = \Lambda F$ that

$$F' = i\omega a \begin{bmatrix} \mathbf{e}'_\pm \\ \mathbf{b}'_\pm \end{bmatrix} e^{-i\omega'\tau} \quad , \tag{4.25}$$

where $\mathbf{b}'_\pm = \mp i\mathbf{e}'_\pm$, and the polarization vector as measured by the comoving inertial observer is given by

$$\mathbf{e}'_\pm = \frac{1}{\sqrt{2}} \begin{bmatrix} \gamma \\ \pm i \\ \pm\beta\gamma i \end{bmatrix} \quad , \tag{4.26}$$

which reduces to $\mathbf{e}_\pm$ for $\beta \to 0$. Here the measured frequency according to the hypothesis of locality is

$$\omega' = \gamma(\omega \mp \Omega) \quad , \tag{4.27}$$

which involves the transverse Doppler effect as well as helicity-rotation coupling. For instance, in a positive-helicity wave the electric and magnetic vectors rotate with frequency ω about the axis of propagation (z-axis). The rotating observer perceives the same type of rotation of the vectors but with frequency $\gamma(\omega - \Omega)$; for $\omega = \Omega$, the fields are perceived to be constant in time as a consequence of the hypothesis of locality. Moreover, the angular momentum of the field about the z-axis is constant, so that a positive (negative) helicity wave is perceived to be right (left) circularly polarized by the rotating observer.

It follows from equations (4.18)-(4.23) that

$$\mathcal{F} = i\omega a \frac{\omega e^{-i\omega'\tau} \mp \Omega}{\omega \mp \Omega} \begin{bmatrix} \mathbf{e}'_\pm \\ \mathbf{b}'_\pm \end{bmatrix} \tag{4.28}$$

This result has three important consequences. The first is that as a result of the nonlocal part of equation (4.1), the measured amplitude of the positive-helicity radiation with $\omega > \Omega$ is enhanced by a factor of $(1 - \Omega/\omega)^{-1}$, while the amplitude of the negative-helicity radiation is diminished by a factor of $(1 + \Omega/\omega)^{-1}$. This prediction of the nonlocal theory could be tested, for example, by considering radio waves with reduced wavelength of $\lambda \sim 1$ cm incident on a system rotating at a frequency of $\sim$ 500 rounds per second; then, $\Omega/\omega = \lambda/\mathcal{L} \sim 10^{-7}$. The second consequence is that for the positive-helicity radiation with $\omega = \Omega$, the nonlocal theory predicts a time-varying field given by

$$\mathcal{F} = i\omega a(1 - i\gamma\omega\tau)\begin{bmatrix} \mathbf{e}'_{\pm} \\ \mathbf{b}'_{\pm} \end{bmatrix} \quad . \tag{4.29}$$

The linear temporal variation of $\mathcal{F}$ implies that the absolute magnitude of the measured field grows indefinitely with proper time; this circumstance is an immediate consequence of the fact that the constant amplitude of the incident monochromatic plane wave is maintained over time. Thus this divergence of the field would be absent for any finite incident *packet* of radiation. The third consequence is that the average of $\mathcal{F}$ over time is nonzero and proportional to the rotation frequency, while the time average of F' (as well as F) is zero.

The electrodynamics of accelerated observers has been presented here in terms of the Faraday tensor $F_{\mu\nu}$; however, the same general results could be obtained from a nonlocal theory of the vector potential A_μ. The nonlocal theory of a vector field is developed in the Appendix.

Electrodynamics has been the focus of this paper; however, the ideas presented here can be extended to other fundamental fields as well. A basic consequence of the general nonlocal theory is that the existence of a basic scalar field is forbidden. This comes about because Λ is unity for a scalar field and hence $\mathcal{R} = 0$, which implies that $K = 0$, so that the scalar field is always local. The same would hold for a pseudoscalar field. Thus the possibility of existence of states with $\omega' = 0$ cannot be avoided. It would therefore be possible for a scalar field to stay completely at rest with respect to an accelerated observer. This contradicts the basic premise of this work.

It is clear from the structure of the nonlocal theory that only when the acceleration is non-uniform would there be a contribution to the measured frequency beyond the hypothesis of locality. The sole exception to this rule would occur, however, when the frequency measured in accordance with the hypothesis of locality vanishes.

The observational consequences of the spin-rotation coupling—evident in equations (4.27) and (4.28)—are explored in the following section.

Chapter 19

Spin-Rotation-Gravity Coupling

Imagine an observer (i.e., a classical measuring device) rotating about the z-axis in an inertial frame. For instance, this could be any observer fixed on the Earth rotating about its proper axis of rotation. Consider a free particle passing by the rotating observer at an instant. What is the energy of the particle as measured by the observer at that event? To answer this question, an assumption is required regarding the measurements of accelerated observers. The standard assumption in the theory of relativity is that the accelerated observer is instantaneously equivalent to a comoving inertial observer. According to this hypothesis, the relationship between the observer and the particle is the same as between two inertial observers at the event in question. Therefore, $E' = \gamma(E - \mathbf{v} \cdot \mathbf{P})$, where $\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r}$. This relation can be written as

$$E' = \gamma(E - \boldsymbol{\Omega} \cdot \mathbf{L}) \quad , \tag{5.1}$$

where $\mathbf{L} = \mathbf{r} \times \mathbf{P}$. This is a relativistic generalization of the well-known relation in classical mechanics relating the Hamiltonian in the inertial frame to that in the rotating frame, $H' = H - \boldsymbol{\Omega} \cdot \mathbf{L}$, which results in the Coriolis and centrifugal forces.

Let us now consider a wave field characterized by a wave function $\psi(t, \mathbf{x})$. The transformation to the rotating frame may be expressed in terms of spherical coordinates as $(r, \theta, \varphi) \to (r', \theta', \varphi' + \Omega t)$, so that if the wave function has temporal and azimuthal dependence of the form $\exp(-iEt/\hbar)\exp(iM\varphi)$ in the inertial frame, then in the rotating frame its time dependence becomes $\exp[-i(E - \hbar M\Omega)t/\hbar]$, where M is the *total* angular momentum parameter along the axis of rotation. Since the observer's standard clock reads τ, $d\tau = dt/\gamma$, we find that in the rotating frame

$$E' = \gamma(E - \hbar M\Omega) \quad , \tag{5.2}$$

where $M = 0, \pm 1, \pm 2, \ldots$, for a scalar or a vector field, and $M = \pm\frac{1}{2} + \mu$, with $\mu = 0, \pm 1, \pm 2, \ldots$, for a Dirac field. To make contact with classical mechanics, we may express equation (5.2) as $H' = \gamma(H - \boldsymbol{\Omega} \cdot \mathbf{J})$.

The hypothesis of locality in the classical particle picture implies that E' should always be positive as in equation (5.1), while equation (5.2) implies that E' can be positive, zero, or negative. It should be clear that this is a consequence of the wave character of the field and is independent of its spin. Clearly, we can write in general $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and thus the wave treatment reveals the existence of a general spin-rotation coupling term given by $-\gamma\boldsymbol{\Omega} \cdot \mathbf{S}$. An explicit connection between equation (5.1) and equation (5.2) can be established if we consider the JWKB approximation; then, $\mathbf{J} = \mathbf{r} \times \mathbf{P} + \mathbf{S}$ and so

$$E' \approx \gamma(E - \mathbf{v} \cdot \mathbf{P}) - \gamma\boldsymbol{\Omega} \cdot \mathbf{S} \quad . \tag{5.3}$$

It follows that the hypothesis of locality in the particle picture is valid in general when the de Broglie frequency ω, $E = \hbar\omega$, is infinite, so that the waves may be considered in the ray approximation. Only in the limit of a ray—whose interactions are pointlike by definition—would the hypothesis of locality acquire validity.

Consider, for the sake of concreteness, an electromagnetic field $F_{\mu\nu}$ in Minkowski spacetime and an accelerated observer following a given worldline. Let the observer's tetrad frame be $\lambda^{\mu}_{(\alpha)}$, then the electromagnetic field measured by the accelerated observer is $F'_{\alpha\beta} = F_{\mu\nu}\lambda^{\mu}_{(\alpha)}\lambda^{\nu}_{(\beta)}$ according to the hypothesis of locality. If τ is the proper time along the path of the observer, the Fourier transform of $F'_{\alpha\beta}(\tau)$ can give the spectrum of frequencies measured by the observer. If this is calculated for the uniformly rotating observer, equation (5.2) is recovered. On the other hand, if we use the standard relativistic Doppler formula in this case, then the result $\omega_{\rm D} = -k_\mu dx^\mu/d\tau$ is equivalent to equation (5.1). The Doppler frequency is time dependent, since the frequency depends on the velocity of the observer at the event of observation, while the spectrum (5.2) is nonlocal and depends on the whole history of the observer. It follows from these considerations that the application of the hypothesis of locality produces different results in the classical particle and wave pictures; however, the former may be obtained from the latter in the eikonal limit. There can be little doubt that equation (5.2) should essentially be the correct result while equation (5.1) is simply the pointlike approximation. On the other hand, the standard theory of relativity is most consistent when it is thought of as a theory of pointlike coincidences. Therefore, it is important to investigate the physical consequences of the validity of equation (5.2) beyond the Doppler effect.

To this end, let us first deal with the question of why E' can become negative. At first sight, this result contradicts the spirit of relativity theory where only the relative motion of observers is of significance. Thus inertial effects, which in classical mechanics provide an absolute distinction between inertial and accelerated observers, would have to be interpreted in terms of the gravitational effects of distant masses. However, recent work on this problem has shown that inertial effects cannot be interpreted in this way within the standard geometric framework of general relativity [5,6]. In fact, the notion of absolute motion is indispensable. On the other hand, any theory based on absolute motion should necessarily incorporate the principle of relativity (i.e., Lorentz invariance). This circumstance has led to the principle of complementarity of absolute and relative motion [5]. On this basis, any conflict with the quantum theory of accelerated observers is thus avoided; therefore, the results presented here supersede our previous treatment of this problem [18]. For the problem under consideration, inertial observers are not ignored; energy is always positive for ideal inertial observers. However, a noninertial observer can find negative energy eigenstates as a result of inertial wave effects. This would be a consequence of acceleration relative to absolute spacetime. It is important to notice that absorption of negative energy by an accelerated device cannot be interpreted as emission of positive energy by the device since then the causal sequence of events in the inertial frame would be different from the causal sequence of events in the accelerated system. There is no basis for making such a far-reaching assumption. In this work, the inertial frame and the ideal inertial observers will always be preserved and the correspondence between observations in the two frames will be of central importance for the interpretation of phenomena by accelerated observers. These views correspond to the way observations are actually interpreted: astronomical observations—performed by observers rotating with the Earth—are in no way limited by the light cylinder (at c/Ω) corresponding to the proper rotation of the Earth; in fact, all inertial effects due to Earth rotation are subtracted out of the data and then the reduced data (referring now to ideal inertial observers) are analyzed.

The general correspondence between accelerated observers and inertial observers—upon which the present study is based—does not place any extra causality restrictions on accelerated observers. The limitations inherent in the accelerated coordinate system [19] are simply due to the limitations of standard measuring devices which are necessary to establish such coordinate systems. These limitations arise from the hypothesis of locality as discussed in Section 2.

The possibility that E' can be zero turns out to have far-reaching consequences. One may think that this possibility cannot be realized in practice since it involves angular momentum eigenstates. However, recent experiments at the Stanford Synchrotron Radiation Laboratory [20] have demonstrated that a coherent beam of X-ray photons can be prepared in the laboratory in a state characterized by (ω, J, M). It is interesting to illustrate $E' = 0$ using a thought experiment: Imagine an observer rotating uniformly with frequency Ω and a beam of positive helicity electromagnetic radiation incident along the axis of rotation. Then $\omega' = \gamma(\omega - \Omega)$ since the observer perceives the electromagnetic field rotating along the direction of propagation with frequency $\omega - \Omega$ with respect to time t or $\gamma(\omega - \Omega)$ with respect to its proper time τ. For a rotating observer along the axis of rotation $\gamma = 1$, but ω' is still $\omega - \Omega$. For $\omega = \Omega$, the observer stays completely at rest with the field. That is, according to all observers that are static in the rotating system, the electromagnetic field has no temporal variation at all. The possibility that $E' = 0$, i.e., observers can stay completely at rest with an electromagnetic wave contradicts the complementarity principle of absolute and relative motion and should be rejected. Classical motion involves particles as well as electromagnetic waves. Classical particle motion is relative while classical wave motion is absolute in the sense that it is independent of any observer. Hence, it should be impossible for an observer to remain at rest with respect to an electromagnetic wave. The nonlocal theory of accelerated observers presented in the previous section incorporates the complementarity principle in a natural way and is consistent with all observational data available at present [17]. The basic idea of the nonlocal theory is to replace the hypothesis of locality with the assumption that the field measured by the accelerated observer is a linear superposition of the measurements of the infinite class of hypothetical inertial observers that the accelerated observer has passed through. This is the most general assumption that is consistent with causality as well as the superposition principle. The nonlocal theory excludes the possibility of existence of any fundamental scalar field in nature (cf. Section 6); this consequence of the theory appears to be consistent with observational data. Furthermore, the nonlocal theory is in agreement with equation (5.2) for a uniformly rotating observer except for $E' = 0$. Under this "resonance" condition, the electromagnetic fields as measured by the uniformly rotating observer vary linearly with proper time. When $E' > 0$, the nonlocal theory implies—for the thought experiment under consideration here—that the measured amplitude of the positive helicity component would increase by a factor of $(1 - \omega/\Omega)^{-1}$. In a similar way, the amplitude of the negative helicity component would decrease by a factor of $(1 + \omega/\Omega)^{-1}$. It would be most interesting to subject this result of the nonlocal theory to a direct laboratory test.

To test the theoretical ideas presented here, one has to verify the existence of the spin-rotation-gravity coupling in its varied forms as well as the consequences of the nonlocal theory. Therefore, let us consider below some of the current experimental possibilities:

(i) It appears from the recent work of Papini and his coworkers [21] that certain depolarization effects that had been observed in circular accelerators [22] could be explained by the spin-rotation coupling. This work has opened up a new path of inquiry regarding polarized beams in high-energy accelerators with possibilities for further theoretical and experimental studies.

(ii) Significant progress has been achieved in the quantum-mechanical study of Newtonian gravity and inertia via interferometry; in particular, neutron and neutral atom interferometers have been especially useful thus far [23]. When the spin of, say, the neutron is considered, the Hamiltonian for a thermal neutron in an interferometer fixed in a frame rotating with frequency $\boldsymbol{\Omega}$ (such as the Earth) includes the term $-\hat{\mathbf{S}} \cdot \boldsymbol{\Omega}$, which is an inertial term proportional to intrinsic spin (rather than the inertial mass). A definite experimental set-up for measuring this effect has been suggested by Werner [23,24] and progress continues towards the development of large-scale interferometers capable of measuring this effect [24].

The observed gravitationally-induced quantum phase of particles is due to the Newtonian "gravitoelectric" contribution; the corresponding gravitomagnetic contribution due to Earth rotation would be ten orders of magnitude smaller [25]. When the spin of the particle is taken into account, the Hamiltonian includes the term $\hat{\mathbf{S}} \cdot \boldsymbol{\Omega}_D$, where $\boldsymbol{\Omega}_D$ is the dragging frequency of the local inertial frames due to Earth rotation. This is a consequence of the spin-rotation coupling and the gravitational Larmor theorem [3], which states that a gravitomagnetic field can be locally replaced by a rotating frame with $\boldsymbol{\Omega} = -\boldsymbol{\Omega}_D$. For the Earth, $\Omega_D/2\pi \sim GJ/2\pi c^2 R^3 \sim 10^{-14}$Hz at the poles; therefore, the gravitational spin-spin coupling is far below the level of sensitivity of current experiments.

The physical consequences of the spin-rotation-gravity couplings for a Dirac particle have been investigated by a number of authors [26]. For a neutrino with a very small inertial mass, Cai and Papini [27] have pointed out that the spin-rotation coupling might have interesting consequences for neutrino astrophysics. Moreover, the consequences of helicity-rotation-gravity couplings for light have been theoretically explored [28] for applications in astronomy (such as differential gravitational deflection of polarized radiation, and polarization-dependent aberration of starlight) as well as laboratory experiments (e.g., helicity-Sagnac effect).

(iii) Silverman [29] has discussed the optical activity induced by spin-rotation coupling for systems (in particular, atomic hydrogen in its ground state) in a rotating frame. He has pointed out that the results could be significant for metrology as well as astronomy. In the presence of a gravitational field, Silverman's results could be carried over to the gravitomagnetic case by the gravitational Larmor theorem [3,28], i.e., $\boldsymbol{\Omega} \to -\boldsymbol{\Omega}_D$, where $\boldsymbol{\Omega}_D$ is the dragging frequency of a local inertial frame. Let us note that for the Earth, $\Omega_\oplus/2\pi \approx 10^{-5}$Hz, while for an extreme Kerr black hole of mass M, $\Omega_D/2\pi \approx 3\times10^4(M_\odot/M)$Hz. Thus Silverman's results could be of interest in connection with the astrophysics of a rotating collapsed system.

(iv) Several recent papers [30] have reported on experiments that have searched for novel spin-dependent interactions in the laboratory. With respect to a frame fixed on the Earth, a spinning particle with $v << c$ (as in the reported experiments [30]) should have an extra interaction Hamiltonian, $\hat{H}_{\text{Earth}} = -\hat{\mathbf{S}}\cdot\boldsymbol{\Omega} + \hat{\mathbf{S}}\cdot\boldsymbol{\Omega}_D$. Since $\hbar\Omega_\oplus \approx 0.5\times10^{-19}$eV and $\hbar\Omega_D \sim \hbar GJ/c^2R^3 \sim 10^{-29}$eV for the Earth, at least the first term should have been detectable in some of the reported experiments. For instance, Venema *et al.* [30] report a sensitivity of $\approx 2\times10^{-21}$eV for a spin-dependent interaction energy. These experiments need to be carefully re-examined in order to determine if the spin-rotation coupling has already been detected in the laboratory.

Chapter 20

Discussion

Phenomenological nonlocal electrodynamics has existed at least since the early work of Hopkinson [31], who followed Maxwell's suggestion—based on Boltzmann's work in elasticity theory—regarding the dielectric properties of glasses. Such nonlocal theories rely upon phenomenological memory effects that usually fade exponentially [32]. However, the nonlocal theory presented here is not phenomenological in nature; in particular, for the uniformly rotating observer there is constant "memory" of the acceleration.

Nonlocal effects are usually extremely small in most experimental conditions; however, the nonlocal theory entails the interesting prediction that basic scalar as well as pseudoscalar fields do not exist. This is a direct consequence of the assumption that the propagation of a fundamental field is independent of the motion of the observer; that is, an arbitrary observer can never be comoving with respect to a basic field.

The classical theory developed here can be simply extended to the quantum regime, since the basic ideas employed in the construction of the nonlocal theory are in fact compatible with quantum mechanics. Once a general quantum field $\hat{F}$ is given in the inertial frame, the quantum field $\hat{\mathcal{F}}$ according to an accelerated observer is determined from the general nonlocal transformation given by the form of equation (4.1). The quantum invariance condition is satisfied in this approach; that is, quanta are not created nor destroyed upon transformation to the accelerated system.

Finally, the general approach adopted in this paper points towards a nonlocal field theory of gravitation that would reduce to general relativity in the eikonal limit. The structure of such a theory is currently under investigation. In this connection, it should be pointed out that nonlocal theories of gravitation have been the subject of recent investigations [33,34].

Chapter 21

Acknowledgement

I am grateful to Mário Novello, Marcelo Schiffer, and Jaime Tiomno for interesting discussions.

Chapter 22

Appendix

The purpose of this appendix is to present the nonlocal theory of a vector field A_μ. In particular, A_μ could be the vector potential associated with an electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; however, the vector potential is determined up to a gauge transformation. Though A_μ has therefore no independent physical significance in the classical theory, this is not generally the case in quantum mechanics.

The hypothesis of locality implies that the measured field according to accelerated observers is

$$A'_\alpha = \lambda^\mu{}_{(\alpha)} A_\mu \quad , \tag{A.1}$$

while the nonlocal theory predicts

$$\mathcal{A}_\alpha(\tau) = A'_\alpha(\tau) + \int_{\tau_0}^{\tau} K^{*}_{\alpha}{}^{\beta}(\tau - \tau') A'_\beta(\tau') d\tau' \quad . \tag{A.2}$$

Imagine now the uniformly rotating observer under consideration in Section 4. Let A represent the column vector $(\mathcal{A}_\alpha)$; then,

$$A' = \Lambda^* A \quad , \tag{A.3}$$

where Λ^* is given by

$$\Lambda^* = \begin{bmatrix} \gamma & -\beta\gamma\sin\varphi & \beta\gamma\cos\varphi & 0 \\ 0 & \cos\varphi & \sin\varphi & 0 \\ \beta\gamma & -\gamma\sin\varphi & \gamma\cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.4}$$

on the basis of equations (4.6)-(4.9). The resolvent kernel can be determined from the general relation (4.4) and the result is

$$\mathcal{R}^*(\tau) = \gamma\Omega \begin{bmatrix} \beta^2\gamma^2\sin\varphi & -\beta\gamma\cos\varphi & -\beta\gamma^2\sin\varphi & 0 \\ -\beta\gamma\cos\varphi & -\sin\varphi & \gamma\cos\varphi & 0 \\ \beta\gamma^2\sin\varphi & -\gamma\cos\varphi & -\gamma^2\sin\varphi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad . \tag{A.5}$$

Using Laplace transforms, it is possible to show that for the uniformly rotating observer the kernel is a constant matrix whose elements are proportional to the scalars associated with the acceleration of the observer. In fact,

$$K^* = \gamma^2\Omega \begin{bmatrix} 0 & \beta & 0 & 0 \\ \beta & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \tag{A.6}$$

where $K^* = (K^*_\alpha{}^\beta)$; hence, the kernel is antisymmetric as before, since

$$K^*_{\alpha\beta} = -\phi_{\alpha\beta} \quad , \tag{A.7}$$

where the acceleration tensor $\phi_{\alpha\beta}$ is given by equation (4.17).

It is now possible to express the nonlocal vector field measured by the rotating observer in terms of the field measured by static inertial observers in Minkowski spacetime. The result is

$$\mathcal{A}_0 = \gamma A_0 + \beta\gamma(-\sin\varphi A_1 + \cos\varphi A_2) + \beta\gamma^2\Omega \int_0^\tau (\cos\varphi' A_1 + \sin\varphi' A_2) d\tau' \quad , \quad \text{(A.8)}$$

$$\mathcal{A}_1 = \cos\varphi A_1 + \sin\varphi A_2 + \gamma\Omega \int_0^\tau (\sin\varphi' A_1 - \cos\varphi' A_2) d\tau' \quad , \quad \text{(A.9)}$$

$$\mathcal{A}_2 = \beta\gamma A_0 + \gamma(-\sin\varphi A_1 + \cos\varphi A_2) + \gamma^2\Omega \int_0^\tau (\cos\varphi' A_1 + \sin\varphi' A_2) d\tau' \quad , \quad \text{(A.10)}$$

$$\mathcal{A}_3 = A_3 \quad . \quad \text{(A.11)}$$

Let us now restrict attention to an electromagnetic vector potential corresponding to the monochromatic plane wave under consideration in Section 4. In a particular gauge, we can set

$$A = a \begin{bmatrix} 0 \\ \mathbf{e}_\pm \end{bmatrix} e^{-i\omega(\gamma\tau - z)} \quad , \quad \text{(A.12)}$$

where a is the complex amplitude introduced in equation (4.24). The hypothesis of locality implies that the vector potential measured by the accelerated observer is

$$A' = \frac{a}{\sqrt{2}} \begin{bmatrix} \pm i\beta\gamma \\ 1 \\ \pm i\gamma \\ 0 \end{bmatrix} e^{-i\omega'\tau} \quad . \quad \text{(A.13)}$$

On the other hand, it follows from equations (A.8)-(A.11) that

$$\mathcal{A} = \frac{\omega \mp \Omega e^{i\omega'\tau}}{\omega \mp \Omega} A' \quad . \quad \text{(A.14)}$$

This result contains the same general features that were discussed for $\mathcal{F}$ in Section 4; in particular, $\mathcal{A}$ varies linearly with proper time when $\omega' = 0$.

REFERENCES

[1] A. Einstein, *The Meaning of Relativity* (Princeton University Press, Princeton, 1955).

[2] B. Mashhoon, Found. Phys. **16** (Wheeler Festschrift), 619 (1986); in *Quantum Gravity and Beyond: Essays in Honor of Louis Witten*, edited by F. Mansouri and J. Scanio (World Scientific, Singapore, 1993), p. 257;
J.D. Norton, Rep. Prog. Phys. **56**, 791 (1993);
R. Neutze and W. Moreau, Phys. Lett. A **183**, 141 (1994).

[3] B. Mashhoon, Phys. Lett. A **173**, 347 (1993).

[4] E.T. Whittaker, *A History of the Theories of Aether and Electricity* (Nelson, London, 1951).

[5] B. Mashhoon, F.W. Hehl and D.S. Theiss, Gen. Rel. Grav. **16**, 711 (1984);
B. Mashhoon, Phys. Lett. A **126**, 393 (1988); Found. Phys. Lett. **6**, 545 (1993); in *Directions in General Relativity: Papers in Honor of Dieter Brill*, edited by B.L. Hu and T.A. Jacobson (Cambridge University Press, Cambridge, 1993), p. 182; Nuovo Cimento B **109**, in press (1994).

[6] W. Rindler and L. Mishra, Phys. Lett. A **173**, 105 (1993);
W. Rindler, preprint (1994).

[7] B. Mashhoon, Phys. Lett. A **145**, 147 (1990);
S.R. Mainwaring and G.E. Stedman, Phys. Rev. A **47**, 3611 (1993).

[8] B. Mashhoon, Phys. Lett. A **122**, 67 (1987); **122**, 299 (1987); **139**, 103 (1989).

[9] W. Fugmann and M. Kretzschmar, Nuovo Cimento B **103**, 389 (1989); **106**, 351 (1991).

[10] W. Moreau, Am. J. Phys. **60**, 561 (1992);
R. Neutze and W. Moreau, Phys. Lett. A **179**, 389 (1993).

[11] B. Mashhoon, Phys. Lett. A **143**, 176 (1990).

[12] E.R. Caianiello, Riv. Nuovo Cimento **15**, no. 4 (1992).

[13] H. Salecker and E.P. Wigner, Phys. Rev. **109**, 571 (1958).

[14] E. Mach, *The Science of Mechanics* (Open Court, La Salle, 1960).

[15] E.P. Wigner, Annals of Math. **40**, 149 (1939).

[16] V. Volterra, *Theory of Functionals and of Integral and Integro-Differential Equations* (Dover, New York, 1959);
F.G. Tricomi, Integral Equations (Interscience, New York, 1957).

[17] B. Mashhoon, Phys. Rev. A **47**, 4498 (1993).

[18] B. Mashhoon, Found. Phys. **16** (Wheeler Festschrift), 619 (1986), Sec. 6, p. 631.

[19] W. Rindler, Am. J. Phys. **34**, 1174 (1966).

[20] D.E. Brown *et al.*, Phys. Rev. Lett. **69**, 699 (1992).

[21] Y.Q. Cai, D.G. Lloyd and G. Papini, Phys. Lett. A **178**, 225 (1993);
G. Papini, in Proc. of the 5th Canadian Conf. on Gen. Rel. and Rel. Astrophysics, University of Waterloo, May, 1993.

[22] J.S. Bell and J. Leinaas, Nucl. Phys. B **212**, 131 (1983); **284**, 488 (1987).

[23] S.A. Werner and H. Kaiser, in *Quantum Mechanics in Curved Space-Time*, edited by J. Audretsch and V. de Sabbata (Plenum, New York, 1990), p. 1;
M. Kasevich and S. Chu, Appl. Phys. B **54**, 321 (1992);

J. Audretsch and C. Lämmerzahl, Appl. Phys. B **54**, 351 (1992);
J. Audretsch, F.W. Hehl and C. Lämmerzahl, in *Relativistic Gravity Research*, edited by J. Ehlers and G. Schäfer (Springer, Berlin, 1992), p. 368;
G.E. Stedman, Contemp. Phys. **26**, 311 (1985);
A.H. Rostomyan and A.M. Rostomyan, Phys. Stat. Sol. (a) **126**, 29 (1991);
G.C. Scorgie, J. Phys. A: Math. Gen. **26**, 3291 (1993);
F.V. Kowalski, Phys. Lett. A **170**, 11 (1992);
F.V. Kowalski, J. Murray and A.C. Head, Phys. Lett. A **174**, 190 (1993);
F. Hasselbach and M. Nicklaus, Phys. Rev. A **48**, 143 (1993).

[24] B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988); **68**, 3812 (1992); Phys. Lett. A **126**, 393 (1988); **145**, 147 (1990).

[25] J.M. Cohen and B. Mashhoon, Phys. Lett. A **181**, 353 (1993);
J. Kuroiwa, M. Kasai and T. Futamase, Phys. Lett. A **182**, 330 (1993).

[26] C.G. de Oliveira and J. Tiomno, Nuovo Cimento **24**, 672 (1962);
E. Schmutzer, Ann. Physik **29**, 75 (1973);
B.M. Barker and R.F. O'Connell, Phys. Rev. D **12**, 329 (1975);
F.W. Hehl and W.T. Ni, Phys. Rev. D **42**, 2045 (1990);
J. Anandan, Phys. Rev. Lett. **68**, 3809 (1992);
J. Huang, Ann. Physik **3**, 53 (1994).

[27] Y.Q. Cai and G. Papini, Phys. Rev. Lett. **66**, 1259 (1991); **68**, 3811 (1992);
G. Papini, in Proc. of the 5th Canadian Conf. on Gen. Rel. and Rel. Astrophysics, University of Waterloo, May, 1993.

[28] B. Mashhoon, Phys. Rev. D **10**, 1059 (1974); Nature **250**, 316 (1974); Ann. Phys. (N.Y.) **89**, 254 (1975); Phys. Rev. D **11**, 2679 (1975); Phys. Lett. A **139**, 103 (1989); **173**, 347 (1993);
T. Damour and R. Ruffini, C.R. Acad. Sci. A **279**, 971 (1974);

P. Carini, L.L. Feng, M. Li and R. Ruffini, Phys. Rev. D **46**, 5407 (1992).

[29] M.P. Silverman, Phys. Lett. A **152**, 133 (1991); Nuovo Cimento D **14**, 857 (1992).

[30] B.J. Venema *et al.*, Phys. Rev. Lett. **68**, 135 (1992);
D.J. Wineland *et al.*, Phys. Rev. Lett. **67**, 1735 (1991);
R.C. Ritter *et al.*, Phys. Rev. Lett. **70**, 701 (1993);
E.G. Adelberger *et al.*, Annu. Rev. Nucl. Part. Sci. **41**, 269 (1991);
T.C.P. Chui and W.T. Ni, Phys. Rev. Lett. **71**, 3247 (1993).

[31] J. Hopkinson, Phil. Trans. Roy. Soc. London **167**, 599 (1877).

[32] A.C. Eringen, J. Math. Phys. **25**, 3235 (1984).

[33] S. Deser and B.E. Laurent, Ann. Phys. (N.Y.) **50**, 76 (1968).

[34] M. Novello and S.L.S. Duque, preprint (1993).