QUANTUM GRAVITY

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1ST. LECTURE

We face the problem of putting the quantum theory and Einstein's theory together. One has here the two greatest theories of the XXth century. Even though there is no experimental motivation for the attempt to unify them, one's likely to learn something profound by making the attempt. One can also give logical arguments about why one should quantize the gravitational field. One can begin by asking the question: if you don't quantize the gravitational field, how do you proceed?

The suggestion that comes naturally and is proposed by everybody is to replace the classical Einstein equations:

$$G_{\mu\nu} = k\, T_{\mu\nu} \qquad (1)$$

by

$$G_{\mu\nu} = k\, \langle \mathcal{T}_{\mu\nu} \rangle \qquad (2)$$

where $\langle \mathcal{T}_{\mu\nu} \rangle$ is an expectation value of the operator stress tensor of all other (quantized) fields in Nature.

D.Page has proposed an experiment, which makes use of a Cavendish-type gravity apparatus, to check the validity of this equation. Suppose we have a quantum process, involving, for example, a photon counter, in which individual quantum events can produce a decision whether the Cavendish balls are place in one position or the other. The <u>expectation value</u> of the stress tensor will represent a kind of smeared out ball that is partly in one position and partly in the other. The result of the experiment, which Page has actually performed, is that the observed gravitational field is <u>not</u> produced by such a source but by a source in which the balls are either in one position or the other. So eq.(2) is no good (see reference (1)). This does not yet fully settle the question whether the <u>dynamical</u> gravitational field should be quantized. All it shows is that the Coulomb part of the gravitational field must be quantized. But this part is "quantized" already by the fact that the source which generates it is quantized. Such a situation is already well known in the case of the electromagnetic field. One cannot argue that the field should be quantized simply because we have atomic physics which involves the Coulomb field of the nucleus. The Coulomb field is not really a separate dynamical entity. It is determined by the dynamics of the nucleus itself.

In order to obtain quantum electrodynamics, one must quantized also the radiation field. Similarly, in quantum gravity we must quantize gravitational radiation (*). There the experimental situation is that we don't really know if gravitational radiation exists. However, there is indirect evidence for it, and Einstein's theory predicts it. Most theoretical

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(*) This was pointed out already by Feynman at the Conference on the Role of Gravitation in Physics, Chapel Hill, USA, 1957.

physicists believe in Einstein's theory. Of course, even if we do observe gravitational radiation the issue of quantization will not really be settled in the experimental sense. Because quantum gravitational effects do not become important until energies of order 10^{19} GeV are reached it seems very unlikely that any experiment will be able to decide this issue. So we have to use indirect arguments. Clearly, eq. (2) does not hold for the Coulomb part of the field. If it does not hold for the Coulomb part it should also not hold for the radiation part because, from a relativistic point of view, the two parts are inseparable. If you want a relativistic theory the two must go together. This is an Einstein argument, not an experimental one.

There are some other things wrong with eq.(2). First of all quantum mechanics is no longer quantum mechanics because, in the expectation value, the wave function appears bilinearly. The wave function itself affects the gravitational field. The gravitational field, in turn, affects the movement of particles and hence their wave function. So if you write down the complete dynamics, the Schrödinger equation is no longer linear. The superposition principle breaks down and the standard interpretation of quantum mechanics (unitarity, conservation of probability, etc). evaporates. Of course, the superposition principle only breaks down very, very slightly. So for most practical purposes the standard interpretation of quantum theory will be O.K. But it would be an ugly theory. Of course, standard quantum theory may eventually be superceded by another theory. But the new theory will be beautiful. In these lectures I shall assume that standard quantum mechanics is both fundamental and essentially complete.

Eq. (2) has been used quite a lot in recent years, particularly in quantum cosmology. Essentially what one does is to take a given cosmological background and calculate $\langle T_{\mu\nu}\rangle$ in some assumed state. Because the spacetime is not flat and the curvature is changing with time, one has a situation similar to an harmonic oscillator with time-varying parameters (mass, spring constant, etc). The oscillator gets excited by a process known as parametric amplification. A time-varying gravitational field (i.e., a time-varying curvature) will excite the vacuum and produce particle pairs. It will produce both real and virtual particles. The virtual particle production is usually called vacuum polarization, although pehaps the term "polarization" is not so good in gravity as in the case of electrodynamics (in QED it is a dipole polarization while in gravity theory it is a quadrupole polarization).

Having computed $\langle T_{\mu\nu}\rangle$ one then tries to alter the background geometry so as to make eq. (2) self-consistent. In some of the early work with eq. (2) only the contribution to $\langle T_{\mu\nu}\rangle$ coming from real particle production was computed. More careful work, done later, has included the vacuum polarization part (see references 2 - 12).

All computations of this type have been confined to free fields propagating in a given background. This corresponds, in the technical language of Feynman graphs to the one-loop approximation. In loop calculations one encounters divergences. In the present context the divergences arise from the fact that the stress tensor of any field is expressed as the sum of products of field operators taken at the same spacetime point. Such operator products have to be renormalized. In the renomalization process one finds that the divergences can be removed only by adding counter terms to the gravitational action that are not like any that appear in the classical Einstein theory. This reflects the fact that the theory is not perturbatively renormalizable. Nevertheless, in certain circumstances, the terms that get subtracted vanish. For example, if the field happens to be conformally invariant, then $\langle T_{\mu\nu}\rangle$ may be reasonably well defined. A lot of work has in fact been done with conformally invariant fields.

Expectation values like $\langle T_{\mu\nu}\rangle$ can be expressed as functional derivatives of a so-called <u>effetive action</u> with respect to an effective field (*). In quantum electrodynamics the current expectation value can be expressed as

$$\langle j^{\mu}\rangle = \frac{\delta W}{\delta A_{\mu}} \tag{3}$$

(*) This was first shown by Schwinger (see reference (13)).

where W is the effective action and A_μ is an effective background field. $<j^\mu>$ has a divergence corresponding to a divergence in effective action. In the case of QED the divergent part of W is just

$$W_{div} = \text{const} \,.\, x \int F_{\mu\nu} F^{\mu\nu} d^4x \tag{4}$$

and by means of charge renormalization this term cancels out; it can be absorbed into the standard Maxwell action. In the case of gravity theory eq. (3) is replaced by

$$<T^{\mu\nu}> = Z \frac{\delta W}{\delta g_{\mu\nu}} \tag{5}$$

and W has a divergent part of the form

$$W_{div} = \Lambda^4 \int g^{1/2} d^4x + \Lambda^2 \int g^{1/2} R \, d^4x + \ell\mu \left[\Lambda \int g^{1/2} (\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}) d^4x \right] \tag{6}$$

where Λ is a high energy cutoff. The first term renormalizes the cosmological constant. The second, quadratically divergent, term, which is like the Einstein action, renormalizes the gravitational constant. The third, logarithmically divergent term, is quadratic in the curvature and has no counterpart in the classical theory. In principle the constants α and β would have to be determined by experiments that go beyong standard gravitational experiments (*). This requires a classical theory for which the field equations are of the 4th differential order instead of the 2nd. The initial data for such equations involve derivatives up to the 3rd order and it is hard to understand the physical meaning of such data. Another difficulty is that things get worse when one goes beyond one-loop. In the case of two loops the first term in the RHS of eq. (6) becomes of 6th order in the cutoff, the second term becomes of the 4th order and the third becomes of the 2nd order. In addition, a logarithmically divergent term appears which is cubic in the Riemann tensor. The higher you go the worse it gets. In the limit one has an infinite number of constants that have to be determined by experiment.

Weinberg (**) has pointed out that these constants, although infinite in number, are related to a set of renormalization group equations, also infinite in number. He speculates that there may be a stable submanifold with an ultraviolet fixed point in the infinite-dimensional manifold of coupling constants. Of course, in practice, implementing this idea would be extremelly difficult, because even in one-loop approximation the calculations are enormous. In my view this is not a practical or even a necessarily correct approach. I think that the ideas of perturbative renormalization theory may be basically wrong, and that we must ultimately go beyong perturbation theory. I shall have more to say about this in may last lecture.

Sakharov has introduced the idea that the gravitational field may be just an epiphenomenon of other fields through the contributions that they make to the effective action. The inverse of the gravity constant measures the resistance of spacetime to bending, and in Sakharov's view this resistance arises from the existence of these other fields. In principle the gravitational constant may thus be calculable.

Michael Duff (15) has pointed out a difficulty in Sakharov's idea. Suppose you have a field in an external gravitational field, i.e., in a given spacetime geometry. If you try to calculate $<T_{\mu\nu}>$ in this setting you will get different answers depending on how you define the field. Suppose you have a scalar field. It can equally well be represented by a scalar ϕ or by a scalar density $\bar{\phi}$, related to ϕ by

$$\bar{\phi} \equiv g^{1/2} \phi \tag{7}$$

(*) Actually α and β satisfy renormalization group equations that relate them to a choice of length scale introduced in the regularization procedure.

(**) See his article in reference (14).

You can write the action either in terms of ϕ or in terms of $\bar{\phi}$. Depending on which form you take for developing the quantum theory you will get different answers. There is no reason why a scalar field should be more fundamental then a scalar density, so you cannot decide which is the "true" action (similar phenomena occur with vector fields). Duff shows that if you also quantize the gravitational field, that is, if you add graviton loops to the other loops, then you'll get the same answer with either formalism. This may be regarded as another argument, albeit a technical one, for quantizing the gravitational field.

I shall conclude this lecture by considering one other irsue that is sometimes raised. Suppose we accept that the gravitational field must be quantized. Is it possible that a fundamental limitation exists, arising from the very process of measuring the gravitational field, that prevents some of the predictions of the quantum formalism from ever being tested? A similar question about the electromagnetic field was raised many years ago by Landau and Peierls. Bohr and Rosenfeld (16) showed in that case that the answer was negative, at least if one is permitted to ignore the actual atomic constitution of matter. Years later a Bohr-Rosenfeld type analysis was carried out for the gravitational field (B.S.DeWitt (17)). Again it was found that the quantum formalism is in principle fully testable with, however, one fundamental qualification: any attempt to measure a field average over a spacetime domain smaller than the Planck length will yield a result that has no prediction value whatever. (The rest of this lecture consisted of a resumé of the article cited below and the reader is referred to it for details).

2ND LECTURE

In classical electromagnetic theory waves propagate linearly and the superposition principle is valid. In quantum electrodynamics, because of the presence of the electron-position field this is no longer true when high field intensities are reached. One photon can scatter another. Although the cross sections is very small, if the waves are of sufficiently great intensity (we assume the wavelength to be much longer than the Compton wavelength so that there is very little pair production). they can scatter each other at a classical level. This scattering arises from the presence of virtual particles and is due to vacuum polarization in the scattering region. The field equations that these high intensity waves satisfy are no longer linear. They deviate from Maxwell's equations.

How does one obtain these non-linear equations? Suppose we have an external electromagnetic field that is switched on only for a finite period of time and extends over only a finite region of space (i.e., it has compact support in spacetime). Consider the probability amplitude that if the initial state is the vacuum, the final state will also be the vacuum. I am going to take Schwinger's point of view and work in the Heisenberg picture in which a "state" is a time-independent quantity. In speaking about the vacuum-to-vacuum amplitude (sometimes called vacuum persistence amplitude) I must distinguish the past or "in" vacuum, which is the state in which no particles are present in the remote past, from the future or "out" vacuum, which is the state in which no particles are present in the remote future. These two vacuum states will be denoted respectively by

$$|\text{in vac}\rangle$$

and

$$|\text{out vac}\rangle \, .$$

The vacuum-to-vacuum amplitude is

$$\langle \text{out vac} \mid \text{in vac}\rangle \equiv e^{iW} \tag{8}$$

Remark: the vacuum-to-vacuum probability, which is the square of this amplitude, is given by

$$|\langle \text{out vac} \mid \text{in vac}\rangle|^2 = e^{-2\,\text{Im}W} \tag{9}$$

If the imaginary part of W differs significantly from zero then the field, even if initially in the vacuum state, will not remain a vacuum field in the future. There is a finite probability that electron-positron pairs get produced. Conversely, if there is negligible particle production then W will be nearly a real quantity. Note that W always satisfies $ImW \geq 0$.

W is a functional of the external field A_μ^{ext}, which is a c-number:

$$W = W[A_\mu^{ext}] \tag{10}$$

Let us assume that there is no external current H-vector. Then A_μ must be regarded as a sourceless field propagating in from infinity. At infinity it is an infinitely weak classical field.

The total quantum field is the sum of A_μ^{ext} (which may be regarded as a background field) and a quantum remainder $\underset{m}{A}_\mu$. Let us introduce the so-called "Schwinger average" of any arbitrary operator $\underset{m}{Q}$:

$$\langle \underset{m}{Q} \rangle \overset{def}{=} \frac{\langle out,vac|\underset{m}{Q}|in,vac\rangle}{\langle out,vac|in,vac\rangle} \tag{11}$$

Define

$$A_\mu \overset{def}{=} \frac{\langle out,vac|(A_\mu^{ext} + \underset{m}{A}_\mu)|in,vac\rangle}{\langle out,vac|in,vac\rangle} \tag{12}$$

A_μ is called the <u>effective field</u>. It depends in a one-to-one fashion on A_μ^{ext}. Therefore instead of regarding W as a functional of A_μ^{ext} we may alternatively regard it as a (different*) functional of A_μ:

$$W[A_\mu^{ext}] \overset{def}{=} \Gamma[A_\mu] \tag{13}$$

It is possible to show that the effective field satisfies the equations

$$\frac{\delta\Gamma}{\delta A_\mu} = 0. \tag{14}$$

These are the nonlinear equations which, for high intensity fields, replace the Maxwell equations of the classical theory. To obtain their explicit form, even only approximately, one must carry out difficult calculations and perform renormalizations. Γ turns out to have the structure

$$\Gamma = S + \Sigma \tag{15}$$

where S is the classical action functional and Σ is the part of Γ that describes purely quantum effects. When Σ is set equal to zero eqs.(11) reduce to the usual Maxwell equations, which are local partial differential equations. The presence of Σ converts eqs.(11) to non-local equations which, in certain presentations, take the form of integro-differential e-quations.

Another relation that may be derived is the following:

$$\langle \underset{m}{j}^\mu \rangle = \frac{\delta W}{\delta A_\mu^{ext}} \tag{16}$$

Here $\underset{m}{j}^\mu$ is the 4-vector current operator and $\langle \underset{m}{j}^\mu \rangle$ describes the vacuum-polarization-plus-pair-production generated by the external field A_μ^{ext}. By taking higher functional derivative one can obtain also other useful Schuringer averages. By suitably probing the vacuum-to-vacuum amplitude in this way one can in fact, obtain all physical amplitudes. Loosely speaking, one may say that the vacuum already contains a complete blueprint for the field dyanmics.

(*) When an external source is present W and Γ are not numerically equal but are connected by a Legendre transformation.

instead of going into the calculation of W at this stage I shall consider a simpler problem in which, by varying the physical background, one can produce nontrivial physical effects. The most famous of these is the Casimir effect (see reference 18). While computing van der Walls forces between very close molecules (forces that arise due to charge density fluctuations), Casimir found that the interaction energy can be expressed as a sum of terms, one of which does not depend on the internal structural details of the molecules. Its presence implies that an atractive force must exist between any two paralel plane conducting surfaces in a facuum. In order to derive this force, let us first consider just one conducting surface. Consider the electromagnetic field in the semi-infinite space outside the conductor. If the conductor is perfect, there are sufficient boundary conditions as to solve Maxwell's equations by decomposition into mode functions. Instead of the conventional decomposition into plane waves one has a decomposition of the form

$$F_{\mu\nu} = \sum_A (a_A u^A_{\mu\nu} e^{-iW_A t} + a^*_A u^{A*}_{\mu\nu} e^{iW_A t}) \tag{17}$$

with $W_A > 0$, where the $u^A_{\mu\nu}$ are monochromatic reflected waves characterized by the index A. If the normalization of the $u^A_{\mu\nu}$ is chosen appropriately then the coefficients a_A, a^*_A will satisfy standard commutation relations for annihilation and cretion operators:

$$[a_A, a^*_B] = \delta_{AB} \tag{18}$$

Associated with these operators there is a vacuum state vector defined by

$$a_A \,|vac\rangle = 0, \quad \text{for all A.} \tag{19}$$

The existence of operators satisfying these equations depends, in the present case, on the fact that the whole field system is invariant under time displacements. When invariance under time displacements does not hold one cannot define a unique vacuum together with particles states, and the whole concept of "particle" becomes somewhat ambiguous.

Consider now some vacuum expectation values outside a plena conducting surface. The value of the electric field $\underset{\sim}{E}$ is zero because the field is still a collection of harmonic oscillators. However, the mean value of the square of the fields strength, $\underset{\sim}{E}^2$, is not zero. Actually it is not well defined because $\underset{\sim}{E}^2$, being the product of two field operators taken at the same spacetime point, contains the usual quantum field theoretical divergences. However, we can ask for the difference between the mean value of $\underset{\sim}{E}^2$ in the half-space vacuum and the mean value of E^2 in the Minkowski vacuum:

$$\langle \underset{\sim}{E}^2 \rangle_{\text{conductor}} - \langle \underset{\sim}{E}^2 \rangle_{\text{Minkowski}} \tag{20}$$

this also is non-zero. The difference could in principle be measured by performing a Lamb-shift experiment in the vazinity of the conducting surface.

Another important quality is the mean value of the stress tensor,

$$\underset{\sim}{T}^{\mu\nu} = \underset{\sim}{F}^{\mu\lambda} F^{\nu}_{\lambda} - 1/4\, \eta^{\mu\nu} \underset{\sim}{F}_{\lambda\tau} \underset{\sim}{F}^{\lambda\tau} \tag{21}$$

$\langle \underset{\sim}{T}^{\mu\nu} \rangle$, like $\langle \underset{\sim}{E}^2 \rangle$, diverges. How do we renormalize it? In classical mechanics the energy zero point is arbitrary. In Einstein's theory the energy zero point is absolute. $\langle \underset{\sim}{T}^{\mu\nu} \rangle$ must vanish in an empty Minkowski space time if quantum field theory is to be consistent with general relativity. This fact yields an unambiguous renormalization.

Many properties of $\langle \underset{\sim}{T}^{\mu\nu} \rangle$ can be inferred without calculation. We note that:

1 - $\underset{\sim}{T}^{\mu\nu}$ is conserved: $\dfrac{\partial T_{\mu\nu}}{\partial x^{\mu}} = 0$.

2 - In the special case of QED, it is also traceless: $\underset{\sim}{T}^{\mu}_{\mu} = 0$.

Other properties are conveniently described by introducing a special coordinate system based on the fact that the conductor defines privileged directions in both space and time. The space coordinates are oriental so that the x^3 axis is perpendicular to the plane x^1x^2 of the conductor (see Figure 1). The time axis x^0 is oriented so that the conductor is at rest.

It is clear, by symmetry, that $\langle T^{\mu\nu} \rangle$ can depend at most only on x^3. Furthermore there can be no energy flux in any preferred direction. This means that $\langle T^{i0} \rangle = 0$. For similar reasons off-diagonal elements like $\langle T^{12} \rangle$, $\langle T^{13} \rangle$ and $\langle T^{23} \rangle$ must vanish. That is to say, $\langle T^{\mu\nu} \rangle$ must be diagonal and independent of x^0, x^1 and x^2.

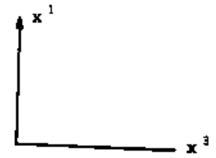

FIGURE 1

Moreover, because a perfect conductor remains a perfect conductor in any state of motion parallel to its surface, the vacuum stresses in its vicinity must look the same no matter how rapidly one is skimming over its surface. That is to say, the ether always keeps its relativistic properties, and hence $\langle T^{\mu\nu} \rangle$ must be invariant under Lorentz transformations that correspond to boosts parallel to the (x^1, x^2)-plane. This means that the first three rows and columns of $\langle T^{\mu\nu} \rangle$ must be proportional to the metric tensor of a (2 + 1)-dimensional Minkowski space, namely diag(-1,1,1). If to this inference, one adds the observations that $T_\mu{}^\mu = 0$ in the case of the electromagnetic field, one concludes that $\langle T^{\mu\nu} \rangle$ has the form

$$\langle T^{\mu\nu} \rangle = f(x^3) \times \mathrm{diag}\,(-1,1,1,-3). \tag{22}$$

But that is not all. The form of the function $f(x^3)$ too may be deduced. For thus one invokes the conservation law $\langle T^{\mu\nu}{}_{.\nu} \rangle = \langle T^{\mu\nu} \rangle_{.\nu} = 0$. In particular

$$0 = \langle T^{3\nu} \rangle_{.\nu} = -3f'(x^3) , \tag{23}$$

which implies that f is a constant, independent of x^3. Now $\langle T^{\mu\nu} \rangle$ has the dimensions of energy density. The only fundamental constants that enter into the theory are h and c. To get a constant having the dimensions of energy density one needs also a unit of length, mass, or time. No natural units with these dimensions exist in the present problem. Therefore one can only conclude that f = 0 and hence $\langle T^{\mu\nu} \rangle = 0$ in an infinite half-space.

This conclusion is, in fact, confirmed by explicit calculation. The Green's function for an infinite half-space is readily constructed from the Minkowski Green's function by the method of images. $\langle T^{\mu\nu} \rangle$ is then obtained by appropriately differentiating this Green's function and bringing into coincidence the spacetime points on which it depends. The result, of course, diverges and must be "renormalized" by subtraction of the corresponding Minkowski result. This is equivalent to subtracting the Minkowski Green's function from the half-space Green's function. Although the resulting "renormalized" Green's function does not itself vanish it nevertheless yields $\langle T^{\mu\nu} \rangle = 0$.

All the above arguments concerning the form of $\langle T^{\mu\nu} \rangle$ hold equally well for the slab manifold, except that there is now a natural unit of length - the separation distance, a, between the parallel conductors. In the region between the conductors, therefore, we expect

$$\langle T^{\mu\nu} \rangle = f(a) \times \mathrm{diag}(-1,1,1,-3). \tag{24}$$

The form of the function f(a) may be determined by considering the work required to separate the conductors adiabatically. From the infinite half-space analysis one knows that the conductors experience no forces from the outside. There is an internal force, however, of amount 3f(a) per unit area, tending to pull them together. If the conductors are moved a distance da farther apart an amount of work dW = 3f(a)da, per unit area must be supplied. This must show up as an increase in the energy per unit area. E = - af(a). Setting dW = dE and integrating, one immediately obtains

$$f(a) = A/a^4 \tag{25}$$

where A is some universal constant.

The form (25) may also be inferred by dimensional analysis. The only combination in which h, c and a can be united to yield an energy density is hc/a^4. We shall henceforth set h = c = 1. The constant A is then a pure number.

A evalution of A requires explicit computation. Again the Green's function can be constructed by the method of images and again the "renormalized" Green's function is obtained by subtracting off the Minkowski Green's function. The renormalized $\langle T^{\mu\nu} \rangle$ no longer vanishes. The antecipated form (24),(25) is confirmed and A is found to have the value $\pi^2/720$. Within expected errors this value is in agreement with experiment.

It will be observed that the energy density in the ether between the conductors is negative. It is a tiny energy, too small by many orders of magnitude to produce a gravitational field that anybody is going to measure. Yet one can easily construct gedanken experiments in which the law of conservation of energy is violated unless this energy is included in the source of the gravitational field. It turns out that the energy density in the quantum ether is often negative. The quantum theory therefore violates the hypotheses of the famous Hawking-Penrose theorems concerning the inevitability of singularities in spacetime, which imply the ultimate breakdown of classical general relativity.

3RD. LECTURE

Topological Effects: in the Casimir effect the energy density between the plates is negative. The same is also true if the electromagnetic field is replaced by a massless conformally invariant scalar field. In order to introduce fermion fields it is convenient to replace plate boundary conditions by periodic boundary conditions. The vacuum energy density for bosson fields is again found to be negative. For permion fields it is found to be positive! With periodic topologies bosons seem to be associated with negative energies and fermions with positive energies.

There is also another type of topological effect (De Witt, (14)). Consider a real scalar field. It may be regarded as the cross section of a fiber bundle in which the fiber is the real line $\mathbb{R}$. If spacetime is not simply connected the fiber bundle may be either trivial or nontrivial. Suppose 3-space is periodic in the x^3-direction. Then the fiber bundle may be twisted, which implies that a scalar field must satisfy antiperiodic boundary conditions. Similarly for spinor fields. In this case it turns out that what is negative in the untwisted case becomes positive in the twisted case and viceversa. E.g., twisted fermion fields have negative vacuum energy.

So far we have confined our attention to plane boundaries. What happens when the boundaries are curved? Consider first the case in which the boundary consists of two non-parallel plane conductors joined along the line of intersection (see fig. 2). The curvature may be regarded as concentrated on this line. The method of images, as always, is available for construction of the Green function. The renormalized $\langle \mathcal{T}^{\mu\nu} \rangle$ is found to depend on the intersection angle and to vary as the inverse of the 4th power of the distance s to the intersection line:

$$\langle \mathcal{T}^{oo} \rangle \sim - \frac{\text{const}}{s^4} \qquad (26)$$

In the case of the smoothly curved conductor (see figure 3), one finds that for points near to the conducting surface,

$$\langle \mathcal{T}^{oo} \rangle \sim - \frac{\text{const}}{s^3 \, r_c} \qquad (27)$$

where r_c is the radius of curvature. As the conductor is approached, the energy density tends to minus infinite ou the concave side and to plus infinite on the convex side. The inverse cubelaw eq. (27) actually represents

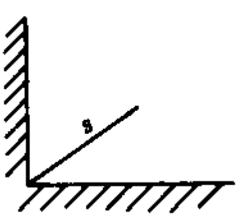

FIGURE 2

a breakdown of the perfect conductor approximation. The real energy-momentum tensor begins to depend on the structure of the conductor. Note that, as in our earlier examples, it is possible to define a precise vacuum in these cases because there is a prefered time direction for which the conductor is at rest and every thing is stationary.

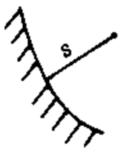

FIGURE 3

Suppose we have a plane conductor undergoing acceleration perpendicular to its surface. Can we define a vacuum in this case? In the previous case, the curvature of the boundary of the relevant incomplete spacetime manifold was purely spatial. In the present case the manifold boundary processes a curvature in time. If the acceleration varies in time the conductor will emit or absorb photons, but if the accelerations is constant then an equilibrium can be established on the concave side of the motion (see figure 4). In this particular case there is a Killing vector field. It is possible to find a system of coordinates in which the Killing vector can be expressed as:

$$K = \partial/\partial\tau \tag{28}$$

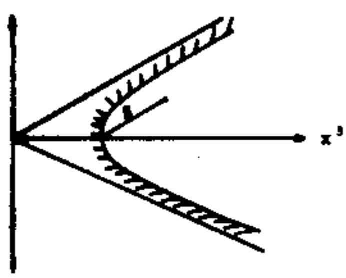

FIGURE 4

In terms of this new variable τ we can decompose the field into positive ($e^{-iw\tau}$) and negative ($e^{iw\tau}$) energy components. There will be coefficients associated with each energy component which satisfy the usual commutation relations and are identified as creation and annihilation operators. So again we can define a vacuum state.

Consider another example to clarify the situation. Suppose we have an elevator filled with gas which is suddenly accelerated. At the beginning a compression shock wave will more from the floor to the ceiling. This wave can be interpreted in terms of phonons. These phonons will gradually diffuse and the temperature of the gas will increase. As the elevator undergoes a uniform acceleration a new stationary state will be reached, and a new non-phonon state can be defined. A similar situation is encountered in the conductor case. As we suddenly accelerate it photons will be produced even though it carries no charge. As the acceleration becomes constant a stationary state will be attained.

Let a be the acceleration of the conductor. One can show that for values of s small compared to a^{-1} the vacuum energy density behaves like

$$\langle T^{oo}\rangle \sim -\frac{\text{const}}{s^3 a^{-1}} \tag{29}$$

the inverse (a^{-1}) of the acceleration in this case plays the role of a radius of curvature.

More interesting is the behaviour far away from the conductor. There, we have

$$\langle T^{oo}\rangle \sim -\frac{\text{const}}{s^4} \tag{30}$$

We can now accelerate the conductor more and more until it approaches, as a limiting path the boundary of a wedge-shaped region (see figure 5). Here spacetime is one quadrant of a Minkowski spacetime. There is a vacuum associated with this quadrant known as the Rindler vacuum (see reference (19)), because there exists a global timelike Killing vector field in the quadrant.

If the region above the elevator were filled with a thermal gas then the temperature would change from point to point. It in fact varies according to the following law:

$$T \sim 1/s \tag{31}$$

Now note that from eqs(30) and (31) we find that the energy density goes like

$$\langle T^{oo} \rangle \sim - \text{const} \cdot T^{-4} \tag{32}$$

that is, the vacuum energy density varies like a thermal radiation density, but it has the wrong sign! How can we understand this negativity?

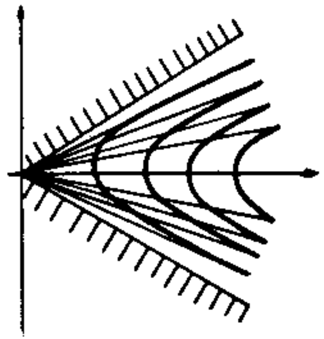

FIGURE 5

It looks as if the ground state of the Rindler vacuum is in some sense below the absolute zero of temperature. One needs to add photons to it to bring its energy up to that of the Minkowski vacuum, and these photons must be added in a thermal distribution. This interpretation is in fact correct. One may consider two kinds of photons - Rindler photons and Minkowski photons. Both are equally "real", but they are different. The Minkowski vacuum contains no Minkowski photons but it is filled with a thermal distribution of Rindler photons.

How could we detect Rindler or Minkowski photons? How does a given particle detector respond in a given circumstance? (See reference 20)). The answer to this question will help us to understand the negative energy phenomenon. A particle detector is certainly not something that measures $T^{\mu\nu}$, for the only way to measure $T^{\mu\nu}$ is by measuring the gravitational or electromagnetic field associated with it.

Usually a photon detector is a dipole detector. For simplicity let us consider a monopole detector designed to detect scalar particles. The interaction Lagrangean between the monopole and the field may be taken in the form

$$\mathcal{L}_{int} = m(\tau)\phi\,[x(\tau)] \tag{33}$$

there ϕ is the scalar field and the functions $x^\mu(\tau)$ define the world line of the detector, idealized to be a pointlike object. The operator $m(\tau)$ represents its monopole moment at the proper time τ. In calculating the detector response we shall need its matrix elements:

$$\langle E|m(\tau)|E'\rangle = \langle E|m(0)|E'\rangle\, e^{i(E-E')\tau} \tag{34}$$

Suppose the detector is initially in its ground state and the field is in a state described by the symbol Ψ. Then the probability amplitude that the monopole gets excited to the level E while the field undergoes a transition to a state Ψ' is given by

$$A(E,\Psi'|0,\Psi) = \langle E,\Psi'|T\left[\exp(i\int_{-\infty}^{\infty} \mathcal{L}_{int}\, d\tau)\right]|0,\Psi\rangle \tag{35}$$

Let us assume that $E > 0$ and neglect radiactive corrections. The amplitude can then be approximated by the first term of the perturbation expansion, namely

$$A(E,\Psi'|0,\Psi) \simeq i\langle E,\Psi'|\int_{-\infty}^{\infty} m(\tau)\phi\,[x(\tau)]\, d\tau|0,\Psi\rangle \tag{36}$$

this amplitude depends both on the path $x^\mu(\tau)$ and on the initial state Ψ of the field. Using eq. (34) we may rewrite it in the form

$$A(E,\Psi'|0,\Psi) = i\langle E|m(0)|0\rangle \int_{-\infty}^{\infty} e^{iE\tau}\langle\Psi'|\phi[x(\tau)]\,|\Psi\rangle d\tau \tag{37}$$

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From eq. (37) of the last lecture we may easily obtain the probability that the detector gets excited to the state $|E\rangle$. It is:

$$P(E) = \sum_{\Psi'} |A(E,\Psi'|0,\Psi)|^2 \approx |\langle E|\underset{\sim}{m}(0)|0\rangle|^2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-iE(\tau-\tau')} \cdot \langle\Psi|\phi[x(\tau)]\phi[x(\tau')]|\Psi\rangle \quad (38)$$

The detector response is seen to depend jointly on the monopole-moment matrix element and the Fourier transform (along the world line) of the Wightman function of the field in the state $|\Psi\rangle$. If the motion is such that the Wightman function depends only on the proper time difference $(\tau-\tau')$ them one of the τ integrations can be omitted so as to yield a simple transition rate.

If the field is in the standard Minkowski vacuum state and the detector moves along a geodesic world line, then the Wightman function has positive frequency only, the Fourier transform vanishes and the detector remains in its groundstate $(P(E)) = 0$. If the detector suffers an acceleration the Wightman function contains negative as well as positive frequencies and P(E) no longer generally vanishes.

Suppose, on the other hand, that the field is in the Rindler vacuum state and the detector moves along a Rindler world line, i.e., along a flow line of the Killing vector field. This is an accelerated motion but the detector does not get excited in this case. The general rule is as follows: unaccelerated detectors respond to Minkowski (i.e., standard) photons; accelerated detectors respond to Rindler photons. An unaccelerated detector gets excited in a Rindler vacuum (*) and an accelerated detector gets iscited in a Minkowski vacuum. A Minkowski vacuum is full of Rindler photons (from both left and right quadrants actually). To build up a a Minkowski vacuum from a Rindler vacuum one must add Rindler photons, and these have to be added in a thermal distribution.

One way to see this is to calculate the Wightman function for uniformly accelerated motion in a Minkowski vacuum:

$$\langle\Psi|\phi[x(\tau)]\,\phi[x(\tau')]|\Psi\rangle = -\frac{(a/2\pi)^2}{4\sinh^2\left[1/2\; a(\tau-\tau'-i0)\right]} \quad (39)$$

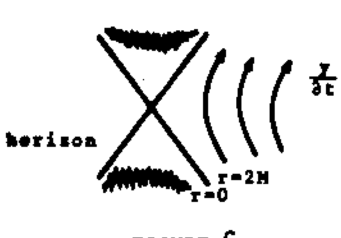
($|\Psi\rangle$ = Minkowski vacuum, a = acceleration, $\hbar = 1$).

This is a thermal Green's function having periodicity i/T where

$$T = a/2\pi \quad (k = 1). \quad (40)$$

The detector gets excited just as if it were at rest in a thermal photon bath with this temperature.

Black Holes: thermal states play a particularly important role in the theory of black holes. This may be seen by comparing Schwarzschild coordinates with Kruskal coordinates. Let t be the standard Schwarzschild time coordinate. It defines a Killing vector field $\partial/\partial t$ which is timelike outside the "horizon" (see fig. 6).



(Each point in the figure represents a 2-sphere of radius r)

FIGURE 6

(*) W.Unreeh was the first to discuss the behavior of accelerated detectors. See reference (20).

The relation between the Schwarzschild and Kruskal coordinate systems is similar to the relation between Rindler and Minkowski coordinate systems. In the right hand quadrant a complete set of mode functions can be found and we can decompose the field into positive and negative parts with respect to the Killing vector $\partial/\partial t$. A vacuum state can thus be defined with respect to this Killing vector. At large distances from the black hole this vacuum is undistinguishable from ordinary Minkowski vacuum. In particular $\langle T^{\mu\nu}\rangle$ vanishes there. Near the horizon, on the other hand, this vacuum shares many of the features of the Rindler vacuum: a particle detector at rest with respect to $\partial/\partial t$ remains in its ground state; furthermore, $\langle T^{00}\rangle$ in a local Lorentz frame becomes negatively infinite on the horizon just as it does on the boundary of the Rindler wedge in the Rindler vacuum. This vacuum was first introduced by Boulware.

There is another vacuum state that may be imposed on the Schwarzschild geometry - a state for which $\langle T^{\mu\nu}\rangle$ in a local frame remains finite on the horizon. This state is fixed by the requirement that a freely falling detector undergoes no stimulated transition in the vicinity of the horizon. This detector has the minimum posible excitation as it crosses the horizon.

This vacuum, known as the Hawking-Hartle vacuum, has a thermal interpretation from the point of view of the Boulware vacuum (as in the Rindler-Minkowski case). Let M be the mass of the black hole. Choose units for which the gravity constant G = 1 and the velocity of light c = 1. Let a detector be placed in the Hawking-Hartle vacuum at rest with respect to $\partial/\partial t$ at a position $r = 2M + \varepsilon$, $\varepsilon << 2M$, where r is the conventional Schwarzschild radial coordinate. In order to stay in this position the detector must experience an absolute acceleration equal to

$$a = \frac{(2M/\varepsilon)^{1/2}}{4M} \tag{41}$$

Because the Hawking-Hartle vacuum has the local properties of the Minkowski vacuum near $r = 2M$, it follows that the detector must react in the high frequency range at least, as if it were immersed in a thermal photon bath at temperature

$$t_\varepsilon = a/2\pi = \frac{(2M/\varepsilon)^{1/2}}{8\pi M} \tag{42}$$

these photons correspond to mode functions based on $\partial/\partial t$. They are real because they carry energy. Furthermore, they are able to escape to infinity where, because of the redshift factor $(\varepsilon/2M)^{1/2}$, they wind up at temperature

$$T_\infty = \frac{1}{8\pi M} \tag{43}$$

The blackhole we have been considering is assumed to be an "eternal" blackhole with an horizon that consists of both future and past parts. A blackhole formed by collapse has only a future horizon. The condition that $\langle T^{\mu\nu}\rangle$ be smooth on this horizon leads (starting from a precollapse state with no particles at inifnity) to a final state for which the photons at infinity are thermal, with temperature given by eq. (43), but are outgoing only. This radiation was first discovered by Hawking. In the case of the eternal blackhole the state at infinity is that of a thermal bath in equilibrium with the blackhole, with as many photons being absorbed as emitted. Associated with this equilibrium we have the usual termodynamics relation

$$dE = TdS, \tag{44}$$

where E = M, from which we can find the entropy:

$$S = 1/4\ A, \tag{45}$$

where $A = 16\pi M^2$ is the area of the blackhole.

It should be noted that the characteristic wavelength of the Hawking radiation is of the order of the blackhole radius itself. So it doesn't make much sense trying to localize the region from which the Hawking radiation originates. As the black hole radiates, an energy flux density proportional to $(1/M^4)$ is emitted. This implies a luminosity for the black hole proportional to $(const/M^2)$. The usual relation between luminosity and the mass rate of change is expressed by

$$\frac{dM}{dt} = - \frac{const}{M^2} \tag{46}$$

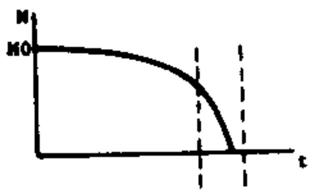


FIGURE 7

. Integrating this formula, we obtain a cubic curve for the time dependence of the mass $M(\tau)$ (see fig. 7).

This is a reasonable approximation as long as the geometry is not varying too fast, for stationarity of the final geometry was assumed in order to get a rate of particle production. As the mass decreases, the curvature increases and hence the geometry varies too. Eq. (46) does not hold in the final instants of the process, for then there are diastic changes in all relevant quantities and thus a highly non-stationary situation. A correct analysis of the final stages of black hole decay will require a full quantum theory of gravity. In my final lecture I shall discuss the formulation of quantum gravity that makes use of the so-called effective action, and I shall describe some properties that the effective action must have and even propose a reasonable approximate expression for it.

5TH LECTURE - THE EFFECTIVE ACTION FOR QUANTUM GRAVITY

(For the contents of this lecture the reader is referred to the following article: B.S.DeWitt Phys.Rev.Lett., 47, 1647 (1981)).

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