NEUTRON STARS IN THEORY AND OBSERVATION

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INTRODUCTION

It is well known that an ordinary star lives because nuclear fuel (hydrogen) provides the pressure to sustain the gravitational pull. Neglecting for a momentum angular momentum and magnetic fields three distinctively different outcomes for the end of an isolated star are possible:

1. If the core of the star was originally less massive than about 1.2 M_θ (1 M_θ = 2.10^{33}g) it will end up as a "<u>white dwarf</u>". Having cooled off for some 10^9 years it will become a "<u>black dwarf</u>";

2. If the core exceeded $\sim$ 1.2 M_θ and was less massive than $\sim$ 2.4 M_θ a <u>neutron star</u> ("n*") will be formed since the binding energy of a n* is $\sim$ 10% of its rest-mass-energy formation of a n* is a catastrophic event observable as a <u>supernova</u>. (Note however that most of this energy $E \sim 0.1 M_0 c^2$ $\sim 2.10^{53}$ ergs goes into neutrinos and eventually gravitational waves, only $\sim 10^{49}$ erg goes into visible light) The supernova remnants will die within some 10^4 years whereas the n* will live up to 10^7 years as a <u>pulsar</u> and for ever as an <u>accreting X-ray source</u>.

3. If the core-mass exceeds $\sim$ 2.4 M_θ no force can stop collapse and according to Einstein's theory a "<u>black hole</u>" is formed. This issue is as yet purely theoretical — based on the theory of nuclear forces and general relativity — just as were n* before the discovery of pulsars.

For star in binary systems (and essentially all may have passed through a binary state) the story is more complicated: the heavier of the two develops faster and will shed mass on the companion. The formation of a n* may but as we know now must not disrupt the system. n* in binary systems have been discovered. The comparison of the so formed n* will in turn develop and shed mass back on the n* without altering much the n* mass (see below) and may itself finally undergo a supernova explosion with the result of two single n* or a binary system composed of two n* which also may have been observed. Logically there is also the possibility of a n*-black hole system.

Things which complicate this picture are angular momentum and magnetic fields and it is these which make the objects observable to us. It is believed that both ordinary pulsars (i.e. pulsating radio sources) and X-ray pulsars get their clock mechanism through an oblique magnetic field whereas angular momentum keeps the clock going. Furthermore accretion from a comparison is a prerequisite to make X-ray radiation, just how this proceeds in detail is unknown. Once the matter has settled on the n* surface it gets compressed by the matter piling up on top of it. It will then be processed through nuclear burning. H burning is innocent and will just heat the n* surface but He burning may be explosive and give rise to short flashes of X-rays. These are possibly observed as "bursters" . Here then a direct determination of the surface area becomes possible which agrees well typical n* dimensions to end this short overview some numerology:

324 pulsars are known with periods between 33msec and 4.2 sec.

96 X-ray point sources are known out of which

~ 24 show "bursters". The bursters rise in a time less than a second last up to 100 seconds and carry a total amount of energy of some 10^{39} ergs. The spectra fit blackbody radiation with a temperature $T \approx 3.10^7$K coming from an area with radius 10 km.

2 pulsars are in binary systems with as yet undiscovered companion

out of the 96 X-ray sources 65 have a precisely established optical counterpart. n* in binary systems are especially valuable since they allow determination of their mass (through theory). The so determined n* masses fall into a very narrow range $1.2\ M_\odot \leq M_{n*} \leq 1.6\ M_\odot$ which is remarkable.

1. *PULSARS: THE STANDARD MODEL*

The standard model describes pulsars as a rotating n* with an inclined magnetic field by some angle χ. If such an object would rotate in vacuo it would emit low frequency waves at a rate

$$\dot{E} = -\frac{2}{3c^3}(\ddot{\mu})^2 = -\frac{2}{3c^3} R_s^6 B_s^2 \Omega^4 \tag{1}$$

Such radiation would slow down the angular momentum at a rate $\dot{E} = I\Omega\dot{\Omega} \simeq MR_s^2\Omega\dot{\Omega}$. This slow-down of n* is in fact observed and equating both energy-loss formulae we obtain

$$\Omega = -\frac{2}{3c^3} M^{-1}\Omega^3\phi^2 \qquad \phi := B_s R_s^2 \tag{2}$$

However if one computes the electric field parallel to the magnetic field at the n* surface one finds that it is orders of magnitude stronger than the gravitational force, so the surface particles will be pulled away. Even if the n* has a solid surface with good binding the vacuum around would be unstable to e^-e^+ pair creation. Therefore wherever possible a magnetonsphere will develop such that the net force on a charged particle

$$\vec{K} = e\,(\vec{E} + \frac{\vec{V}}{c} \times \vec{B}) = 0 \tag{3}$$

To this belongs a minimum charge density $4\pi\rho = \mathrm{div}\vec{E}$ which can be computed if we known in $\vec{E} = -\frac{\vec{V}}{c} \times \vec{B}$ the velocity $\vec{V}$. For rigid conotation $\vec{V} = \vec{\Omega} \times \vec{R}$ and this gives

$$\rho = -\frac{\vec{\Omega}\,\vec{B}}{2\pi c} \qquad n = \frac{\Omega B}{2\pi c e} \tag{4}$$

From (2) we can obtain the magnetic field strength at the surface ($\sim 10^{12}$ Gauss for most $\dot{\Omega}$ and Ω taking $R_s = 10^6$cm) and for $\Omega = 200$ (the value of the fastest pulsar) we get $n_e \simeq 10^{12}\ \text{cm}^{-3}(\Omega/200)B_{12}$. However rigid corotation is possible only up to $|\vec{\Omega} \times \vec{R}| = c$ the so called velocity of light cylinder. There the density will be for a dipole magnetic field throughout $n = n_s(\Omega R_s/c)^3 \simeq 10^6\ \text{cm}^{-3}\ (\Omega/200)^4$ corresponding to a plasma frequency $\omega_p^2 = 4\pi e^2 n_e/m_e \quad \omega_p \simeq 10^7(\Omega/200)^2$. Such a plasma is much too dense to let a wave of frequency Ω propagate but in our case this argument does not hold. The low frequency waves are so powerful that they will simply blow the plasma away accelerating if until energy balance is achieved:

$$m_e n_e c^2 \gamma_e \simeq \frac{B^2}{4\pi} \simeq \frac{B^2}{4\pi} \quad \text{dipole at light cylinder} \tag{5}$$

With our above given numbers we obtain $\gamma = 10^6 B_{s12}^2(\Omega/200)^{-3}$. A relativistic wind plus low frequency wave with $\gamma \simeq 10^6$ will emerge from the velocity of light cylinder carrying $\sim 10^{38}$ergs sec^{-1}. What will these electrons (or positrons) do if they enter the Crab nebula which has an estimated magnetic field of $\sim 10^{-3}$ Gauss ?

2. *THE SUPERNOVA REMNANT*

Particles in a magnetic field B emit synchrotron radiation at a frequency

$$\omega_c = \frac{3}{2}\frac{eB}{mc}\gamma^2 = 10^{16}(\text{Hertz})\ B_{-3}\gamma_6^2 \tag{6}$$

and at a rate

$$\dot{\varepsilon} = \frac{\varepsilon}{\tau} = \frac{mc^2\gamma}{\tau} = 0.510^{-14}(\text{erg/sec})\ B_{-3}^2\gamma_2^2 \tag{6a}$$

with

$$\tau = \frac{0.0167}{B_0^2\ \gamma_3}\ \text{years} \tag{6b}$$

i.e. in the ultraviolett in our case. It is reassuming that this is in fact observed with the Crab nebula. Since τ is only $\sim$ 20 years but the nebula $\sim$ 920 years old the radiation from the nebula is direct evidence that energetic particles are fed into the nebula from the pulsar. As crude as the picture may be it does allow for a prediction which can be checked: if the energy is transported they low-frequency electromagnetic waves, as given by eqn. (1), the braking index

$$n := \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2}$$

should be 3 since $\dot{\Omega} \sim -\Omega^3$. This number has been measured (only) for the Crab pulsar and found to be $n \approx 2.5$. With a time independent braking index

$$\Omega = \Omega_o \left[1 - (n-1)\frac{\dot{\Omega}_o}{\Omega_o}(t-t_o)\right]^{\frac{1}{1-n}} \qquad (7)$$

so that in 1054 when the Crab pulsar was born it had Ω/Ω_o = 1.9, 1.7, 1.68 for n = 3,25 and 2.4 respectively and the pulsar rotated roughly twice as fast as it does now ($\Omega_o \approx 200\ sec^{-1}$). The "time" $\Omega_o/\dot{\Omega}_o = :\tau \approx 2500$ years for the Crab pulsar and the total energy it would have pumped into the nebula would have been for constant n = 3

$$\Delta E = \int_o^{t_o} \dot{E}\, dt = \dot{E}_o \int \left(\frac{\Omega}{\Omega_o}\right)^4 dt = \dot{E}_o \frac{\tau}{2}\left(1 - \frac{2t_o}{\tau}\right)^{-1} >> \dot{E}_o t_o$$

Without braking the total energy would have been $\dot{E}_o t_o$ = $3 10^{48}$ ergs ($E_o = 10^{38}$ ergs) and with braking 5 times as much i.e. $\Delta E \sim 1.5 10^{49}$ ergs. This is comparable to the total kinetic energy in the nebula if its mass is $\sim M_\theta$ i.e. $1.5 10^{49}$ ergs kinetic and roughly as much magnetic energy. The velocity of the expanding gaseous filaments is V ≈ 1500 km/sec which gives $E_{kin} = \frac{1}{2} M_\theta V^2 = 1.5 10^{49}$ erg and with $R \sim 3 10^{18}$ cm an age of $\Gamma \sim 800$ years whereas the nebula is 920 years old. The average acceleration must have been $g = 1.6\ 10^{-3}$ cm sec^{-2} which is more than accounted for by the pulsar. What happens to the electrons in the nebula ? The formulae collected in (6) tell us that for $\langle B\rangle = 10^{-3}$ Gauss and $\langle\gamma\rangle = 10^6$ corresponding to $\varepsilon \approx 1$ erg we find that $\sim 10^{49}$ electrons (or more) should be present. They get degraded in energy and would have piled up with a constant $\langle B\rangle = 10^{-3}$ Gauss at $\gamma_3 = 0.0167.10^6/920 = 20\gamma_3$ giving rise to radio waves with $\nu_m = 10^3(\text{Hertz})\gamma^2 B_{-3}$, i.e. $\nu_m \approx 10^{11}$.

This is not whas is observed. The bulk of the emission is at the lowest yet observed frequency $\nu \sim 10^7$-10^8 Hertz corresponding to $\gamma \approx 300$ and a lifetime $t \approx 6$–10^4 years. It may be that the dynamics of the S-N-remnant resolves this puzzle. Using a Sedov-Taylor similarity solution for the expansion of the remnant into the surrounding interstellar medium of density ρ_1

$$R(t) = \left(\frac{2.2E_o}{\rho_1}\right)^{1/5} \sim 10^{15}\text{cm } E_{50}^{1/5}\ n_o^{-1/5}\ t_o^{2/5} \tag{8}$$

$$R = \frac{5}{2}\ v\ t \tag{8a}$$

$$n = 4n_1 = \text{const} \tag{8b}$$

$$kT = \frac{3}{25}\ \frac{2.2E_o}{n_1}\ R^{-3} \tag{8c}$$

E_o is the explosion energy. If the magnetic flux is constant $\phi = BR^2$ = const B will change as $(t_o/t)^{4/5}$ which shows that the magnetic field was large enough to degrade electrons down to $\gamma \sim 300$ in the first 10 years which is still a little short but not completely hopeless. After some 10^4 years the remnant will have died off and we are left with a n* travelling through interstellar space. Is the n* able to accrete interstellar matter ? Let us first consider single particle accretion. In this case a particle is unable to give away angular momentum so the quantity $\vec{L} = m\vec{V} \times \vec{R}$ is a constant. At the surface of the n* the free fall velocity is $v_{ff} \approx \sqrt{2GM/R} \sim 0.3c$ and the corresponding angular momentum $L \lesssim mv_{ff}R_*$. The impact radius ℓ for particles which are accreted is therefore $\ell = R_* \frac{v_{ff}}{v_\infty}$ when v_∞ is the average velocity of the particles for away from the star. The current will therefore be $I = \pi \ell^2 n_\infty v_\infty$ corresponding to an accretion Mass-rate $\dot{M} = mI \sim 10^3$ g sec^{-1} $n_{0\infty}$ M_θ $v_{6\infty}$ and to a luminosity $L = 0.1\ \dot{M}c^2 \approx 10^{23}$ erg sec^{-1} $n_{0\infty}$ M_θ $v_{6\infty}$. Even if the n* is travelling at 1000 km/sec ($v_6 = 10^2$) through a dense cloud $n_\infty = 10^4$ this would give only 10^{29} erg sec^{-1} (and the radiation would peak in X-range) hopelessly undetectable with *present instruments. There is a chance* only if there are enough collisions among the particles to that angular momentum can be removed i.e. the particles must behave as a gas. This leads us to the simplest picture of an accreting X-ray source.

3. *ACCRETING n**

3.1. *Spherical Accretion*

We get a first feeling for the problems involved if we approximate the gas by an ideal gas which gets adiabatically compressed as it falls forwards the n*. The equation $p = R\rho T$ $R = C_p - C_v$ gives with $s = \frac{3}{2}\,\ell n\; p\rho^{-\gamma} = \text{const}$ the equation of state

$$p = p_\infty \left(\frac{\rho}{\rho_\infty}\right)^\gamma \quad T = T_\infty \left(\frac{\rho}{\rho_\infty}\right)^{\gamma-1} \tag{9}$$

For stationary (i.e. $\partial/\partial t = 0$) spherically symmetric accretion the momentum equation (neglecting radiation) reads

$$\rho v \frac{dv}{dr} = -\frac{dp}{dr} - \rho \frac{M}{r^2} \tag{10}$$

and particle conservation $\dot{\rho} + \text{div}\,\rho v = 0$ gives

$$dI = \text{const} = \rho v r^2 d\Omega \tag{11}$$

with the first integrals

<u>Bernoulli' equation</u>

$$\frac{1}{2} v^2 - \frac{GM}{r} + \frac{1}{\gamma-1}\left(\frac{dp}{d\rho}\right) = \text{const}\;\frac{1}{\gamma-1} C_{s\infty}^2 \qquad C_s^2 := \frac{dp}{d\rho} \tag{12}$$

$$v = \frac{\dot{M}}{4\pi}\frac{1}{\rho r^2} \tag{13}$$

Eliminating p and $\rho = \rho_\infty (C_s/C_{s\infty})^{2/\gamma-1}$ in favour of C_s we obtain equivalently

$$v = \frac{\dot{M}}{2\pi\rho_\infty} + 2\ (\frac{C_{s\infty}}{C_s})^{2/\gamma-1} \tag{13a}$$

$$\frac{v^2}{2} + \frac{C_s^2}{\gamma-1} = \frac{GM}{r} + \frac{1}{\gamma-1} C_{s\infty}^2 \tag{12a}$$

We are looking for a simultaneous solution of (13a) and (12a). Going look a moment to the differential relations $d/dr(r^2\rho v) = 0$ and eqn. (10) we find (equating dv/dr in both equations)

$$\frac{d\rho}{dr}\ (\frac{C_s^2}{\rho} - \frac{v^2}{\rho}) = \frac{2v^2}{r} - \frac{GM}{r^2} \tag{14}$$

so that the set of equations is over determined if $v = C_s$ at some points r_s (sonic point). In this case the accretions rate is no longer a free parameter but is determined $v^2 = C_s^2 = \frac{1}{2}\frac{GM}{r_s}$ from (12a)

$$\begin{aligned} C_s &= v = C_{s\infty} \sqrt{2/5-3\gamma} \\ r_s &= \frac{5-3\gamma}{4} \frac{GM}{C_{s\infty}^2} \\ \dot{M} &= 4\pi \cdot (GM)^2 \cdot C_{s\infty}^{-3} \cdot \lambda_c \qquad \lambda_c = 1.2 \end{aligned} \tag{15}$$

Typical values are $\dot{M} = 10^{17}$ g $sec^{-1} = 10^{-9}\ M_\theta$/year, v_{n*}^2 at n* surface $v_{n*}^2 = 0.1c^2$ so that $\rho_{n*} = 10^{-6}$ gcm^{-3} or $n_{n*} = 10^{18} cm^{-3}$ much denser than a pulsar magnetosphere. Starting with $T_\infty =$ 10^4 ^oK we obtain with $C_{s\infty}$ as determined from (15) $n_\infty = 3.105^6 cm^{-3}$. This it taken literally would imply $T_{n*} = T_\infty\ (\rho_{n*}/\rho_\infty)^{\gamma-1}$ ~ ~ $10^4.(3.10^{12})^{\gamma-1}$ which even for a γ as low as 4/3 would give the absurd temperature of $T_{n*} > 10^8\ ^oK$ corresponding to a black-body luminosity $L = 4\pi R^2 aT^4$ ($a = c\,\frac{\pi^2}{60}\,(\hbar c)^{-3}\,k_B^4 = 5{,}6.10^{-5}$ cqs units) $L \sim 6.10^{40}(\text{erg/sec})T_8{}^4 R_6{}^2$ many orders of magnitude larger than the total energy set free by accretion. The final in fall is therefore no longer adiabatic.

3.2. *Influence of Magnetic Fields on Accretion*

In our somewhat idealized description the flux of gas becomes supersonic at $r_s = 0.2\,\frac{GM}{C_{s\infty}^2} \approx 2.10^{13}$ cm i.e. very far from the n* surface. Magnetic fields become important when $\epsilon_{kin} \approx \frac{B^2}{8\pi}$ which gives for a dipolar field

$$r_c = \left(\frac{\sqrt{2}\ B_{os}^2 R_s^6 C_s^3}{8\pi\lambda_c \rho_\infty m_p GM^{5/2}}\right)^{2/7} \approx 3.10^8\text{cm} \cdot B_{12}^{4/7} \cdot R_6^{12/7} \cdot n_5^{-2/7} \cdot M_\theta^{-5/7} \tag{16}$$

The speed of light cylinder is tippically at $r_L \approx 10^{10}$cm so that we have

$$r_s \gg r_L \gg r_c \gg r_{n*}$$

We expect that at $r = r_c$ the material gets decelerated and shocked. From there on (inwards) the magnetic field will dominate the infall dynamics. For free fall along the magnetic field lines one would still get $v = 0.3c$ at the surface of the n* corresponding to a single particle energy $0.1\ m_p c^2 = 100$ MeV. At the latest at the n* surface this energy goes into heat $L = 0.1\dot{M}c^2$ implying a steady-state temperature

$$T_{bb} = \left(\frac{0.1\dot{M}c^2}{4\pi R^2 a}\right)^{1/4} = 25.10^7\ {}^oK \cdot \left(\frac{\dot{M}}{10^{17}gsec^{-1}}\right)^{1/4} \cdot R_6^{-1/2} \qquad (17)$$

with photon-energy $k_BT \sim 1$ keV << $0.1m_pc^2 \approx 100$ MeV. It is this large factor by which the two energies differ which makes it at present so difficult to tell how the X-radiation is generated.

3.3. *The Eddington Limit for Spherical Accretion*

With luminosity $L = \dot{E} \approx 0.1\ c^2\dot{M}$ the merging photons exert a force on the infalling electrons and ions. The force is $K = m\dot{p}_{photon}$ and $\dot{p}_{photon} = \frac{2E_{photon}}{c}\ \frac{\sigma}{F}$ where σ is the cross-section for Thomson scattering. The electrons are morf effective and have

$$\sigma_{Th} = \pi(\frac{c^2}{m_ec^2}) = 6.7\ 10^{-24}cm^2$$

so the force on the infalling electrons is

$$F_e = \sigma_{Th}\ \frac{L}{2\pi r^2 C} \qquad (18)$$

it has the same radial dependence as does the gravitational pull $-\ GMm/r^2$. Equating both forces (which stops the infall completely) gives the critical luminosity

$$L_{Edd.} = G\ \frac{2\pi mM}{\sigma_{Th}} = 1.26\ 10^{38}erg/sec\ M_\theta\ m_p \qquad (19)$$

$$M_{Edd.} = 4\pi cRm_p\sigma_{Th}^{-1} = 1.38\ 10^{18}gsec^{-1} \qquad (20)$$

$$T_{bb\ Edd.} = 4.4.10^7\ {}^0K\ R_6^{-1/2}\ M_\theta^{1/4} \qquad (21)$$

In all observations so far one has concluded so far that indeed $L_X \lesssim L_{Edd.}$ (although our derivation is some only for spherical accretion)

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Linha 1: tado estacionario que ......

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Linha 6: Junto con $2U + W \approx 0$ conduce ....

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Linha 8: la ecuación (2) toma la forma

Linha 9: $1 + \eta = \frac{\lambda-1}{\ell n\lambda}\left[\left|\frac{W}{2U}\right| + h^2(\lambda+1)\right]$ (4)

Linha 17: $h^2 = 0.3$ y $\eta = 0.7$

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TABLA III : sustituir w^2 por h^2

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ADDENDA et ERRATA

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Una expresión menos simplista del límite inferior para 1+z en la época de formación de las galaxias es

$$1 + z \geq 0.71 \frac{(1+2\eta)^{1/2}}{(1-\eta)^{1/4}}$$

de donde sale la siguiente tabla

$1-\eta$	$1+z \geq$	$z \geq$
0.9999	12.25	11.25
0.999	6.88	5.88
0.99	3.86	2.86
0.9	2.10	1.10
0.8	1.70	0.70
0.7	1.48	0.48

Nótese que, según estos resultados, la formación de galaxias espirales ($1-\eta \approx 0.99$) tendría lugar en $z \approx 3$, corrimiento al rojo característico de los QSS.

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Es posible simplificar aún más la expresión (4) observando que el virial con rotación ($2U+W_r \approx 0$) equivale a

$$\left|\frac{W}{2U}\right| = 1 - h^2$$

de donde la (4) se reduce a

$$1 + \eta = \frac{\lambda-1}{\ell n\lambda} \left[1 + h^2 \lambda\right]$$