Awaking the vacuum in relativistic stars (and putting it to sleep)

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In collaboration with Daniel Vanzella, George Matsas, André Landulfo, and Raissa Mendes.

Vacuum instability in relativistic stars

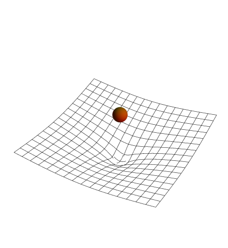
Hypothesis

The physical system:

Static spherically symmetric star made of perfect fluid

+

A free quantum scalar field $\hat{\Phi}$



The space-time metric:

$$ds^2 = -f(\chi)\left(dt^2 - d\chi^2\right) + r^2(\chi)\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

Field equation: the Klein-Gordon equation.

$$-\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\hat{\Phi}) + (m^2 + \xi R)\hat{\Phi} = 0.$$

WCCL and D. Vanzella, *Phys. Rev. Lett.* **104**, 161102 (2010)
WCCL, G. Matsas, and D. Vanzella, *Phys. Rev. Lett.* **105**, 151102 (2010)

Vacuum instability in relativistic stars

Field modes

The field modes can be written as

$$g_{\lambda l \mu} = T_\lambda(t) \frac{F_{\lambda l}(\chi)}{r(\chi)} Y_{l\mu}(\theta, \varphi),$$

where $Y_{l\mu}$ are the spherical harmonics, while T_λ and $F_{\lambda l}$ satisfy the following equations:

$$\frac{d^2}{dt^2} T_\lambda + \lambda T_\lambda = 0 \qquad -\frac{d^2}{d\chi^2} F_{\lambda l} + V_{\text{eff}}^{(l)} F_{\lambda l} = \lambda F_{\lambda l},$$

with the effective potential given by

$$V_{\text{eff}}^{(l)} = f\left[\xi R + \frac{l(l+1)}{r^2}\right] + \frac{1}{r}\frac{d^2 r}{d\chi^2}.$$

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Field modes

When the effective potential $V_{\text{eff}}^{(l)} > 0$, the time-independent Schrödinger-type equation

$$-\frac{d^2}{d\chi^2}F_{\lambda l} + V_{\text{eff}}^{(l)}F_{\lambda l} = \lambda F_{\lambda l}$$

only have solutions with $\lambda > 0$, which leads to the usual oscillatory modes

$$v_{\varpi l\mu} \propto e^{-i\varpi t}, \text{ with } \varpi = \sqrt{\lambda}.$$

Whenever $V_{\text{eff}}^{(l)}$ is negative enough in a region big enough we will also have solutions with $\lambda < 0$ and will need the following tachyonic modes:

$$w_{\lambda l\mu} \propto e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}, \text{ with } \Omega = \sqrt{-\lambda}.$$

Hence, the field operator can be expanded as

$$\hat{\Phi} = \sum_{l\mu}\int d\varpi(\hat{b}_{\varpi l\mu}v_{\varpi l\mu} + \text{H.c.}) + \sum_{\Omega l\mu}(\hat{c}_{\Omega l\mu}w_{\Omega l\mu} + \text{H.c.}).$$

Vacuum instability in relativistic stars

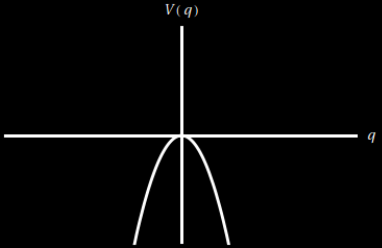
Hamiltonian operator

The Hamiltonian operator associated with the quantum field is given by

$$\begin{aligned} \hat{H} &\equiv \int_{\Sigma_t} \frac{d\Sigma}{\sqrt{f}} \hat{T}_{00} \\ &= \sum_{l\mu} \int d\varpi \frac{\varpi}{2} (\hat{b}_{\varpi l\mu} \hat{b}^{\dagger}_{\varpi l\mu} + \hat{b}^{\dagger}_{\varpi l\mu} \hat{b}_{\varpi l\mu}) - \sum_{\Omega l\mu} \frac{\Omega}{2} (\hat{c}_{\Omega l\mu} \hat{c}_{\Omega l\mu} + \hat{c}^{\dagger}_{\Omega l\mu} \hat{c}^{\dagger}_{\Omega l\mu}). \end{aligned}$$

The contribution from the unstable sector to the Hamiltonian can be written as

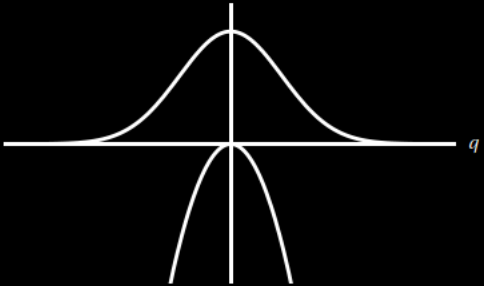
$$\begin{aligned} \hat{H}_{\Omega l\mu} &= -\frac{\Omega}{2} (\hat{c}_{\Omega l\mu} \hat{c}_{\Omega l\mu} + \hat{c}^{\dagger}_{\Omega l\mu} \hat{c}^{\dagger}_{\Omega l\mu}) \\ &= \frac{1}{2} \hat{p}^2 - \frac{\Omega^2}{2} \hat{q}^2. \end{aligned}$$



Vacuum instability in relativistic stars

Hamiltonian operator

$$\hat{H}_{\Omega l\mu} = \frac{1}{2}\hat{p}^2 - \frac{\Omega^2}{2}\hat{q}^2$$



Vacuum instability in relativistic stars

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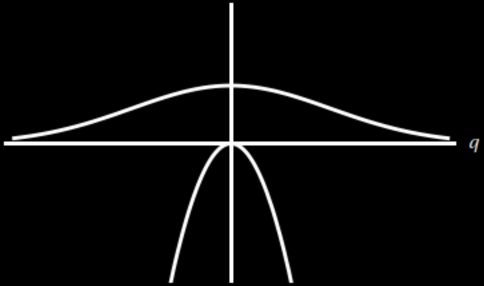

Vacuum instability in relativistic stars

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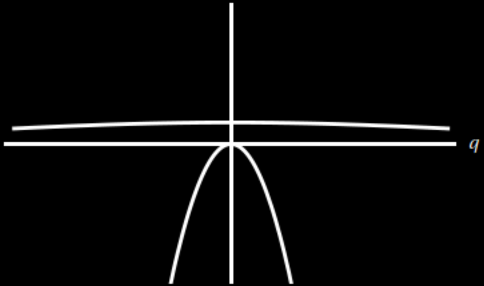


Vacuum instability in relativistic stars

Probing the instability with particle detectors

The fact that

$$i\partial_t w_{\Omega l\mu} \neq \text{const} \times w_{\Omega l\mu}$$

implies

$$[\hat{H}, \hat{c}^\dagger_{\Omega l\mu}\hat{c}_{\Omega l\mu}] \neq 0$$

and the states of the quantum field cannot be associated to any particle content when the field is unstable.

In fact, for a two-state Unruh-DeWitt particle detector which remains turned on for a period of coordinate time $\Delta t \gg \bar{\Omega}^{-1}$ the excitation probability is given by

$$P_{\text{exc}} \sim g(\Delta E, \Delta t, \bar{\Omega}, r_{\text{d}}) e^{2\bar{\Omega}\Delta t},$$

with $\bar{\Omega}$ the highest value for Ω. This increasing excitation rate is due to the fact that during the unstable phase the energy of the field is not bounded from below.

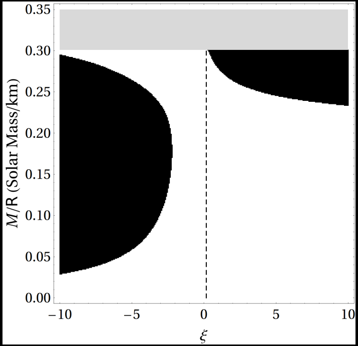
A. Landulfo, WCCL, G. Matsas, and D. Vanzella, *Phys. Rev. D* **86**, 104025 (2012)

Vacuum instability in relativistic stars

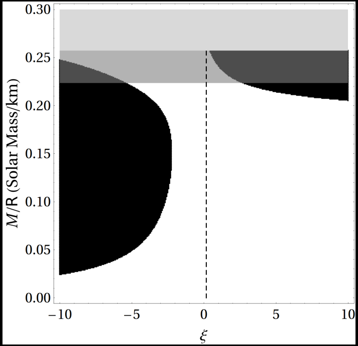
Search for tachyonic modes

The diagrams show the existence of tachyonic modes for different values of the mass-radius ratio of the star and ξ when $m = 0$.

Uniform-density star



Parabolic density profile

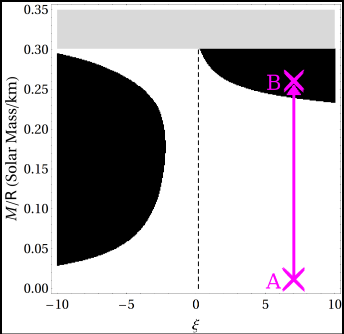


WCCL, G. Matsas, and D. Vanzella, *Phys. Rev. Lett.*, **105**, 151102 (2010).

Awaking the vacuum in relativistic stars

Star formation

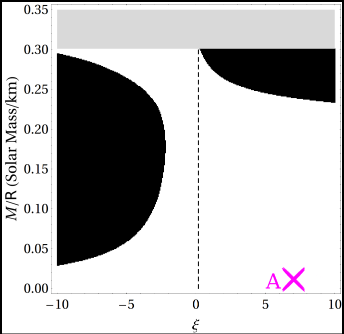
Consider the formation of a relativistic star from matter initially scattered throughout space.



$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & \text{(A)} \\ -f(dt^2 - d\chi^2) + r^2 d\Omega & \text{(B)} \end{cases}$$

Awaking the vacuum in relativistic stars

Quantum field in region A



The field operator can be expanded as

$$\hat{\Phi} = \int d^3k(\hat{a}_{\mathbf{k}} u_{\mathbf{k}} + \text{H.c.}),$$

where

$$u_{\mathbf{k}} \overset{A}{\propto} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}} t)}$$

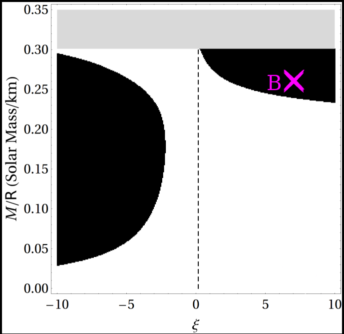
and $\omega_{\mathbf{k}} = \|\mathbf{k}\|$. Then, the in-vacuum is defined as

$$\hat{a}_{\mathbf{k}}|0_{\text{in}}\rangle = 0$$

for all $\mathbf{k}$.

Awaking the vacuum in relativistic stars

Quantum field in region B



The field operator can also be expanded as

$$\hat{\Phi} = \sum_{l\mu} \int d\varpi \hat{b}_{\varpi l \mu} v_{\varpi l \mu} + \sum_{\Omega l \mu} \hat{c}_{\Omega l \mu} w_{\Omega l \mu} + \text{H.c.},$$

with

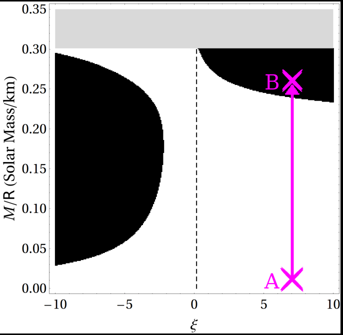
$$v_{\varpi l \mu} \overset{\text{B}}{\propto} e^{-i\varpi t}$$

and

$$w_{\Omega l \mu} \overset{\text{B}}{\propto} e^{\Omega t - i\pi/4} + e^{-\Omega t + i\pi/4}.$$

Awaking the vacuum in relativistic stars

Vacuum awakening effect



The fluctuations in the in-vacuum can be shown to behave as

$$\langle 0_{\rm in}|\hat{\Phi}^2|0_{\rm in}\rangle \overset{\rm B}{\sim} \frac{\kappa e^{2\bar{\Omega}t}}{2\bar{\Omega}}\left(\frac{F_{\bar{\Omega}0}}{r}\right)^2[1+\mathcal{O}\left(e^{-\epsilon t}\right)],$$

while the components of the energy-momentum tensor grow as

$$\rho_{\rm v} \propto \langle 0_{\rm in}|\hat{T}_{00}|0_{\rm in}\rangle \overset{\rm B}{\sim} \frac{\bar{\Omega}}{\mathsf{R}^3} h(r/\mathsf{R}) e^{2\bar{\Omega}t},$$

$$j^i_{\rm v} \propto \langle 0_{\rm in}|\hat{T}_{0i}|0_{\rm in}\rangle \overset{\rm B}{\sim} -\frac{\bar{\Omega}}{\mathsf{R}^3} V_i(r/\mathsf{R}) e^{2\bar{\Omega}t}.$$

The vacuum energy density and energy current satisfy

$$\partial_t \rho_{\rm v} + D_i j^i_{\rm v} = 0.$$

Falling asleep of the vacuum and particle creation

Quantum fluctuations versus relativistic stars

Consider a star of radius R. From the energy-momentum tensor we can estimate the typical energy density associated to quantum fluctuations:

$$\rho_{\rm v} \sim \frac{\hbar c}{\mathsf{R}^4} \approx \frac{3 \times 10^{-46}\ \mathrm{g/cm^3}}{(\mathsf{R}/1\ \mathrm{m})^4}.$$

For R $\approx 10^4$ m and $\rho_{\rm s} \approx 10^{15}$ g/cm^3 (typical size and density of neutron stars)

$$\frac{\rho_{\rm v}}{\rho_{\rm s}} \sim 3 \times 10^{-77}.$$

Apparently, quantum fluctuations will not play a important role in the context of relativistic stars.

Falling asleep of the vacuum and particle creation

Quantum fluctuations versus relativistic stars

The Again, a star of radius R. From the energy-momentum tensor,

$$\rho_{\rm v} \stackrel{\rm B}{\sim} e^{\frac{2ct}{\rm R}} \frac{\hbar c}{\rm R^4} \sim \exp\left\{\frac{t/10^{-9}\ {\rm s}}{{\rm R}/1\ {\rm m}}\right\} \frac{3 \times 10^{-46}\ {\rm g/cm^3}}{({\rm R}/1\ {\rm m})^4}.$$

For $\rm R \approx 10^4\ m$ and $\rho_{\rm s} \approx 10^{15}\ {\rm g/cm^3}$

$$\frac{\rho_{\rm v}}{\rho_{\rm s}} \stackrel{\rm B}{\sim} 3 \times 10^{-77} \times \exp\left\{\frac{t}{10^{-5}\ {\rm s}}\right\}.$$

Therefore

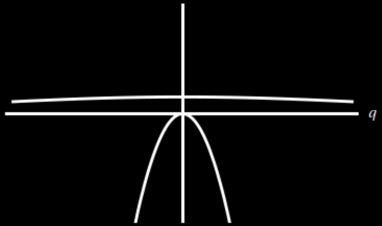
$$t \approx 2 \times 10^{-3}\ {\rm s} \quad \Rightarrow \quad \frac{\rho_{\rm v}}{\rho_{\rm s}} \sim 1.$$

In conclusion, backreaction effects must be taken into account.

Falling asleep of the vacuum and particle creation

Particle creation

The backreaction process will lead the system to some new stable configuration.



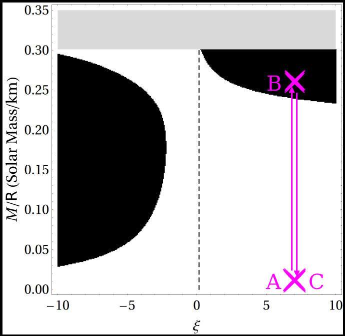
At the end of the unstable phase, the quantum fluctuations will be too big to fit into the new vacuum state. The “accommodation” of these fluctuations will lead then to a particle creation process.

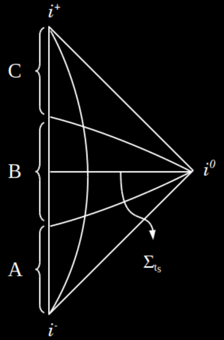
A. Landulfo, WCCL, G. Matsas, and D. Vanzella, *Phys. Rev. D* **86**, 104025 (2012)

Falling asleep of the vacuum and particle creation

Particle creation

For the sake of calculation, consider a spacetime which is symmetric by time reflection with respect the Cauchy surface Σ_{t_s}.





$$ds^2 = \begin{cases} -dt^2 + d\mathbf{x}^2 & \text{(A)} \\ -f(dt^2 - d\chi^2) + r^2 d\Omega & \text{(B)} \\ -dt^2 + d\mathbf{x}^2 & \text{(C)} \end{cases}$$

Falling asleep of the vacuum and particle creation

Particle creation

With the supposition that the spacetime will become flat when the system stabilize, we can expand the quantum field in terms of a third set of modes:

$$\hat{\Phi} = \int_{\mathbb{R}^3} d^3p(\hat{d}_{\mathbf{p}}\nu_{\mathbf{p}} + \text{H.c.}),$$

with

$$\nu_{\mathbf{p}} \overset{\text{C}}{\propto} e^{i(\mathbf{p}\cdot\mathbf{x} - \omega_{\mathbf{p}}t)}.$$

Then, we can estimate the number of particles in the mode $g_{\mathbf{p}}$ produced in C:

$$\langle 0_{\text{in}}|\hat{N}_{\mathbf{p}}|0_{\text{in}}\rangle \overset{\text{C}}{\sim} e^{2\bar{\Omega}T} \int_{\mathbb{R}^3} d^3k|\zeta_{\mathbf{kp}}|^2,$$

where

$$\zeta_{\mathbf{kp}} \equiv -i[(i\alpha^*_{\bar{\Omega}\mathbf{k}} + \beta_{\bar{\Omega}\mathbf{k}})\alpha_{\bar{\Omega}-\mathbf{p}} + (\alpha^*_{\bar{\Omega}\mathbf{k}} - i\beta_{\bar{\Omega}\mathbf{k}})\beta^*_{\bar{\Omega}-\mathbf{p}}]$$

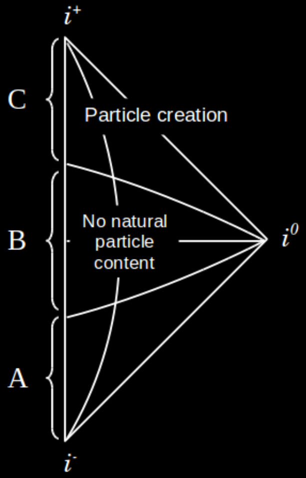
with the Bogoliubov coefficients

$$\alpha_{\bar{\Omega}\mathbf{k}} \equiv (u_{\mathbf{k}}, w_{\bar{\Omega}})_{\text{KG}} \qquad \beta_{\bar{\Omega}\mathbf{k}} \equiv -(u^*_{\mathbf{k}}, w_{\bar{\Omega}})_{\text{KG}}$$

and T the time duration of the unstable phase.

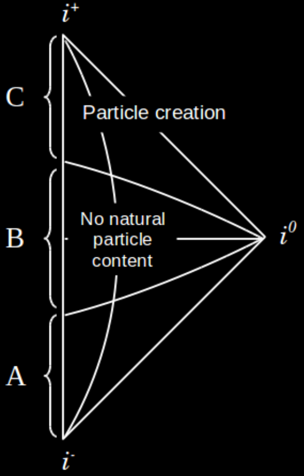
Falling asleep of the vacuum and particle creation

Particle creation



Falling asleep of the vacuum and particle creation

Particle creation



Classical analyses have shown that when the system reaches its new stable configuration, about 10% of the energy of the system is released as scalar waves.

J. Novak, *Phys. Rev. D* **58**, 064019 (1998)
M. Ruiz et al, *Phys. Rev. D* **86**, 104044 (2012)

Deviations from sphericity

Spacetime metric

It is interesting to know whether the main features of the effect discussed so far are preserved when some symmetries are relaxed. As a laboratory, we have used thin static spheroidal shells. The most general metric for the external spacetime is

$$ds_+^2 = -e^{2\lambda(\rho,z)}dt^2 + e^{2[\nu(\rho,z)-\lambda(\rho,z)]}(d\rho^2 + dz^2) + \rho^2 e^{-2\lambda(\rho,z)}d\varphi^2,$$

where, due to the vacuum Einstein equation, λ and ν satisfy

$$\partial_\rho^2\lambda + \partial_z^2\lambda + \rho^{-1}\partial_\rho\lambda = 0 \qquad \partial_\rho\nu = \rho[(\partial_\rho\lambda)^2 - (\partial_z\lambda)^2],$$

$$\partial_z\nu = 2\rho\partial_\rho\lambda\partial_z\lambda.$$

We assume that the shell $\mathcal{S}$ satisfies $\lambda|_{\mathcal{S}} = \lambda_0 \equiv \text{const.}$ The internal portion of the spacetime is flat and described by

$$ds_-^2 = -e^{2\lambda_0}dt^2 + e^{-2\lambda_0}(d\bar{\rho}^2 + d\bar{z}^2 + \bar{\rho}^2 d\varphi^2).$$

Deviations from sphericity

Spacetime metric

Taking $\rho^2 = a^2(x^2-1)(1-y^2)$ and $z = axy$, the (Laplace) equation for λ becomes

$$\partial_x[(x^2-1)\partial_x\lambda] + \partial_y[(1-y^2)\partial_y\lambda] = 0,$$

with $y \in [-1,1]$ and $x \in [1,\infty[$. The solution which is regular in $y = \pm 1$ and and well behaved when $x \to \infty$

$$\lambda = \sum_{j=0}^{\infty} A_j Q_j(x) P_j(y),$$

where P_j and Q_j are zero-order associated Legendre functions of first and second kinds. In our analysis we have restricted ourselves tho the following class of static spheroidal shells: $A_0 = -\beta > 0$, $A_j = 0$ for $j > 0$, and

$$\lambda = -\frac{\beta}{2}\ln\left(\frac{x+1}{x-1}\right).$$

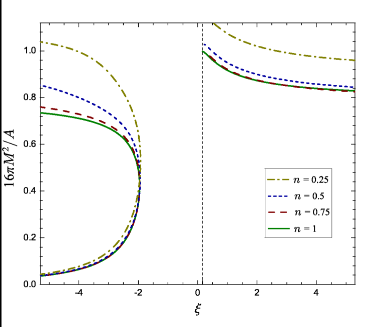
J.D. McCrea, *J. Phys. A: Math. Gen.* **9**, 697 (1976)

Deviations from sphericity

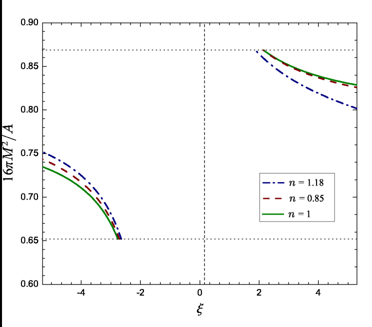
Search for tachyonic modes

The diagrams show the existence of tachyonic modes for a particular class of spheroidal shells and a massless field. In the diagrams below, $n \equiv L_{\text{equatorial}}/L_{\text{meridional}}$.

Prolate shells



Oblate shells



WCCL, R. Mendes, G. Matsas, and D. Vanzella, arXiv:1304.0582

Conclusions

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- Semiclassical gravity may have much to say about relativistic stars depending on the existence of proper scalar fields.

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- Semiclassical gravity may have much to say about relativistic stars depending on the existence of proper scalar fields.
- Relativistic stars have much to say about whether non-minimally coupled free scalar fields exist in nature.

Prospects for further research

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- Field-theoretic analysis of the Fock space for unstable fields in curved spacetimes (in preparation).

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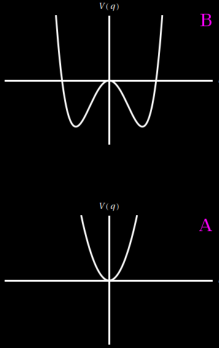
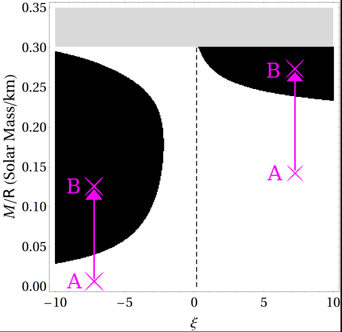
- Field-theoretic analysis of the Fock space for unstable fields in curved spacetimes (in preparation).
- Backreaction analysis and its relation to the classical regime (work in progress).
- Vacuum awakening in cosmological scenarios (in discussion).
- Instability in spacetimes generated by rotating sources (under analysis — R. Mendes and G. Matsas).

Thank you for your attention!

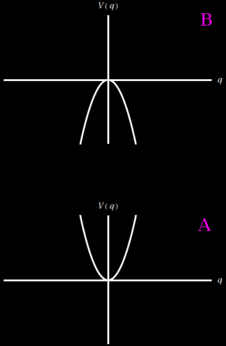
Some comments on the backreaction issue

Scalarization energetically favored





Scalarization NOT energetically favored



T. Damour and G. Esposito-Farèse, *Phys. Rev. Lett.* **70**, 2220 (1993)
P. Pani et al, *Phys. Rev. D* **83**, 081501 (2011)