Super-Hubble Waves and the Size of the Universe
(arXiv:1304.1181)

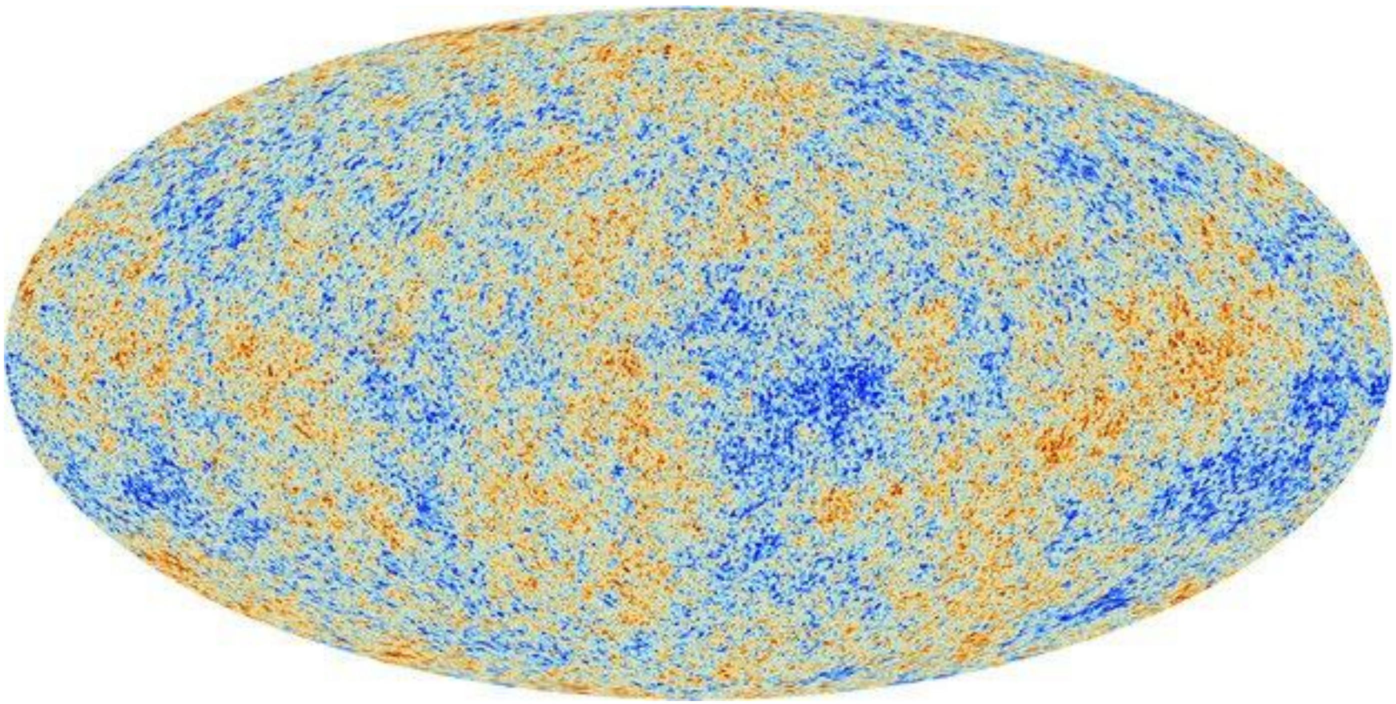
Thiago Pereira & Luis Gustavo T. Silva
Universidade Estadual de Londrina

V Workshop Challenges of New Physics in Space
Rio de Janeiro -- 2013

Outline

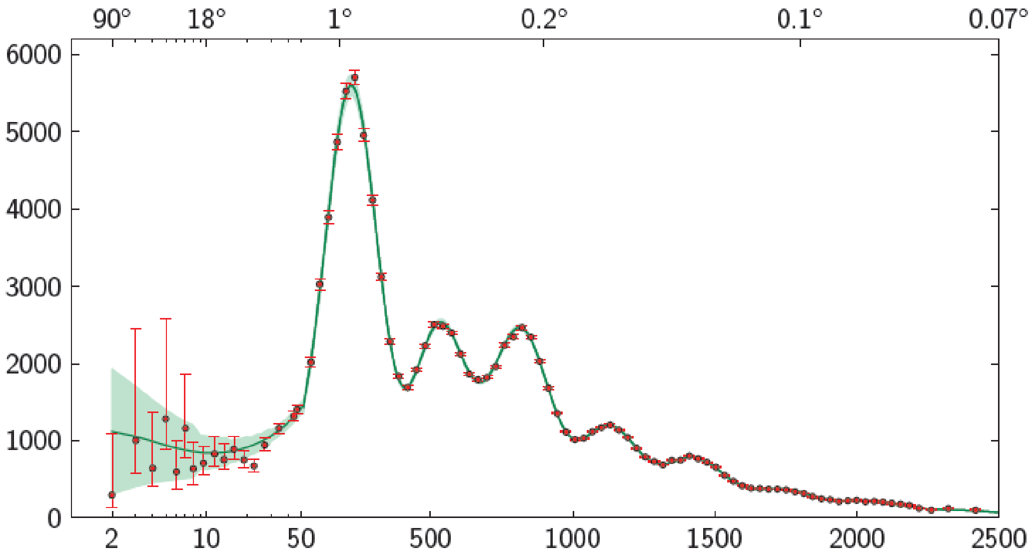
- Real versus harmonic space CMB
- Correlation function in real space
- Super-Hubble stochastic waves and bounds on the size of the universe
- Conclusions

Planck Results



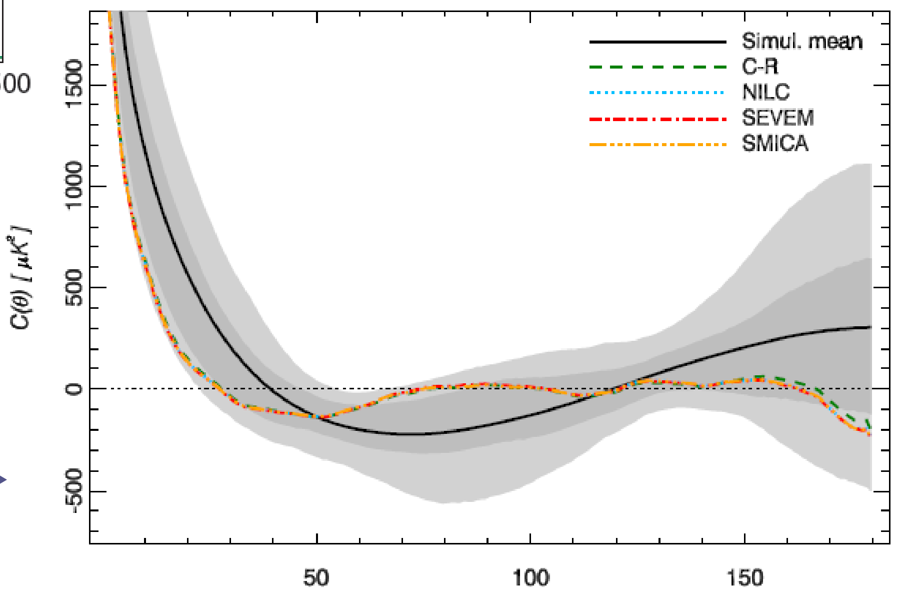
$$\Delta T(\hat{\mathbf{n}}) = \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\hat{\mathbf{n}}) \qquad C(\theta) = \langle \Delta T(\hat{\mathbf{n}}_1) \Delta T(\hat{\mathbf{n}}_2) \rangle$$

Temperature power spectrum



$$C_\ell = 2\pi \int_{-1}^{1} C(\mu) P_\ell(\mu) d\mu$$

$$C(\theta) = \sum_{\ell=2}^{\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

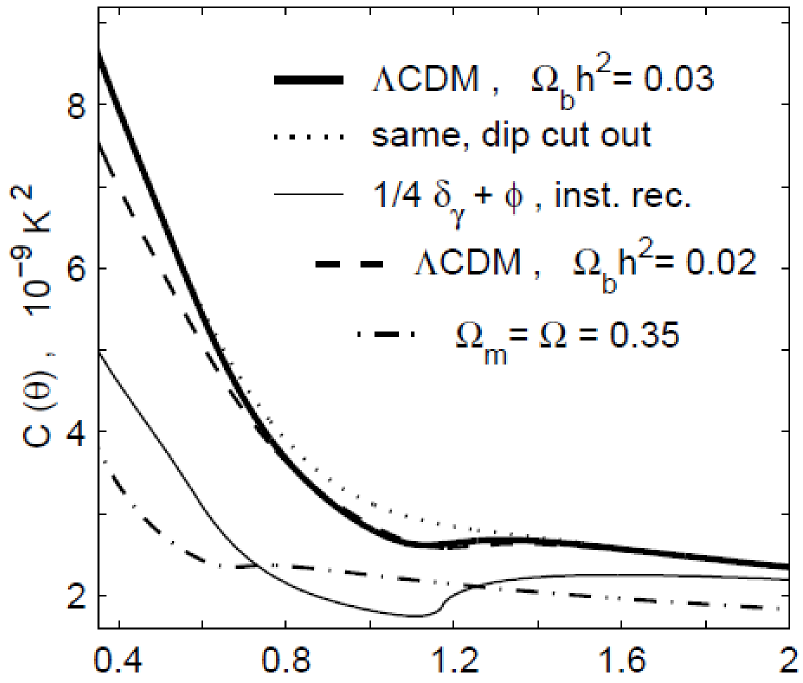


Why worry about real-space CMB?

- Clear connection between pre-recombination physics and data.
  (Bashinsky & Bertschinger, PRL 2001)

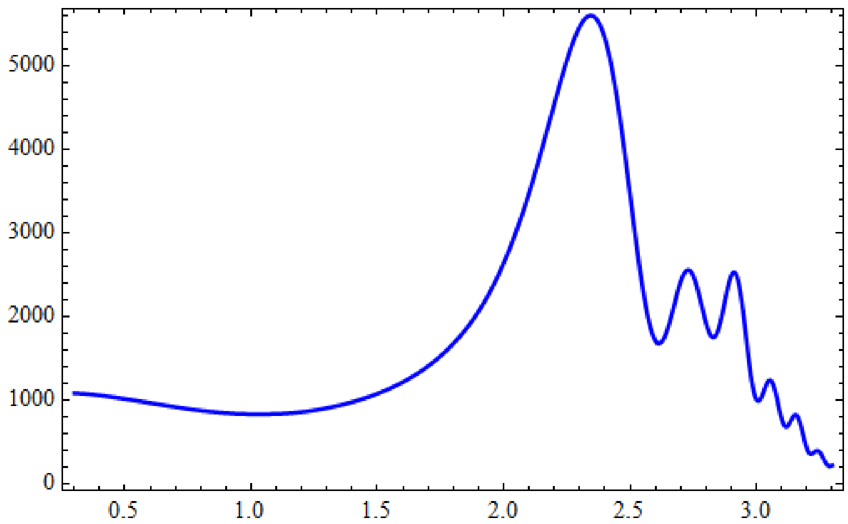
- Clear interpretation of causality in physical processes.
  (Abramo et. al, PRD 2010)

- Arguably the most important source of large-angle anomalies.



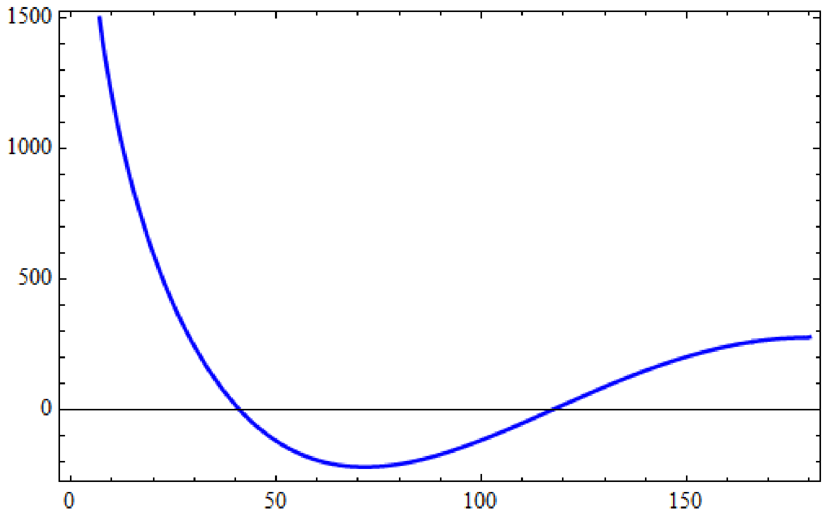
CMB in different representations

Harmonic Space



- Cls decoupled at different ls.
- All scales contribute to a given l.

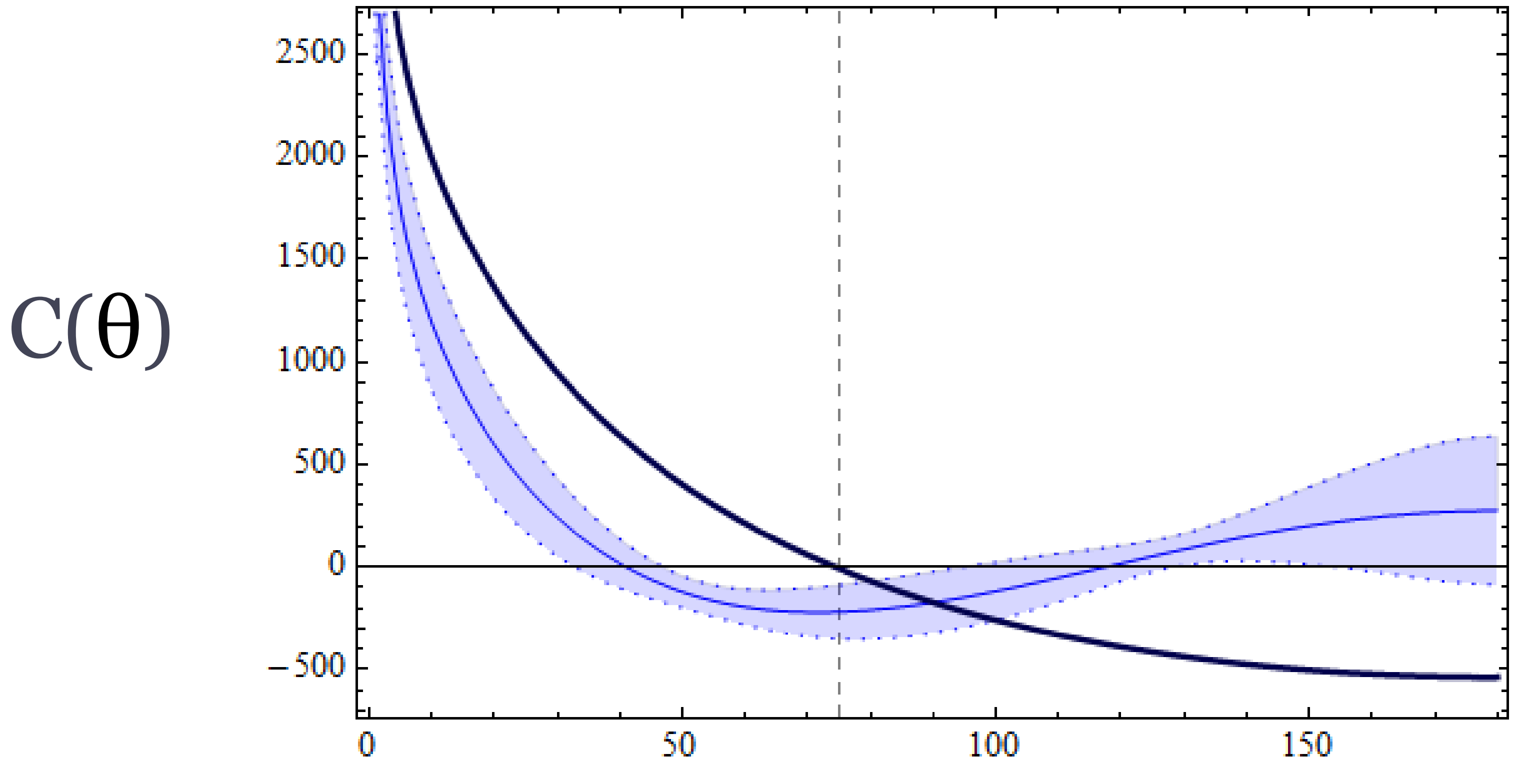
Real Space



- C(θ) coupled at different θ.
- Physics is interpreted locally.

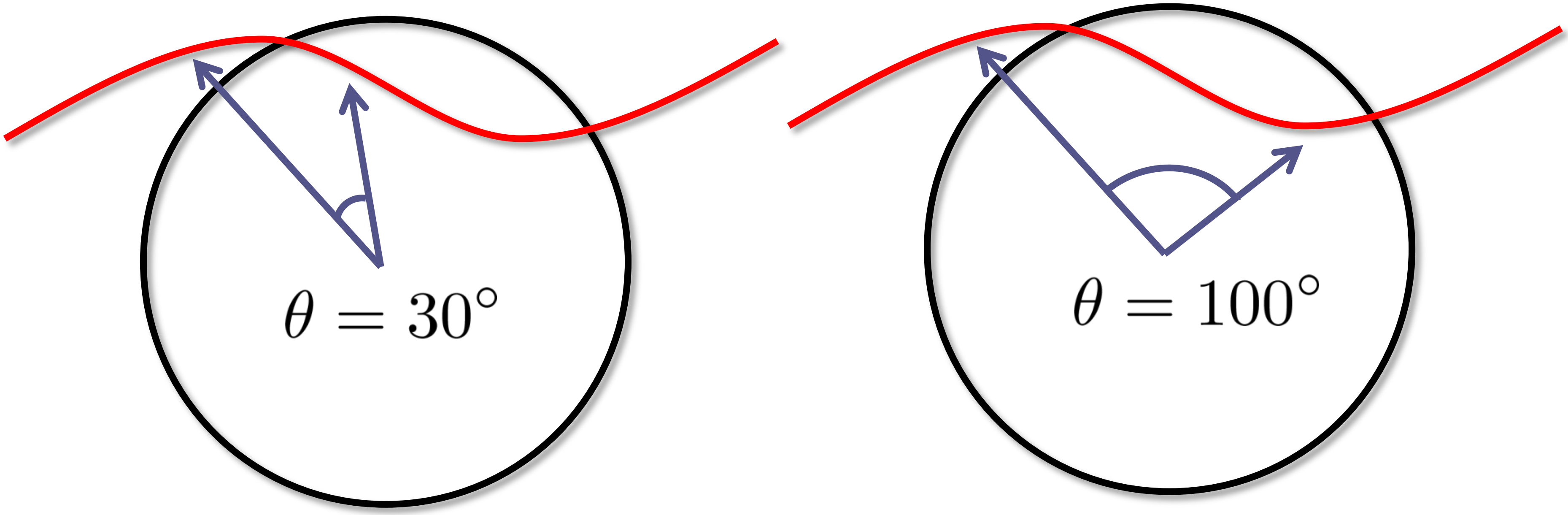
Both representations convey the same physics!!

Why C(θ)=0 at large scales?



$$\ell(\ell+1)C_\ell = \text{cst.} \longrightarrow 2C_1 = 2\times 3C_2$$

Why C(θ)=0 at large scales?



$$C(30^\circ) = \langle(\pm)(\pm)\rangle \geq 0 \qquad C(100^\circ) = \langle(\pm)(\mp)\rangle \leq 0$$

What about larger and shifted super-Hubble waves?

Sachs-Wolfe effect: $\Delta T = \frac{1}{3}\Phi$

$$C(\theta) = \langle \Delta T(\mathbf{n}_1)\Delta T(\mathbf{n}_2)\rangle$$
$$= \frac{1}{9}\int d^3k d^3q \langle \Phi(\mathbf{k})\Phi(\mathbf{q})\rangle e^{i(\mathbf{k}\cdot\mathbf{x}+\mathbf{q}\cdot\mathbf{y})}$$
$$= \frac{A}{9}\int_0^\infty \frac{dk}{k} k^{n_s-1}\frac{\sin kr}{kr}$$

where

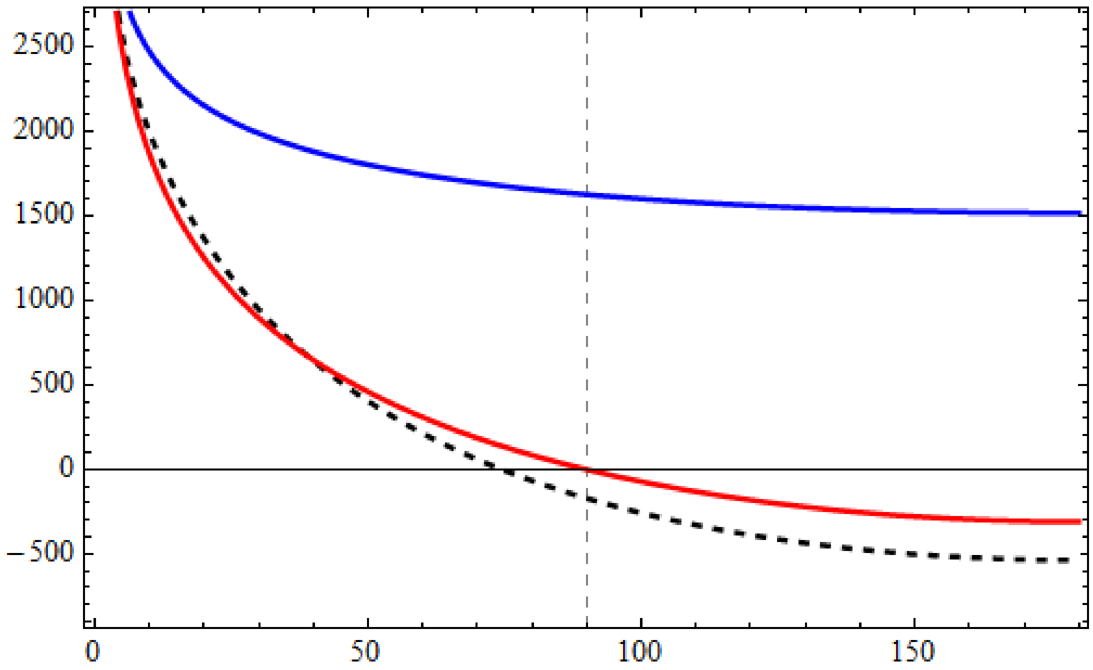
$$\langle \Phi(\mathbf{k})\Phi(\mathbf{q})\rangle = 2\pi^2 k^{-3} P(k)\delta(\mathbf{k}+\mathbf{q})$$

$$P(k) = Ak^{ns-1}$$

$$r = 2\Delta\tau\sqrt{1-\cos\theta}$$

Sachs-Wolfe effect

$$C(\theta) \propto \begin{cases} \frac{1}{(1-\cos\theta)^{\frac{1-n_s}{2}}} & n_s > 1 \\ \ln\left(\frac{1}{1-\cos\theta}\right)^{1/2} & n_s = 1 \end{cases}$$



Blue spectra

Harrison-Zel'dovich

Given ns, how large a super-Hubble can be?

Filtered stochastic waves

$$\mathrm{d}s^2 = a^2[-(1+2\Phi)\mathrm{d}\tau^2 - (1-2\Phi)\delta_{ij}\mathrm{d}x_i\mathrm{d}x_j]$$

$$\Phi(\tau, \mathbf{k}) = \psi(\mathbf{k})\ \phi(\tau, k)$$

Random Initial Conditions:

$$\langle\psi(\mathbf{k})\psi(\mathbf{q})\rangle = P(k)\delta(\mathbf{k}+\mathbf{q})$$

Gravitational Dynamics:

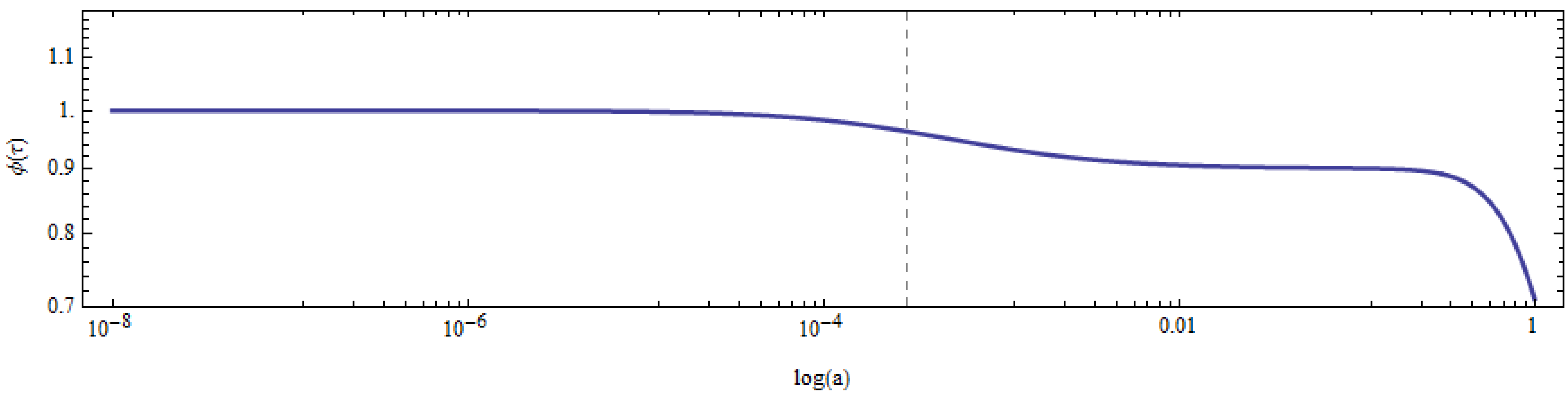
$$\phi'' + 3\mathcal{H}(1+c_s^2)\phi' + [2\mathcal{H}' + \mathcal{H}^2(1+3c_s^2)]\phi + k^2c_s^2\phi = 0$$

Filtered stochastic waves

Large scale approximation:

0

$$\phi'' + 3\mathcal{H}(1+c_s^2)\phi' + [2\mathcal{H}' + \mathcal{H}^2(1+3c_s^2)]\phi + k^2 c_s^2 \phi = 0$$



Ansatz for initial conditions:

$$\psi_{\mathbf{k}} = \sqrt{P(k)}e^{i\mathbf{k}\cdot\mathbf{x}} \qquad P(k) = Ak^{n_s-4}$$

One realization!

Filtered stochastic waves

Large scale gravitational wave:

$$\Phi(\tau, \mathbf{x}) = \phi(\tau)\psi(\mathbf{x}) = \phi(\tau) \sum_{\ell,m} \varpi_{\ell m}(x) Y_{\ell m}(\hat{\mathbf{x}})$$

$$\varpi_{\ell m} \equiv \varpi_\ell(x)\varphi_{\ell m} \qquad \langle \varphi_{\ell m}\varphi_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}$$

Initial conditions:

$$\varpi_\ell(k) = (-i)^\ell \sqrt{P(k)}\, j_\ell(kL) \quad \xrightarrow{\text{Hankel transform...}} \quad \varpi_\ell(x)$$

Super-Hubble limit: $x/L \ll 1$

$$\varpi_\ell(x) = 2^{\frac{n_s}{2}-1} A \frac{\Gamma(\ell + \frac{2+n_s}{4})}{\Gamma(\ell + \frac{3}{2})\Gamma(\frac{n_s-4}{4})} \left(\frac{x}{L}\right)^\ell$$

CMB @ large scales

$$\Delta T(\tau_0, \hat{\mathbf{n}}) = \left[\frac{\delta_r}{4} + \Phi\right](\tau, \mathbf{x}_{\rm dec}) + 2\int_{\tau_{\rm dec}}^{\tau_0} \frac{\partial \Phi}{\partial \tau} + \hat{\mathbf{n}} \cdot \nabla v|_{\tau_{\rm dec}}^{\tau_0}$$

SW ISW Doppler

$$\Delta T(\tau_0, \hat{\mathbf{n}}) = \sum_{\ell,m} a_{\ell,m} Y_{\ell m} \qquad a_{\ell m} = \phi(\tau_{\rm dec}) \varphi_{\ell m} (\mathcal{S}_\ell + \mathcal{I}_\ell + \mathcal{D}_\ell) \left(\frac{x_{\rm dec}}{L}\right)^\ell$$

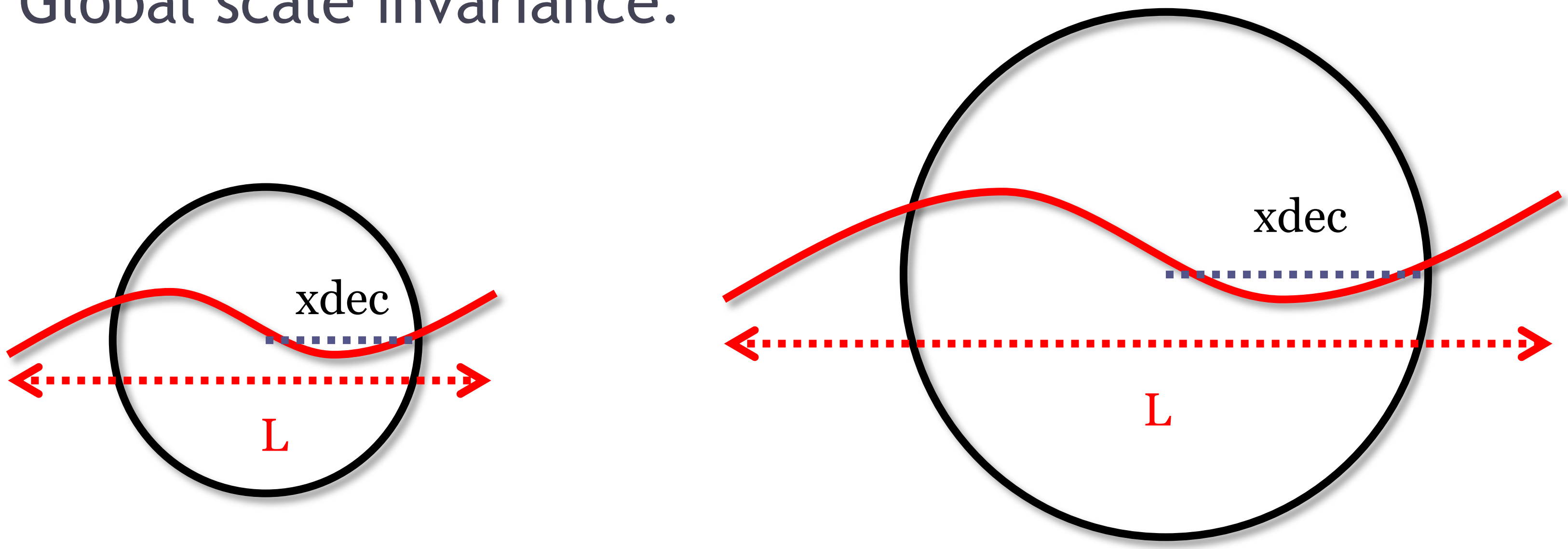
$$\mathcal{S}_\ell = \frac{2}{5} g_\ell(n_s)$$

$$\mathcal{I}_\ell = -2 g_\ell(n_s) \left[1 - \int_{\tau_{\rm dec}}^{\tau_0} \frac{\phi(\tau)}{\phi(\tau_{\rm dec})} \frac{d}{dx} \left(\frac{x}{x_{\rm dec}}\right)^\ell d\tau\right]$$

$$\mathcal{D}_\ell = \frac{g_\ell(n_s)}{x_{\rm dec}\phi(\tau_{\rm dec})} [f(\tau_0)\delta_{\ell 1} - f(\tau_{\rm dec})\ell] \qquad f(\tau) = (a\phi)^{\cdot} 4\pi G a^3 \rho (1 + w)$$

Fixing free constants

Global scale invariance:



$$P(k) = A k^{n_s - 4}$$ Fixes → A

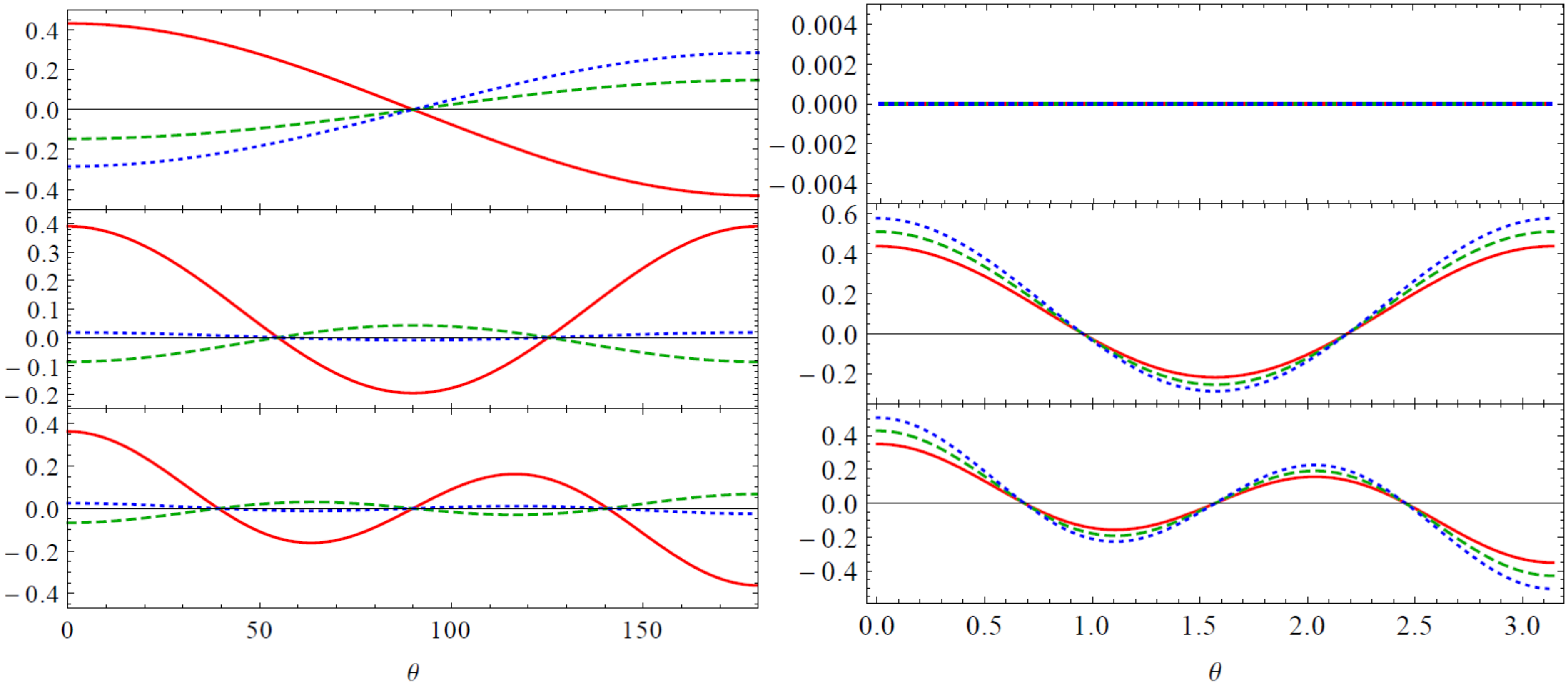
Linear regime:

$$\frac{\delta\rho}{\rho} \lesssim 1$$ Fixes → $\phi(\tau_{\rm dec})$

Multipolar contribution

Primordial dipole identically zero!

(Erickcek et. al, PRD 2008)



Observational constraints

Temperature power spectrum

$$C_\ell = \phi(\tau_{\rm dec})^2(\mathcal{S}_\ell + \mathcal{I}_\ell + \mathcal{D}_\ell)^2 \left(\frac{x_{\rm dec}}{L}\right)^{2\ell}$$

$$C_2 \propto \left(\frac{x_{\rm dec}}{L}\right)^4$$

→ Grishchuk-Zel'dovich effect

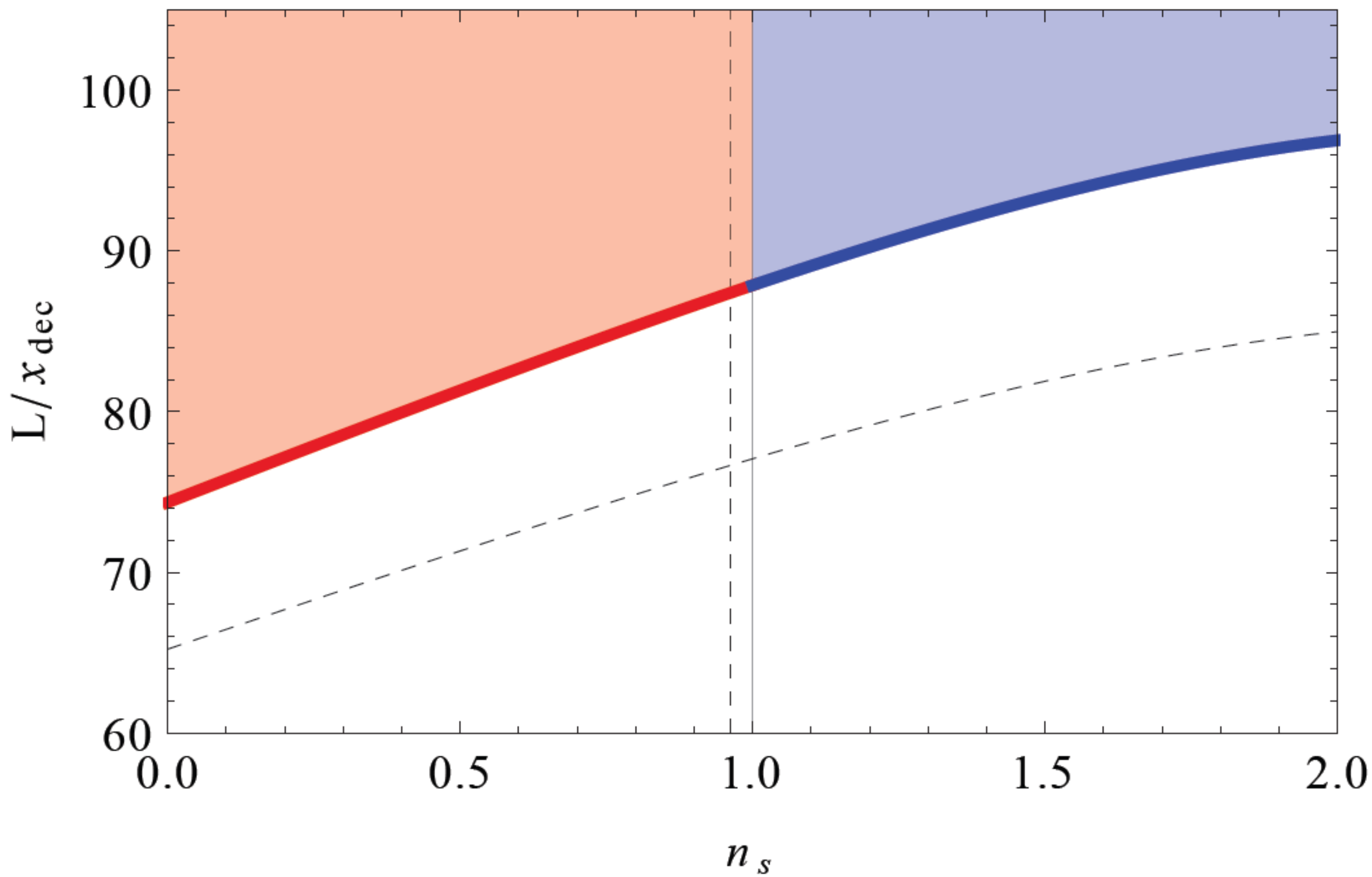
Planck quadrupole

$$\frac{2(2+1)}{2\pi}C_2 = (299.495^{+797.980}_{-159.596}) \times 10^{-12}$$

Planck's best fit ns=0.9624 → $\frac{L}{x_{\rm dec}} \lesssim 87$

Lower bound on e-folds: → $N \gtrsim N_{min} + \ln 87 \approx N_{\rm min} + 4$

Observational constraints



Conclusions

- Translating harmonic CMB to real space is not trivial…
- Large scale fluctuations severely constrain early inhomogeneous/anisotropic signatures.
- In principle, the running of ns could be associated with the C(θ) anomaly.

Obrigado!