Equivalence principle, fundamental constants, spatial isotropy

Jean-Philippe UZAN

Overview of the lectures

Equivalence principle and the fundamental constants

- lecture 1: equivalence principle
  constants and gravity

- lecture 2: Observational constraints on the constancy of constants

Test of local isotropy

- lecture 3: Weak lensing as a test of local spatial isotropy

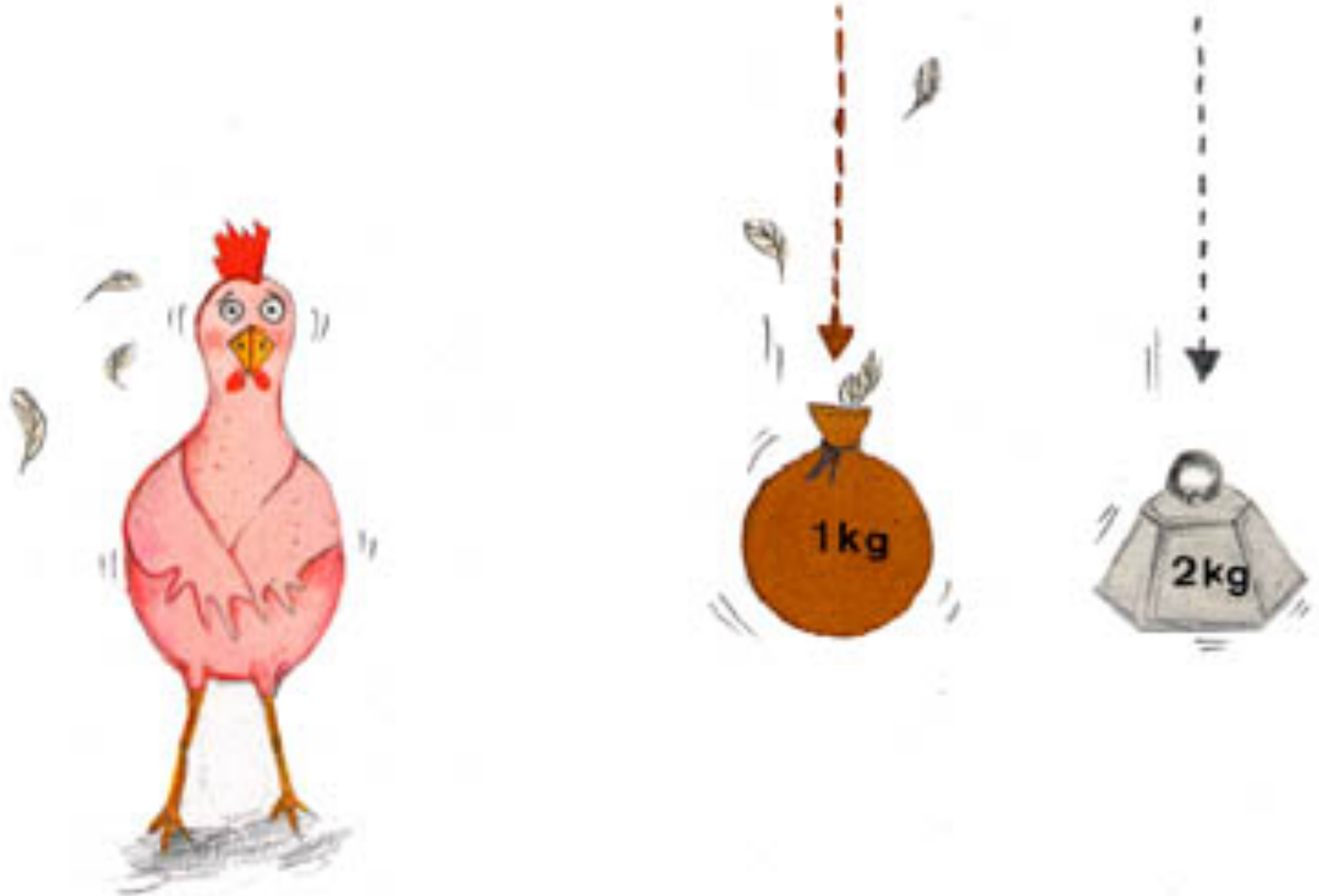
complementary to Chris' lectures on Copernican principle

Equivalence principle and fundamental constants

Jean-Philippe UZAN

Universality of free fall

Universality of free fall



The equivalence principle is not a basic principle of physics but an empirical fact.

Outline of lecture 1

This lecture will address:

- What is the Equivalence Principle and how can we test it locally
- What is the relation between constants and the equivalence principle
- Examples of theories with varying constants (more technical)
- Constants and units (more cultural, if we have time)

What is the equivalence principle?

- Universality of free fall in Galilean and Newtonian Gravity
- How well is it constrained?
- Importance for General Relativity
- Need to test it on astrophysical scales

Equivalence principle

« C'est alors, considérant ces faits, qu'il me vint à l'esprit que si l'on supprimait totalement la résistance du milieu, tous les corps descendraient avec la même vitesse. »

Galilée, *in Discours concernant deux sciences nouvelles*, 1638
Traduction de Maurice Clavelin, PUF, 1995.

Equivalence principle

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Galilée, *in Discours concernant deux sciences nouvelles*, 1638
Traduction de Maurice Clavelin, PUF, 1995.

« Il y a une puissance de la gravité, qui concerne tous les corps, proportionnelle aux différentes quantités de matière qu'ils contiennent. »

« Cette force est toujours proportionnelle à la quantité de matière des corps, & elle ne diffère de ce qu'on appelle l'inertie de la matière que par la manière de la concevoir. »

« La force de la pesanteur entre les différentes particules de tout corps est inversement proportionnelle au carré des distances des positions des particules. »

Isaac Newton, *in Principia*, Londres, 1687
Traduction d'Émilie du Châtelet, Paris, 1759.

The equivalence principle in Newtonian physics

Inertial mass is the mass that appears in Netwon's law of motion.

$$F = m_I\, a\,,$$

Passive gravitational mass is the mass that characterizes the response to a gravitational field (notion of we

$$F = m_G\, g$$

Active gravitational mass characterizes the strength of the gravitational field created by an object

$$F_{AB} \propto m_{G,A}^{act} m_{G,B}^{pass}.$$

Action-reaction law implies th $m_{G,A}^{act} m_{G,B}^{pass} = m_{G,B}^{act} m_{G,A}^{pass}$

And thus, m_G^{act}/m_G^{pass} is a constant, that can be chosen to be 1.

The equivalence principle in Newtonian physics

The deviation from the universality of free fall is characterized by

$$\eta \equiv 2\frac{|a_1 - a_2|}{|a_1 + a_2|}$$

Second law: $F = m_I a$

Definition of weight $F = m_G g$

$$\left.\begin{array}{l} F = m_I a \\ F = m_G g \end{array}\right\} a = (m_G/m_I)g,$$

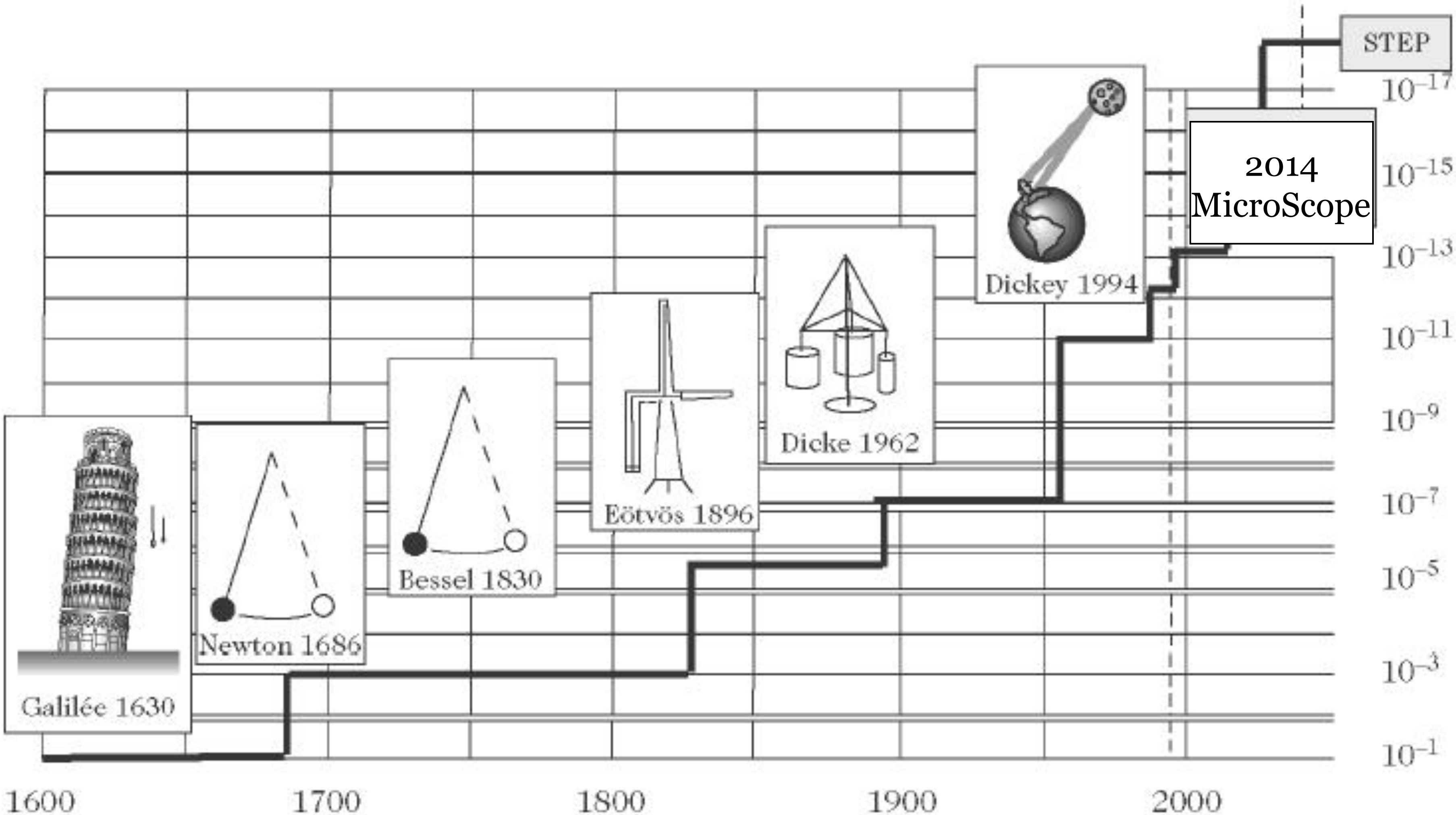
So that $$\eta = 2\frac{|m_G^1/m_I^1 - m_G^2/m_I^2|}{m_G^1/m_I^1 + m_G^2/m_I^2}$$

Consider a pendulum of length L in a gravitational field g,

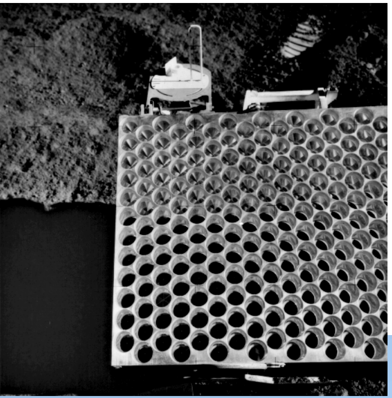
$$\ddot{\theta} + \omega^2\theta = 0 \quad \text{où} \quad \omega \equiv \omega_0\sqrt{\frac{m_G}{m_I}} \quad \text{et} \quad \omega_0 \equiv \sqrt{\frac{g}{L}}.$$

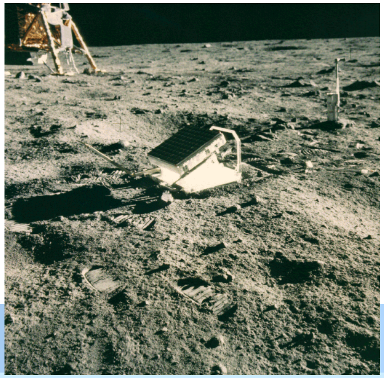
Then $$\eta \approx 2\frac{|\omega_B - \omega_A|}{\omega_0}$$

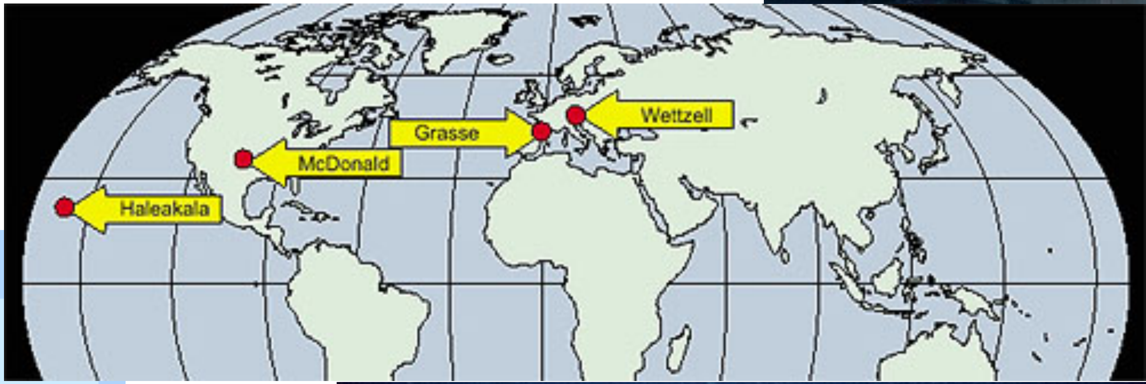
Tests on the universality of free fall

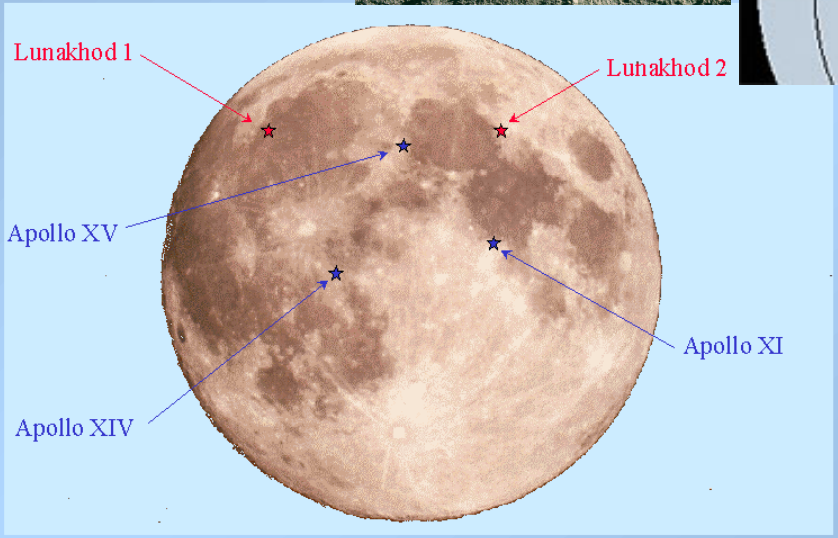


Lunar laser ranging

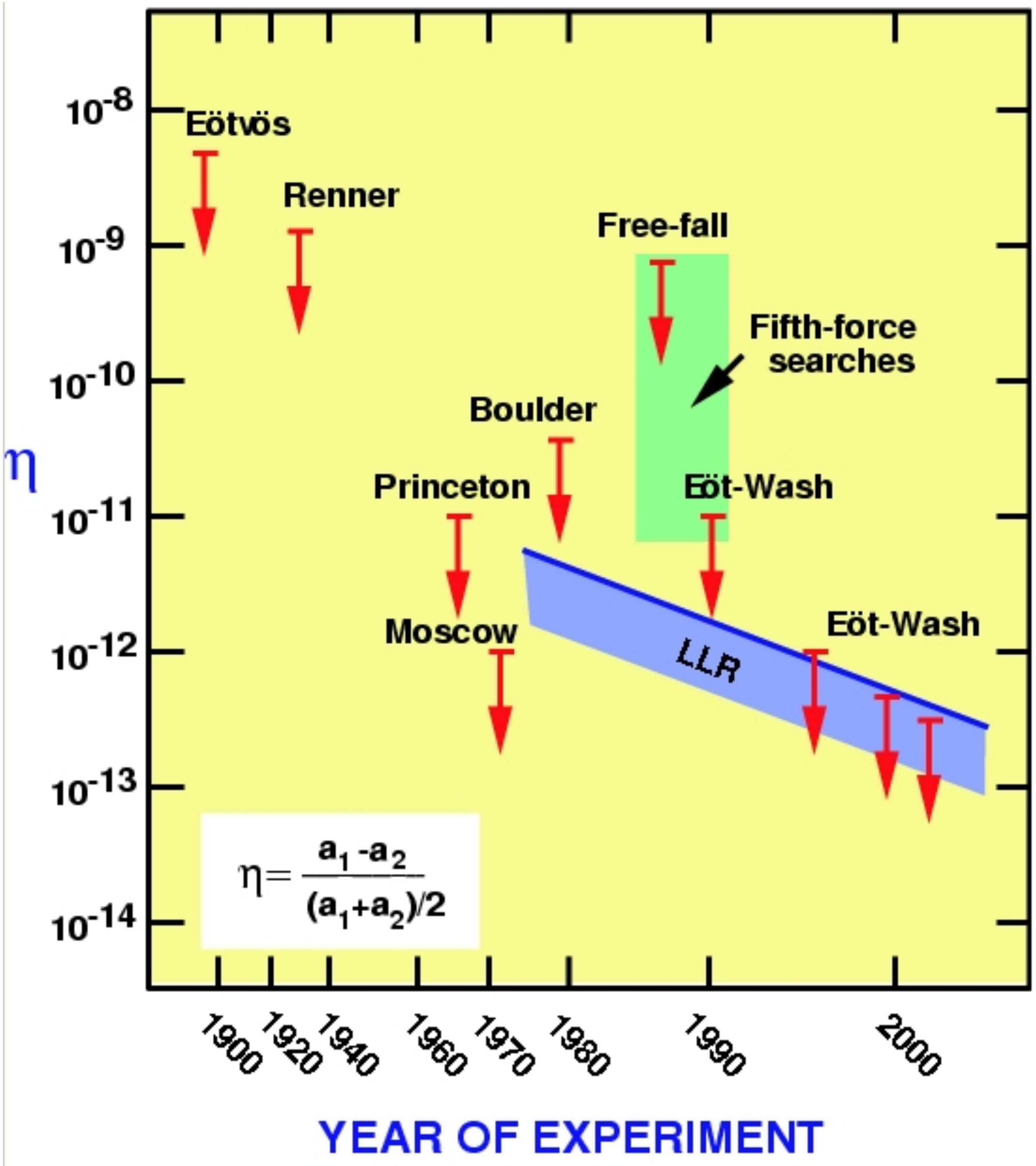








Solar system



$$\eta_{\mathrm{Te,Bi}} = (0.3 \pm 1.8) \times 10^{-13}.$$

[Schlamminger, 2008]

Holds to a very high precision in the Solar system

On the basis of general relativity

The equivalence principle takes much more importance in general relativity

It is based on **Einstein equivalence principle**

universality of free fall
local Lorentz invariance
local position invariance

The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

If this principle holds then gravity is a consequence of the geometry of spacetime

This principle has been a driving idea in theories of gravity from Newton to Einstein

Implication of the Equivalence principle

Principle is very efficient in building general relativity

Absolute & rigid spacetime		Dynamical & « elastic » spacetime
$\eta_{\mu\nu}$	$\longrightarrow$	$g_{\mu\nu}$

But all constants of local (special relativistic) physics remains absolute and rigid.

GR in a nutshell

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

Physical metric

$$S_{matter}(\psi, g_{\mu\nu})$$

gravitational metric

Dynamics for metric theories

$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}\,R_*\,d^4x$$

Relativity

$$g_{\mu\nu} = g^*_{\mu\nu}$$

Equivalence principle and test particles

Action of a test mass:

$$S = -\int mc\sqrt{-g_{\mu\nu}v^\mu v^\nu}\,dt \qquad \text{with} \qquad \begin{aligned} v^\mu &= dx^\mu/dt \\ u^\mu &= dx^\mu/d\tau \end{aligned}$$

$$\delta S = 0$$

$$a^\mu \equiv u^\nu \nabla_\nu u^\mu = 0$$

(geodesic)

$$g_{00} = -1 + 2\Phi_N/c^2$$

(Newtonian limit)

$$\dot{\mathbf{v}} = \mathbf{a} = -\nabla\Phi_N = \mathbf{g}_N$$

Parameter space

Tests of general relativity on astrophysical scales are needed

- *galaxy rotation curves: low acceleration*
- *acceleration: low curvature*

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_\odot^2}$$

Cosmology:

$$R = 3H_0^2\{\Omega_m(1+z)^3 + 4\Omega_\Lambda\}$$

Dark energy:

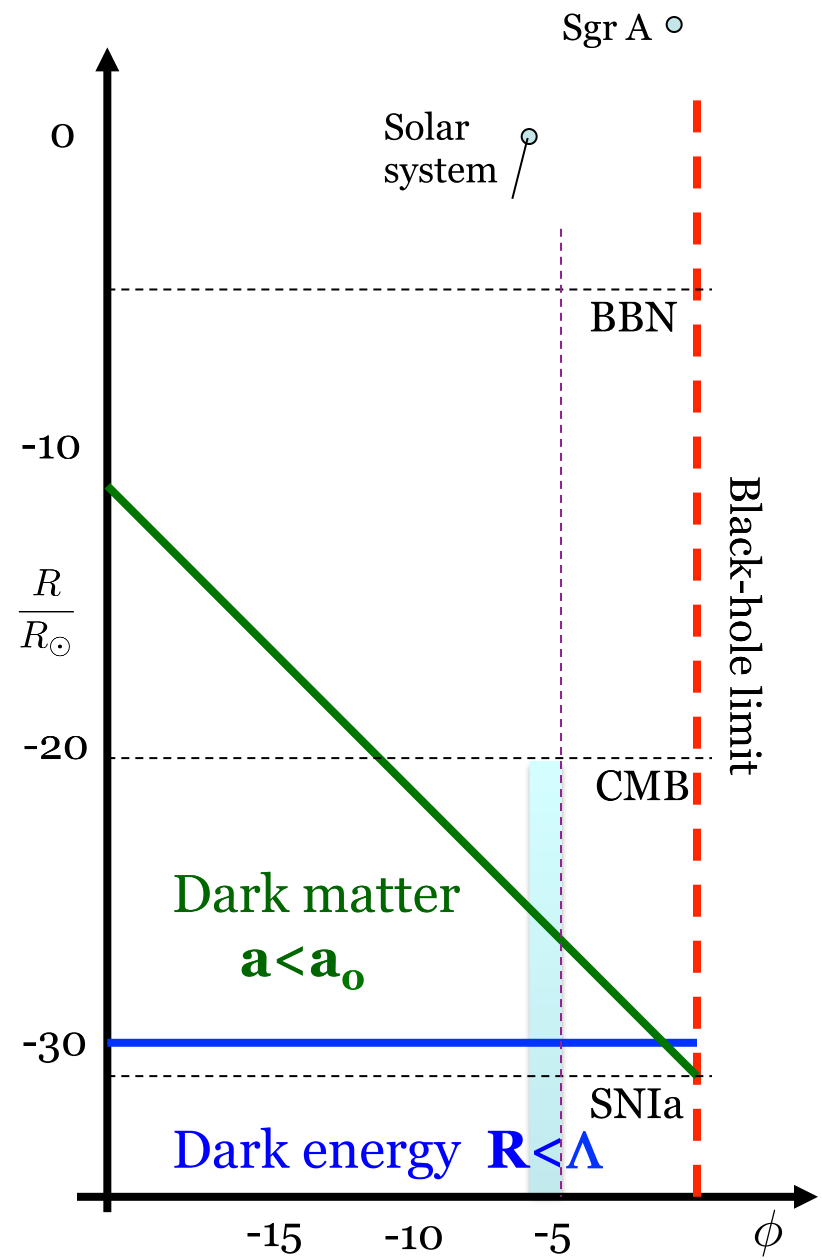
$$R < R_\Lambda = 12H_0^2\Omega_\Lambda$$

Dark matter:

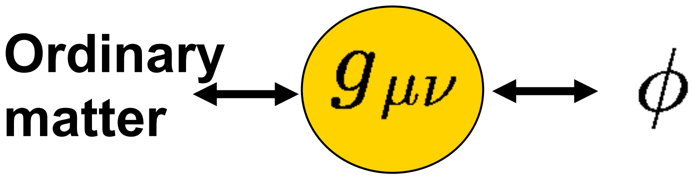
$$a < a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2$$

[Psaltis, 0806.1531]



Universality classes of extensions

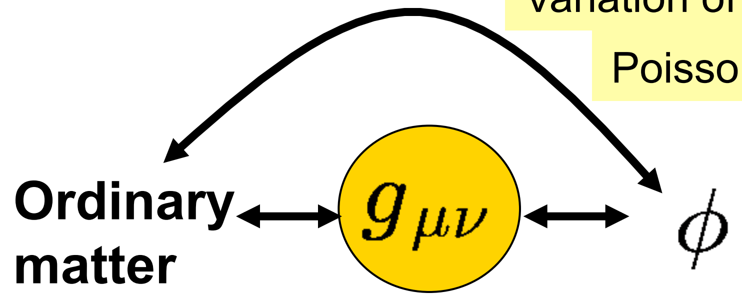


Ex : quintessence, ...

$S_{\rm de}[{\rm de}; g_{\mu\nu}]$

Variation of constants

Poisson equation

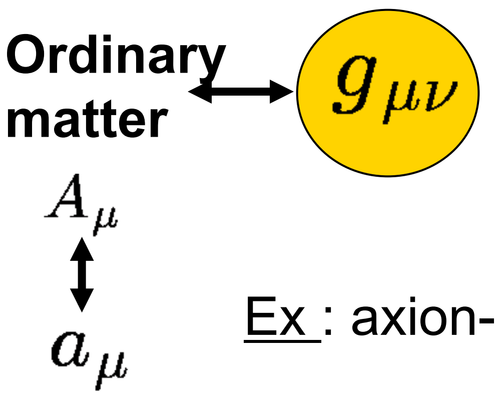


Ex : scalar-tensor, TeVeS ....

$S_\varphi[\varphi; g_{\mu\nu}]$

$S_m[{\rm mat}; g_{\mu\nu}] \rightarrow S_m[{\rm mat}_i; A_i^2(\varphi) g_{\mu\nu}].$

$S_{\rm em}[A_\mu; g_{\mu\nu}] \rightarrow S_{\rm em}[A_\mu, a_\mu; g_{\mu\nu}].$



Ex : axion-photon mixing

Distance duality

Always need NEW fields

[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Constants and
the equivalence principle?

Equivalence principle and constants

Imagine some constants are space-time dependent

1- Local position invariance is violated.

2- Universality of free fall has also to be violated

Mass of test body = mass of its constituants + binding energy

In Newtonian terms, a free motion implies $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = \vec{0}$

But, now

$$\frac{d\vec{p}}{dt} = \vec{0} = m\vec{a} + \underbrace{\frac{dm}{d\alpha}\dot{\alpha}\vec{v}}_{m\vec{a}_{\text{anomalous}}}$$

The same relativistically

Action of a test mass:

$$S = -\int m_A[\alpha_i] c \sqrt{-g_{\mu\nu} v^\mu v^\nu}\, dt$$

with $v^\mu = dx^\mu/dt$, $u^\mu = dx^\mu/d\tau$

Dependence on some constants

$$\delta S = 0$$

$f_{A,i}$

$$a^\mu_A = -\sum_i \left(\frac{\partial \ln m_A}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial x^\beta} \right) (g^{\beta\mu} + u^\beta u^\mu)$$

(NOT a geodesic)

$$g_{00} = -1 + 2\Phi_N/c^2$$

(Newtonian limit)

$$\mathbf{a} = \mathbf{g}_N + \delta\mathbf{a}_A$$

$$\delta\mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla\alpha_i + \dot{\alpha}_i \frac{\mathbf{v}}{c^2} \right)$$

Anomalous force
Composition dependent

Constants as a test of the equivalence principle

The constancy of constants is related to

- the local position invariance

- the universality of free fall

Can we test whether they have kept the same value during the evolution of the Universe?

Fundamental constants

JPU, Rev. Mod. Phys. **75**, 403 (2003); Liv. Rev. Relat. (2010)
JPU, [astro-ph/0409424, arXiv:0907.3081]
R. Lehoucq, JPU, *Les constantes fondamentales* (Belin, 2005)
G.F.R. Ellis and JPU, Am. J. Phys. **73** (2005) 240
JPU, B. Leclercq, De *l'importance d'être une constante* (Dunod, 2005)
translated as "*The natural laws of the universe*" (Praxis, 2008).

A debate that (re)started in 1999

1999 : an Australian team of astrophysicists lead by John Webb claims that the *fine structure constant* was smaller in the past!

This constant is defined from
- the speed of light
- the charge of the electron
- the Planck constant…

…It should not vary !!

1937 : Dirac develops his *Large Number hypothesis*.

Assumes that the gravitational constant was varying as the inverse of the age of the universe.

$$F_{grav}/F_{elec} = \frac{G m_e m_p}{e^2/4\pi\varepsilon_0} \sim 10^{-40} \sim \frac{H_0 e^2/4\pi\varepsilon_0}{m_e c^3} = (t_U/\text{atomic units})^{-1}$$

This hypothesis was quickly ruled out (Teller / Gamow).

What is a constant?

Constant : PHYS., Numerical value of ***some*** quantity that allows to characterize a body. Quantity whose value is fixed (***e.g*. **mass and charge of the electron, speed of light) and that plays a ***central*** role in physical theories.

This definition asks more questions than it gives answers!

- How many constants?
- Are they all on the same footing?
- What role do they play in laws of physics?
- Can they vary? *(according to the dictionary, NO!)*

Making a list of constants

Let us start to look in a book of physics (probably the best place to find constants)
depends on *when* and by *who* the book was written

Any parameter not determined by the theories at hand

It has to be assumed constant (no equation/ nothing more fundamental)

Reproductibility of experiments.

It does not show our *knowledge* but our *ignorance*

Studying the constant of a theory =
To study the limits of this theory

Today : gravitation = general relativity
matter = standard model

} **22 constants**

Reference theoretical framework

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [*General Relativity* + *SU(3)xSU(2)xU(1)*]:

- G : Newton constant (**1**)

- **6** Yukawa coupling for quarks
- **3** Yukawa coupling for leptons

- mass and VEV of the Higgs boson: **2**

- CKM matrix: **4** parameters
- Non-gravitational coupling constants: **3**
- Λ_{uv}: **1**

- c, ħ : **2**

- cosmological constant

(grouped by a brace:) **22** constants
19 parameters

The number of constants may change

This number can change with our knowledge of physics.

+: *Example:* Neutrino masses

Add **3** Yukawa couplings + **4** CKM parameters = **7** more

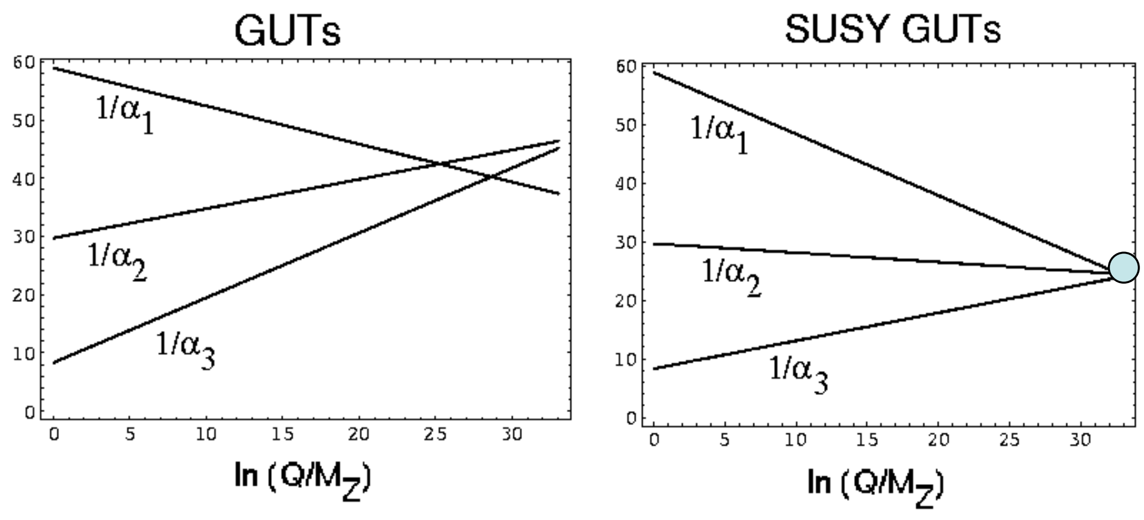
-: *Example:* Unification

$$\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{E}$$

$$\text{SM}: \quad b_i = (41/10, -19/6, -7)$$

$$\text{MSSM}: \quad b_i = (33/5, 1, -3)$$

$$\alpha^{-1} = \tfrac{5}{3}\alpha_1^{-1} + \alpha_2^{-1}$$



Constants: why are they interesting?

Physical theories involve constants

These parameters cannot be determined by the theory that introduces them; we can only measure them: *limit of what we can explain!*

These arbitrary parameters have to be assumed constant:

- *experimental validation*
- *no evolution equation*

By testing their constancy, we thus test the laws of physics in which they appear

A physical measurement is always a comparison of two quantities, one can be thought as a unit

- *it only gives access to dimensionless numbers*
- *we consider variation of dimensionless combinations of constants*

Theories with varying constants

A (now) famous example

In particle physics, one needs to determine the mass spectrum. One can indeed set the masses by hand (from measurements).

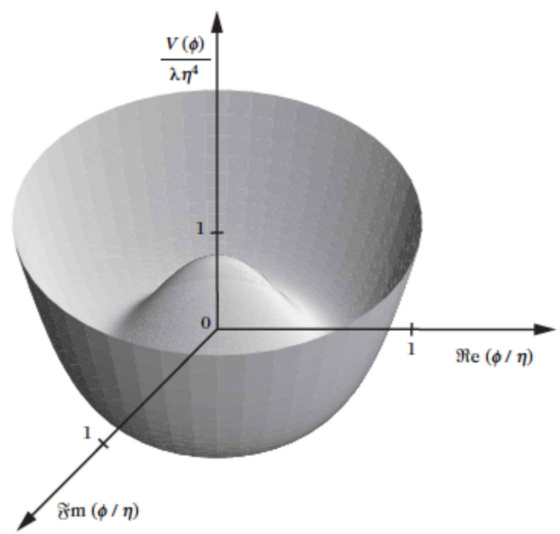
To ensure symmetries, particles need to be massless and get their mass from a symmetry braking mechanism: Higgs mechanism.

One:
- adds a new dynamical degree of freedom
- It has a dynamics

$$V_{\text{Higgs}}(\phi) = \lambda \left(|\phi|^2 - \eta^2\right)^2$$

High T → Low T

m=0 → m= Youkawa x VEV



Masses are no more constants and have changed during the couling of the universe.

They are replaced by Yukawa couplings + parameters of the Higgs potential.

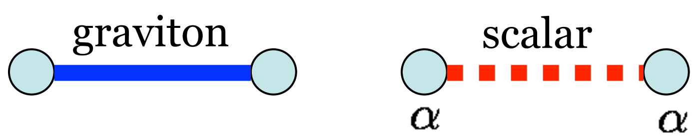
Famous example: Scalar-tensor theories

spin 2

spin 0

$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\} + S_m\{\text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$

$$G_{\rm cav} = G(1+\alpha^2)$$



$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$

Motion of massive bodies determines $G_{cav}M$ **not** GM.

G_{cav} is a priori space-time dependent

Extra-dimensions

Such terms arise when compactifying a higher-dimensional theory

Example:

$$S = \frac{1}{12\pi^2 G_5} \int \bar{R}\sqrt{|\bar{g}|}\, \mathrm{d}^5 x$$

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{M^2}\phi^2 A_\mu A_\nu & \frac{1}{M}\phi^2 A_\mu \\ \frac{1}{M}\phi^2 A_\nu & \phi^2 \end{pmatrix}$$

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g}\,\phi \left(R - \frac{\phi^2}{4M^2} F_{\alpha\beta} F^{\alpha\beta} \right)$$

$$G = \frac{3\pi \bar{G}_5}{4V_{(5)}} \qquad V_{(5)} = \int \mathrm{d}y$$

Example of varying fine structure constant

It is a priori « **easy** » to design a theory with varying fundamental constants

Consider

$$S = \int\left\{\frac{1}{16\pi G}R - 2(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{4}B(\phi)F^2_{\mu\nu}\right\}\sqrt{-g}\,d^4x$$

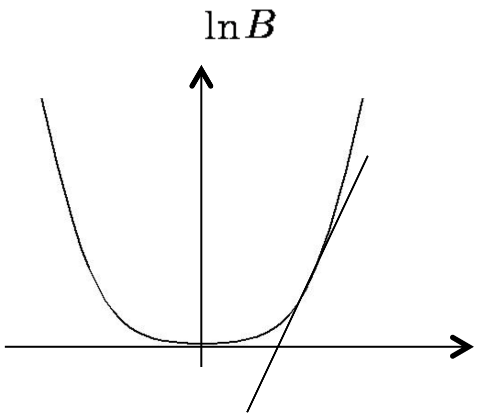
But that may have dramatic implications.

$$m_A(\phi) \supset 98.25\alpha\frac{Z(Z-1)}{A^{1/3}}\text{MeV} \longrightarrow f_i = \partial_\phi \ln m_i \sim 10^{-2}\frac{Z(Z-1)}{A^{4/3}}\alpha'(\phi)$$

It is of the order of

$$\eta_{12} \sim 10^{-9}\underbrace{\mathrm{X}_{1,2,\mathrm{ext}}(A,Z)}_{\mathcal{O}(0.1-10)} \times (\partial_\phi \ln B)^2_0$$

Requires to be close to the minimum



Theoretical aspects

They are related to the equivalence principle and allow tests of GR on Astrophysical scales *[dark matter/dark energy vs modified gravity debate]*

If a constant is varying, it has to be replaced by a dynamical field

This has 2 consequences:

1- the equations derived with this parameter constant will be modified

one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

Most high-energy extensions of general relativity contain « varying constants»

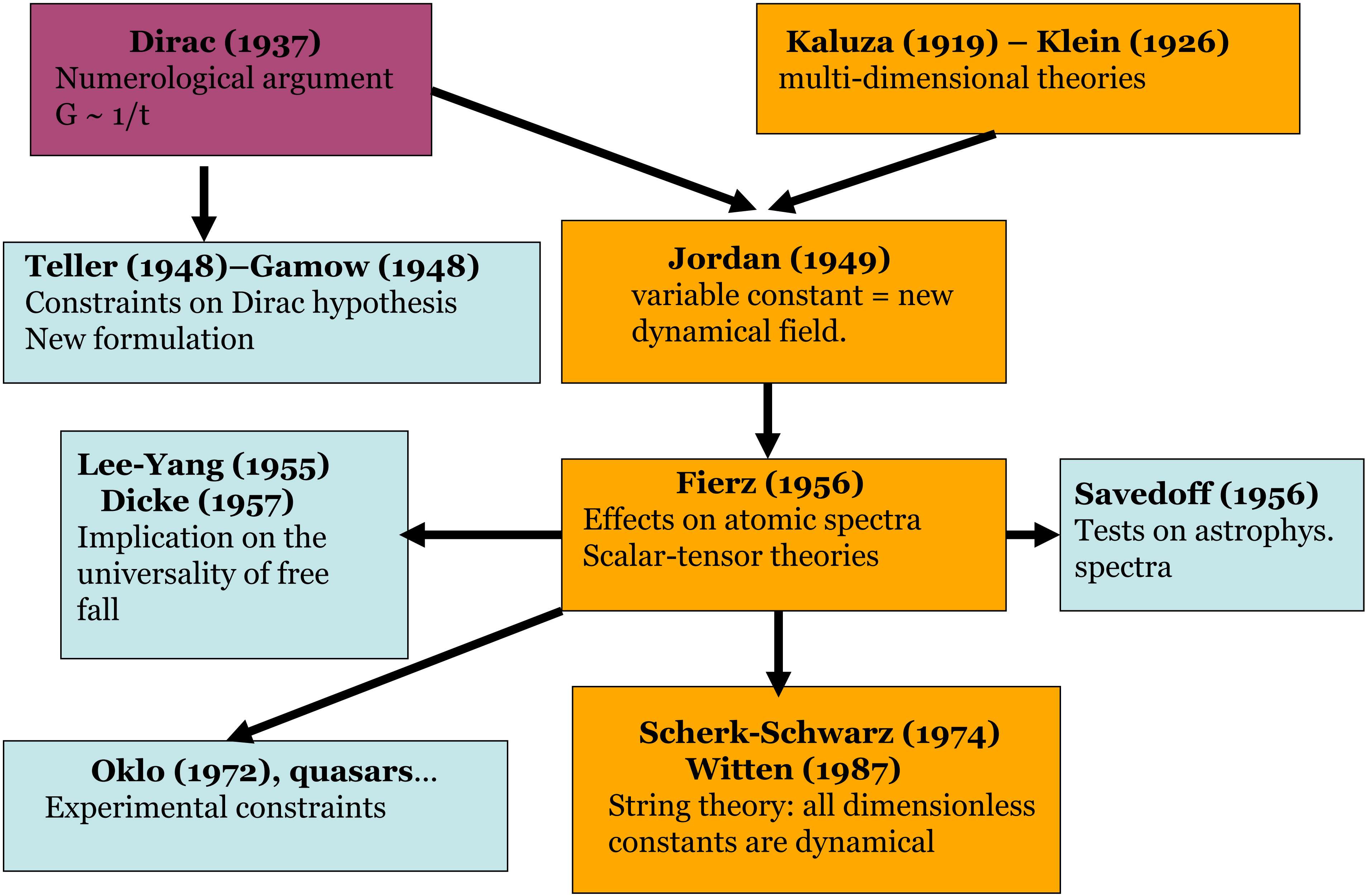
In string theory, the value of any (dimensionless) constant is effective

- it depends on the geometry and volume of the extra-dimension
- it depends on the dilaton

It opens a window on extra-dimensions
Why do the constants vary so little ?

Newton	**Einstein**	**String theory**
Fixed spacetime	**Dynamical spacetime**	**Dynamical spacetime**
Fixed constants	Fixed constants	**Dynamical constants**

« *Why have the constants the value they have ?* »
need to go to cosmological considerations. But we can start to adress this question [Coincidence / fine tuning / Landscape /...]


Dirac (1937)
Numerological argument
G ~ 1/t
Kaluza (1919) – Klein (1926)
multi-dimensional theories
Teller (1948)–Gamow (1948)
Constraints on Dirac hypothesis
New formulation
Jordan (1949)
variable constant = new dynamical field.
Lee-Yang (1955)
Dicke (1957)
Implication on the universality of free fall
Fierz (1956)
Effects on atomic spectra
Scalar-tensor theories
Savedoff (1956)
Tests on astrophys. spectra
Oklo (1972), quasars…
Experimental constraints
Scherk-Schwarz (1974)
Witten (1987)
String theory: all dimensionless constants are dynamical

Constants and units

(cultural, but important, after speech, if we have time)

What is a constant?

Constant : PHYS., Numerical value of ***some*** quantity that allows to characterize a body. Quantity whose value is fixed (***e.g*.** mass and charge of the electron, speed of light) and that plays a ***central*** role in physical theories.

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Three classes of constants

Does this mean that all constants are to be put on the same footing?

- **Class A** : characterizes a *given* physical system,

  e.g. : mass of the electron

- **Class B** : characterizes a *class* of phenomena,

  e.g.: charge of the electron

- **Class C** : universal constant,

  e.g.: speed of light, Planck constant, gravitation constant

The classification depends on time!

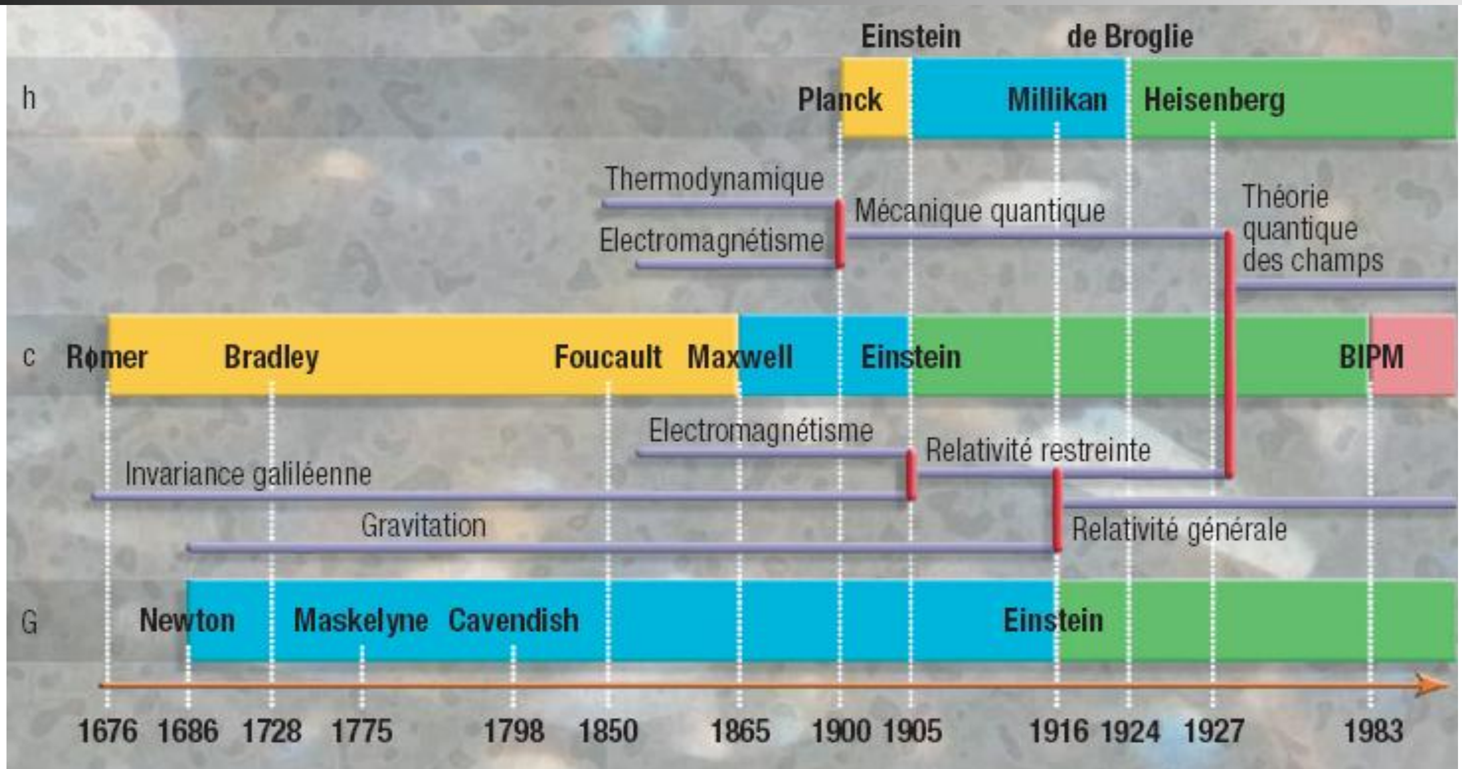
The 3 fundamental constants played a role of ***concept synthesizers***:

they created bridges between concept that were incompatible before

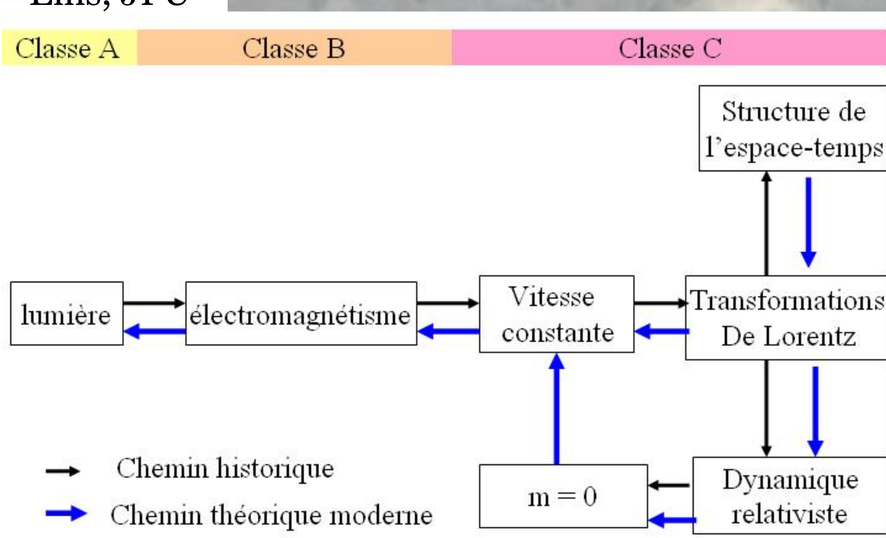
space & time ⟶ spacetime

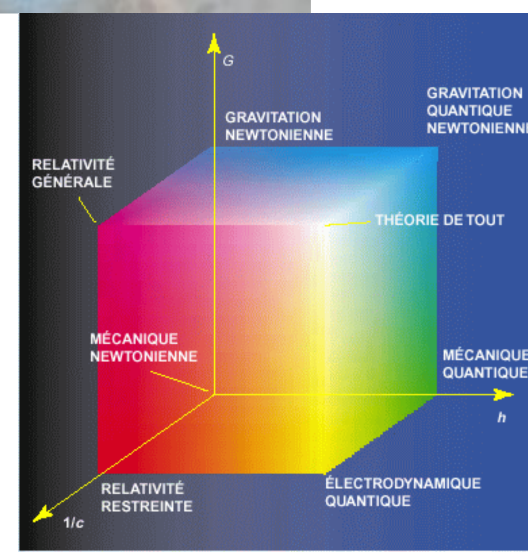
particle & waves ⟶ wave function

Change of classes and history of physics



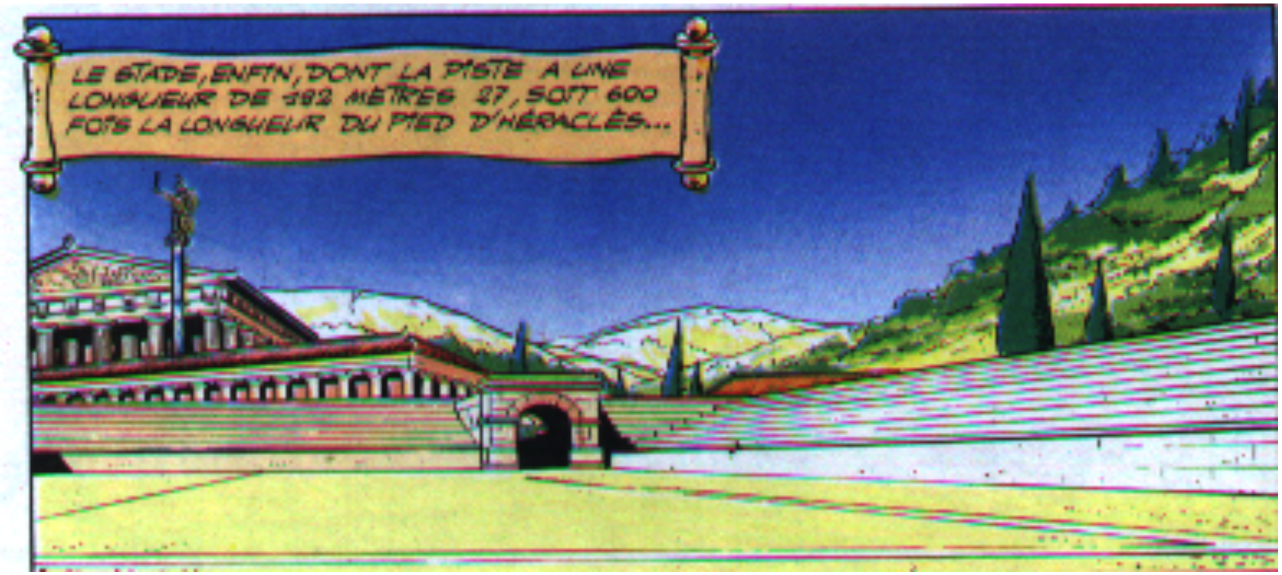
Ellis, JPU

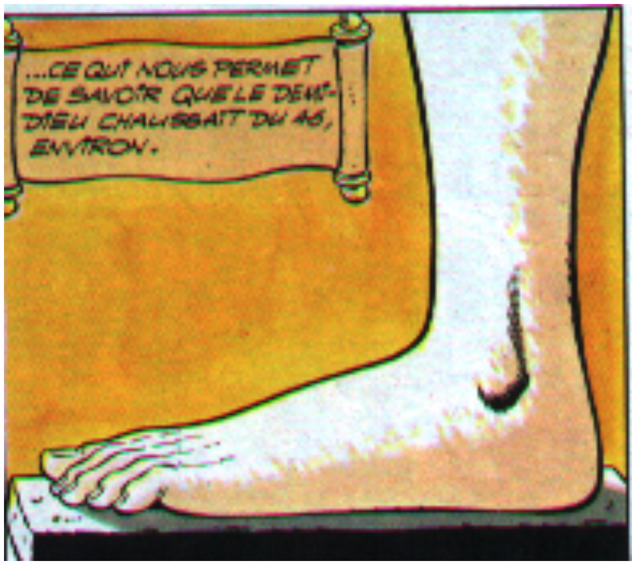




From units to constants

Units systems were initially very *anthropomorphic*





They depend on some reference person

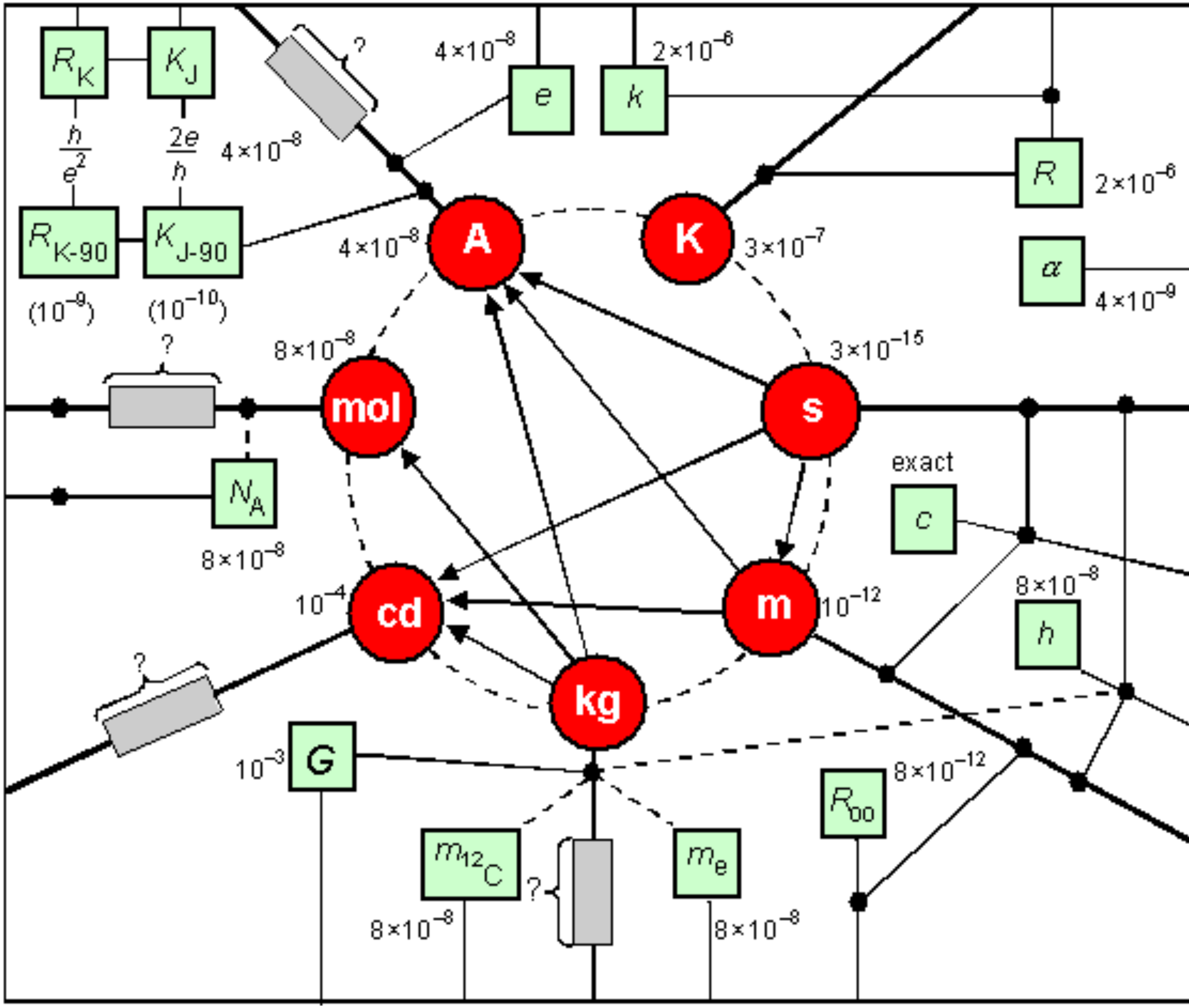
Vary from a region to another, confusion of names etc...

French revolution

26 March 1791, pushed by Charles Maurice Talleyrand, "le **mètre**" is defined as 1/40,000,000 as the length of a meridian

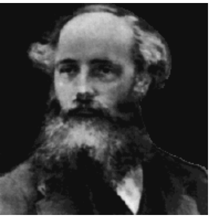
The metre

SI (>1983)



R_K
K_J
$\frac{h}{e^2}$
$\frac{2e}{h}$
R_{K-90}
K_{J-90}
(10^{-9})
(10^{-10})
?
4×10^{-8}
e
k
2×10^{-6}
R
α
4×10^{-9}
A
K
3×10^{-7}
3×10^{-15}
mol
8×10^{-8}
s
N_A
exact
c
cd
10^{-4}
m
10^{-12}
h
kg
G
10^{-3}
$m_{^{12}C}$
m_e
R_∞
8×10^{-12}

From units to constants

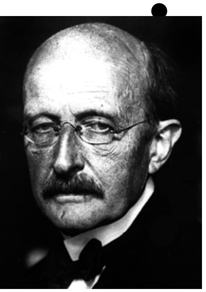


J.C. Maxwell (1870)

« If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or motion or the mass of our planet, but in the wavelength, the period of vibration, and absolute mass of these imperishable and unalterable and perfectly similar molecules. »

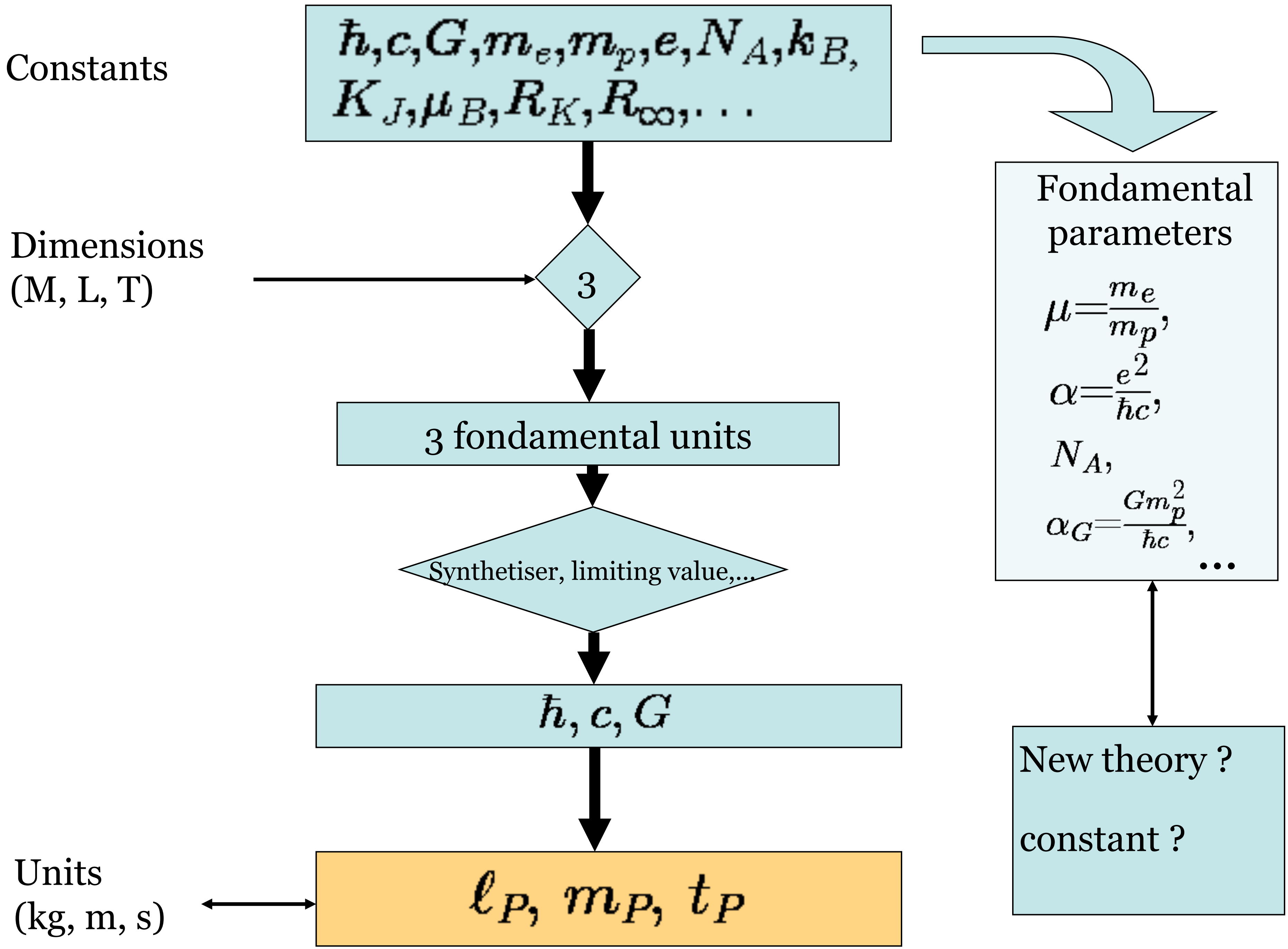
G. Johnstone-Stoney (1881)

« Nature presents us with 3 such units »

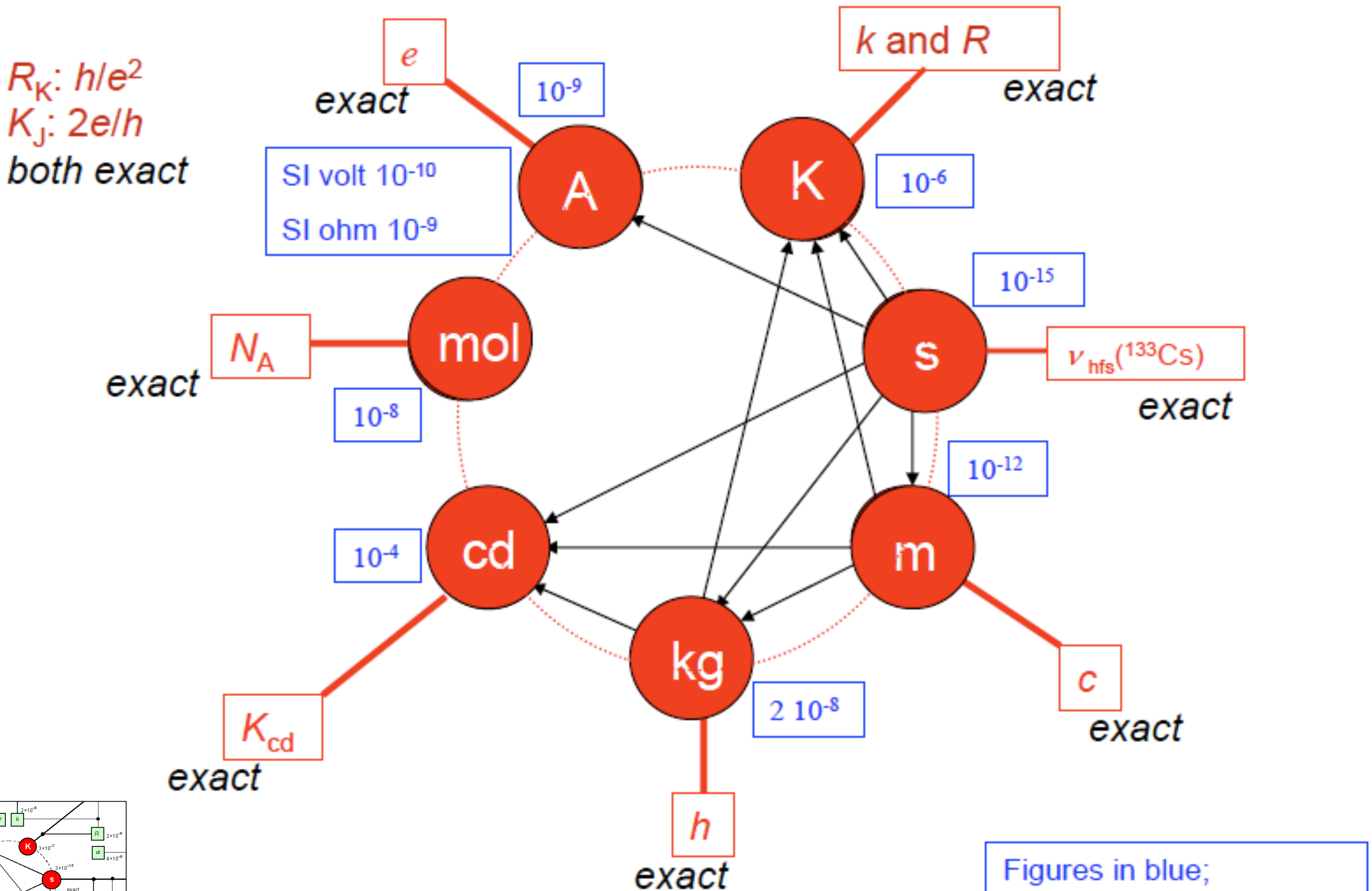


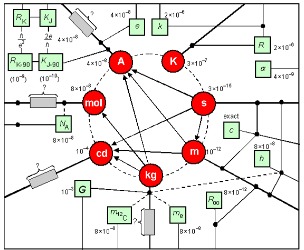
- ## Planck (1900)

« It offers the possibility of establishing units for length, mass, time and temperature which are independent of specific bodies or materials and which necessarily maintain their meaning for all time and for all civilizations, even those which are extraterrestrial and nonhuman, constants which therefore can be called fundamental physical units of measurement »


Constants
ħ, c, G, m_e, m_p, e, N_A, k_B, K_J, μ_B, R_K, R_∞, . . .
Dimensions
(M, L, T)
3
3 fondamental units
Synthetiser, limiting value,...
ħ, c, G
Units
(kg, m, s)
ℓ_P, m_P, t_P
Fondamental parameters
μ = m_e / m_p,
α = e^2 / ħc,
N_A,
α_G = G m_p^2 / ħc,
...
New theory ?
constant ?

New SI





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