An all-scale exploration of alternative theories of gravity

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General Outline

- Beyond GR: motivation and pitfalls
- Alternative theories of gravity: theory and phenomenology
- Testing gravity with compact objects

Testing General Relativity with compact objects

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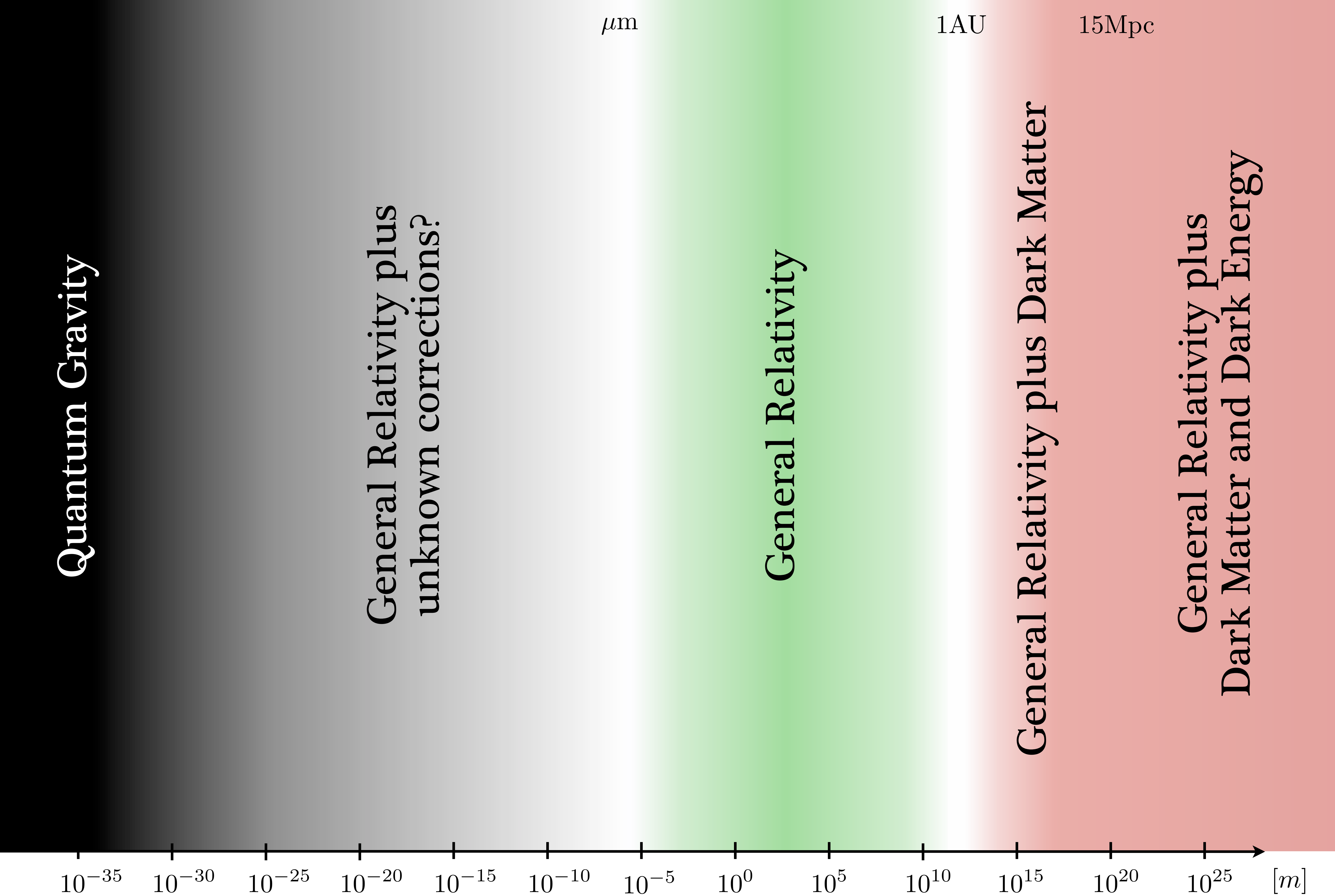
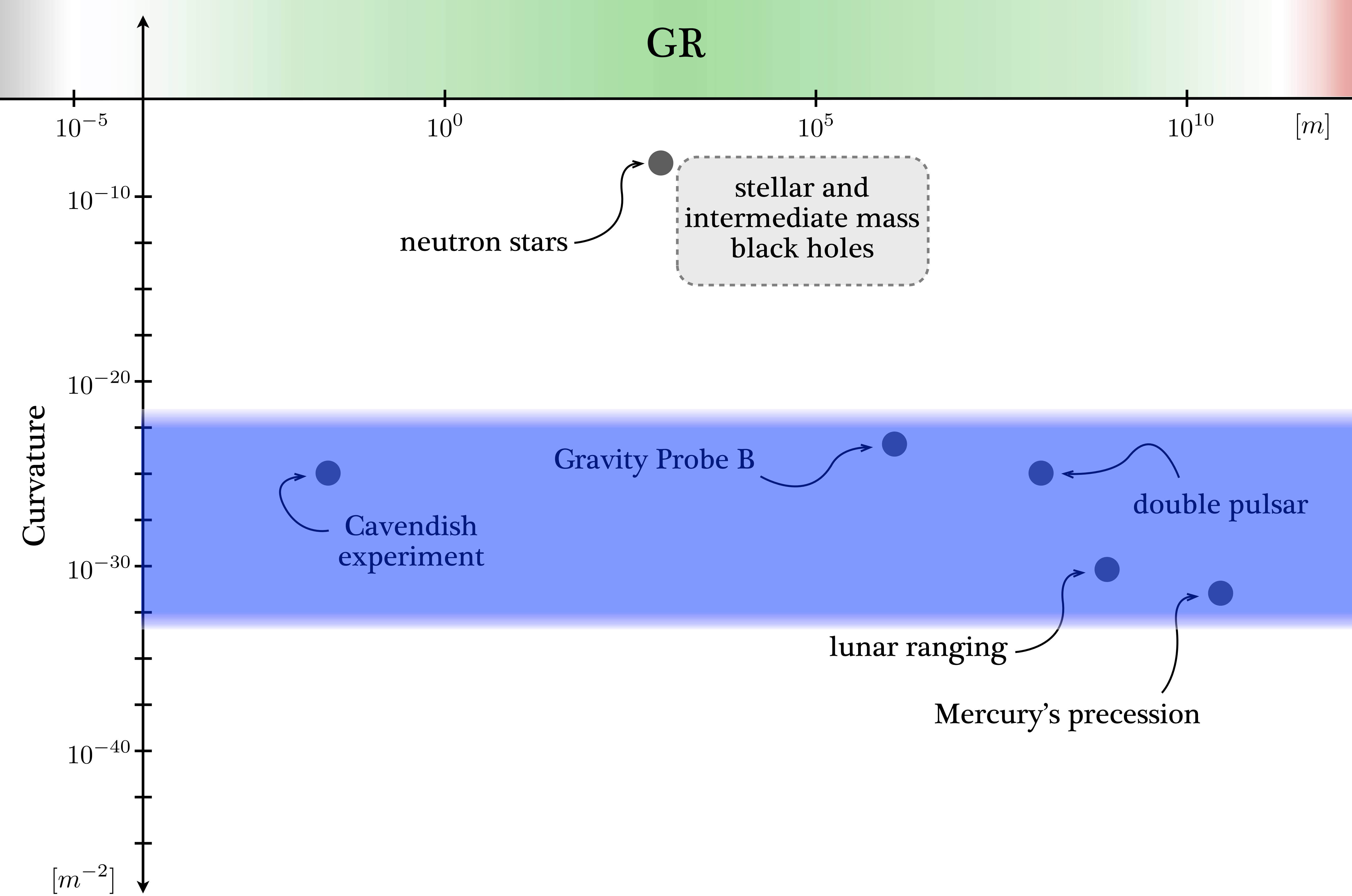

μm
1AU
15Mpc
Quantum Gravity
General Relativity plus unknown corrections?
General Relativity
General Relativity plus Dark Matter
General Relativity plus Dark Matter and Dark Energy
10^{-35} 10^{-30} 10^{-25} 10^{-20} 10^{-15} 10^{-10} 10^{-5} 10^{0} 10^{5} 10^{10} 10^{15} 10^{20} 10^{25} $[m]$


GR
10^{-5}
10^{0}
10^{5}
10^{10}
$[m]$
Curvature
10^{-10}
10^{-20}
10^{-30}
10^{-40}
$[m^{-2}]$
neutron stars
stellar and intermediate mass black holes
Gravity Probe B
Cavendish experiment
double pulsar
lunar ranging
Mercury's precession

Neutron Stars as Nuclear Physics labs

Neutron star physics description requires

1. gravity (Einstein's equations)
2. Magnetohydrodynamics (MHD equations)
3. Microphysics (equation of state)

There is too much uncertainty in 3. to start meddling with 1.

Better strategy: Assume GR and try to get insight into the microphysics, i.e.

- Nuclear physics
- Superfluidity
- ...

Black holes as gravity labs

Black holes are fully described by just gravity!

- No matter
- No microphysics

Simple enough yet full GR solutions: perfect!

Additional motivation

- They contain horizons: causal structure probes
- They contain singularities: this is exactly were GR is supposed to be breaking down!

Are black holes different?

Not necessarily! Consider scalar-tensor theory (or f(R) gravity)

$$S_{\rm st} = \int d^4x\sqrt{-g}\Big(\varphi R - \frac{\omega(\varphi)}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - V(\varphi) + L_m(g_{\mu\nu},\psi)\Big)$$

Black holes that are

- Endpoints of collapse, i.e. stationary
- isolated, i.e. asymptotically flat

are GR black holes!

T. P. S. and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012)

(generalizes the 1972 proof for Brans-Dicke theory)

S.W. Hawking, Comm. Math. Phys. 25, 167 (1972).

No difference from GR?

Actually there is...

- Perturbations are different!

  E. Barausse and T.P.S., Phys. Rev. Lett. 101, 099001 (2008).

- They even lead to new effects, e.g. floating orbits

  V. Cardoso et al., Phys. Rev. Lett. 107, 241101 (2011).

- Cosmic evolution could also lead to scalar "hair"

  T. Jacobson, Phys. Rev. Lett. 83, 2699 (1999);
  M. W. Horbatsch and C. P. Burgess, arXiv:1111.4009 [gr-qc].

- More general couplings lead to "hairy" solutions

  P. Kanti et al., Phys. Rev. D 54, 5049 (1996).

Black holes with matter

What if there is matter around a black hole?

$$\tilde{\Box}\phi - U'(\phi) + A^3(\phi)\, A'(\phi) T = 0$$

- In general no $\phi =$constant solutions when $T \neq 0$ unless

$$A'(\phi_0) = 0 \qquad\qquad U'(\phi_0) = 0$$

- Hint that a small amount of matter might perhaps drastically change the solutions

Even when this conditions are satisfied

- Spontaneous scalarization
- Superradiant instability

V. Cardoso, I. P. Carucci, P. Pani and T. P. S., to appear

Why LV gravity?

Lorentz-violating effects are severely constrained in the matter sector. However,

- Observational constraints are far weaker in the more weakly coupled gravitational sector
- A low energy effective theory of Lorentz-violating gravity is needed for such tests (e.g. Einstein-aether theory)

The might be an additional pay-off:

- Recently it has been claimed that some models of Lorentz-violating gravity are power-counting renormalizable (Hořava-Lifshitz gravity)

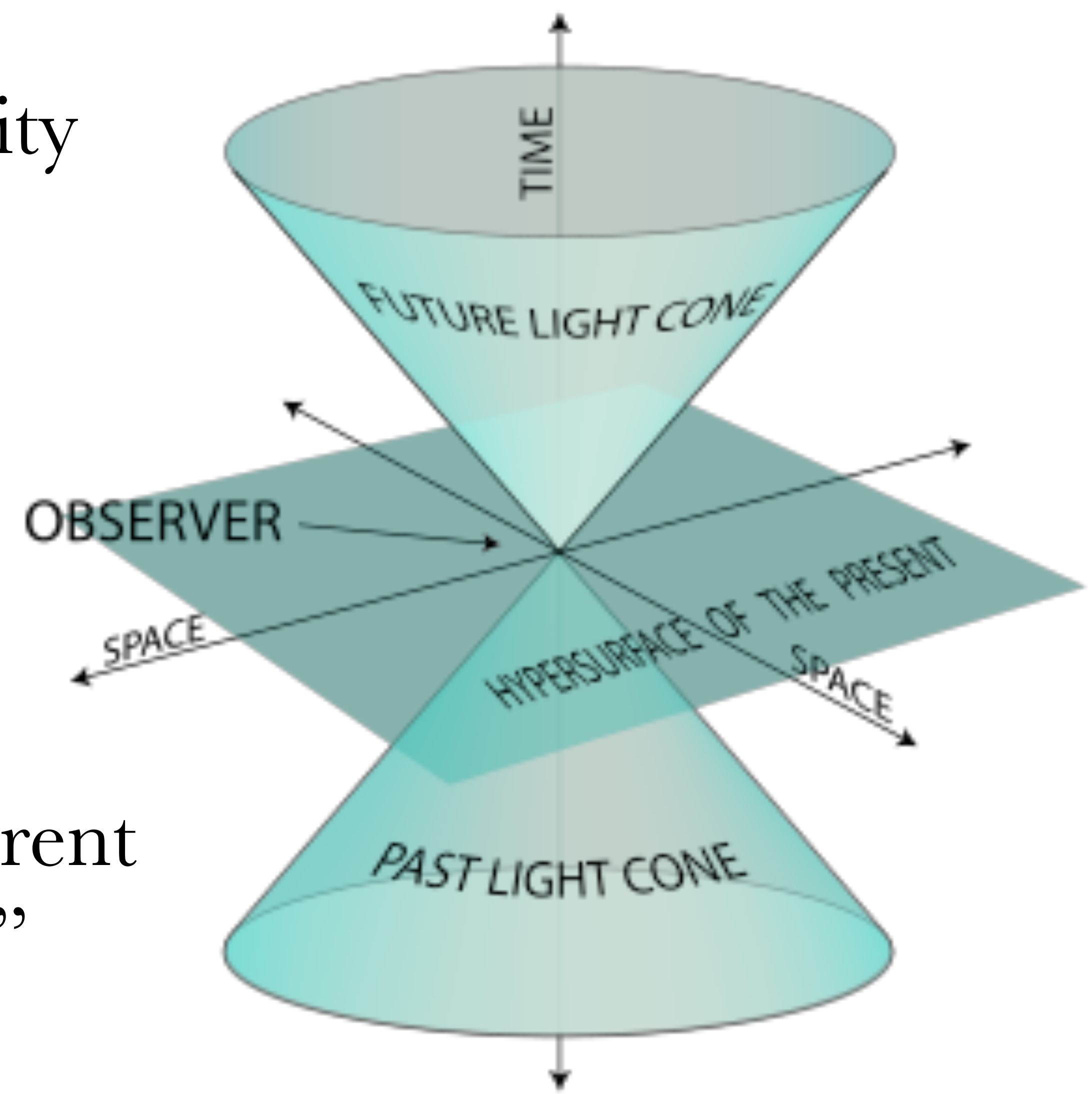
LV and black hole structure

Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

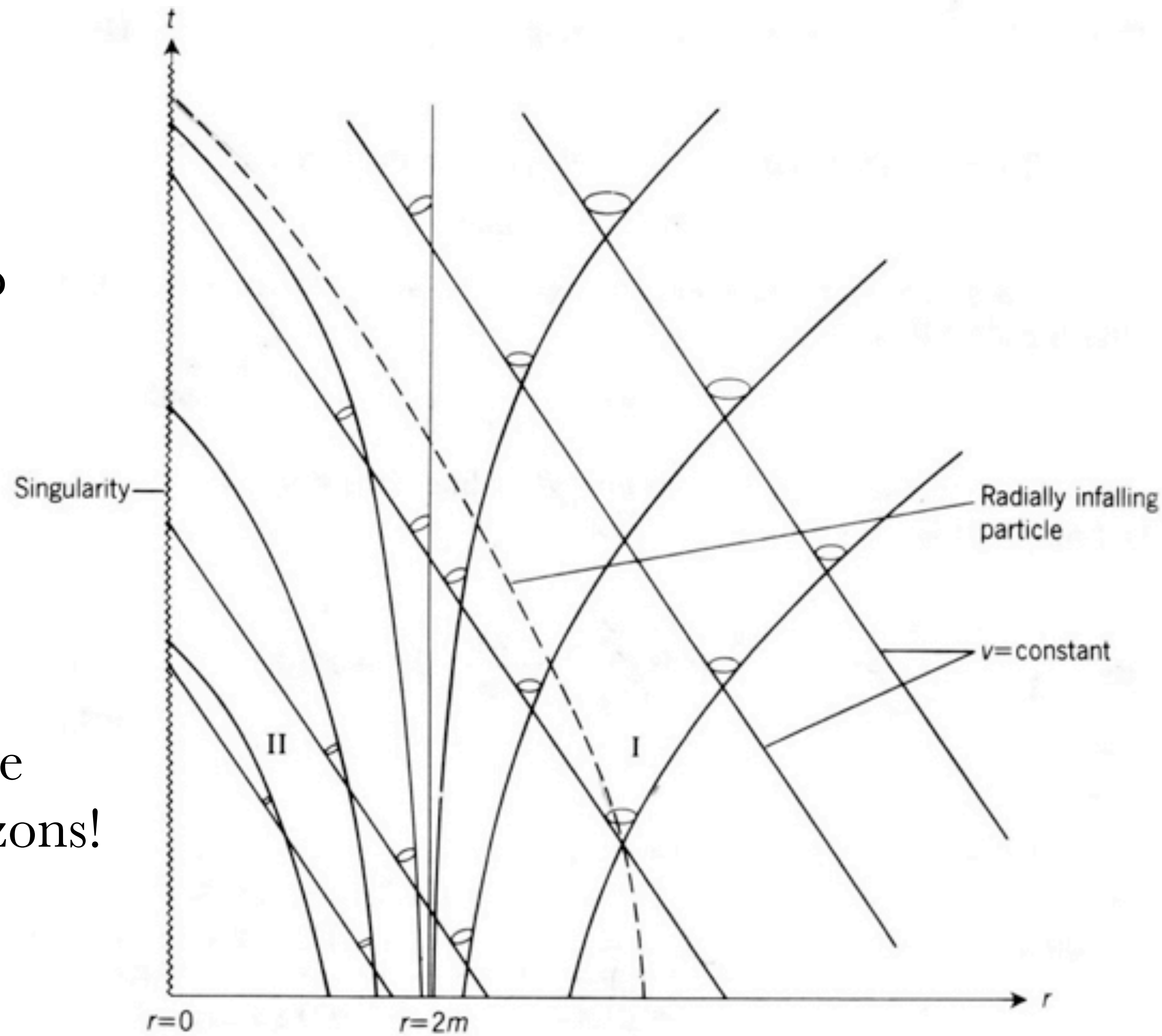
- Different modes have different speeds and different "light" cones
- But there are still "light" cones!



LV and black hole structure

What happens to black holes?

- They will have multiple horizons!



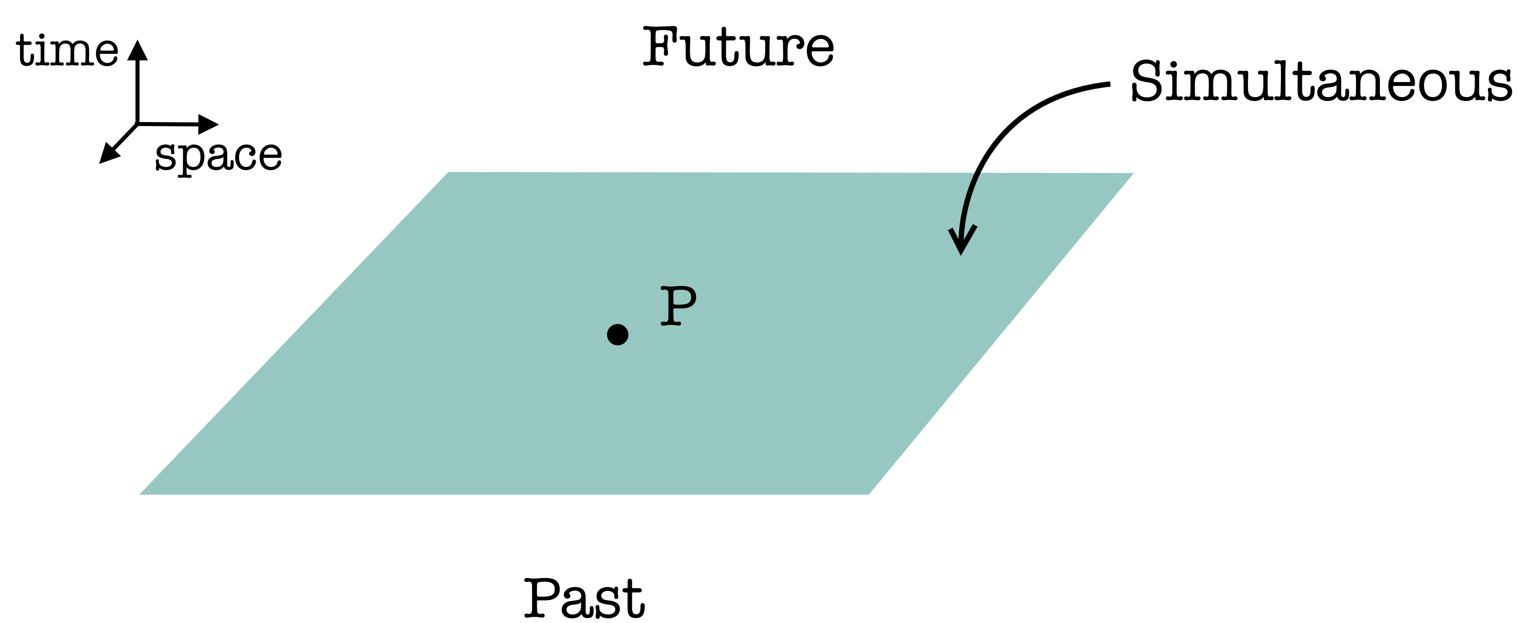
LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \ldots$$

- No light cones!

Causal structure without relativity



No black holes at all??

Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x\sqrt{-g}(-R - M^{\alpha\beta\mu\nu}\nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).

Einstein-aether theory

- Extensively tested and still viable
- It propagates a spin-2, a spin-1 and spin-0 mode.
- Linear dispersion relations.
- These modes travel at different speeds.
- We expect multiple horizons!
- Requires a UV-completion (which would likely modify the dispersion relations and lead to arbitrarily higher speeds for all modes).

Hypersurface orthogonality

Now assume $$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{æ}^{ho} = \frac{1}{16\pi G_H}\int dT d^3x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a^i a_i\right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_{æ}} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).

Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6\right)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi\,{}^{(3)}R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)

Horava-Lifshitz gravity

- Higher order terms contain higher order spatial derivatives: higher order dispersion relations!
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- "Low energy limit" is h.o. Einstein-aether theory!
- We expect no causal boundaries!

Our goal

We are interested in vacuum black hole solutions which are

- spherically symmetric (so, also h.o. aether)
- static
- asymptotically flat
- everywhere regular apart from the central singularity

Finding such solutions analytically seems unfeasible, so we find them numerically

- There is a one-parameter family of such solutions
- I suppress all the (complicated and challenging) details about how to prove that and about how to find these solutions

Parameter space

We impose the following viability constraints

- Classical and quantum-mechanical stability
- Avoidance of vacuum Cherenkov radiation by matter
- Exact agreement with Solar system experiments (vanishing preferred frame parameters)

The last constraint is more restrictive than actually required. However,

- It reduces the dimensions of the parameters space to 2
- It provided an important simplification with little given away

Characteristic quantities

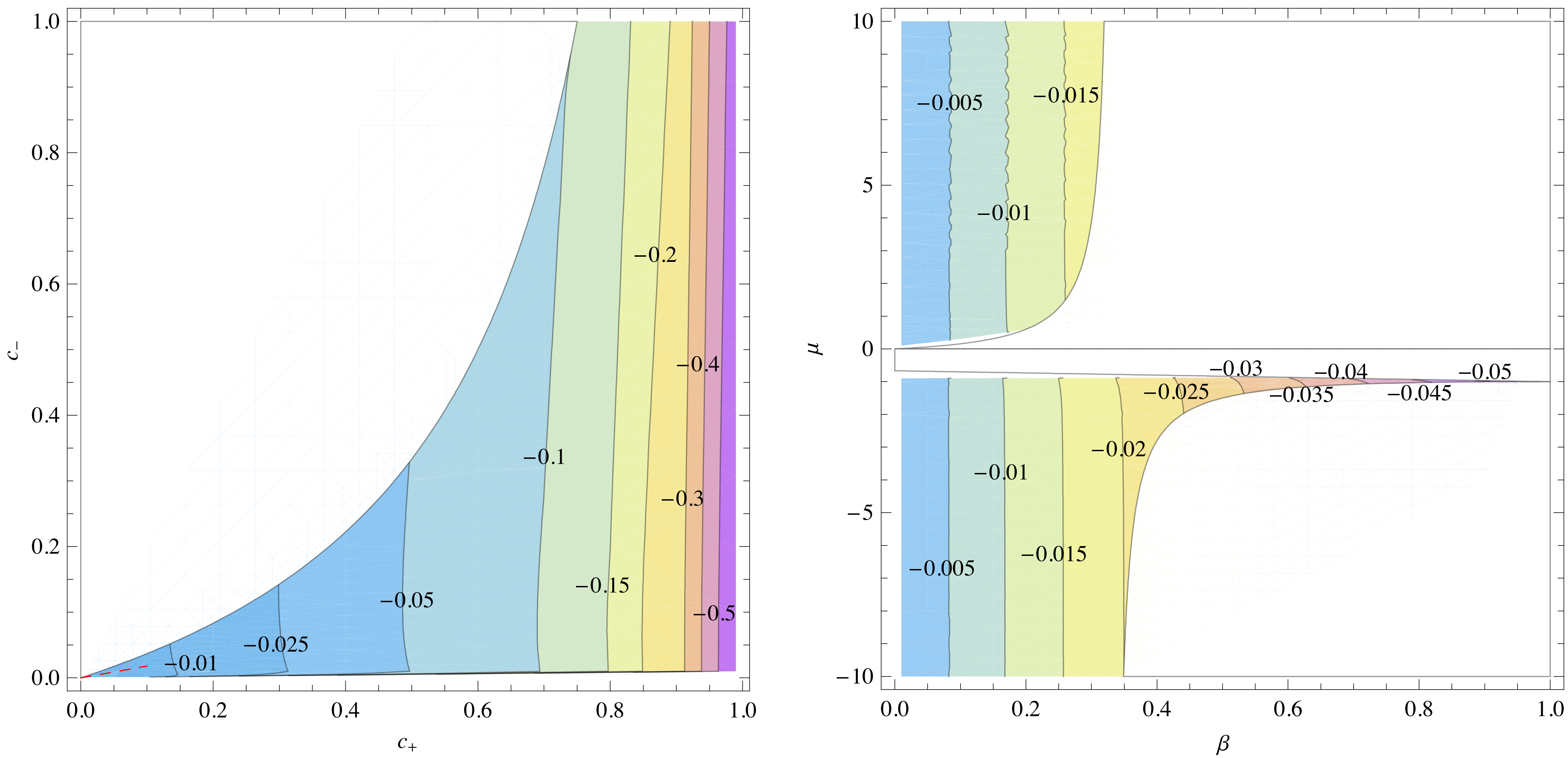
$\omega_{\rm ISCO}\, r_g$: orbital frequency at the ISCO times gravitational radius. Can be measured using X-ray spectra from accretion or gravitational waves from EMRIs

$z_{\rm max}$: $(= \nu_{\rm emitted}/\nu_{\rm measured} - 1)$ the maximum redshift for a photon emitted at the ISCO. Can be measured using iron-Kα line

$b_{\rm ph}/r_g$: impact parameter for circular photon orbit/grav. radius. Can be measured by gravitational lensing or in the future via black hole quasinormal modes

r_g/r_H : grav. radius/horizon radius. Not measurable but gives info about near horizon region

Results: exterior



$$\omega_{\rm ISCO}\, r_g$$

Interior solution

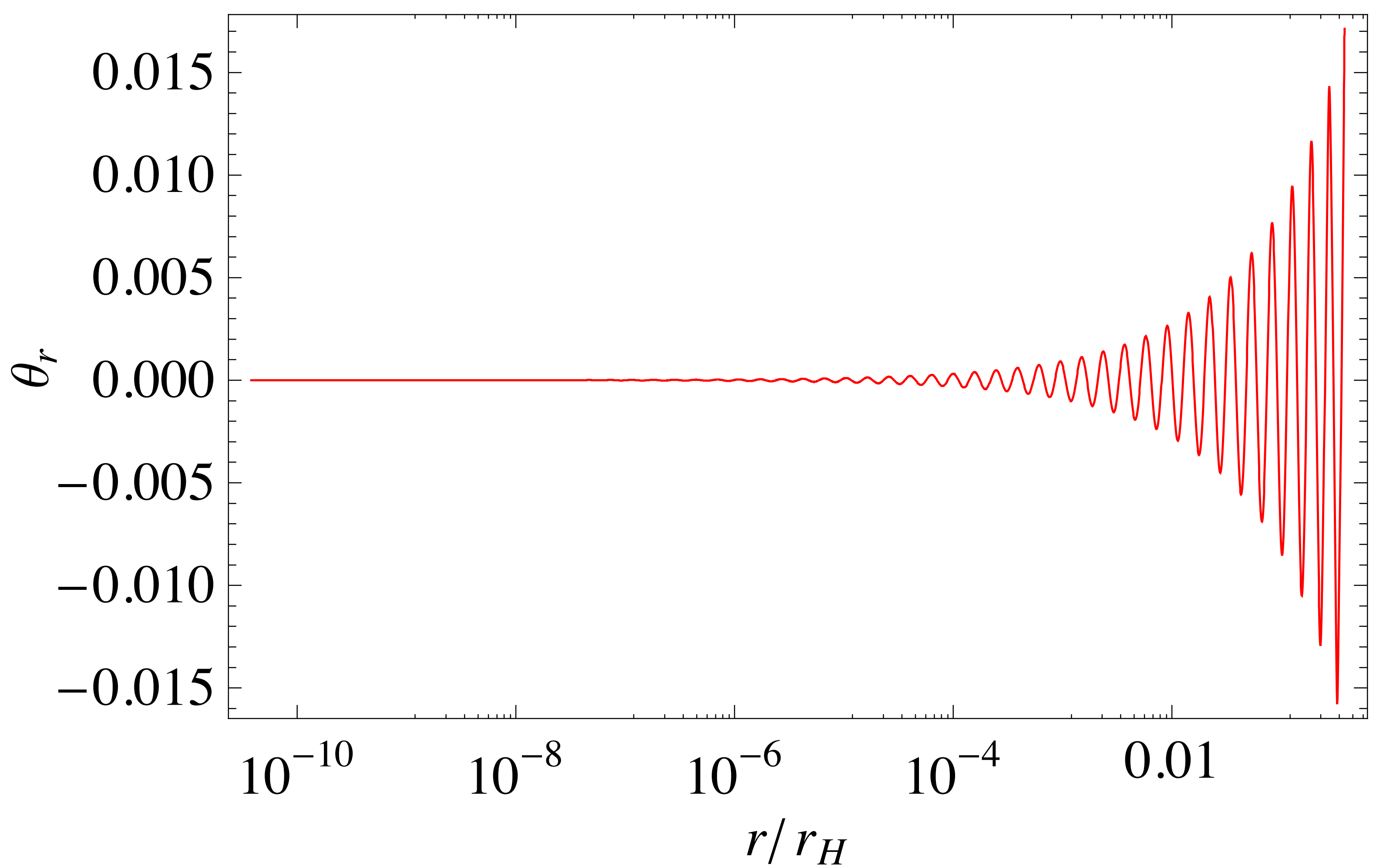
- Curvature singularity at the centre

Lorentz factor of the aether as measured by the future directed observer orthogonal to $r = \text{const.}$ hypersurfaces

$$\gamma_r \equiv u^\alpha_{\text{obs}} u_\alpha$$

and the boost angle

$$\theta_r = \text{arccosh}\gamma_r$$



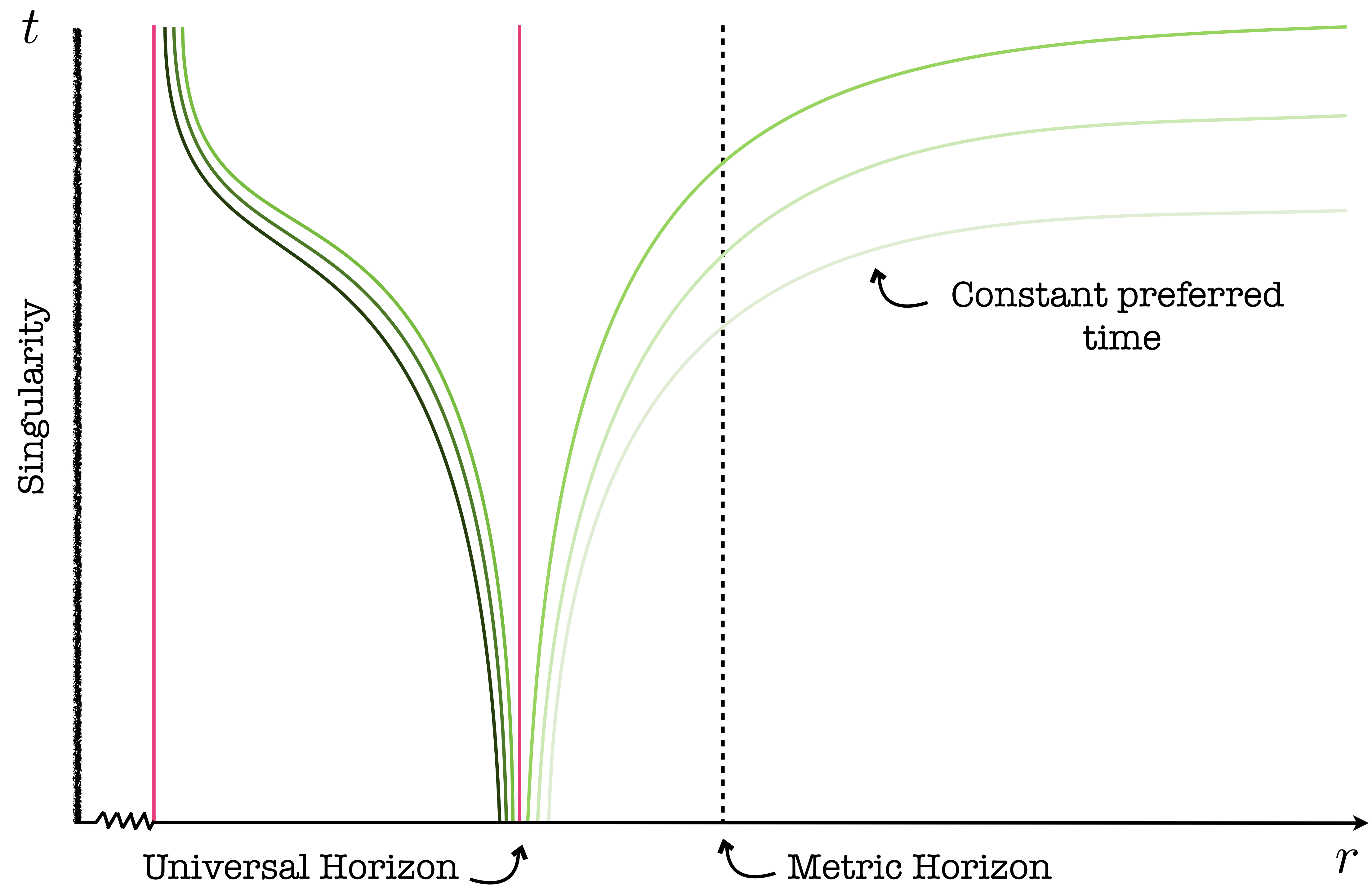
Interior solution

- Signals cannot travel backwards in time
- Future and past direction are locally defined by the aether
- The aether is orthogonal to constant time hypersurfaces in the preferred foliation
- When the boost angle vanishes the aether is orthogonal to constant r hypersurfaces as well!
- Ultimate causal boundary for all signals!

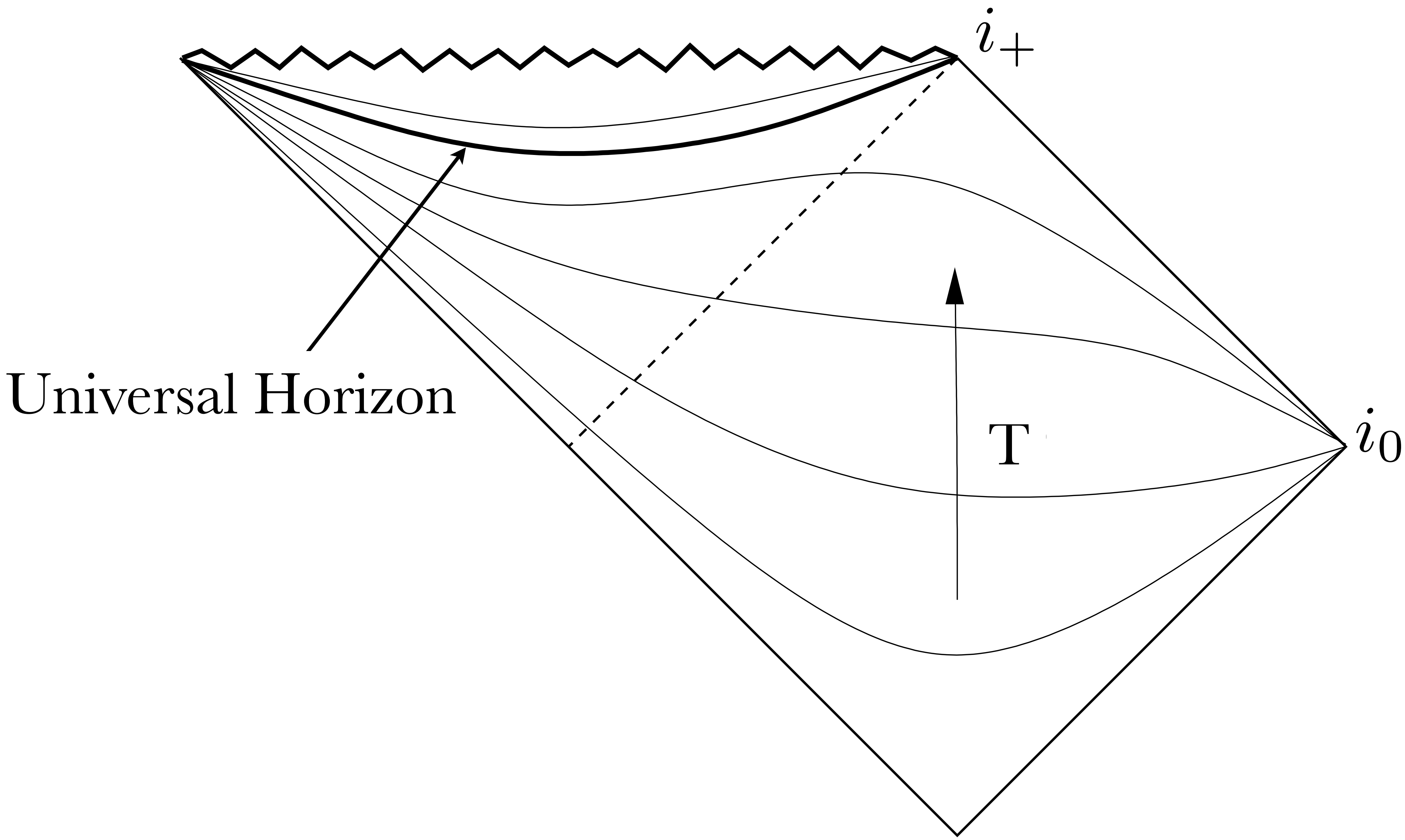
The same result found at decoupling in Horava gravity. However, this horizons seems to be unstable!

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Spacetime diagram



Penrose diagram



Taken from D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Slowly rotating BHs

- What about rotating black holes?
- Difficult to find them, easier to focus on slow rotation

Most general slowly rotating, stationary, axisymmetric metric

$$\begin{aligned} ds^2 &= f(r)dt^2 - \frac{B(r)^2}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &\quad +\epsilon r^2 \sin^2\theta\, \Omega(r,\theta)dtd\varphi + \mathcal{O}(\epsilon^2) \end{aligned}$$

- $f(r)$ and $B(r)$ are the “seed” solutions, so they are known
- $\Omega(r,\theta)$ is to be determined at the next order in ϵ

Slowly rotating BHs in GR

For the Schwarzschild solution as a "seed", for which

$$f(r) = 1 - \frac{2M}{r} \qquad\qquad B(r) = 1$$

at $\mathcal{O}(\epsilon)$ one has only one equation

$$-(r-2M)\left[4\partial_r\Omega + r\,\partial_r^2\Omega\right] = \partial_\theta^2\Omega + 3\cot\theta\partial_\theta\Omega$$

with the known solutions

- $\Omega(r,\theta) = \Omega_H(2M/r)^3$, the slowly rotation Kerr BH
- $\Omega(r,\theta) = \Omega_0 =$ constant, which is the "seed" solution after the transformation $\varphi \to \varphi + \Omega_0\, t$

Slow rotation and the aether

Symmetries require that

$$\partial_t u_\mu = \partial_\phi u_\mu = 0$$

If the aether is hypersurface orthogonal then

$$\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma = 0$$

and then one obtains

$$u_\phi = 0$$

So, one has for the aether at

$$\boldsymbol{u} = \frac{1 + fA^2}{2A} dt + \frac{B}{2A}\left(\frac{1}{f} - A^2\right) dr + \mathcal{O}(\epsilon)^2$$

i.e. nothing to determine at $\mathcal{O}(\epsilon)$!

Solutions

If one plugs the ansaetze in the equations of Einstein-aether theory one gets

$$\Omega(r,\theta) = \text{constant}$$

i.e. no h.o., slowly-rotating solution. But one can have $u_\phi \neq 0$

If one plugs the ansaetze in the equations of HL gravity one gets

$$\Omega(r,\theta) = \Omega(r) = -12J \int_{r_{\rm H}}^{r} \frac{B(\rho)}{\rho^4} d\rho + \Omega_0$$

E. Barausse and T.P.S., Phys. Rev. Lett. 109, 181101 (2012);
E. Barausse and T.P.S., Phys. Rev. D

Solutions

- The two theories share the spherical solutions, but not the slowly rotating one!
- There are no slowly rotating solutions with a preferred foliation in Einstein-aether theory. Can there be a universal horizon then?
- In HL gravity the foliation remain the same in slow rotation

Conclusions and Perspectives

- Black holes are great gravity labs!
- Even in theories whose black hole solutions are the same as in GR there is new black hole phenomenology!
- Black holes are of particular interest in Lorentz-violating theories.
- There is actually a new kind of black hole in this theory! ("universal horizon")
- The exterior is similar to GR black holes. What about rotating black holes?
- Is this horizon stable?
- Will this horizon (and black holes) exist if one has less symmetry?

Interplay of scales