An all-scale exploration of alternative theories of gravity

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General Outline

- Beyond GR: motivation and pitfalls
- Alternative theories of gravity: theory and phenomenology
- Testing gravity with compact objects

Alternative theories of gravity: theory and phenomenology

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General Relativity

The action for general relativity is

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,(R-2\Lambda) + S_m(g^{\mu\nu},\psi)$$

- S_m has to match the standard model in the local frame and minimal coupling with the metric is required by the equivalence principle.
- Gravitational action is uniquely determined thanks to:
  1. Diffeomorphism invariance
  2. Requirement to have second order equations
  3. Requirement to have 4 dimensions
  4. Requirement to have no other fields

Scalar-tensor theory

The action for scalar-tensor theory is

$$S_{\rm ST} = \int d^4x\sqrt{-g}\Big(\varphi R - \frac{\omega(\varphi)}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - V(\varphi) + L_m(g_{\mu\nu},\psi)\Big)$$

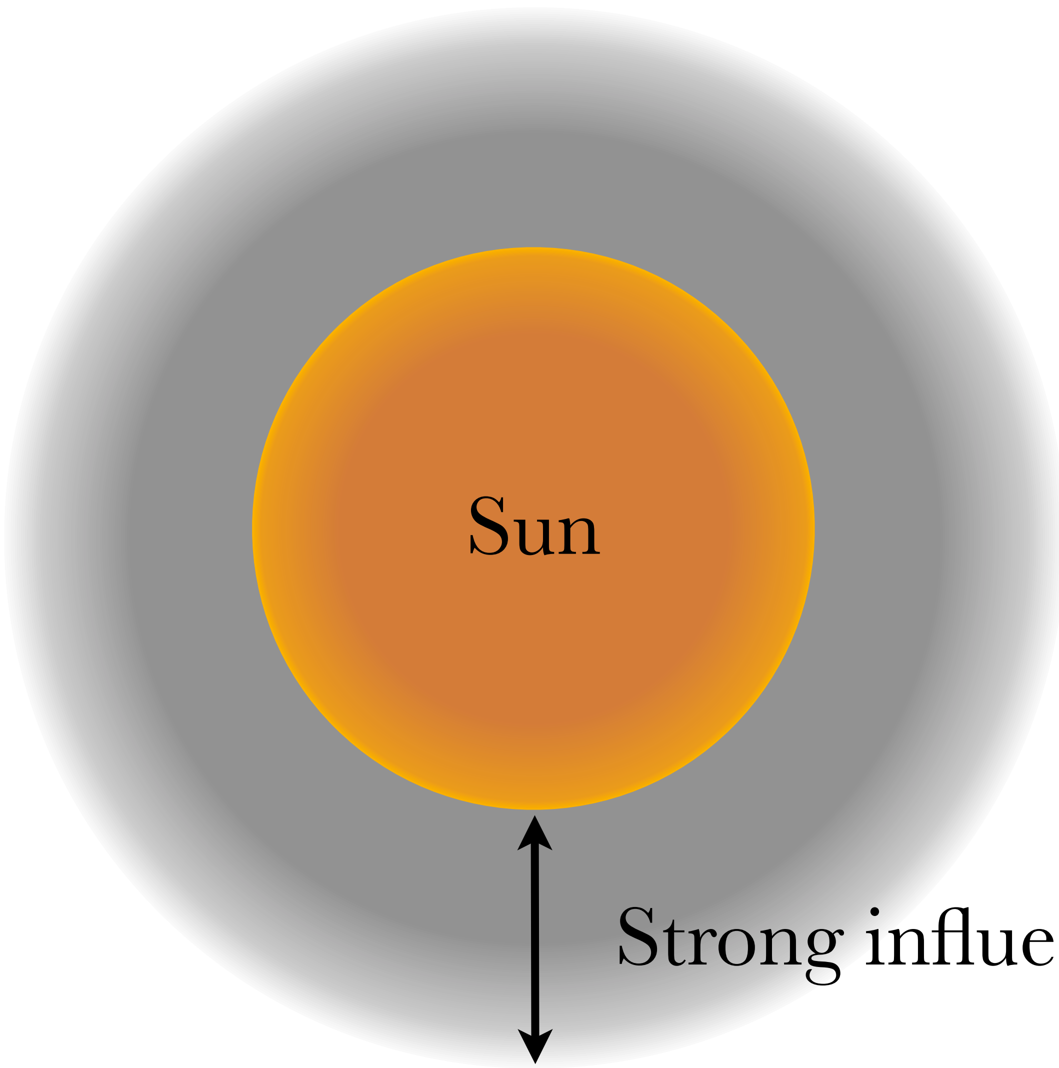
- Makes sense as an EFT for a scalar coupled nonminimally to gravity
- The generalization $\omega_0 \to \omega(\varphi)$ does not really solve the problem with the range of the force.
- If the mass of the scalar is large then no large scale phenomenology
- There are interesting theories with large or zero mass though.

Chameleons

The scalar field satisfies the following equation

$$(2\omega + 3)\Box\varphi = -\omega'(\nabla\varphi)^2 + \varphi V' - 2V + T$$

φ is sourced by matter



- The mass is affected by the presence of matter
- Potentials can be designed to have large mass locally

Strong influence by the boundary conditions

J. Khoury and A. Weltman, Phys. Rev. Lett. 93, 171104 (2004)

Einstein frame

Under a conformal transformation and a scalar field redefinition

$$g_{\mu\nu} = A^2(\phi)\tilde{g}_{\mu\nu} \qquad \phi = \phi[\varphi]$$

the action of scalar tensor theory becomes

$$S_{\rm ST} = \int d^4x\sqrt{-\tilde{g}}\Big(\tilde{R} - \frac{1}{2}\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi - U(\phi) + L_m(g_{\mu\nu},\psi)\Big)$$

and the equation of motion for ϕ is

$$\tilde{\Box}\phi - U'(\phi) + A^3(\phi)\,A'(\phi)T = 0$$

so there is an effective potential

$$U_{\rm eff} \equiv U(\phi) - A^4(\phi)T/4$$

Symmetrons

In spherical symmetry

$$\frac{d^2}{dr^2}\phi + \frac{2}{r}\frac{d}{dr}\phi = U' + A^3 A' \rho$$

Assume that

$$U(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad A(\phi) = 1 + \frac{1}{2M^2}\phi^2$$

Then the effective potential is

$$U_{\rm eff} = \frac{1}{2}\left(\frac{\rho A^3}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4$$

- Spontaneous symmetry breaking when $\rho = 0$
- Vanishing VEV when $\rho A^3 > M^2\mu^2$

K. Hinterbichler and J. Khoury, Phys. Rev. Lett. 104, 231301 (2010)

Generalized Galileons

One can actually have terms in the action with more than 2 derivatives and still have second order equations:

$$\delta((\partial\phi)^2\Box\phi) = 2[(\partial^\mu\partial^\nu\phi)(\partial_\mu\partial_\nu\phi) - (\Box\phi)^2]\delta\phi$$

- Inspired by galileons: scalar that enjoy galilean symmetry
- Most general action given by Horndeski in 1974!
- It includes well-know terms, such as

$$(\nabla_\mu\phi)(\nabla_\nu\phi)G^{\mu\nu} \qquad \phi\left(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)$$

- The new terms are highly non-linear and can lead to Vainshtein screening

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D 79, 064036 (2009)
G. W. Horndeski, Int. J. Theor. Phys. 10, 363 (1974)

Ostrogradksi instability

$$L = L(q, \dot{q}, \ddot{q}) \quad \rightarrow \quad \frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

- Four pieces of initial data, four canonical coordinates

$$Q_1 = q, \quad P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}; \quad Q_2 = \dot{q}, \quad P_2 = \frac{\partial L}{\partial \ddot{q}}$$

The corresponding hamiltonian is

$$H = P_1 Q_2 + P_2 \ddot{q} + L(Q_1, Q_2, \ddot{q}) \qquad \ddot{q} = \ddot{q}(Q_1, Q_2, P_2)$$

- Linear in P_1
- Unbound from below

f(R) gravity as an exception

$$S = \int d^4x \sqrt{-g} f(R)$$

can be written as

$$S = \int d^4x \sqrt{-g} \left[f(\phi) + \varphi (R - \phi) \right]$$

Variation with respect to ϕ yields

$$\varphi = f'(\phi)$$

One can then write

$$S = \int d^4x \sqrt{-g} \left[\varphi R - V(\varphi) \right] \qquad V(\varphi) = f(\phi) - f'(\phi)\phi$$

Brans-Dicke theory with $\omega_0 = 0$ and a potential

Symmetries and fields

Consider the equation (in flat spacetime)

$$\Box^{\rm flat}\phi = 0$$

and the set of equations

$$\Box\phi = 0 \qquad R_{\mu\nu\kappa\lambda} = 0$$

- The first equation is invariant only for inertial observers
- The second set is observer independent

However,

$$R_{\mu\nu\kappa\lambda} = 0 \quad \Rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu}$$

All solutions for the new field break the symmetry!

Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x \sqrt{-g}(-R - M^{\alpha\beta\mu\nu}\nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).

Hypersurface orthogonality

Now assume $$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S_{æ}^{ho} = \frac{1}{16\pi G_H}\int dT d^3x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a^i a_i\right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_{æ}} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

T. Jacobson, Phys. Rev. D 81, 101502 (2010).

Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6\right)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)
T.P.S., J. Phys. Conf. Ser. 283, 012034 (2011), arXiv:1010.3218 [hep-th]

Horava-Lifshitz gravity

- Higher order terms contain higher order spatial derivatives: higher order dispersion relations!
- They modify the propagator and render the theory power-counting renormalizable
- All terms consistent with the symmetries will be generated by radiative corrections
- This version of the theory is viable so far
- “Low energy limit” is h.o. Einstein-aether theory!

Covariance and fine tuning

Equivalence restricted to low energy action, but extendible

- Construct the most general action for ae-theory at a given order
- Lower order equivalence gives you the prescription of how to introduce the preferred foliation
- Go to the next order, identify the terms that match, set the rest of the couplings to zero/desired value
- Effectively, one can "covariantize" Horava gravity

But seen as a covariant theory, it would look severely fine tuned!

T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. D 83, 124021 (2011)

Constraints

- Classical stability and positivity of energy (no ghosts)

$$\lambda < \frac{1}{3} \quad \vee \quad \lambda > 1\,, \qquad\qquad 2\xi > \eta > 0$$

- Avoidance of vacuum Cherenkov radiation by matter

$$\Longrightarrow \quad \text{superluminal excitations in gravity sector}$$

- Agreement with Solar system experiments (vanishing preferred frame parameters)

$$\alpha_1 = \alpha_1(\lambda,\xi,\eta) < 10^{-4}, \qquad \alpha_2 = \alpha_1(\lambda,\xi,\eta) < 10^{-7}$$

D. Blas, O. Pujolas and S. Sibiryakov, JHEP 1104, 018 (2011)

Strong coupling

The low energy action exhibits strong coupling at energy

$$M_{\rm sc} = f(|\lambda - 1|, \eta) M_{\rm pl}$$

Can be a large energy scale, but problem with renormalizability!

A. Papazoglou and T. P. S., Phys. Lett. B 685, 197 (2010)
I. Kimpton and A. Padilla, JHEP 1007, 014 (2010)

Strong coupling problem can be circumvented if

$$M_{\rm sc} > M_\star$$

D. Blas, O. Pujolas and S. Sibiryakov, Phys.Lett. B 688, 350 (2010)

But then potential tension with observations!

$$10^{16}{\rm GeV} > M_\star > M_{\rm obs}$$

A. Papazoglou and T. P. Sotiriou, Phys. Lett. B 685, 197 (2010)

Percolation of LV

But what about the matter sector and lower order operators?

- Different speeds for different fields in the IR, with logarithmic running!

  R. Iengo, J. G. Russo and M. Serone, JHEP 0911, 020 (2009)

Possible ways out:

- Some extra symmetry, e.g. supersymmetry

  S. Groot Nibbelink and M. Pospelov, Phys. Rev. Lett. 94, 081601 (2005)

- Assume Lorentz symmetry in matter and let the weak coupling to gravity (the Lorentz-violating sector) do the rest

  M. Pospelov and Y. Shang, Phys. Rev. D 85, 105001 (2012)

Hierarchy of scales

Consider the dispersion relation

$$E^2 = m^2 + p^2 + \eta_4 \frac{p^4}{M_{LV}^2} + \mathcal{O}(\frac{p^6}{M_{LV}^4})$$

Assume that there is a universal LV scale, so

$$M_{LV} \sim M_\star$$

Constraint from synchrotron radiation from the Crab Nebula:

$$M_{\rm obs} > 2 \times 10^{16} \text{GeV}$$

$M_\star$ cannot be a universal scale!

S. Liberati, L. Maccione and T. P. S., Phys. Rev. Lett. 109, 151602 (2012)

Summary

- Higher order equation means extra degrees of freedom
- Less symmetry means extra degrees of freedom
- There are innovative ways to hide these degrees of freedom (at least for scalars)
- It is hard to guess fundamental symmetries from an effective theory!
- Combined constraints can be very, very efficient!