

# Equivalence principle, fundamental constants, spatial isotropy

Jean-Philippe UZAN



## Equivalence principle and the fundamental constants

- lecture 1: equivalence principle  
constants and gravity

- lecture 2: **Observational constraints on the variation of constants**

## Test of local isotropy

- lecture 3: Weak lensing as a test of local spatial isotropy

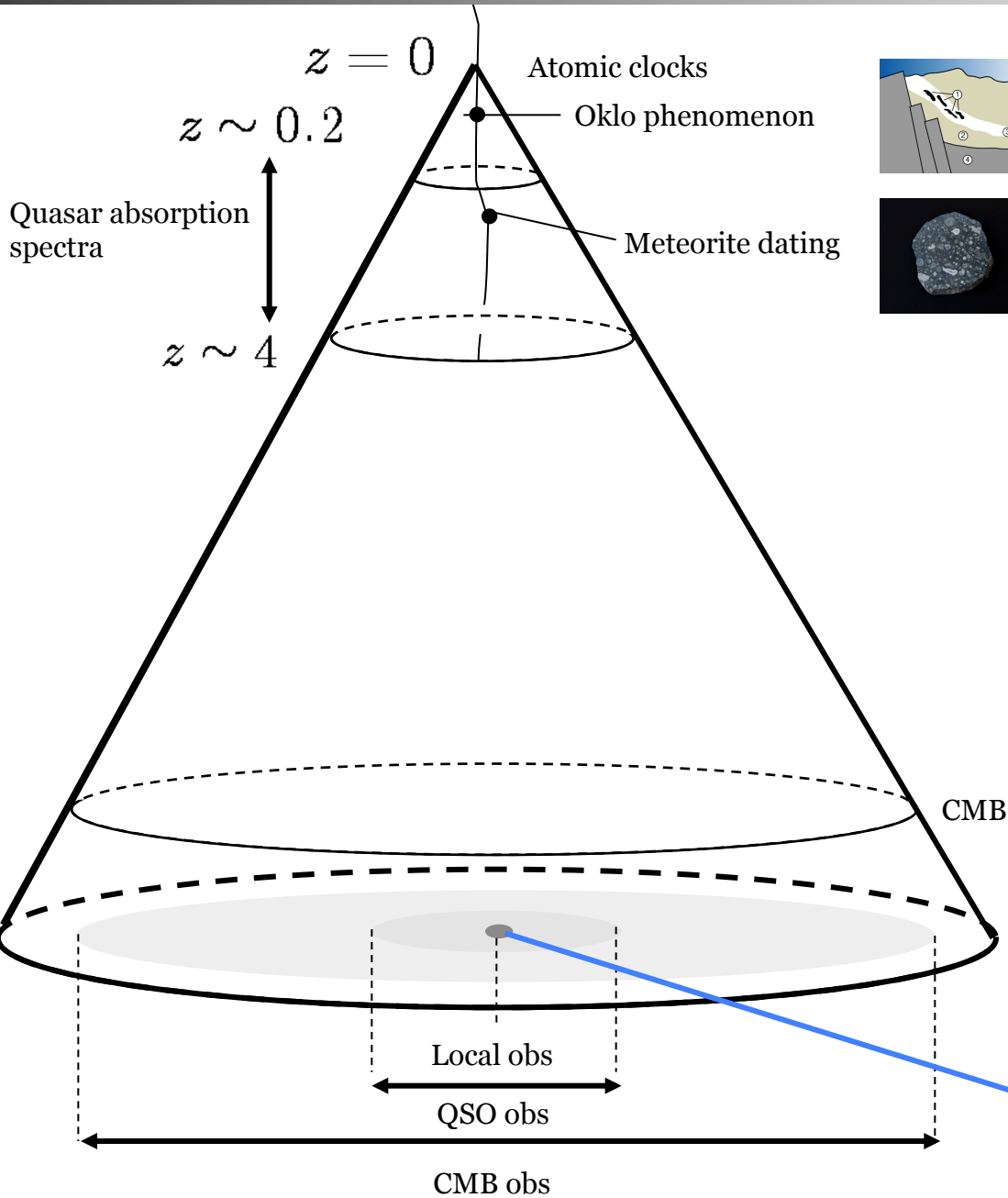
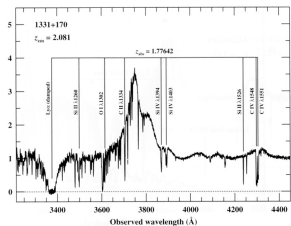
*complementary to Chris' lectures on Copernican principle*

# Observational constraints on the variation of fundamental constants

Jean-Philippe UZAN



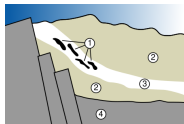
# Physical systems



Atomic clocks

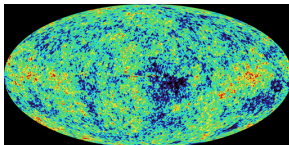
Oklo phenomenon

Meteorite dating

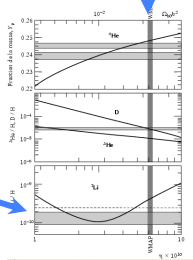


$z = 0.14$

$z = 0.43$



BBN



Local obs

QSO obs

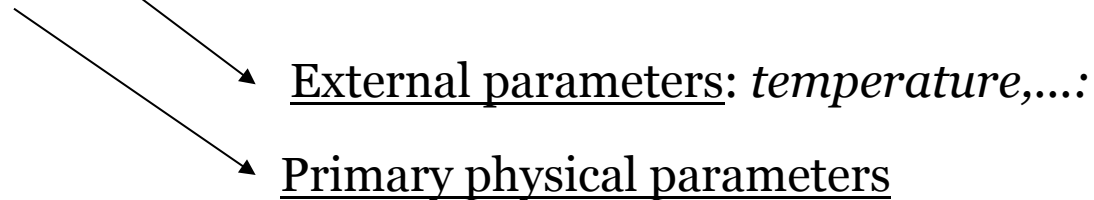
CMB obs

CMB

# Observables and primary constraints

A given physical system gives us an observable quantity

$$O(G_k, X)$$



From a physical model of our system we can deduce the sensitivities to the primary physical parameters

$$\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k}$$

The primary physical parameters are usually not fundamental constants.

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i$$

# Physical systems

| System           | Observable               | Primary constraint                 | Other hypothesis          |
|------------------|--------------------------|------------------------------------|---------------------------|
| Atomic clocks    | Clock rates              | $\alpha, \mu, g_i$                 | -                         |
| Quasar spectra   | Atomic spectra           | $\alpha, \mu, g_p$                 | Cloud physical properties |
| Oklo             | Isotopic ratio           | $E_r$                              | Geophysical model         |
| Meteorite dating | Isotopic ratio           | $\lambda$                          | Solar system formation    |
| CMB              | Temperature anisotropies | $\alpha, \mu$                      | Cosmological model        |
| BBN              | Light element abundances | $Q, \tau_n, m_e, m_N, \alpha, B_d$ | Cosmological model        |

# Atomic clocks

# Atomic clocks

Based the comparison of atomic clocks using different transitions and atoms

*e.g.* hfs Cs vs fs Mg :  $g_p \mu$  ;

hfs Cs vs hfs H:  $(g_p/g_I)\alpha$

*Examples*  $\frac{\nu_{Cs}}{\nu_{Rb}} \propto \frac{g_{Cs}}{g_{Rb}} \alpha^{0.49}$   $\frac{\nu_{Cs}}{\nu_H} \propto g_{Cs} \mu \alpha^{2.83}$

High precision / redshift o (local)

| Clock 1                        | Clock 2   | Constraint (yr <sup>-1</sup> )     | Constants dependence                       | Reference      |
|--------------------------------|---|------------------------------------|--|----------------|
|                                | $\frac{d}{dt} \ln \left( \frac{\nu_{\text{clock1}}}{\nu_{\text{clock2}}} \right)$ |                                    |  |                |
| <sup>87</sup> Rb               | <sup>133</sup> Cs   | $(0.2 \pm 7.0) \times 10^{-16}$    | $\frac{g_{Cs}}{g_{Rb}} \alpha_{EM}^{0.49}$ | Marion (2003)  |
| <sup>87</sup> Rb               | <sup>133</sup> Cs   | $(-0.5 \pm 5.3) \times 10^{-16}$   |  | Bize (2003)    |
| <sup>1</sup> H                 | <sup>133</sup> Cs   | $(-32 \pm 63) \times 10^{-16}$     | $g_{Cs} \mu \alpha_{EM}^{2.83}$            | Fischer (2004) |
| <sup>199</sup> Hg <sup>+</sup> | <sup>133</sup> Cs   | $(0.2 \pm 7) \times 10^{-15}$      | $g_{Cs} \mu \alpha_{EM}^{6.05}$            | Bize (2005)    |
| <sup>199</sup> Hg <sup>+</sup> | <sup>133</sup> Cs   | $(3.7 \pm 3.9) \times 10^{-16}$    |  | Fortier (2007) |
| <sup>171</sup> Yb <sup>+</sup> | <sup>133</sup> Cs   | $(-1.2 \pm 4.4) \times 10^{-15}$   | $g_{Cs} \mu \alpha_{EM}^{1.93}$            | Peik (2004)    |
| <sup>171</sup> Yb <sup>+</sup> | <sup>133</sup> Cs   | $(-0.78 \pm 1.40) \times 10^{-15}$ |  | Peik (2006)    |
| <sup>87</sup> Sr               | <sup>133</sup> Cs   | $(-1.0 \pm 1.8) \times 10^{-15}$   | $g_{Cs} \mu \alpha_{EM}^{2.77}$            | Blatt (2008)   |
| <sup>87</sup> Dy               | <sup>87</sup> Dy  |                                    |  | Cingöz (2008)  |
| <sup>27</sup> Al <sup>+</sup>  | <sup>199</sup> Hg <sup>+</sup>  | $(-5.3 \pm 7.9) \times 10^{-17}$   | $\alpha_{EM}^{-3.208}$                     | Blatt (2008)   |



# Atomic clocks

The gyromagnetic factors can be expressed in terms of  $g_p$  and  $g_n$  (shell model).

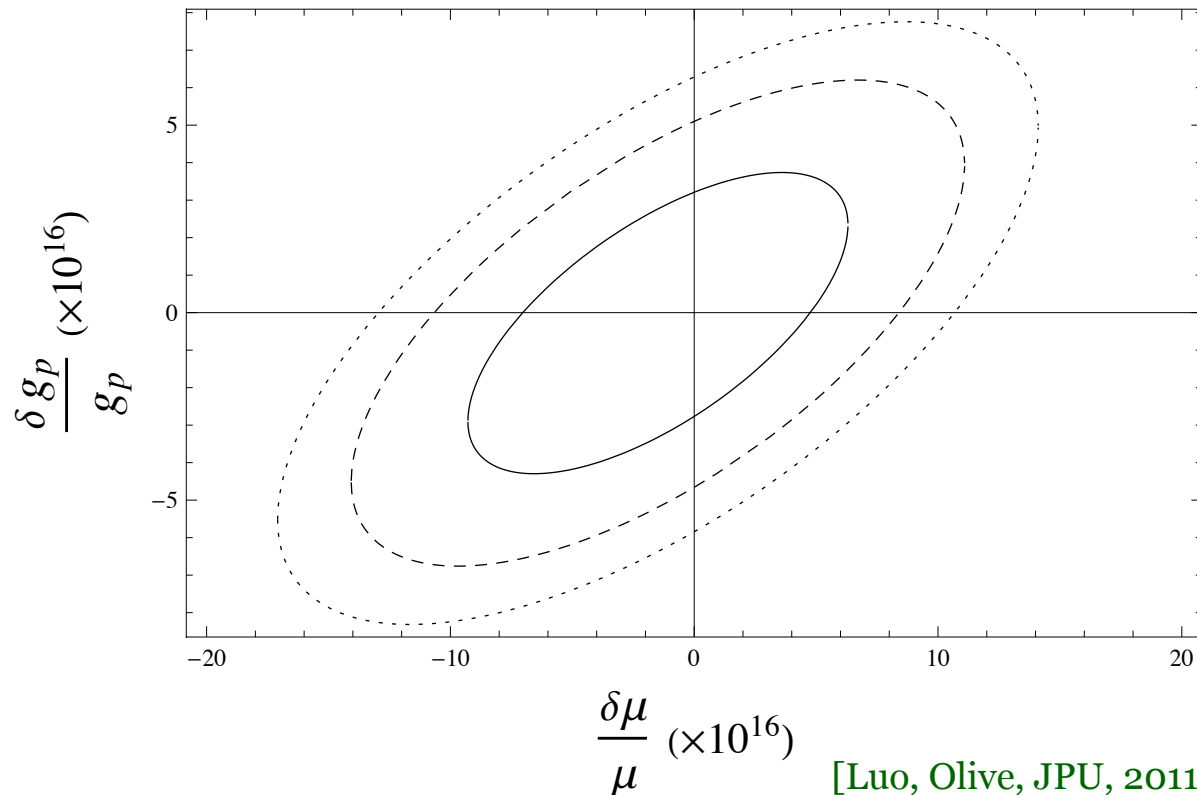
$$\frac{\delta g_{\text{Cs}}}{g_{\text{Cs}}} \sim -1.266 \frac{\delta g_p}{g_p} \quad \frac{\delta g_{\text{Rb}}}{g_{\text{Rb}}} \sim 0.736 \frac{\delta g_p}{g_p}$$

All atomic clock constraints take the form  $\frac{\dot{\nu}_{AB}}{\nu_{AB}} = \lambda_{g_p} \frac{\dot{g}_p}{g_p} + \lambda_\mu \frac{\dot{\mu}}{\mu} + \lambda_\alpha \frac{\dot{\alpha}}{\alpha}$ :

Using Al-Hg to constrain  $\alpha$ , the combination of other clocks allows to constraint  $\{\mu, g_p\}$ .

Note: one actually needs to include the effects of the polarization of the non-valence nucleons and spin-spin interaction.

[Flambaum, 0302015,...



[Luo, Olive, JPU, 2011]

# Importance of unification

**Unification**  $\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{E}$

*Variation of  $\alpha$  is accompanied by variation of other coupling constants*

**QCD scale**  $\Lambda_{QCD} = E \left( \frac{m_c m_b m_t}{E^3} \right)^{2/27} \exp \left[ -\frac{2\pi}{9\alpha_s(E)} \right]$

*Variation of  $\Lambda_{QCD}$  from  $\alpha_s$  and from Yukawa coupling and Higgs VEV*

**Theories in which EW scale is derived by dimensional transmutation**  $v \sim \exp \left[ -\frac{8\pi^2}{h_t^2} \right]$

*Variation of Yukawa and Higgs VEV are coupled*

**String theory** All dimensionless constants are dynamical – their variations are all correlated.

**These effects cannot be ignored in realistic models.**

# Atomic clocks

One then needs to express  $m_p$  and  $g_p$  in terms of the quark masses and  $\Lambda_{\text{QCD}}$  as

$$\frac{\delta g_p}{g_p} = \kappa_u \frac{\delta m_u}{m_u} + \kappa_d \frac{\delta m_d}{m_d} + \kappa_s \frac{\delta m_s}{m_s} + \kappa_{\text{QCD}} \frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}}$$

$$\frac{\delta m_p}{m_p} = f_{T_u} \frac{\delta m_u}{m_u} + f_{T_d} \frac{\delta m_d}{m_d} + f_{T_s} \frac{\delta m_s}{m_s} + f_{T_g} \frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}}$$

$$m_i = h_i v$$

Assuming unification.

$$\frac{\dot{\nu}_{AB}}{\nu_{AB}} = \lambda_{g_p} \frac{\dot{g}_p}{g_p} + \lambda_\mu \frac{\dot{\mu}}{\mu} + \lambda_\alpha \frac{\dot{\alpha}}{\alpha} \longrightarrow \frac{\dot{\nu}_{AB}}{\nu_{AB}} = C_{AB} \frac{\dot{\alpha}}{\alpha}$$

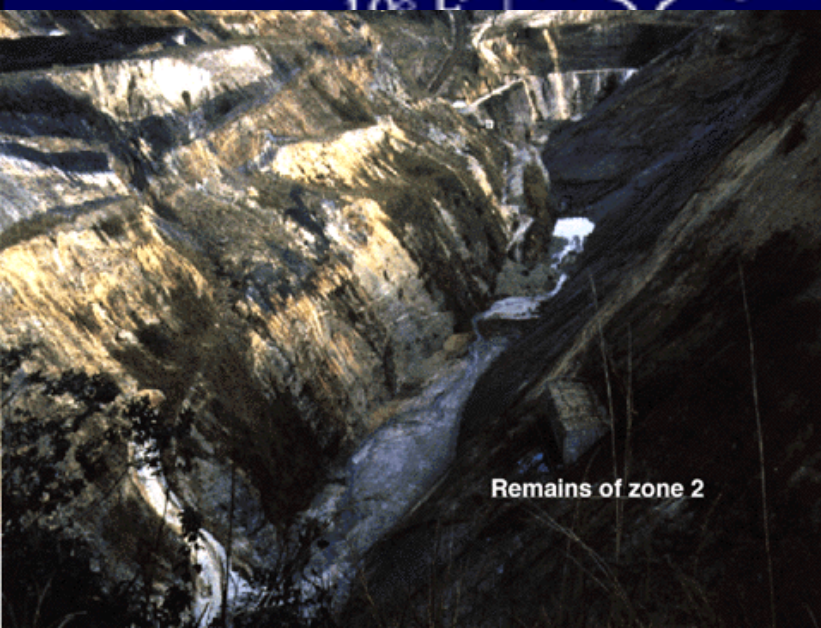
$C_{AB}$  coefficients range from 70 to 0.6 typically.

Model-dependence remains quite large.

# Nuclear methods

*(Oklo / meteorite dating)*

# Oklo- a natural nuclear reactor



It operated 2 billion years ago, during 200 000 years !!

# Oklo: why?

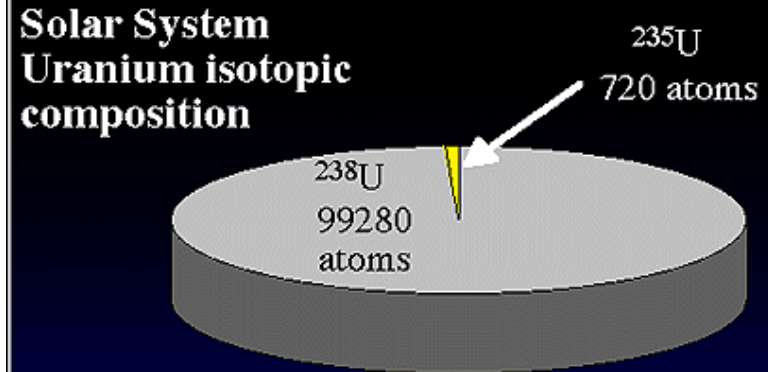
## 4 conditions :

1- Naturally high in  $U^{235}$ ,

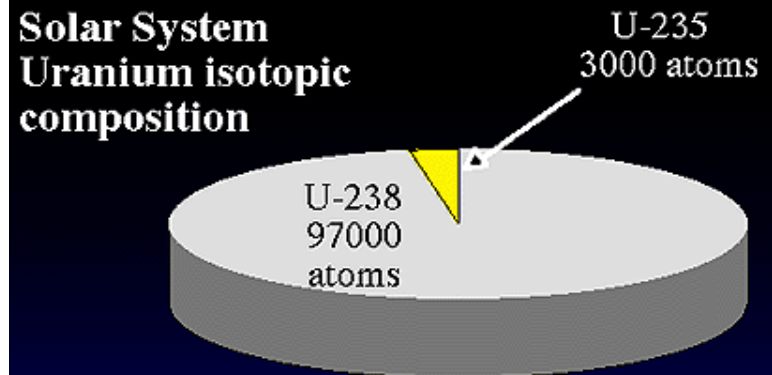
2- moderator : water,

3- low abundance of neutron absorber,

4- size of the room.



For every 100 000  
Solar System Uranium atoms

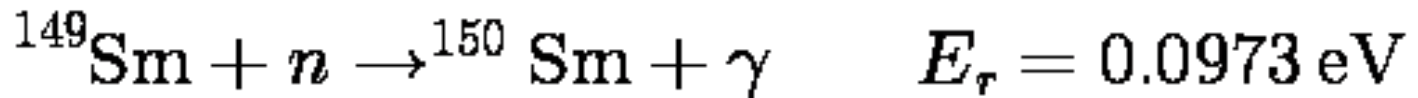


For every 100 000  
Solar System Uranium atoms:  
2 billion years ago

# Oklo-constraints

Natural nuclear reactor in Gabon,  
operating 1.8 Gyr ago ( $z \sim 0.14$ )

Abundance of Samarium isotopes



From isotopic abundances of Sm, U and Gd, one can  
measure the cross section averaged on the thermal neutron flux

$$\hat{\sigma}_{149}(T, E_\gamma) = 91 \pm 6 \text{ kb}$$

From a model of Sm nuclei, one can infer

$$s = \Delta E_\gamma / \Delta \ln \alpha$$

$s \sim 1 \text{ MeV}$  so that

$$\Delta \alpha / \alpha \sim 1 \text{ MeV} / 0.1 \text{ eV} \sim 10^{-7}$$

$$\Delta \alpha / \alpha = (0.5 \pm 1.05) \times 10^{-7}$$

Damour, Dyson, NPB **480** (1996) 37

Shlyakhter, Nature **264** (1976) 340  
Damour, Dyson, NPB **480** (1996) 37  
Fujii et al., NPB **573** (2000) 377  
Lamoreaux, torgerson, nucl-th/0309048  
Flambaum, shuryak, PRD **67** (2002) 083507

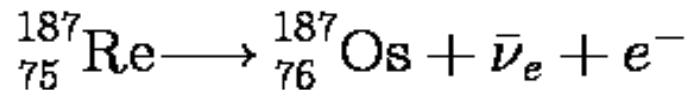
Fujii et al., NPB **573** (2000) 377 2 branches.

# Meteorite dating

Bounds on the variation of couplings can be obtained by constraints on the lifetime of long-lived nuclei ( $\alpha$  and  $\beta$  decayers)

For  $\beta$  decayers, 
$$\lambda \sim \Lambda(\Delta E)^p \propto G_F^2 \alpha^s$$

**Rhenium:**



Peebles, Dicke, PR **128** (1962) 2006

$$\Delta E \sim 2.5 \text{ keV}, \quad s \sim -18000$$

Use of laboratory data + meteorites data

$$-24 \times 10^{-7} < \Delta\alpha/\alpha < 8 \times 10^{-7}$$

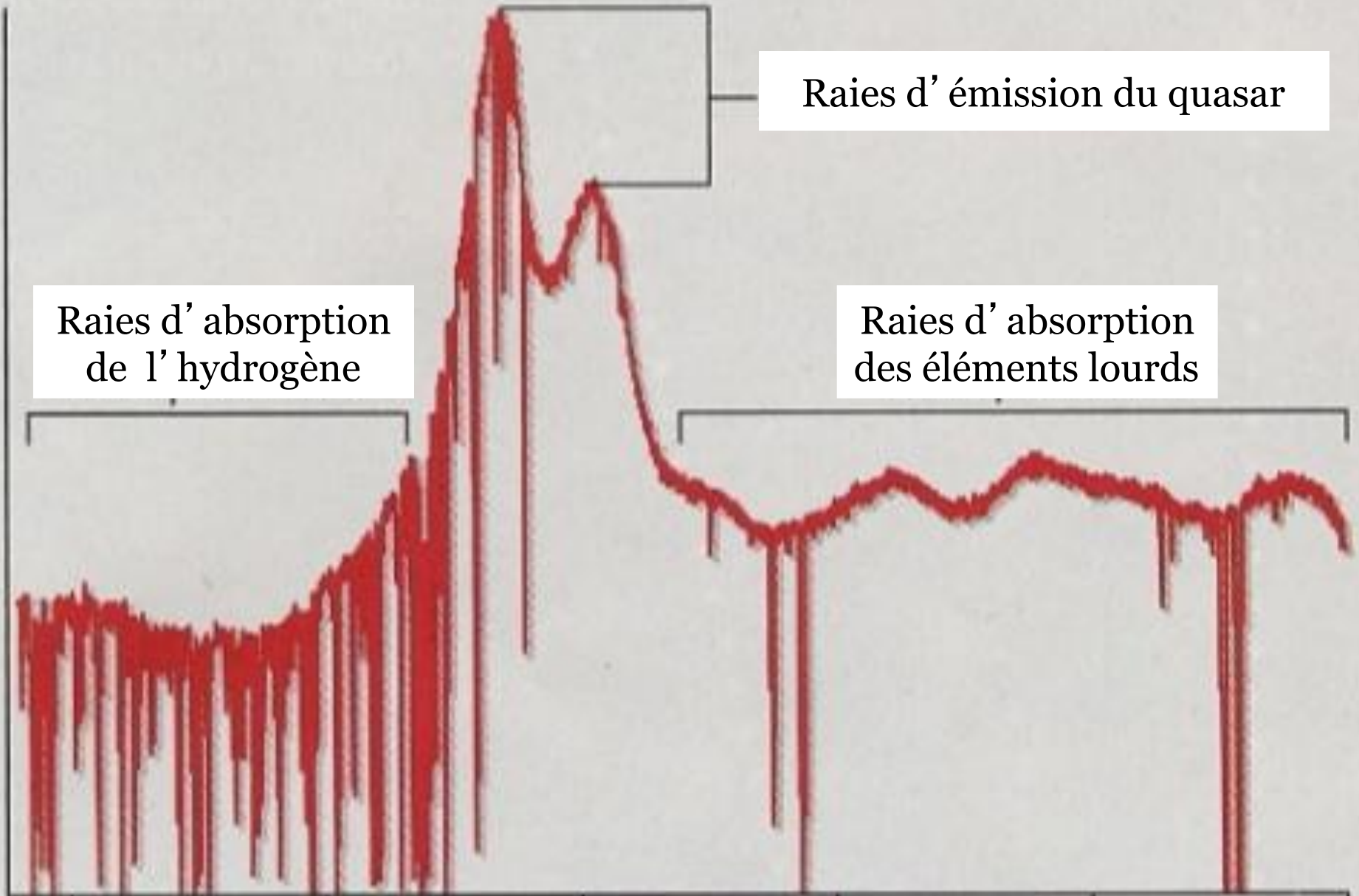
Olive et al., PRD **69** (2004) 027701

Caveats: meteorites datation / averaged value

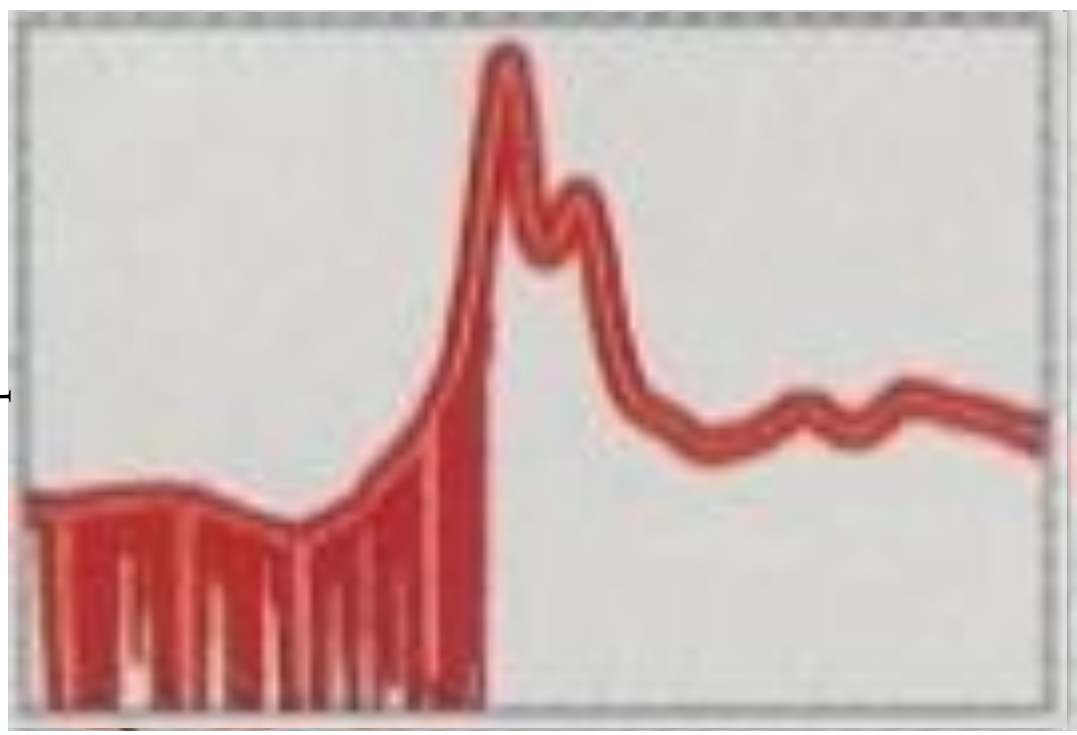


# Quasar absorption spectra

# Spectres d'absorption de quasars



# Absorption spectra

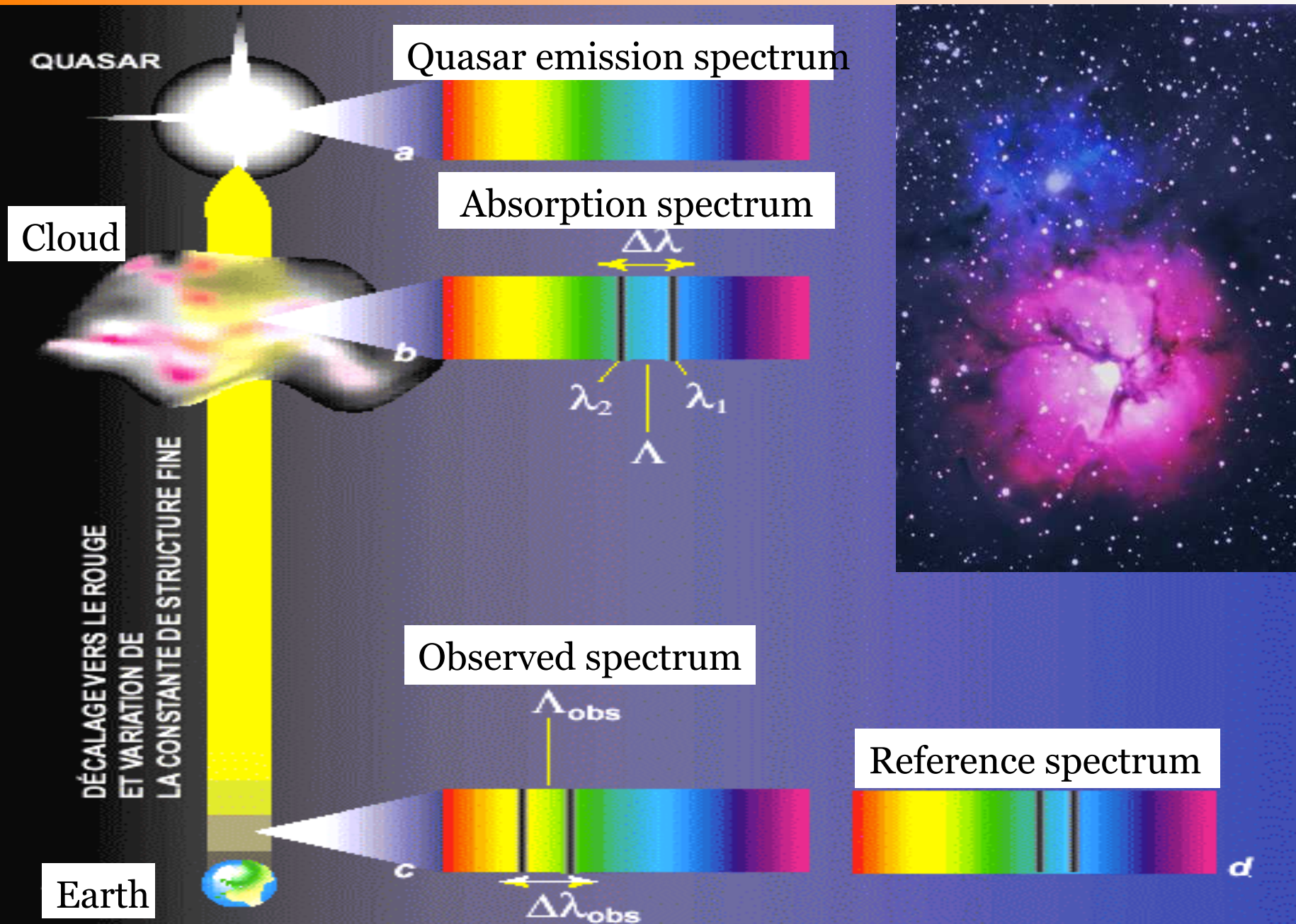


Blue wavelength red

Cosmic expansion redshift all spectra (achromatic)

We look for achromatic effects

# Paleo-spectra



# Generalities

The method was introduced by Savedoff in 1956, using Alkali doublet

Most studies are based on optical techniques due to the profusion of strong UV transitions that are redshifted into the optical band

*e.g. SiIV @  $z > 1.3$ , FeIII1608 @  $z > 1$*

Radio observations are also very important

e.g. hyperfine splitting (HI21cm), molecular rotation, lambda doubling, ...

- offer high spectral resolution ( $< 1 \text{ km/s}$ )
- higher sensitivity to variation of constants
- isotopic lines observed separately (while blending in optical observations)

Shift to be detected are small

e.g. a change of  $\lambda$  of  $10^{-5}$  corresponds to

- a shift of  $20 \text{ m}\text{\AA}$  (i.e. of  $0.5 \text{ km/s}$ ) at  $z \sim 2$
- $\frac{1}{3}$  of a pixel at  $R = 40000$  (Keck/HIRES, VLT/UVES)

Many sources of uncertainty

- absorption lines have complex profiles (inhomogeneous cloud)
- fitted by Voigt profile (usually not unique: require lines not to be saturated)
- each component depends on  $z$ , column density, width



# QSO absorption spectra

## 3 main methods:

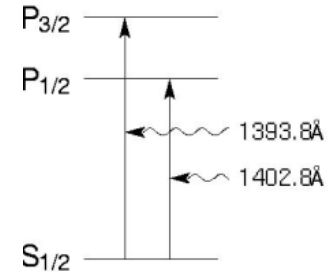
Alkali doublet (AD) Savedoff 1956

Fine structure doublet,  $\Delta\lambda/\lambda \propto \alpha^2$

Single atom

Rather weak limit

Si IV alkali doublet



VLT/UVES: Si IV in 15 systems,  $1.6 < z < 3$

$$\frac{\Delta\alpha}{\alpha} = (0.15 \pm 0.43) \times 10^{-5}$$

Chand et al. 2004

HIRES/Keck: Si IV in 21 systems,  $2 < z < 3$

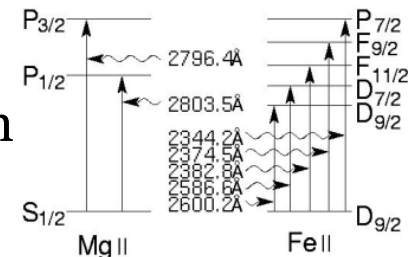
$$\frac{\Delta\alpha}{\alpha} = (-0.5 \pm 1.3) \times 10^{-5}$$

Murphy et al. 2001

Many multiplet (MM) Webb et al. 1999

Compares transitions from multiplet and/or atom  
s-p vs d-p transitions in heavy elements

Better sensitivity



Single Ion Differential  $\alpha$  Measurement (SIDAM)

Levshakov et al. 1999

Analog to MM but with a single atom / FeII

# QSO: many multiplets

The many-multiplet method is based on the correlation of the shifts of different lines of different atoms.

Dzuba et al. 1999-2005

Relativistic N-body with varying  $\alpha$ :

$$\omega = \omega_0 + 2q \frac{\Delta\alpha}{\alpha}$$

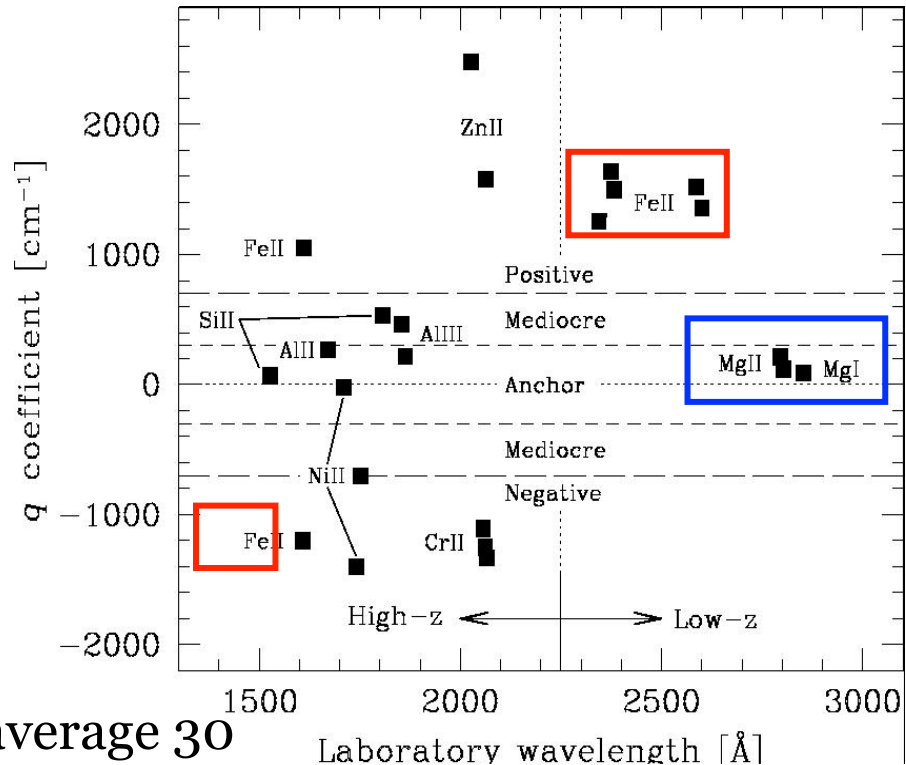
First implemented on 30 systems with MgII and FeII

Webb et al. 1999

R=45000,  
S/N per pixels between 4 & 240, with average 30  
Wavelength calibrated with Thorium-Argon lamp

HIRES-Keck, 143 systems,  $0.2 < z < 4.2$

$$\frac{\Delta\alpha}{\alpha} = (-0.57 \pm 0.11) \times 10^{-5}$$



Murphy et al. 2004

**5σ detection !**

# QSO: uncertainties

- Error in the determination of laboratory spectra
- Different atoms may not be located in the same part of the cloud (relative Doppler)
- Lines may be blended by transitions in another system
- Variation of velocity of the Earth during integration can induce a differential Doppler shift
- Atmospheric dispersion
- Magnetic fields in the clouds
- Temperature variation during the integration
- Instrumental effects (e.g. variation of the intrinsic profile of the instrument)

## Isotopic abundance of MgII (used as an anchor)

- affects the value of the effective rest-wavelengths
- assumed to be close to terrestrial  $^{24}\text{Mg}:^{25}\text{Mg}:^{26}\text{Mg}=79:10:11$
- $r=(26+25)/24$  cannot be measured directly
- from molecular absorption of MgH:  $r$  decreases with metallicity
- But  $r$  found to be high in giant stars in NGC6752
- Ashenfelter et al proposed an enhancement of  $r$  from stars in  $(2-8)M_{\text{sun}}$  in their asymptotic giant branch phase
- If  $r=0.62$  instead of  $r=0.27$ , then no variation of  $\alpha$
- But overproduction of P, Si, Al



# QSO: VLT/UVES analysis

Selection of the absorption spectra:

- lines with similar ionization potentials  
*most likely to originate from similar regions in the cloud*
- avoid lines contaminated by atmospheric lines
- at least one anchor line is not saturated  
*redshift measurement is robust*
- reject strongly saturated systems

Only 23 systems

lower statistics / better controlled systematics

R > 44000, S/N per pixel between 50 & 80

VLT/UVES

$$\frac{\Delta\alpha}{\alpha} = (-0.01 \pm 0.15) \times 10^{-5}$$

Srianand et al. 2007

**DOES NOT CONFIRM HIRES/Keck DETECTION**

# Going further

## Other transitions:

- HI21cm vs UV of heavy element transitions:  $\alpha^2 g_p / \mu$
- HI vs molecular transitions (CO, HCO+, HCN):  $g_p \alpha^2$
- OH18cm: ground state  ${}^2\Pi_{3/2} J=3/2$  of OH is split in 2 levels further split in 2 hyperfine states,  
It constrains  $g_p (\alpha^2 \mu)^{1.57}$
- FIR fine-structure lines (CO)  $\alpha^2 \mu$
- Conjugate OH lines (emission+absorption lines with same shape):  $g_p (\alpha \mu)^{1.85}$
- Molecular lines (H<sub>2</sub>, NH<sub>3</sub>, HD):  $\mu$

**Table 10:** Summary of the latest constraints on the variation of fundamental constants obtained from the analysis of quasar absorption spectra. We recall that  $y \equiv g_p \alpha_{\text{EM}}^2$ ,  $F \equiv g_p (\alpha_{\text{EM}}^2 \mu)^{1.57}$ ,  $x \equiv \alpha_{\text{EM}}^2 g_p / \mu$ ,  $F' \equiv \alpha_{\text{EM}}^2 \mu$  and  $\mu \equiv m_p / m_e$ ,  $G = g_p (\alpha \mu)^{1.85}$ .

| Constant             | Method               | System | Constraint ( $\times 10^{-5}$ )                        | Redshift     | Ref.  |
|----------------------|----------------------|--------|--|--------------|-------|
| $\alpha_{\text{EM}}$ | AD                   | 21     | $(-0.5 \pm 1.3)$                                       | 2.33–3.08    | [366] |
|                      | AD                   | 15     | $(-0.15 \pm 0.43)$                                     | 1.59–2.92    | [87]  |
|                      | AD                   | 9      | $(-3.09 \pm 8.46)$                                     | 1.19–1.84    | [339] |
|                      | MM                   | 143    | $(-0.57 \pm 0.11)$                                     | 0.2–4.2      | [356] |
|                      | MM                   | 21     | $(0.01 \pm 0.15)$                                      | 0.4–2.3      | [86]  |
|                      | SIDAM                | 1      | $(-0.012 \pm 0.179)$                                   | 1.15         | [351] |
|                      | SIDAM                | 1      | $(0.566 \pm 0.267)$                                    | 1.84         | [351] |
| $y$                  | HI - mol             | 1      | $(-0.16 \pm 0.54)$                                     | 0.6847       | [364] |
|                      | HI - mol             | 1      | $(-0.2 \pm 0.44)$                                      | 0.247        | [364] |
|                      | CO, CHO <sup>+</sup> |        | $(-4 \pm 6)$   | 0.247        | [519] |
| $F$                  | OH - HI              | 1      | $(-0.44 \pm 0.36 \pm 1.0_{\text{syst}})$               | 0.765        | [266] |
|                      | OH - HI              | 1      | $(0.51 \pm 1.26)$                                      | 0.2467       | [134] |
| $x$                  | HI - UV              | 9      | $(-0.63 \pm 0.99)$                                     | 0.23–2.35    | [479] |
|                      | HI - UV              | 2      | $-(0.17 \pm 0.17)$                                     | 3.174        | [457] |
| $F'$                 | CII - CO             | 1      | $(1 \pm 10)$   | 4.69         | [316] |
|                      | CII - CO             | 1      | $(14 \pm 15)$  | 6.42         | [316] |
| $G$                  | OH                   | 1      | $< 1.1$  | 0.247, 0.765 | [91]  |
|                      | OH                   | 1      | $< 1.16$   | 0.0018       | [91]  |
|                      | OH                   | 1      | $(-1.18 \pm 0.46)$                                     | 0.247        | [268] |
| $\mu$                | H <sub>2</sub>       | 1      | $(2.78 \pm 0.88)$                                      | 2.59         | [417] |
|                      | H <sub>2</sub>       | 1      | $(2.06 \pm 0.79)$                                      | 3.02         | [417] |
|                      | H <sub>2</sub>       | 1      | $(1.01 \pm 0.62)$                                      | 2.59         | [281] |
|                      | H <sub>2</sub>       | 1      | $(0.82 \pm 0.74)$                                      | 2.8          | [281] |
|                      | H <sub>2</sub>       | 1      | $(0.26 \pm 0.30)$                                      | 3.02         | [281] |
|                      | H <sub>2</sub>       | 1      | $(0.7 \pm 0.8)$  | 3.02, 2.59   | [475] |
|                      | NH <sub>3</sub>      | 1      | $< 0.18$   | 0.685        | [355] |
|                      | NH <sub>3</sub>      | 1      | $< 0.38$   | 0.685        | [343] |
|                      | HC <sub>3</sub> N    | 1      | $< 0.14$   | 0.89         | [243] |
|                      | HD                   | 1      | $< 9$  | 2.418        | [398] |
|                      | HD                   | 1      | $(0.56 \pm 0.55_{\text{stat}} \pm 0.27_{\text{syst}})$ | 2.059        | [332] |

# Cosmic microwave background

It changes the recombination history

- 1- modifies the optical depth
- 2- induces a change in the hydrogen and helium abundances ( $x_e$ )

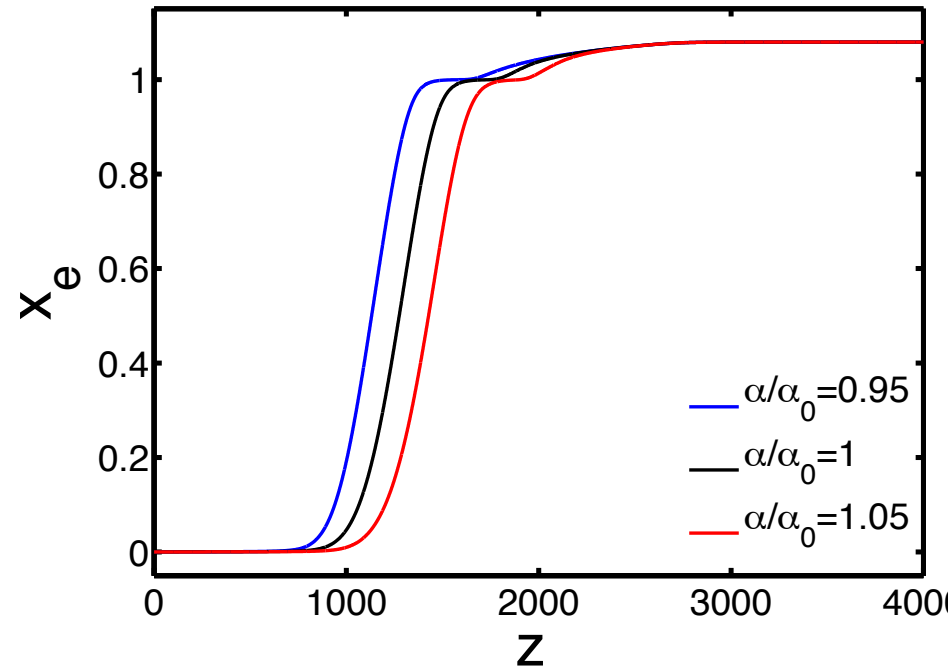
$$\dot{\tau} = x_e n_e C \sigma_T$$

Effect on the position of the Doppler peak  
on polarization (reionisation)

Degeneracies:

- cosmological parameters
- electron mass
- origin of primordial fluctuations

$$\sigma_T \propto \alpha^2 / m_e$$

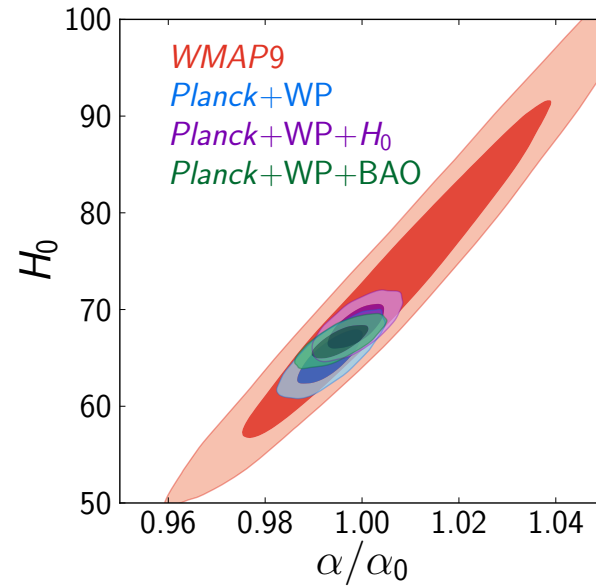
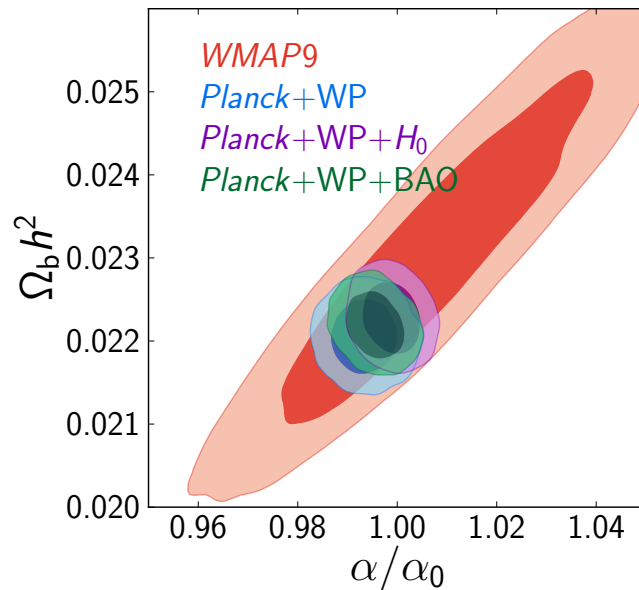
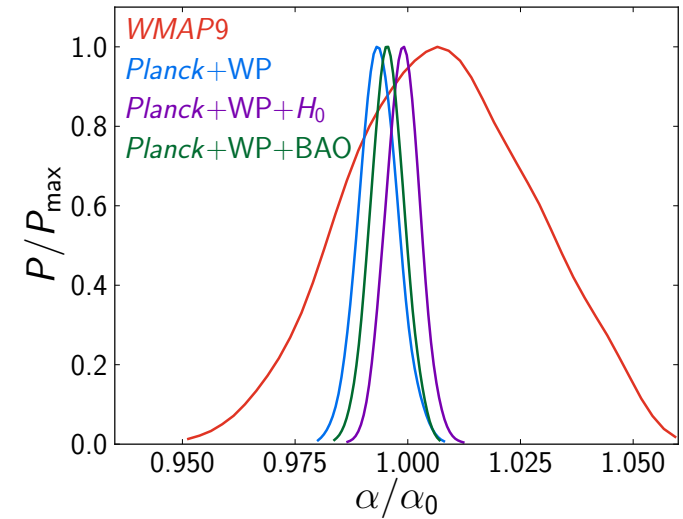


# Planck analysis

Parameters of the fit

$$(\alpha, \Omega_b, \Omega_c, H_0, n_s, A_s, \tau)$$

Marginalized distribution on  $\alpha$



# Planck analysis

|                     | Planck+WP             | Planck+WP+Lensing     | WMAP9                |
|---------------------|-----------------------|-----------------------|----------------------|
| $\Omega_b h^2$      | $0.02206 \pm 0.00028$ | $0.02220 \pm 0.00027$ | $0.02309 \pm 0.0013$ |
| $\Omega_c h^2$      | $0.1174 \pm 0.0030$   | $0.1161 \pm 0.0027$   | $0.1148 \pm 0.0048$  |
| $\tau$              | $0.0949 \pm 0.0143$   | $0.0949 \pm 0.0145$   | $0.089 \pm 0.014$    |
| $H_0$               | $65.2 \pm 1.8$        | $66.0 \pm 1.7$        | $73.9 \pm 10.9$      |
| $n_s$               | $0.9651 \pm 0.0128$   | $0.9768 \pm 0.0116$   | $0.9732 \pm 0.0137$  |
| $\log(10^{10} A_s)$ | $3.106 \pm 0.029$     | $3.102 \pm 0.028$     | $3.09 \pm 0.039$     |
| $\alpha/\alpha_0$   | $0.9936 \pm 0.0043$   | $0.9940 \pm 0.0043$   | $1.008 \pm 0.020$    |

|                     | Planck+WP+HST         | Planck+WP+HighL      | Planck+WP+BAO        |
|---------------------|-----------------------|----------------------|----------------------|
| $\Omega_b h^2$      | $0.02228 \pm 0.00027$ | $0.02210 \pm 0.0027$ | $0.02220 \pm 0.0025$ |
| $\Omega_c h^2$      | $0.1166 \pm 0.0030$   | $0.1185 \pm 0.0031$  | $0.1161 \pm 0.0028$  |
| $\tau$              | $0.096 \pm 0.014$     | $0.094 \pm 0.015$    | $0.097 \pm 0.014$    |
| $H_0$               | $68.3 \pm 1.5$        | $66.2 \pm 1.6$       | $66.7 \pm 1.1$       |
| $n_s$               | $0.9695 \pm 0.0115$   | $0.9666 \pm 0.0114$  | $0.9748 \pm 0.0118$  |
| $\log(10^{10} A_s)$ | $3.097 \pm 0.028$     | $3.10 \pm 0.029$     | $3.10 \pm 0.029$     |
| $\alpha/\alpha_0$   | $0.9989 \pm 0.0037$   | $0.9965 \pm 0.037$   | $0.9955 \pm 0.038$   |

# Big bang nucleosynthesis



# BBN: generality

BBN predicts the primordial abundances of D, He-3, He-4, Li-7

Mainly based on the balance between

1- expansion rate of the universe

2- weak interaction rate which controls  $n/p$  at the onset of BBN

**Example:** helium production

$$Y = \frac{2(n/p)_N}{1+(n/p)_N}$$

$$(n/p)_f \sim e^{-Q/k_B T_f} \quad (B_D, \eta)$$
$$(n/p)_N \sim (n/p)_f e^{-t_N/\tau_n}$$

freeze-out temperature is roughly given by  $G_F^2 (k_B T_f)^5 = \sqrt{GN} (k_B T_f)^2$

Coulomb barrier:  $\sigma = \frac{S(E)}{E} e^{-2\pi\alpha Z_1 Z_2 \sqrt{\mu/2E}}$

Predictions depend on

$$G_k = (G, \alpha, \tau_n, m_e, Q, B_D, \sigma_i)$$
$$X = (\eta, h, N_\nu, \dots)$$

# Scalar-tensor theories

Most general theories of gravity that include a scalar field beside the metric

Mathematically **consistent**

Motivated by **superstring**

**dilaton** in the graviton supermultiplet,

**moduli** after dimensional reduction

Consistent field theory to satisfy WEP

Useful extension of GR (simple but general enough)

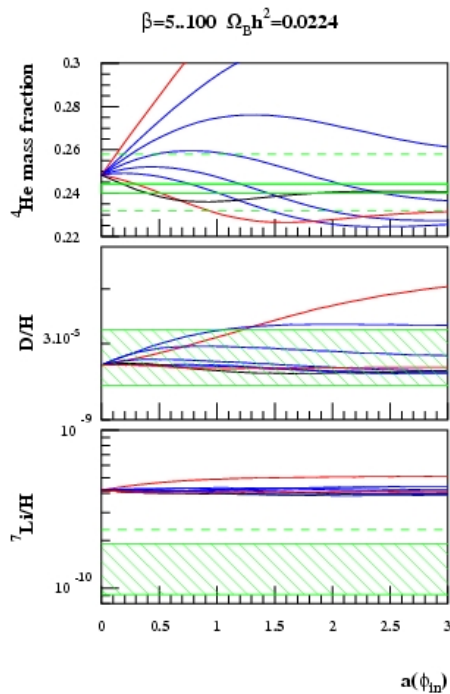
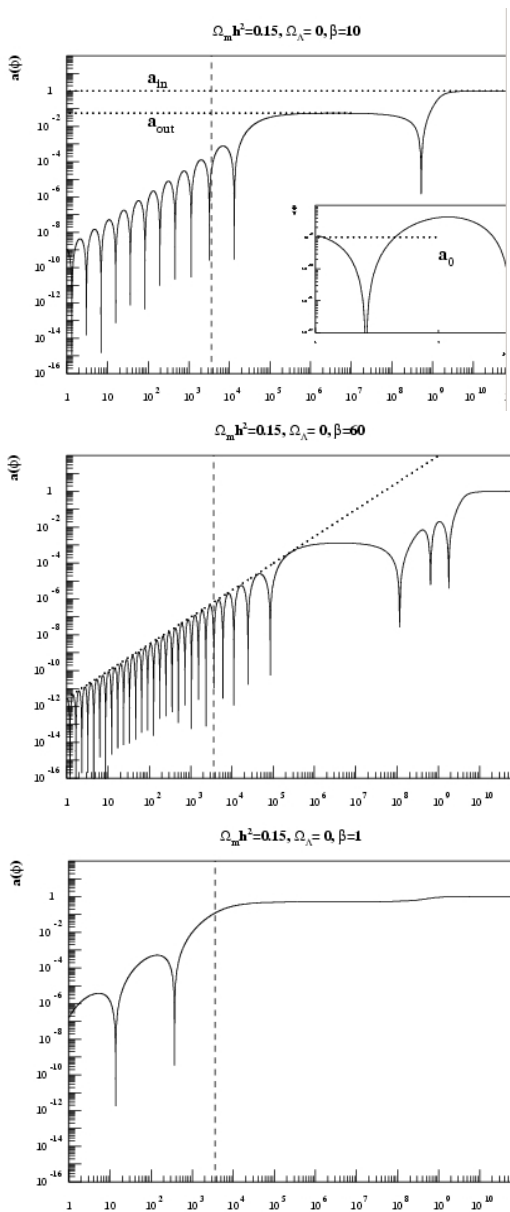
$$S = \frac{c^3}{16\pi G} \int \sqrt{-g} \{ R - 2(\partial_\mu \phi)^2 - V(\phi) \} + S_m \{ \text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \}$$

$$\alpha = d \ln A / d\phi$$

$$\beta = d\alpha / d\phi$$

# BBN constraints

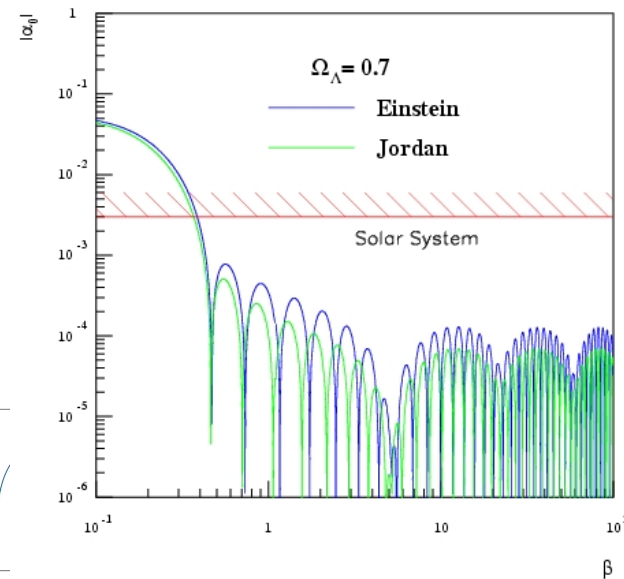
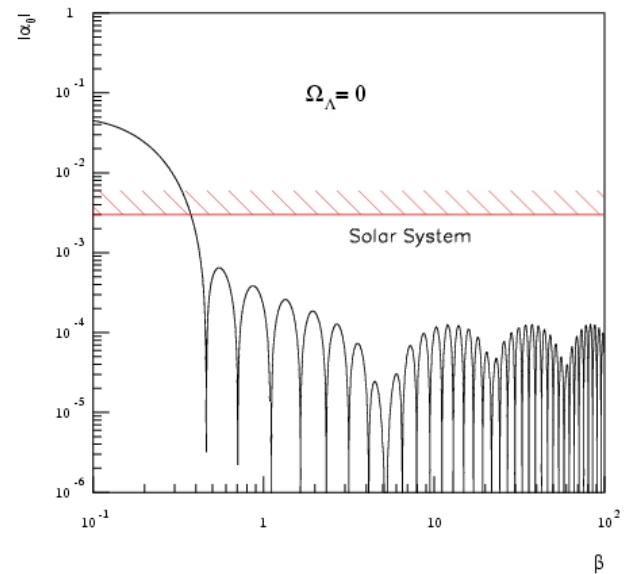
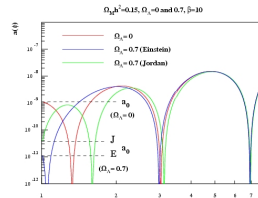
Full dynamics → Abundances → constraints



$a_{in}/a_{out}$  relation not injective

Max value of  $\alpha_0$  but not all  $\alpha_0$  smaller are acceptable

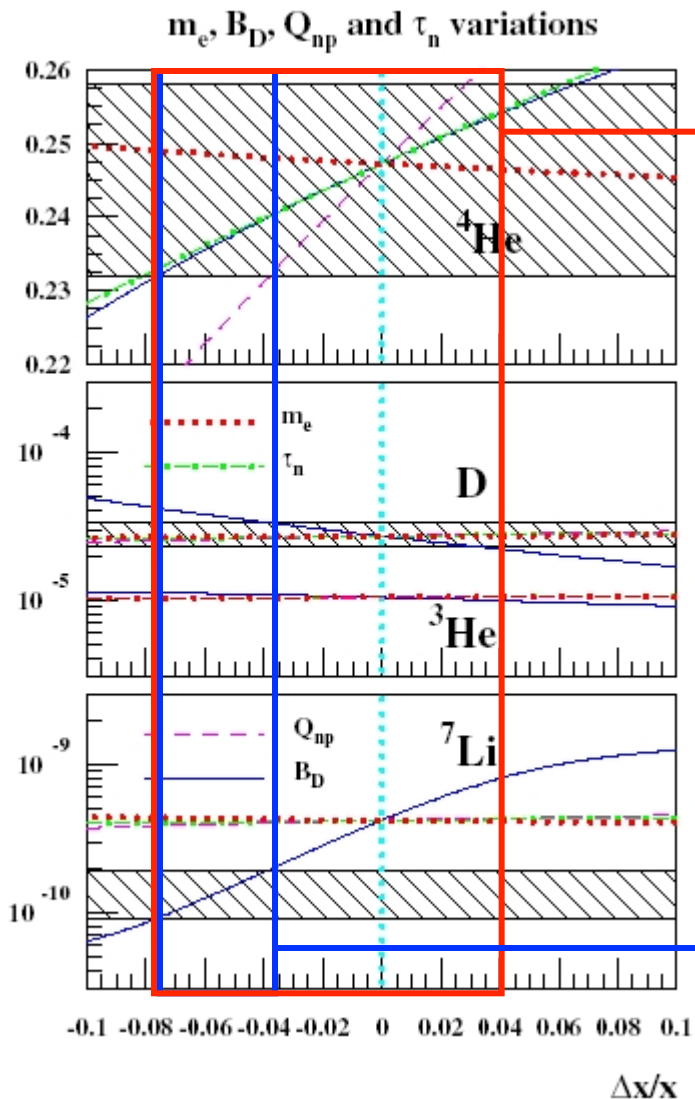
Late time dynamics modified



Coc et al. 2006  
Pichon, Damour

# BBN: effective BBN parameters

Independent variations of the BBN parameters



$$\begin{aligned}
 -7.5 \times 10^{-2} < \frac{\Delta B_D}{B_D} < 6.5 \times 10^{-2} \\
 -8.2 \times 10^{-2} < \frac{\Delta \tau_n}{\tau_n} < 6 \times 10^{-2} \\
 -4 \times 10^{-2} < \frac{\Delta Q}{Q} < 2.7 \times 10^{-2}
 \end{aligned}$$

Abundances are very sensitive to  $B_D$ .  
 Equilibrium abundance of D and the reaction rate  $p(n,\gamma)\text{D}$  depend exponentially on  $B_D$ .

These parameters are not independent.

**Difficulty:** QCD and its role in low energy nuclear reactions.

$$-7.5 \times 10^{-2} < \frac{\Delta B_D}{B_D} < -4 \times 10^{-2}$$

# BBN: fundamental parameters (1)

**Neutron-proton mass difference:**

$$Q = m_n - m_p = a\alpha\Lambda + (h_d - h_u)v$$

$$\frac{\Delta Q}{Q} = -0.6 \left( \frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right) + 1.6 \left( \frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v} \right)$$

**Neutron lifetime:**

$$\tau_n^{-1} = G_F^2 m_e^5 f(Q/m_e) \quad m_e = h_e v$$
$$G_F = 1/\sqrt{2} v^2$$

$$\frac{\Delta\tau_n}{\tau_n} = -4.8 \frac{\Delta v}{v} + 1.5 \frac{\Delta h_e}{h_e} - 10.4 \frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8 \left( \frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right)$$

# BBN: fundamental parameters (2)

## D binding energy:

Use a potential model  $V_{nuc} = \frac{1}{4\pi r}(-g_s^2 e^{-rm_\sigma} + g_v^2 e^{-rm_\omega})$

$$\frac{\Delta B_D}{B_D} = -48 \frac{\Delta m_\sigma}{m_\sigma} + 50 \frac{\Delta m_\omega}{m_\omega} + 6 \frac{\Delta m_N}{m_N}$$

Flambaum, Shuryak 2003

Most important parameter beside  $\Lambda$  is the strange quark mass.  
One needs to trace the dependence in  $m_s$ .

$$\frac{\Delta m_\sigma}{m_\sigma} \sim 0.54 \frac{\Delta m_s}{m_s}$$

$$\frac{\Delta m_\omega}{m_\omega} \sim 0.15 \frac{\Delta m_s}{m_s}$$

$$\frac{\Delta m_N}{m_N} \sim 0.12 \frac{\Delta m_s}{m_s}$$

$$\frac{\Delta B_D}{B_D} = 18 \frac{\Delta \Lambda}{\Lambda} - 17 \left( \frac{\Delta v}{v} + \frac{\Delta h_s}{h_s} \right)$$

This allows to determine all the primary parameters in terms of  $(h_i, v, \Lambda, \alpha)$

# BBN: assuming GUT

## GUT:

The low-energy expression for the QCD scale

$$\Lambda = \mu \left( \frac{m_c m_b m_t}{\mu^3} \right)^{2/27} \exp \left( -\frac{2\pi}{9\alpha_3(\mu)} \right)$$

We deduce

$$\frac{\Delta\Lambda}{\Lambda} = R \frac{\Delta\alpha}{\alpha} + \frac{2}{27} \left( 3 \frac{\Delta v}{v} + \sum_{i=c,b,t} \frac{\Delta h_i}{h_i} \right)$$

The value of  $R$  depends on the particular GUT theory and particle content  
Which control the value of  $M_{\text{GUT}}$  and of  $\alpha(M_{\text{GUT}})$ .

Typically  $R=36$ .

Assume (for simplicity)  $h_i=h$

$$\frac{\Delta B_D}{B_D} = -13 \left( \frac{\Delta v}{v} + \frac{\Delta h}{h} \right) + 18R \frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta Q}{Q} = 1.5 \left( \frac{\Delta v}{v} + \frac{\Delta h}{h} \right) - 0.6(1+R) \frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta \tau_n}{\tau_n} = -4 \frac{\Delta v}{v} - 8 \frac{\Delta h}{h} + 3.8(1+R) \frac{\Delta\alpha}{\alpha}$$

$$(\alpha, v, h)$$

# Stellar physics



# Stellar carbon production

## Triple $\alpha$ coincidence (Hoyle)

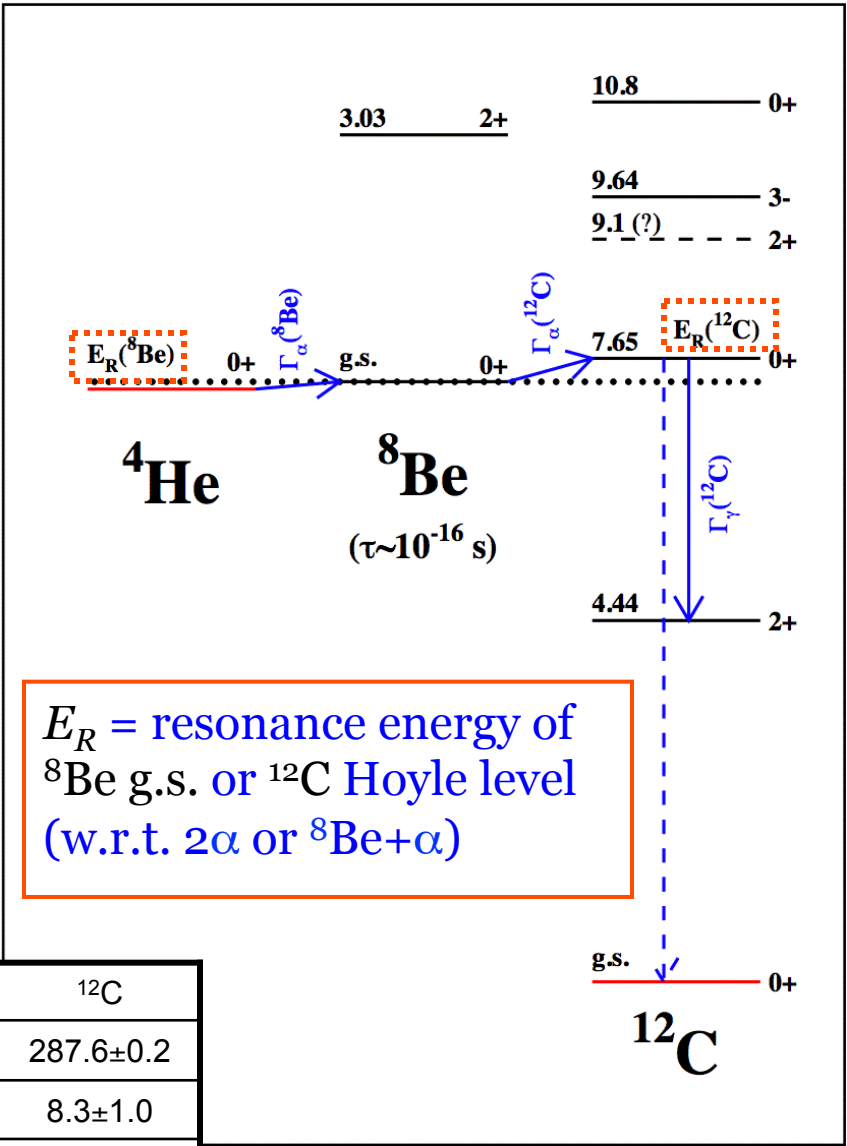
1. Equilibrium between  ${}^4\text{He}$  and the short lived ( $\sim 10^{-16}$  s)  ${}^8\text{Be}$  :  $\alpha\alpha \leftrightarrow {}^8\text{Be}$
2. Resonant capture to the ( $l=0, J^\pi=0^+$ ) Hoyle state:  ${}^8\text{Be} + \alpha \rightarrow {}^{12}\text{C}^* (\rightarrow {}^{12}\text{C} + \gamma)$

Simple formula used in previous studies

1. Saha equation (thermal equilibrium)
2. Sharp resonance analytic expression:

$$N_A^2 \langle \sigma v \rangle^{\alpha\alpha\alpha} = 3^{3/2} 6 N_A^2 \left( \frac{2\pi}{M_\alpha k_B T} \right)^3 \hbar^5 \gamma \exp\left( \frac{-Q_{\alpha\alpha\alpha}}{k_B T} \right)$$

with  $Q_{\alpha\alpha\alpha} = E_R({}^8\text{Be}) + E_R({}^{12}\text{C})$  and  $\gamma \approx \Gamma_\gamma$

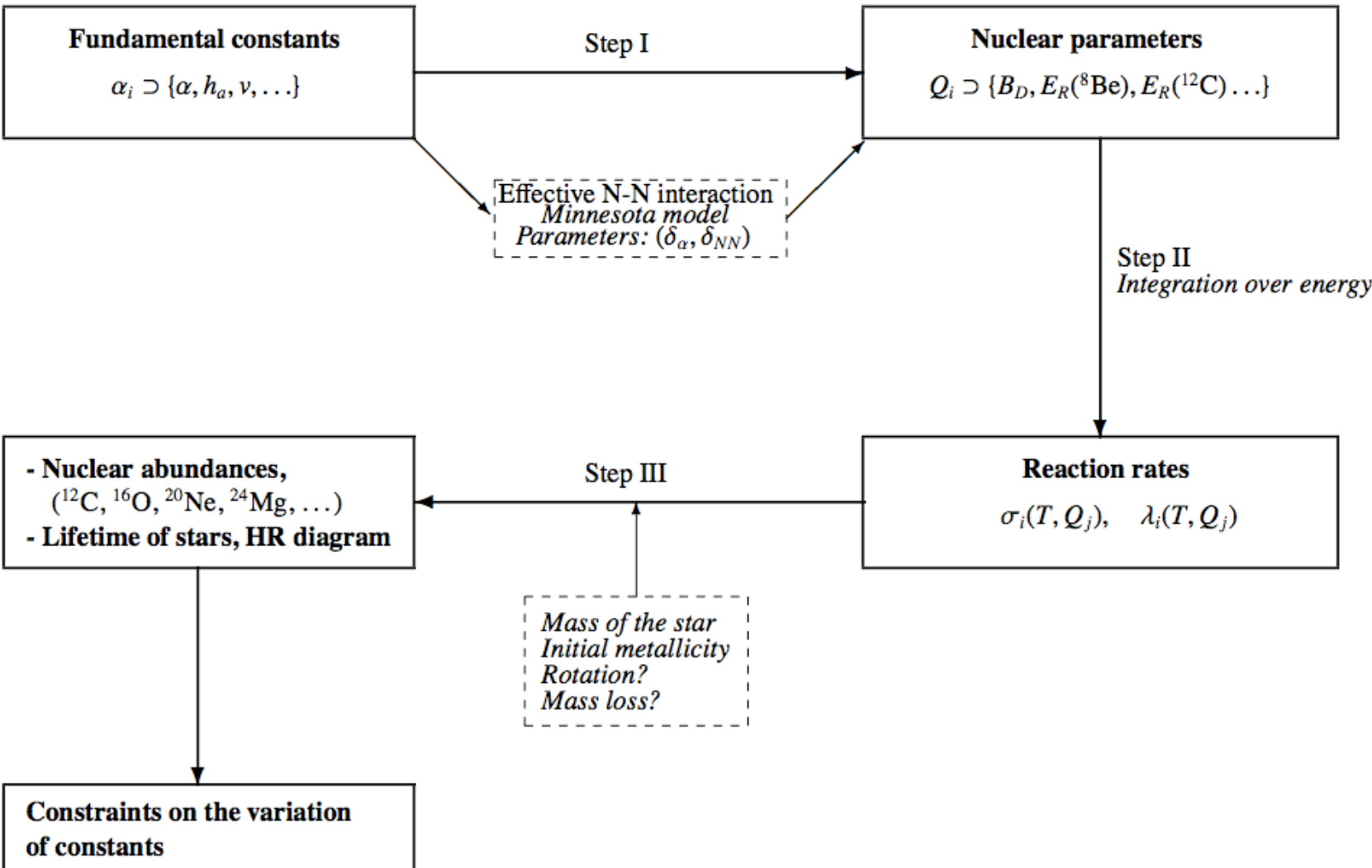


$E_R$  = resonance energy of  ${}^8\text{Be}$  g.s. or  ${}^{12}\text{C}$  Hoyle level (w.r.t.  $2\alpha$  or  ${}^8\text{Be} + \alpha$ )

| Nucleus               | ${}^8\text{Be}$  | ${}^{12}\text{C}$ |
|-----------------------|------------------|-------------------|
| $E_R$ (keV)           | $91.84 \pm 0.04$ | $287.6 \pm 0.2$   |
| $\Gamma_\alpha$ (eV)  | $5.57 \pm 0.25$  | $8.3 \pm 1.0$     |
| $\Gamma_\gamma$ (meV) | -                | $3.7 \pm 0.5$     |

[Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni, 2009]

# Modelisation



# Microscopic calculation

## □ Hamiltonian:

$$H = \sum_{i=1}^A T(r_i) + \sum_{i < j=1}^A (V_{Coul.}(r_{ij}) + V_{Nucl.}(r_{ij}))$$

Where  $V_{Nucl.}(r_{ij})$  is an effective Nucleon-Nucleon interaction

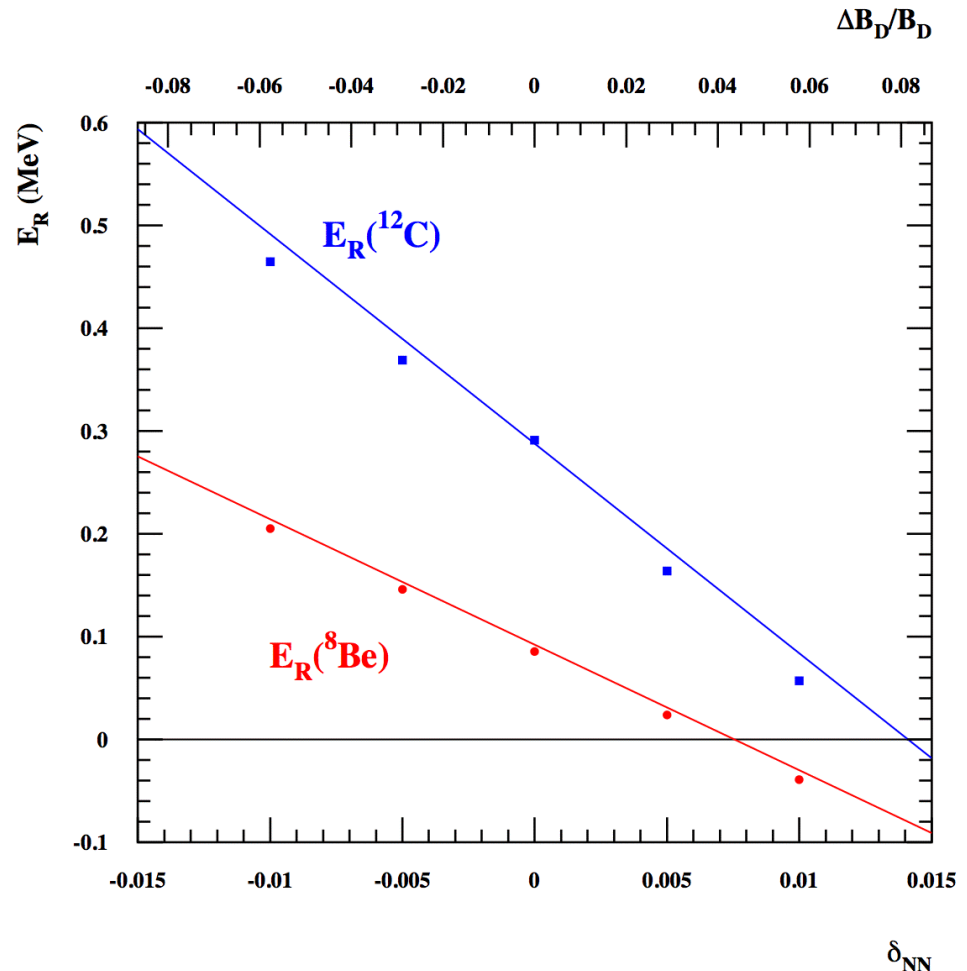
□ Minnesota N-N force [Thompson et al. 1977] optimized to reproduce low energy N-N scattering data.

□  $\alpha$ -cluster approximation for  ${}^8\text{Be}^{\text{g.s.}}$  ( $2\alpha$ ) and the Hoyle state ( $3\alpha$ ) [Kamimura 1981]

## □ Scaling of the N-N interaction

$$V_{Nucl.}(r_{ij}) \rightarrow (1 + \delta_{NN}) \times V_{Nucl.}(r_{ij})$$

to obtain  $B_D$ ,  $E_R({}^8\text{Be})$ ,  $E_R({}^{12}\text{C})$  as a function of  $\delta_{NN}$ :

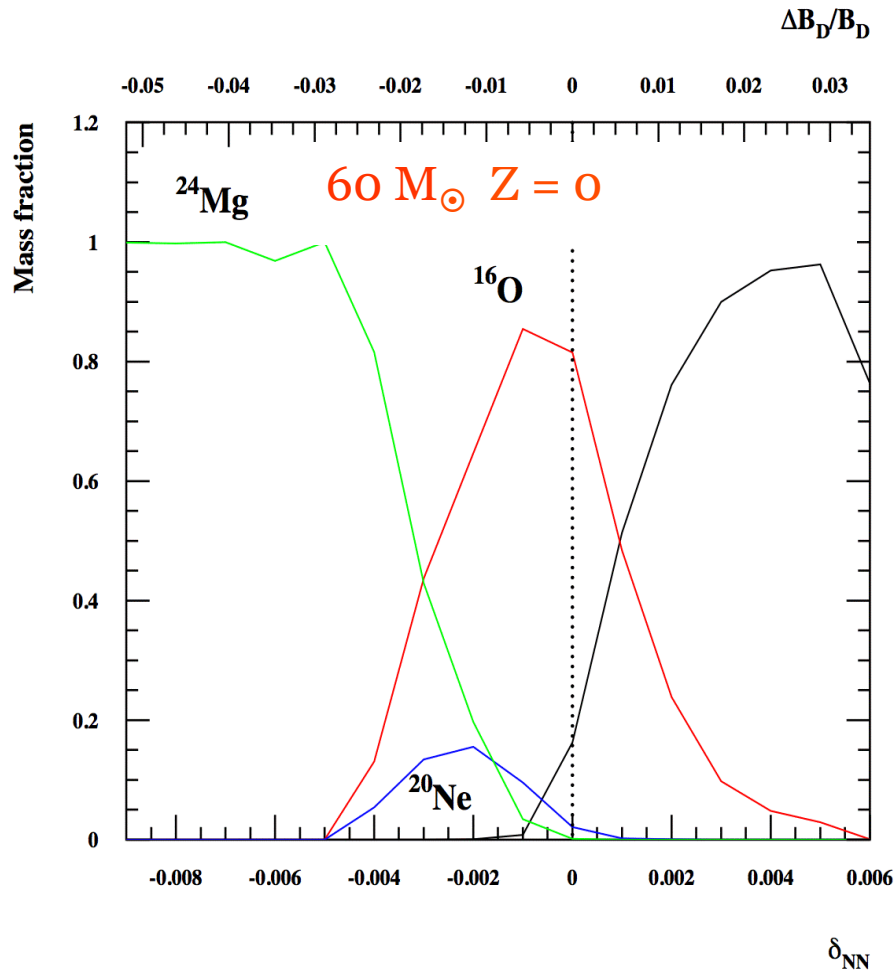


□ Link to fundamental couplings through  $B_D$  or  $\delta_{NN}$

# Composition at the end of core He burning

Stellar evolution of massive Pop. III stars

We choose *typical* masses of  $15$  and  $60 M_{\odot}$  stars/  $Z=0 \Rightarrow$  *Very specific stellar evolution*



- **The standard region:** Both  $^{12}\text{C}$  and  $^{16}\text{O}$  are produced.
- **The  $^{16}\text{O}$  region:** The  $3\alpha$  is slower than  $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$  resulting in a higher  $T_C$  and a conversion of most  $^{12}\text{C}$  into  $^{16}\text{O}$
- **The  $^{24}\text{Mg}$  region:** With an even weaker  $3\alpha$ , a higher  $T_C$  is achieved and  $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$  transforms  $^{12}\text{C}$  into  $^{24}\text{Mg}$
- **The  $^{12}\text{C}$  region:** The  $3\alpha$  is faster than  $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$  and  $^{12}\text{C}$  is not transformed into  $^{16}\text{O}$

# Constraints

From stellar evolution of zero metallicity 15 and 60  $M_{\odot}$  at redshift  $z = 10 - 15$

- Excluding a core dominated by  $^{24}\text{Mg} \Rightarrow \delta_{NN} > -0.005$   
or  $\Delta B_D/B_D > -0.029$
- Excluding a core dominated by  $^{12}\text{C} \Rightarrow \delta_{NN} < 0.003$   
or  $\Delta B_D/B_D < 0.017$
- Requiring  $^{12}\text{C}/^{16}\text{O}$  close to unity  $\Rightarrow -0.0005 < \delta_{NN} < 0.0015$   
or  $-0.003 < \Delta B_D/B_D < 0.009$

$$\Delta B_D/B_D \approx 5.77 \times \delta_{NN}$$

Conservative constraint on Nucleosynthesis

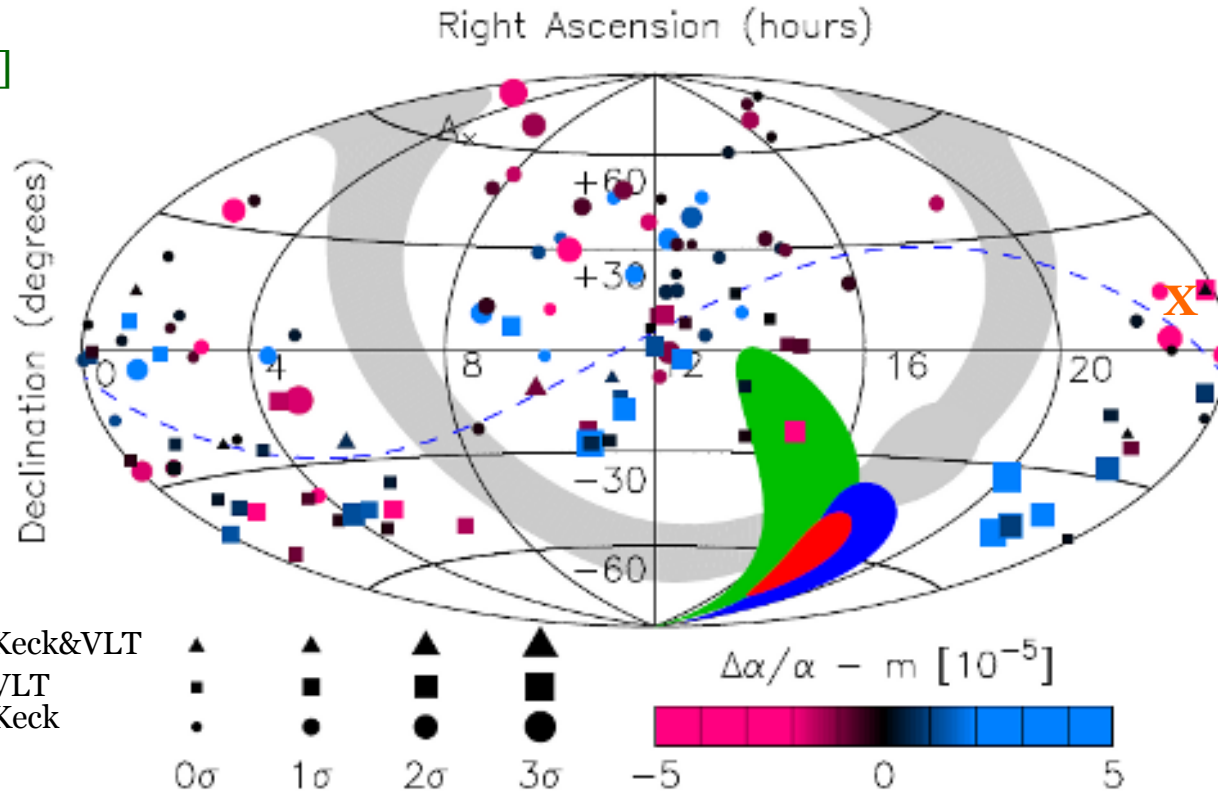
$$^{12}\text{C}/^{16}\text{O} \sim 1 \Rightarrow -0.0005 < \delta_{NN} < 0.0015$$

$$\text{or } -0.003 < \Delta B_D/B_D < 0.009$$

# Spatial variations

# To vary or not to vary

[Webb et al., 2010]



$$(\Delta\alpha_{EM}/\alpha_{EM})_{VLT; z < 1.8} = (-0.06 \pm 0.16) \times 10^{-5}$$

$$(\Delta\alpha_{EM}/\alpha_{EM})_{VLT; z > 1.8} = (+0.61 \pm 0.20) \times 10^{-5}$$

$$(\Delta\alpha_{EM}/\alpha_{EM})_{Keck; z < 1.8} = (-0.54 \pm 0.12) \times 10^{-5}$$

$$(\Delta\alpha_{EM}/\alpha_{EM})_{Keck; z > 1.8} = (-0.74 \pm 0.17) \times 10^{-5}$$

Claim: Dipole in the fine structure constant [« Australian dipole »]

Indeed, this is a logical possibility to reconcile VLT constraints and Keck claims of a variation.

# Planck analysis

$$\Theta(n) = \bar{\Theta}[n, c_a(n)] \leftarrow c_a(n, z) = c_{0a}(z) + \sum_{i=-1}^1 \delta c_a^{(i)}(z) Y_{1i}(n).$$

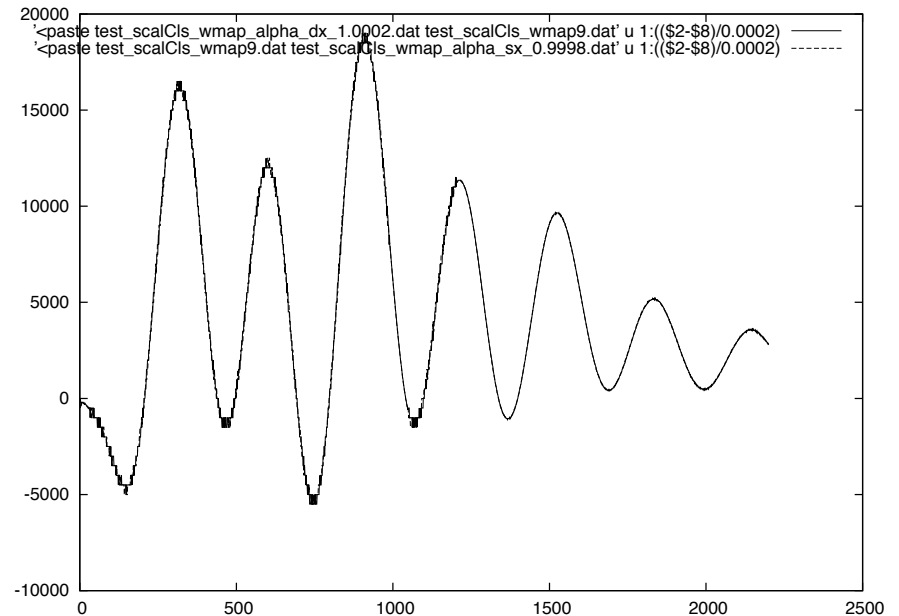
$$= \bar{\Theta} \left[ n, c_{0a} + \sum_{i=-1}^1 \delta c_a^{(i)}(z) Y_{1i}(n) \right]$$

$$\simeq \bar{\Theta}[n] + \sum_a \sum_{i=-1}^{+1} \frac{\partial \bar{\Theta}[n]}{\partial c_a} \delta c_a^{(i)}(z) Y_{1i}(n)$$

Mode coupling:  $D_{\ell m}^{(i)} \equiv \langle a_{\ell m} a_{\ell+1 m+i}^* \rangle$

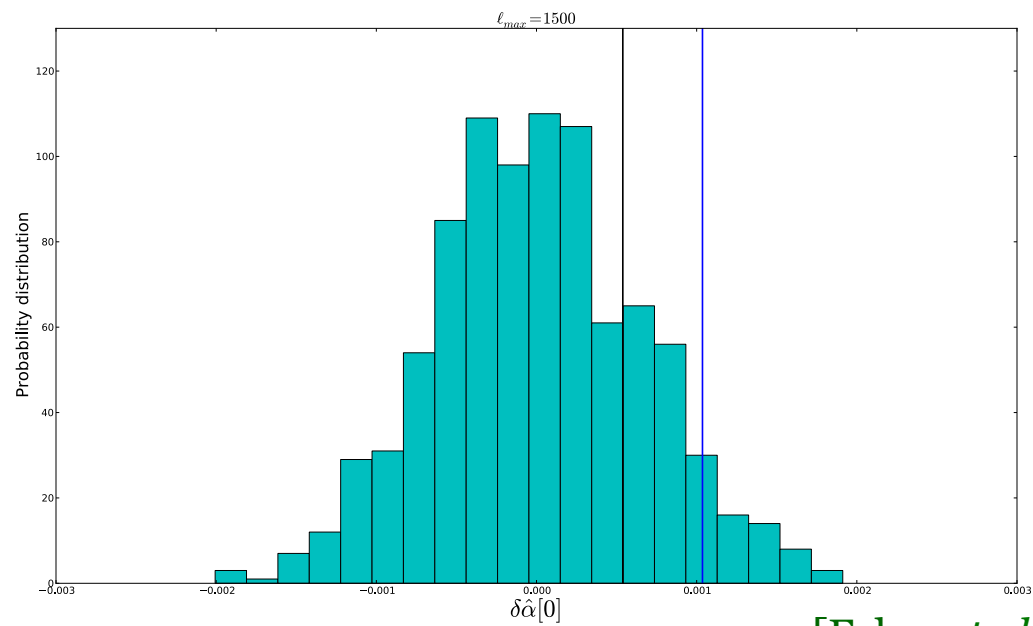
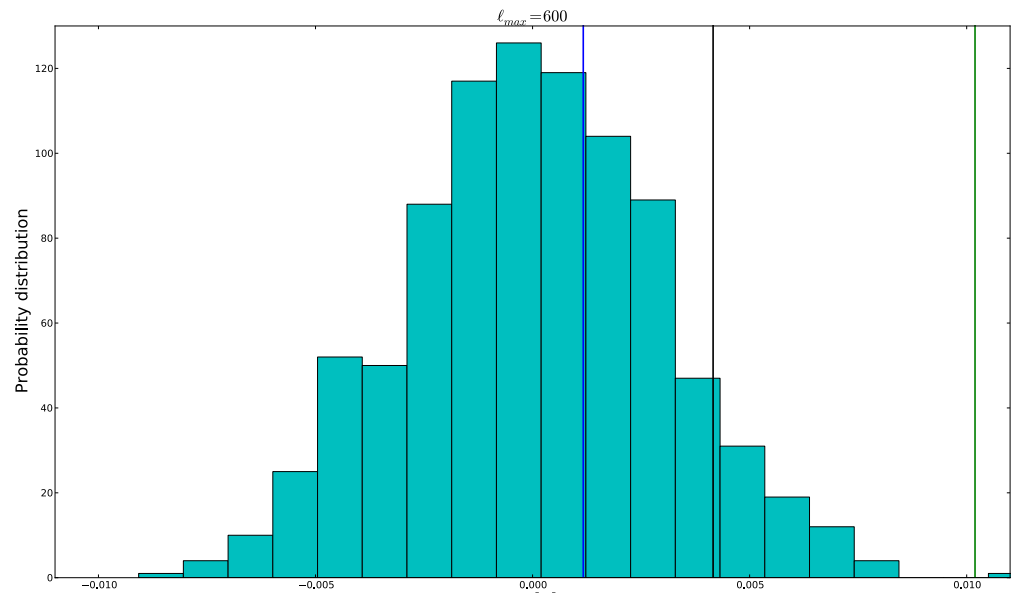
$$D_{\ell m}^{(i)} = f_i(\ell, m) \sum_a \delta c_a^{(i)} \Gamma_{\ell}^{(a)}$$

$$\Gamma_{\ell}^{(a)} \equiv \frac{1}{2} \left( \frac{\partial \bar{C}_{\ell}}{\partial c_a} + \frac{\partial \bar{C}_{\ell+1}}{\partial c_a} \right)$$





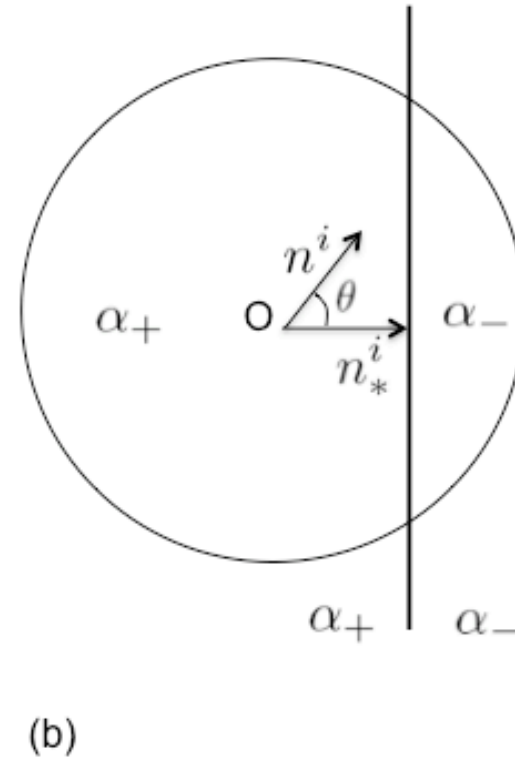
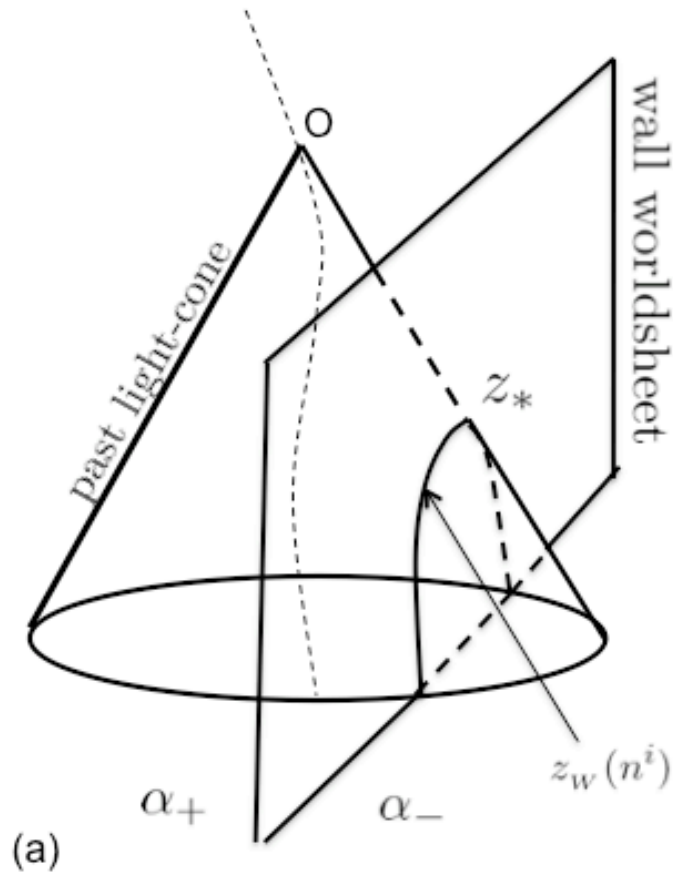
# Planck analysis



# Wall of fundamental constant

[Olive, Peloso, JPU, 2010]

**Idea:** Spatial discontinuity in the fundamental constant due to a domain wall crossing our Hubble volume.



# Wall of fundamental constants

$$S = \int \left[ \frac{1}{2} M_p^2 R - \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) + \frac{1}{4} B_F(\phi) F_{\mu\nu}^2 - \sum_j i \bar{\psi}_j \not{D} \psi_j - B_j(\phi) m_j \bar{\psi}_j \psi_j \right] \sqrt{-g} d^4x,$$

$$B_i(\phi) = \exp\left(\xi_i \frac{\phi}{M_*}\right) \simeq 1 + \xi_i \frac{\phi}{M_*}$$

$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - \eta^2)^2$$

-Parameters  $(\lambda, M_*, \eta, \xi_F, \xi_i)$

- We assume only  $\xi_F$  is non vanishing BUT the scalar field couples radiatively to

nucleons  $\xi_N = m_N^{-1} \langle N | (\xi_F/4) \hat{F}_{\mu\nu}^2 | N \rangle$

$$\xi_p = -0.0007 \xi_F \quad \xi_n = 0.00015 \xi_F$$

$$V_{\text{eff}} = V(\phi) + \xi_N \frac{\phi}{M_*} \rho_{\text{baryon}}$$

# Constraints

- Constraints from atomic clocks / Oklo / Meteorite dating are trivially satisfied

- To reproduce the «observations»

$$\frac{\Delta\alpha}{\alpha} \simeq 2\xi_F \frac{\eta}{M_*} \sim \text{few} \times 10^{-6}$$

- The contribution of the walls to the background energy is

$$\Omega_{\text{wall}} \equiv \frac{U_{\text{wall}} H_0}{\rho_0} \simeq \left( \frac{\eta}{100 \text{ MeV}} \right)^3$$

Assume  $\eta = \mathcal{O}(\text{MeV})$

- CMB constraints  $\left( \frac{\delta T}{T} \right)_{\text{CMB}} \sim 10^{-6} \left( \frac{\eta}{1 \text{ MeV}} \right)^3$

- Valid field theory up to an energy scale  $M_*/\xi_F \sim 10^6 \text{ MeV}$

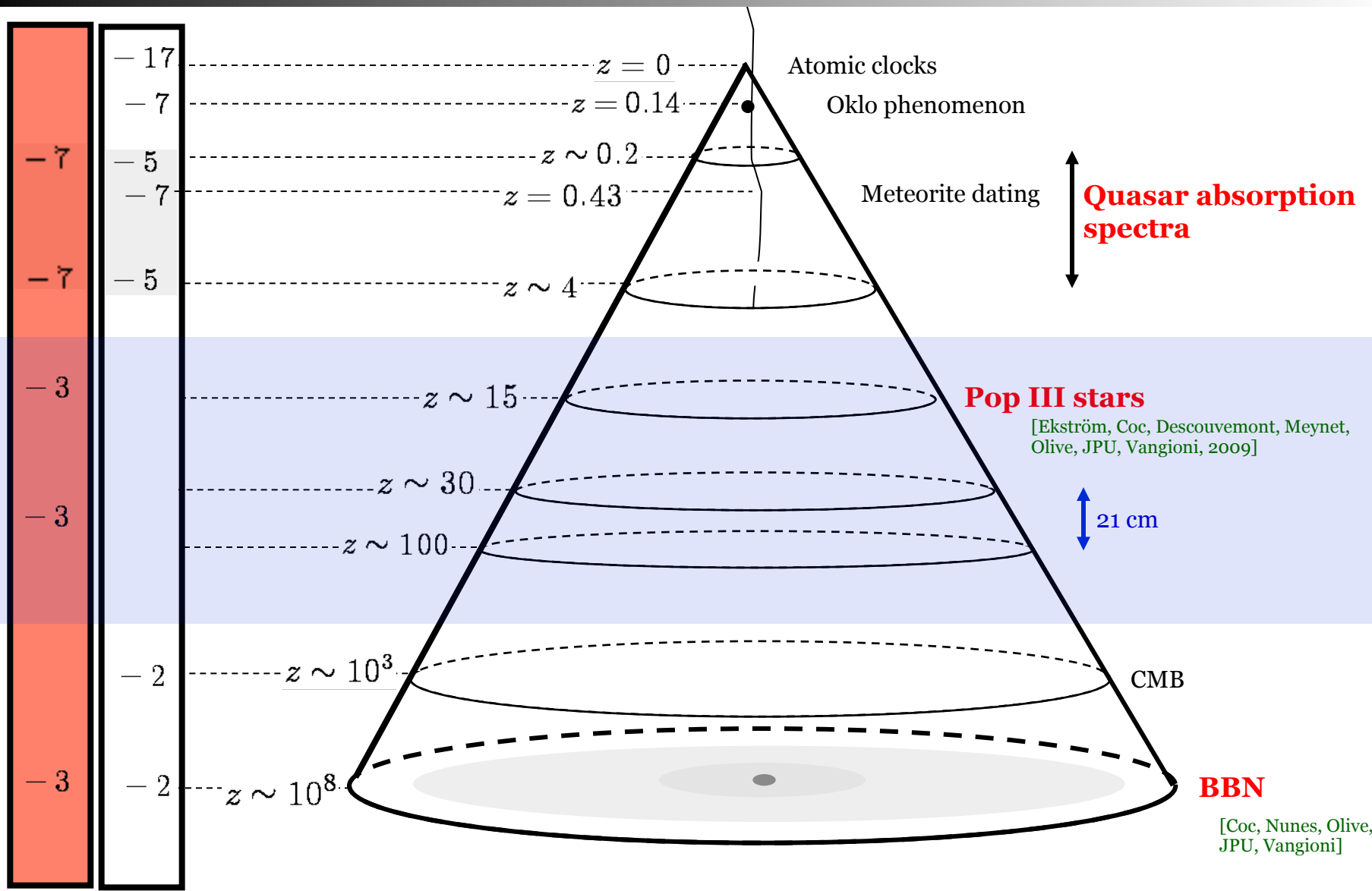
- Astrophysical constraints

- Tunelling to the true vacuum

- Walls form at a redshift of order  $8 \times 10^9$

Future

# Physical systems: new and future



# CODEX: COsmic Dynamics EXperiment

## Time drift of the redshifts

$$\Delta\lambda = \frac{\Delta t}{1+z} [H_0(1+z) - H(z)] \lambda_0$$

Given the cosmological parameters  
shift of  $10^{-6}/\text{an}$

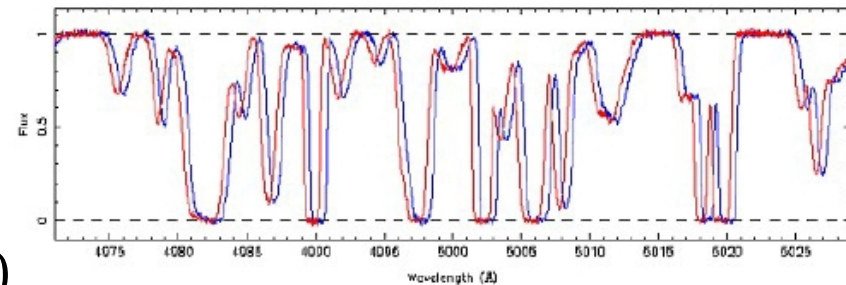
## CODEX:

spectral domain: 400-680 nm

R=150000

10-20 times HARPS on 10 years!

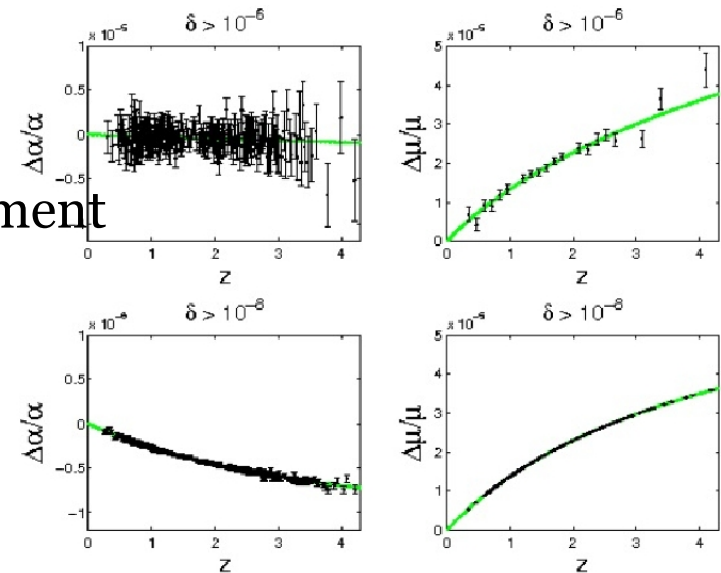
long term calibration (atomic clocks...)



## Constants

The accuracy of a variability measurement  
is determined by the precision of measurement  
of the line positions.

Precision on  $\alpha$  et  $\mu$ :  $10^{-8}$   
2 order of magnitude better  
than VLT/UVES



# Conclusions

The constancy of fundamental constants is a **test of the equivalence principle**.

*The variation of the constants, violation of the universality of free fall and other deviations from GR are of the same order.*

« Dynamical constants » are **generic** in most extensions of GR (extra-dimensions, string inspired model).

*Need for a stabilisation mechanism (least coupling principle/chameleon)*

*Why are the constants so constant?*

*Variations are expected to be larger in the past (cosmology)*

*All constants are expected to vary (unification)*

*In the case of quintessence:* time variations linked to the equation of state and allow to constrain the dynamics of the scalar field even when not dominant.

Observational developments allow to set **strong constraints** on their variation

*New systems [Stellar physics] / new observations*