



An all-scale exploration of alternative theories of gravity

Thomas P. Sotiriou

SISSA - International School for Advanced Studies, Trieste





General Outline

- Beyond GR: motivation and pitfalls
- Alternative theories of gravity: theory and phenomenology
- Testing gravity with compact objects



Beyond GR: motivation and pitfalls

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Quantum Gravity

General Relativity plus
unknown corrections?

General Relativity

General Relativity plus Dark Matter

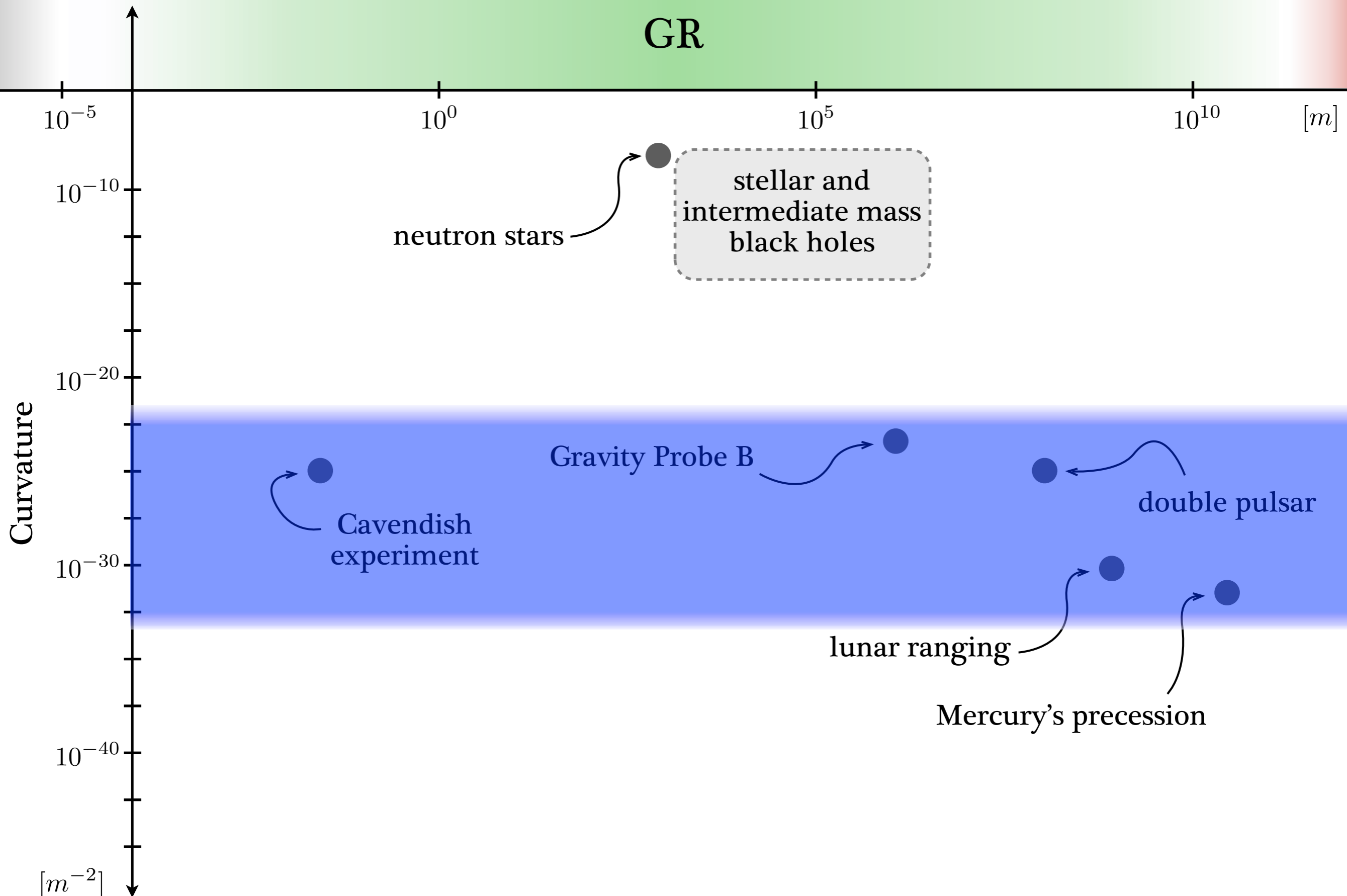
General Relativity plus
Dark Matter and Dark Energy

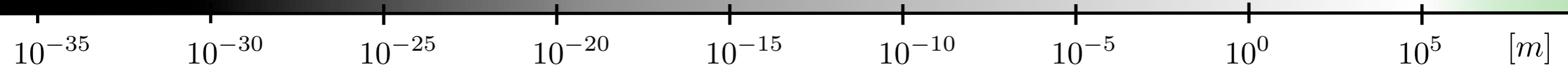
μm

1AU

15Mpc

10^{-35} 10^{-30} 10^{-25} 10^{-20} 10^{-15} 10^{-10} 10^{-5} 10^0 10^5 10^{10} 10^{15} 10^{20} 10^{25} [m]





Why do we need Quantum Gravity?

1. Breakdown of GR: black holes, early universe, ...
2. Conceptual clash with Quantum Field Theory

Not a purely theoretical problem! The lack of predictions is a serious issue!

But GR is not a renormalizable theory - corrections are inevitable!

What should we give up and would that leave a remnant in the IR?

GR + ??

GR

GR + DM + DE



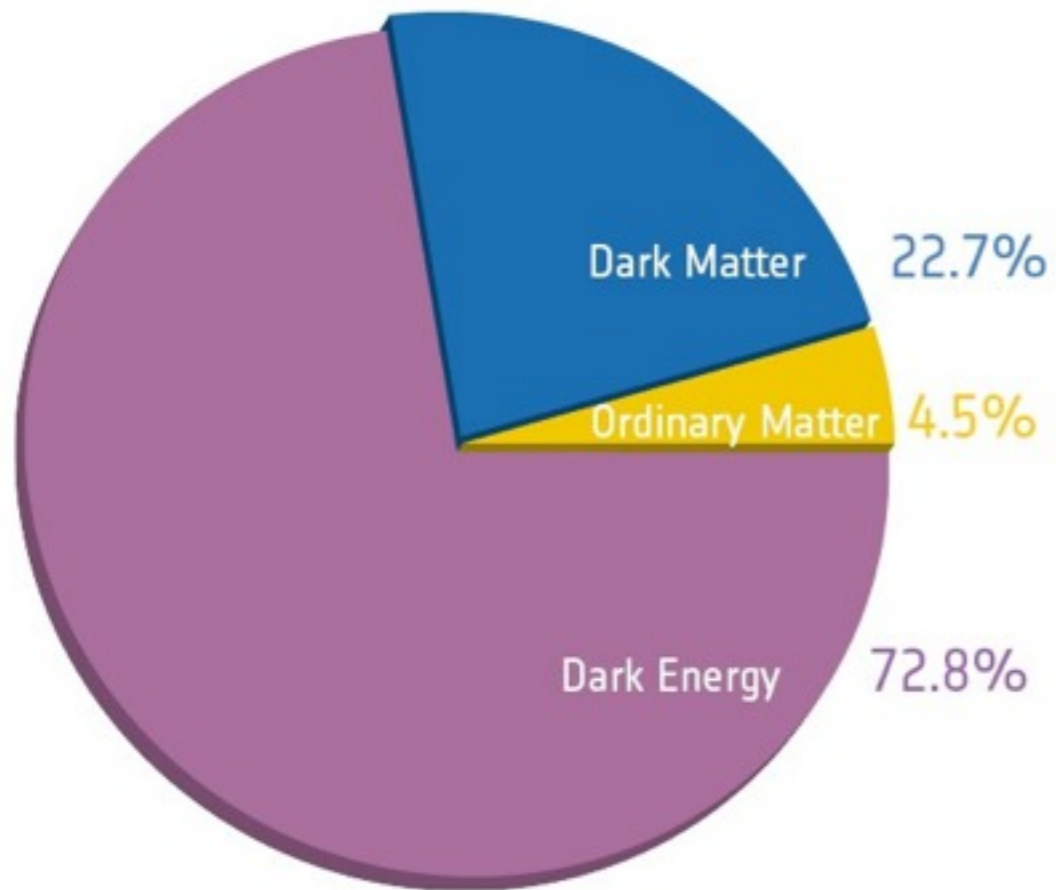
We should not just wait for the right effective action to come from fundamental theory!

Instead we should:

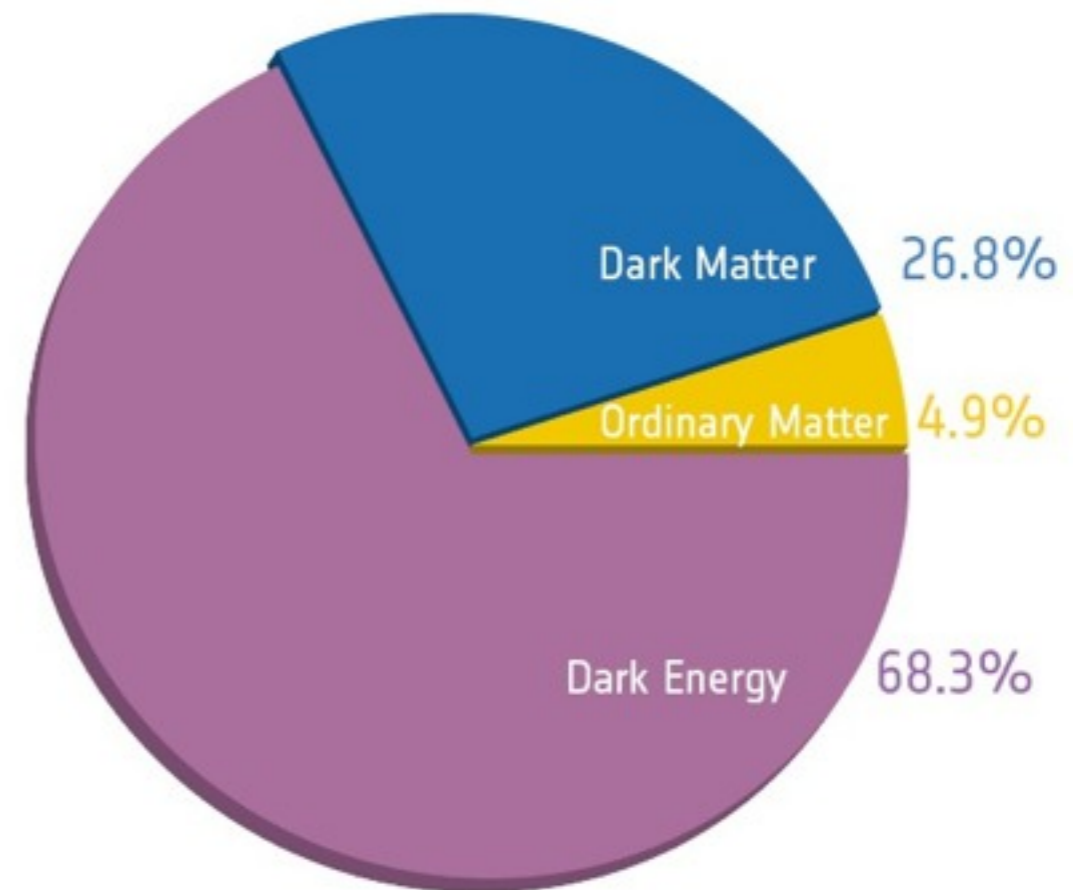
- Interpret experiments
- Combine them with theory (technical naturalness, effective field theory, ...)
- Construct interesting toy theories (classical alternatives to GR)
- Use them to “re-interpret” experiments and give feedback to QG model building

There are also large scale puzzles that call for an answer! Can it be gravity?

Cosmic Budget



Before Planck



After Planck



Dark matter

- ✎ Pressure-less dust fits the data at many different scales (galaxies, clusters, cosmology)
- ✎ Ordinary matter is not pressureless, it interacts and emits
- ✎ Departure from the standard model is needed to accommodate the existence of particles with such characteristics
- ✎ These particles should be the dominant matter component of the universe!

So, one does need new physics anyway, why should it be the standard model that changes?

Gravity as dark matter

The story with Urbain Le Verrier (or better half of it...)

- He was the first to point out that Mercury's precession was not explained by Newtonian theory (1859)
- He conjectured the existence of an extra, innermost planet, called Vulcan, based on that.
- Vulcan was never found! The phenomenon was explained by general relativity.



Vulcan was a form of dark matter which turned out to be modified gravity...



Gravity as dark matter

Dark matter encountered at many different scales

- Galaxies, Clusters, Universe

It also behaves too much like non-interacting matter

- Structure formation

Difficult to explain it with modified gravity in all regimes!

Nonetheless, MOdified Newtonian Dynamics

$$F = \mu m a \quad a = -\frac{GM}{r^2} \quad \begin{array}{ll} \mu = 1 & \text{if } a \gg |a_0| \\ \mu = a/a_0 & \text{if } a \ll |a_0| \end{array}$$

fits the data remarkably well!



MOND and TeVeS

MOdified Newtonian Dynamics:

$$F = \mu m a \quad a = -\frac{GM}{r^2} \quad \begin{array}{ll} \mu = 1 & \text{if } a \gg |a_0| \\ \mu = a/a_0 & \text{if } a \ll |a_0| \end{array}$$

Tensor-Vector-Scalar theory:

$$S = \int d^4x \left[\frac{1}{16\pi G} (R - 2\Lambda) - \frac{1}{32\pi G} \{ K \mathcal{F}^{\alpha\beta} \mathcal{F}_{\alpha\beta} - 2\lambda (U_\mu U^\mu + 1) \} \right. \\ \left. - \frac{1}{2} \{ \sigma^2 (g^{\mu\nu} - U^\mu U^\nu) \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \} \right] (-g)^{1/2}$$

$$\tilde{g}_{\alpha\beta} = e^{-2\phi} g_{\alpha\beta} - U_\alpha U_\beta (e^{2\phi} - e^{-2\phi})$$

Gravity as dark matter

The other half of the story with Urbain Le Verrier

- ✧ Considered perturbations in the orbits of Uranus to explain discrepancies with Newtonian predictions
- ✧ Predicted that the discrepancies are due to the presence of another, unseen planet (1846)
- ✧ Predicted its position and it was found there within 1 degree! (1846)



Before being found it was a form of dark matter but no modification of Newton's theory was needed...



Dark Energy

- Perfect fluid with $p = -\rho$
- Modeled excellently by a cosmological constant!

Problems:

- Why so small? (New)

$$\rho_{\Lambda}^{\text{obs}} \sim (10^{-3} \text{eV})^4$$

- Why so big? (Old)

$$\rho_{\Lambda}^{\text{vac}} \sim M_{\text{cutoff}}^4 \quad M_{\text{cutoff}} \sim 10^{19} \text{GeV}$$

- Why now? (Coincidence problem)

$$\rho_{\Lambda}^{\text{obs}} \sim \rho_{\text{matter}}$$



Quintessence

For a scalar field one has

$$\frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

so, when $V(\phi) \gg \dot{\phi}^2$ then $p/\rho \rightarrow -1$

Naturalness problems:

- Extremely low mass: $m_\phi \sim 10^{-33} \text{eV}$
- No coupling to other matter fields

Tracker solutions might help with the coincidence problem, but that's about it (without extra symmetries).



Dynamical DE models

Usual shortcomings:

- They usually don't solve the old CC problem. Powerful no-go theorem by Weinberg! (modulo exceptions)

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)

- They usually do not solve the new CC problem either! Turning a tiny cosmological constant to a tiny mass of a field, a tiny coupling for some extra terms, a strange potential, etc. is not really a solution

It is only meaningful to “rename” the scale of the CC if this makes it (technically) natural!

(all of the above apply to modified gravity as well)



Quantum gravity and DE

Two birds with one stone but

- QG requires modifications at small scales/high momenta
- DE requires modifications at large scales/small momenta

Usual effective field theory/separation of scales arguments seem to imply that the two are not related!

Additionally GR works excellently in between...



General Relativity

The action for general relativity is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_m(g^{\mu\nu}, \psi)$$

- \mathfrak{S} • S_m has to match the standard model in the local frame and minimal coupling with the metric is required by the equivalence principle.
- \mathfrak{S} • Gravitational action is uniquely determined thanks to:
 1. Diffeomorphism invariance
 2. Requirement to have second order equations
 3. Requirement to have 4 dimensions
 4. Requirement to have no other fields



Beyond GR

To go beyond GR one can

- ✎ Add extra fields
- ✎ Allow for more dimensions
 - ↳ EFT equivalent: adding extra fields
- ✎ Allow for higher order equations
 - ↳ Classically equivalent to adding extra fields
- ✎ Give up diffeomorphism invariance
 - ↳ Classically equivalent to adding extra fields



Taming the extra fields

Extra fields can lead to problems

- Classical instabilities
- Quantum mechanical instabilities (negative energy)

These are particularly hard problems when the fields are nonminimally coupled to gravity.

Supposing that these have been tamed, the major problem becomes to

- hide the extra fields in regimes where GR works well
- make them re-appear when GR seems to fail



Brans-Dicke theory

The action of the theory is

$$S_{\text{BD}} = \int d^4x \sqrt{-g} \left(\varphi R - \frac{\omega_0}{\varphi} \nabla^\mu \varphi \nabla_\mu \varphi - V(\varphi) + L_m(g_{\mu\nu}, \psi) \right)$$

and the corresponding field equations are

$$G_{\mu\nu} = \frac{\omega_0}{\varphi^2} \left(\nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \varphi \nabla_\lambda \varphi \right) + \frac{1}{\varphi} (\nabla_\mu \nabla_\nu \varphi - g_{\mu\nu} \square \varphi) - \frac{V(\varphi)}{2\varphi} g_{\mu\nu}$$

$$(2\omega_0 + 3) \square \varphi = \varphi V' - 2V$$

Solutions with constant φ are admissible and are GR solutions.



Brans-Dicke theory

However, they are not the only ones. E.g. for

$$V = m^2 (\varphi - \varphi_0)^2$$

around static, spherically symmetric stars a nontrivial configuration is necessary and

$$\gamma \equiv \frac{h_{ii}|_{i=j}}{h_{00}} = \frac{2\omega_0 + 3 - \exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}{2\omega_0 + 3 + \exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}$$

So, hiding the scalar requires, either a very large mass (short range) or a very large Brans-Dicke parameter



Summary

- General Relativity is not the end of the story
- It is sensible to attempt to constrain modifications and give feedback to quantum gravity
- It is tempting to assume that modifications account for dark matter or dark energy
- ...but hard to make things work...
- ...and there is no reason that it should!
- major problem in alternative gravity theories: taming the extra degrees of freedom