Enhanced constraints from multi-tracer surveys

or

How to beat cosmic variance

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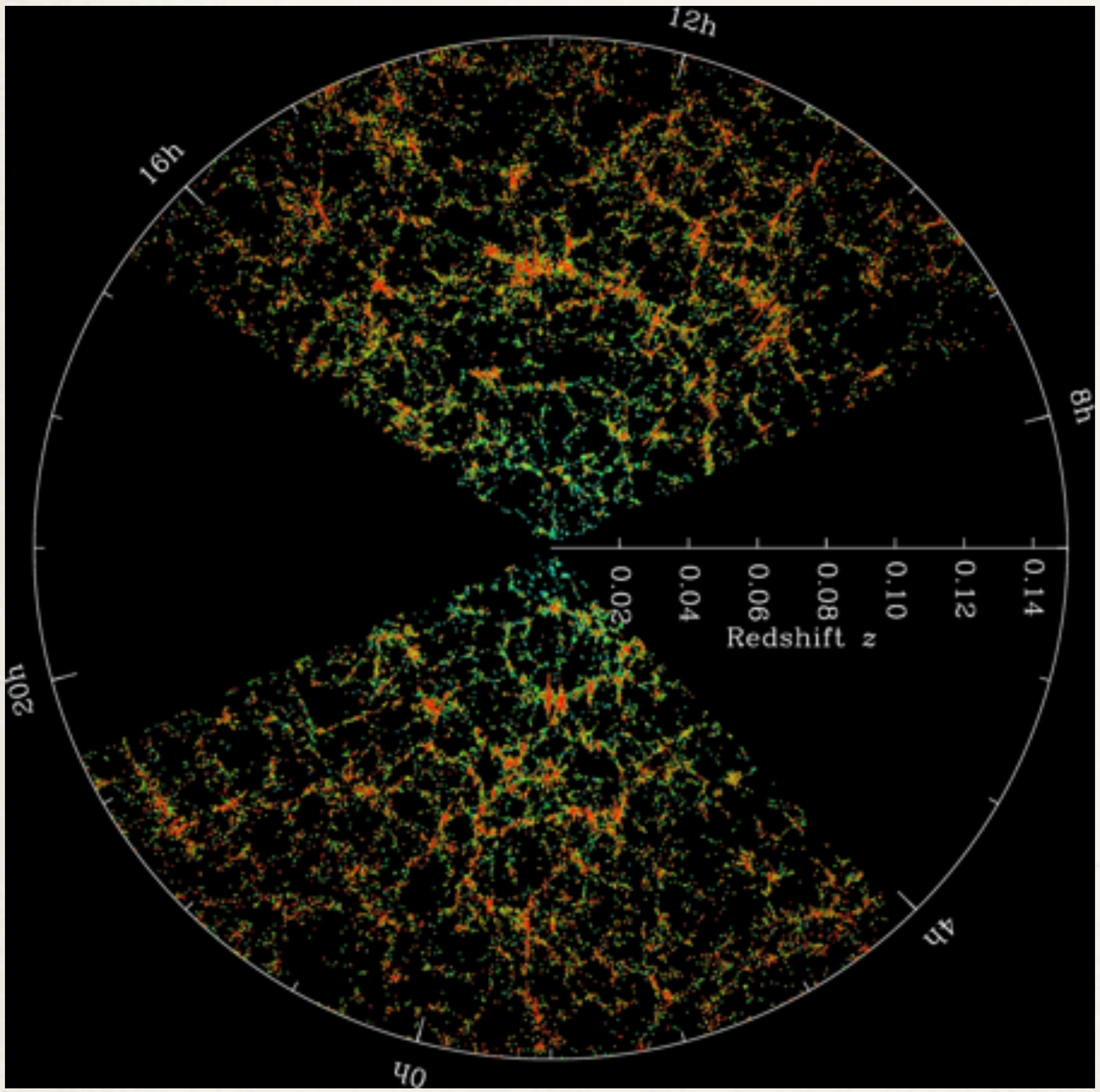
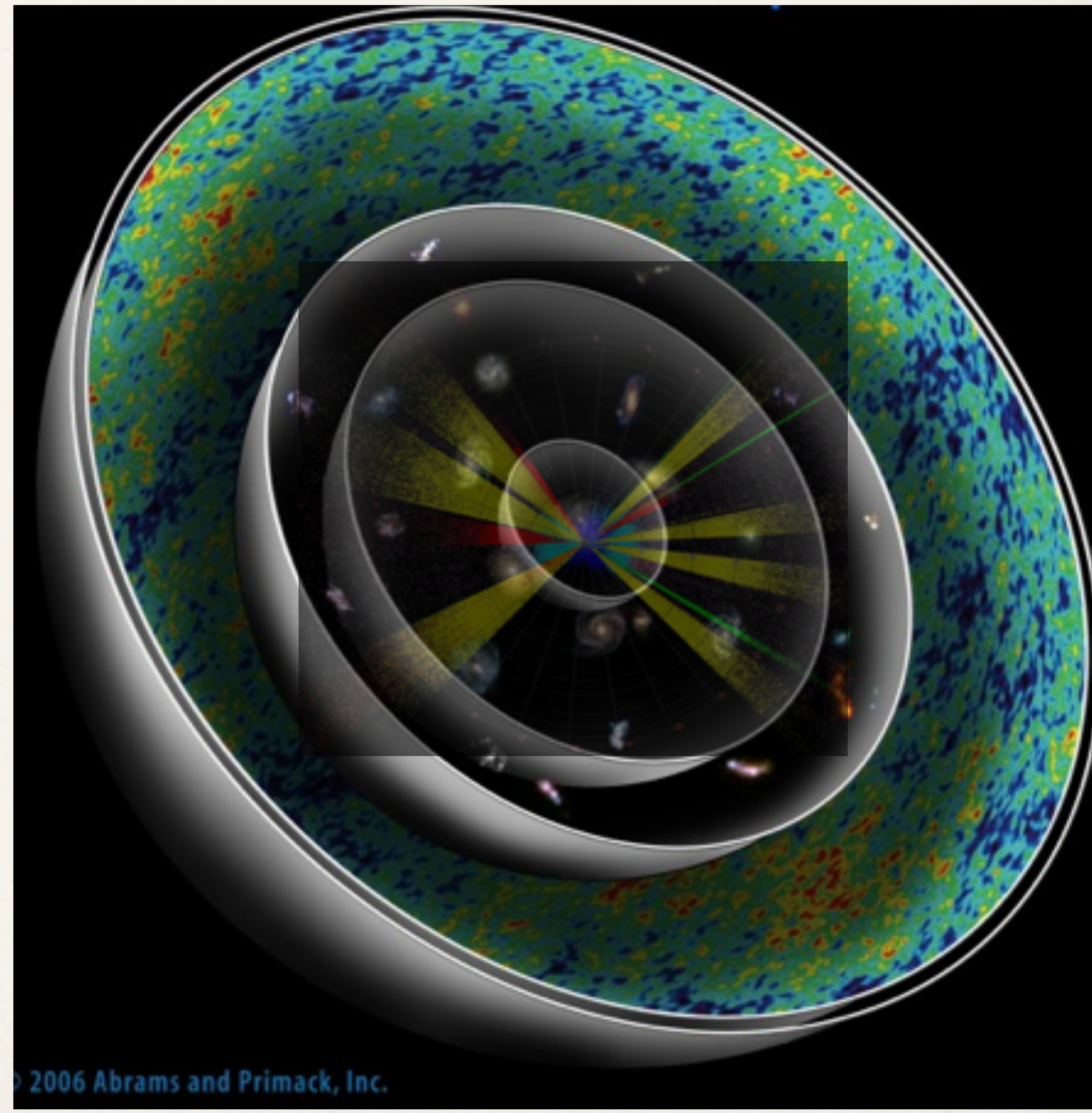
Galaxy surveys are evolving

We used to live in an
era of shot noise



[**Finding galaxies** was the limiting factor]

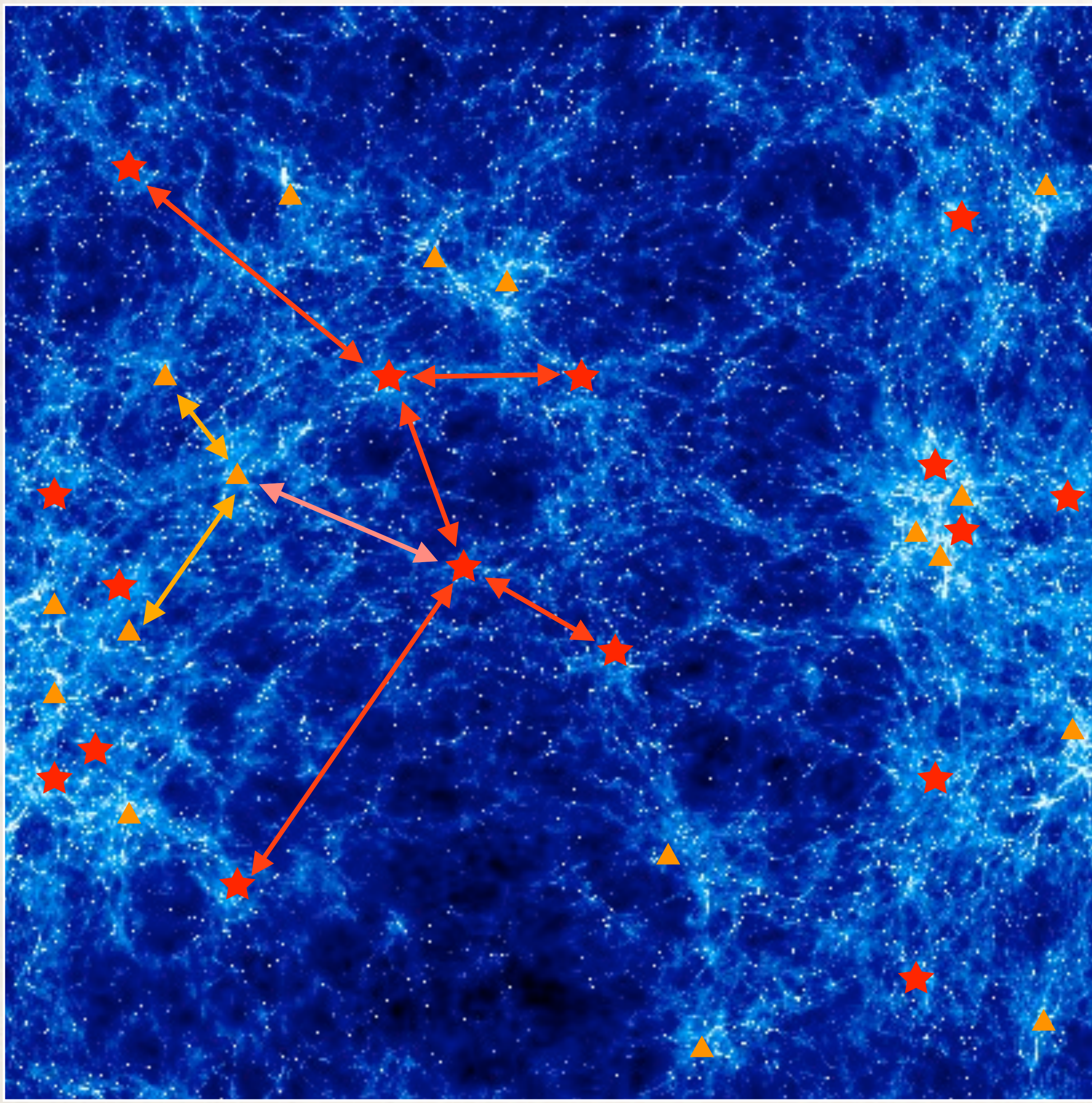
We are now starting an
era of cosmic variance



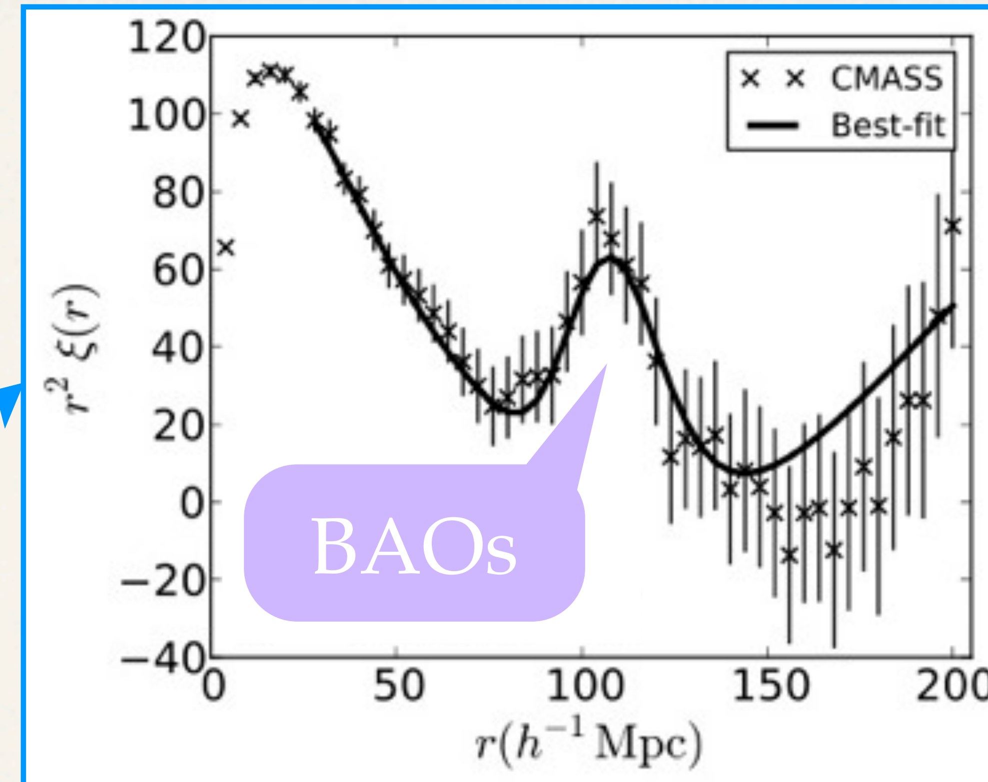
[**Gaining volume** is the limiting factor]

Cosmology from galaxy clustering

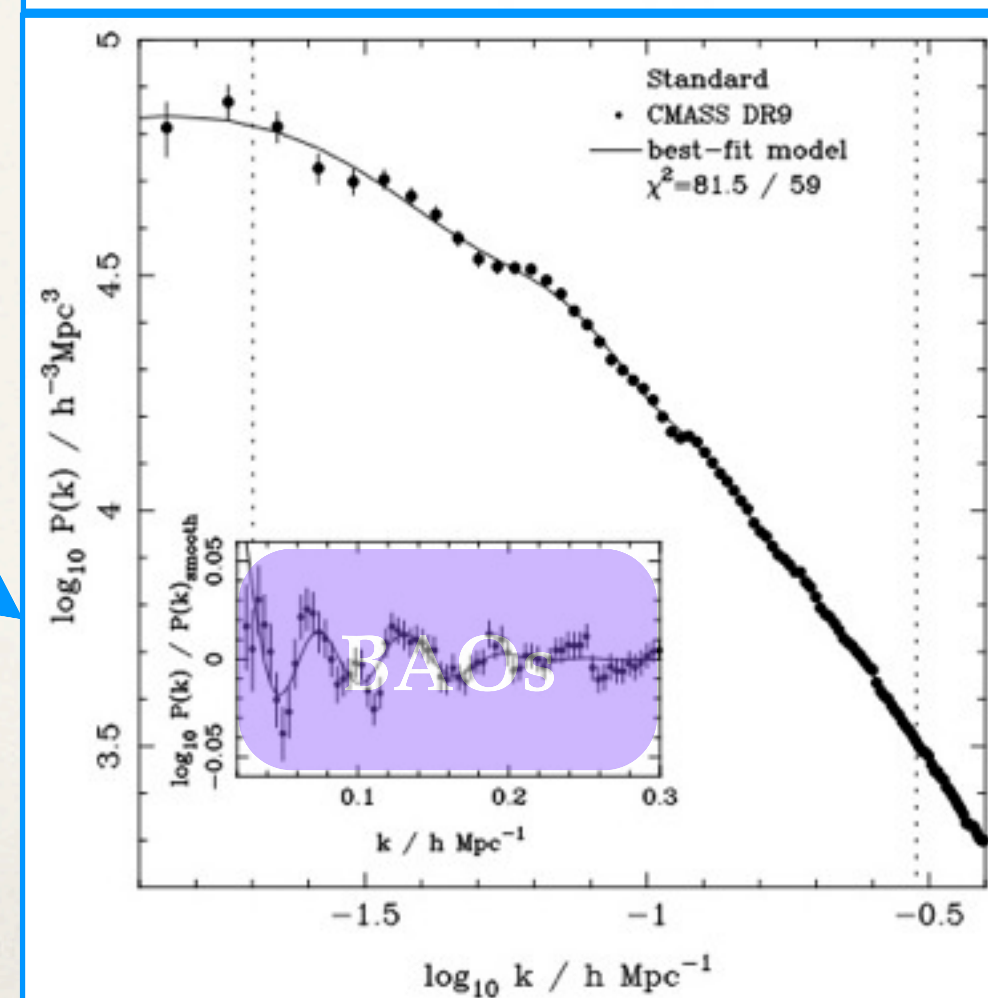
$$\rho_g(\vec{x}) = n_g(\vec{x})\,[1+\delta_g(\vec{x})] \rightarrow n_g(\vec{x})\,[1+b_g(\vec{x})\,\delta(\vec{x})]$$



Clustering in position space



Clustering in Fourier space



Bias:

$$\xi_1(r) = b_1^2\,\xi(r) \longleftrightarrow P_1(k) = b_1^2\,P(k)$$

$$\xi_2(r) = b_2^2\,\xi(r) \longleftrightarrow P_2(k) = b_2^2\,P(k)$$

Fisher information matrix of galaxy surveys

FKP - Feldman, **K**aiser & **P**eacock (1994)
Tegmark et al. (1997), R.A. (2012)

The **galaxy power spectrum in redshift space**, for any galaxy survey, can be **expressed in units of its shot noise** ($1/n_g$). This defines the survey's **effective power spectrum** (which is **adimensional**):

$$f = -\frac{d\log G}{d\log z}$$

$$\mu_k = \frac{k_{||}}{k}$$

$$\mathcal{P}_g(\vec{k},\vec{x}) \to n_g(\vec{x}) \left[b_g(\vec{x}) + f(\vec{x})\,\mu_k^2\right]^2 G^2(\vec{x})\,P(\vec{k})$$

- $n_g(\vec{x})$: average density of galaxies in the survey
- $b_g(\vec{x})$: galaxy bias (function of z, at least)
- $f(\vec{x})$: matter growth rate = redshift distortion parameter
- μ_k: angle of Fourier mode w.r.t. line-of-sight
- $G(\vec{x})$: matter growth function
- $P(\vec{k})$: **matter power spectrum**

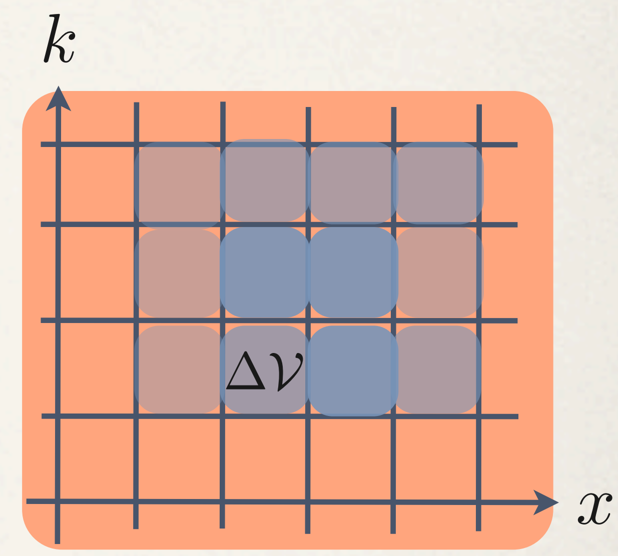
The **Fisher information** for the (**log** of the) **effective power spectrum** is:

$$F[\log \mathcal{P}_g] = \frac{1}{2}\left(\frac{\mathcal{P}_g}{1+\mathcal{P}_g}\right)^2$$

FKP Fisher matrix

Fisher information in phase space

On each **cell of phase space volume** there is a certain **amount of information** about the spectrum (and other quantities), given by:



$$F[\log \mathcal{P}_g] \times \frac{\Delta V_x \, \Delta V_k}{(2\pi)^3} = \frac{1}{2} \left(\frac{\mathcal{P}_g}{1 + \mathcal{P}_g} \right)^2 \times \frac{\Delta V_x \, \Delta V_k}{(2\pi)^3}$$

phase space **density of information < 1/2**

phase space **volume =** $\Delta\mathcal{V}$.

The **precision** with which we can **estimate** the effective power spectrum from the information in **each cell of phase space** is:

$$\frac{\sigma(\mathcal{P}_g)}{\mathcal{P}_g} = \frac{1}{\sqrt{F[\log \mathcal{P}_g] \, \Delta\mathcal{V}}} = \frac{1 + \mathcal{P}_g}{\mathcal{P}_g} \times \sqrt{\frac{2}{\Delta\mathcal{V}}}$$

The Fisher information is **additive**, so **integrating** over the phase space **volume** gives the **total information**.

Fisher information, effective volume and cosmic variance

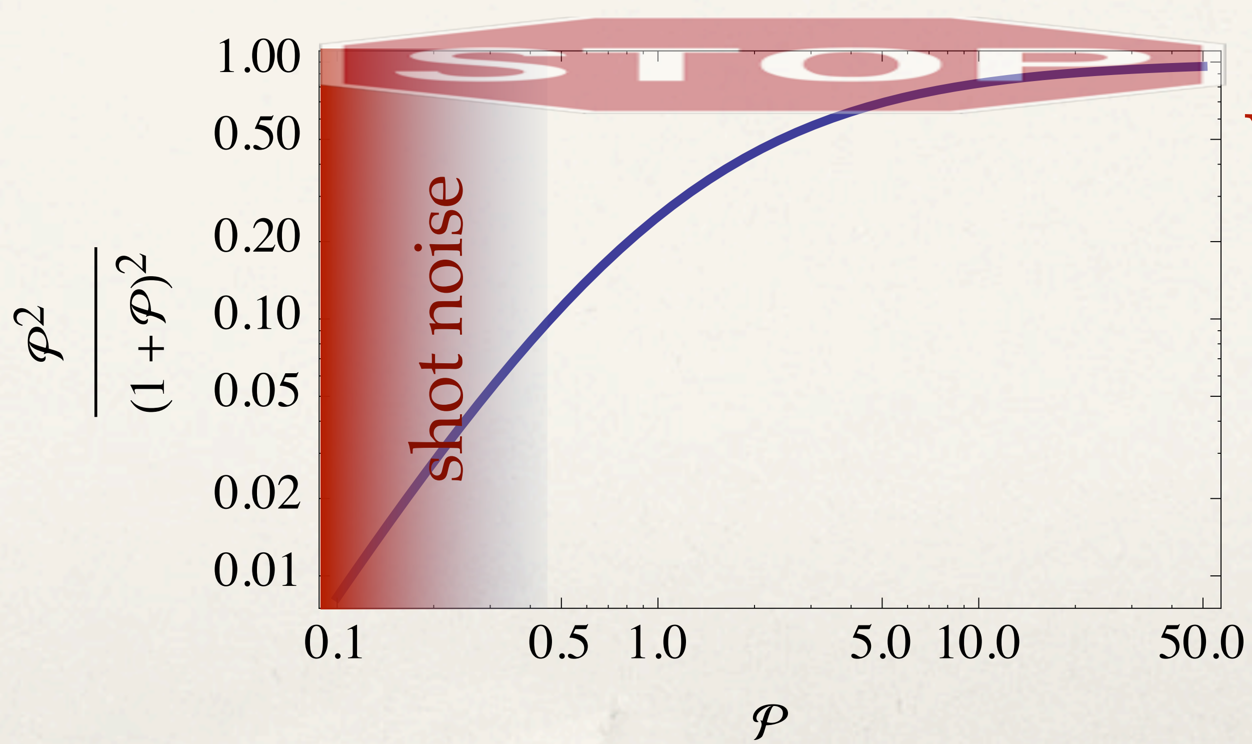
$$p^a \to \theta^i \quad : \quad F_{ij}[\theta] = \sum_{ab} \frac{\partial p^a}{\partial \theta^i} \, F_{ab}[p] \, \frac{\partial p^b}{\partial \theta^j}$$

$$p^a = \log \mathcal{P}_g \to \theta^i \qquad \Rightarrow \; F_{ij} = \int \frac{d^3k \, d^3x}{(2\pi)^3} \frac{d \log \mathcal{P}_g}{d\theta^i} \times \frac{1}{2} \left(\frac{\mathcal{P}_g}{1+\mathcal{P}_g} \right)^2 \times \frac{d \log \mathcal{P}_g}{d\theta^j}$$

Power of a survey:
~ effective volume

$$V_{eff}(\vec{k}) = \int d^3x \left(\frac{\mathcal{P}_g}{1+\mathcal{P}_g} \right)^2 < V$$

STOP



COSMIC VARIANCE

$$\mathcal{P}_g \sim n_g(z) \, P(k)$$

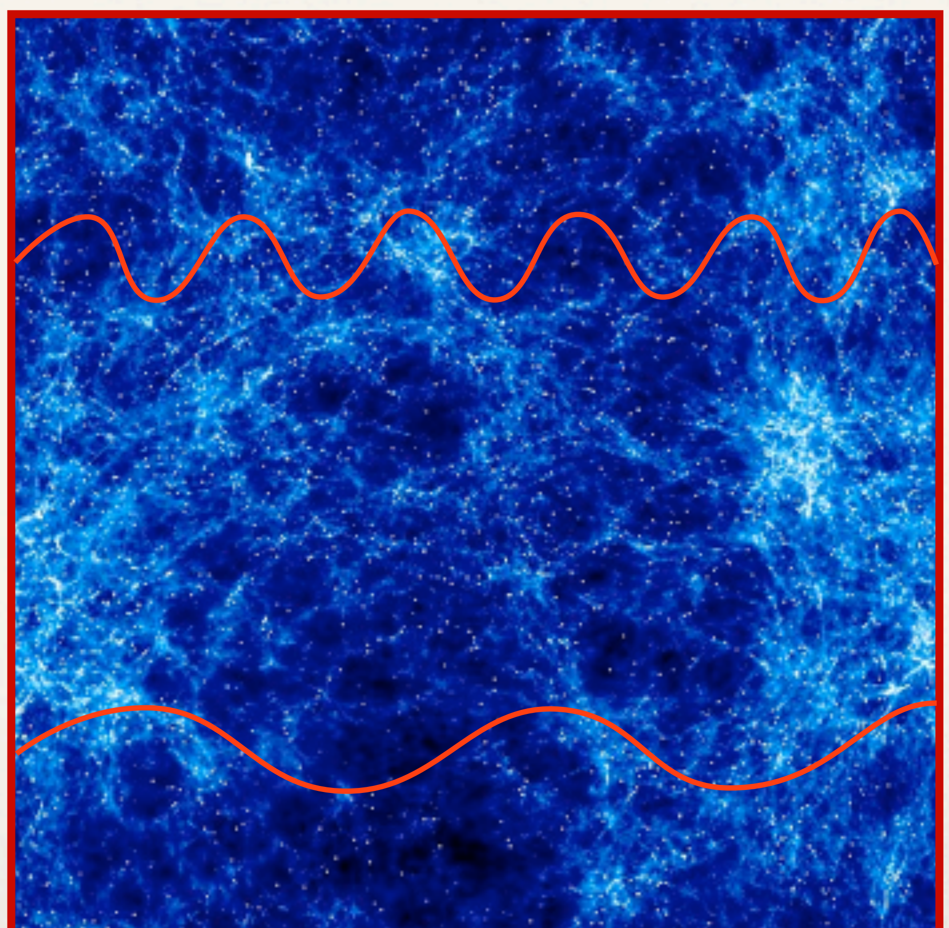
Why?

Much of the cosmological information resides in the **power spectrum**

$$\langle \delta(\vec{k})\, \delta^*(\vec{k}')\rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P(\vec{k})$$

In a **finite volume**, even if we map a **huge number** of tracers, the **precision** with which we can measure the **modes,** and ***P*(*k*)**, is **limited**

$$\frac{\sigma_{P(k)}}{P(k)} \propto \frac{1}{\sqrt{\# \text{ of modes}}}$$



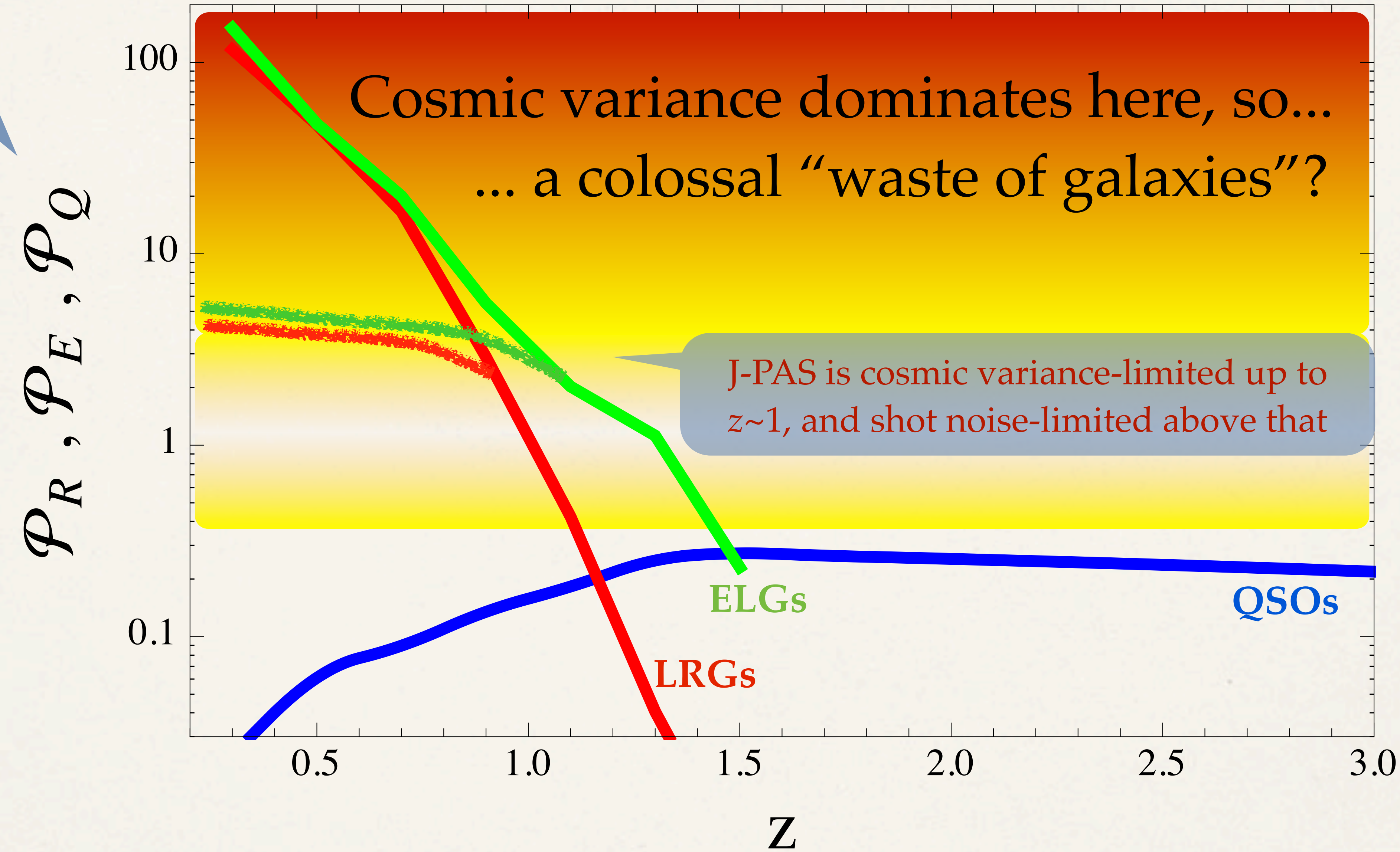
$$\frac{\sigma_{P(k)}}{P(k)}$$

$$\frac{\sigma_{P(k)}}{P(k)}$$

COSMIC VARIANCE

Implications for **cosmology**
Example: J-PAS

@ k=0.1 h Mpc^{-1}



"sweet spot"

Are these **many millions** of $z \lesssim 1$ galaxies really "wasted", from the point of view of cosmology?

NO!

How to “beat” cosmic variance

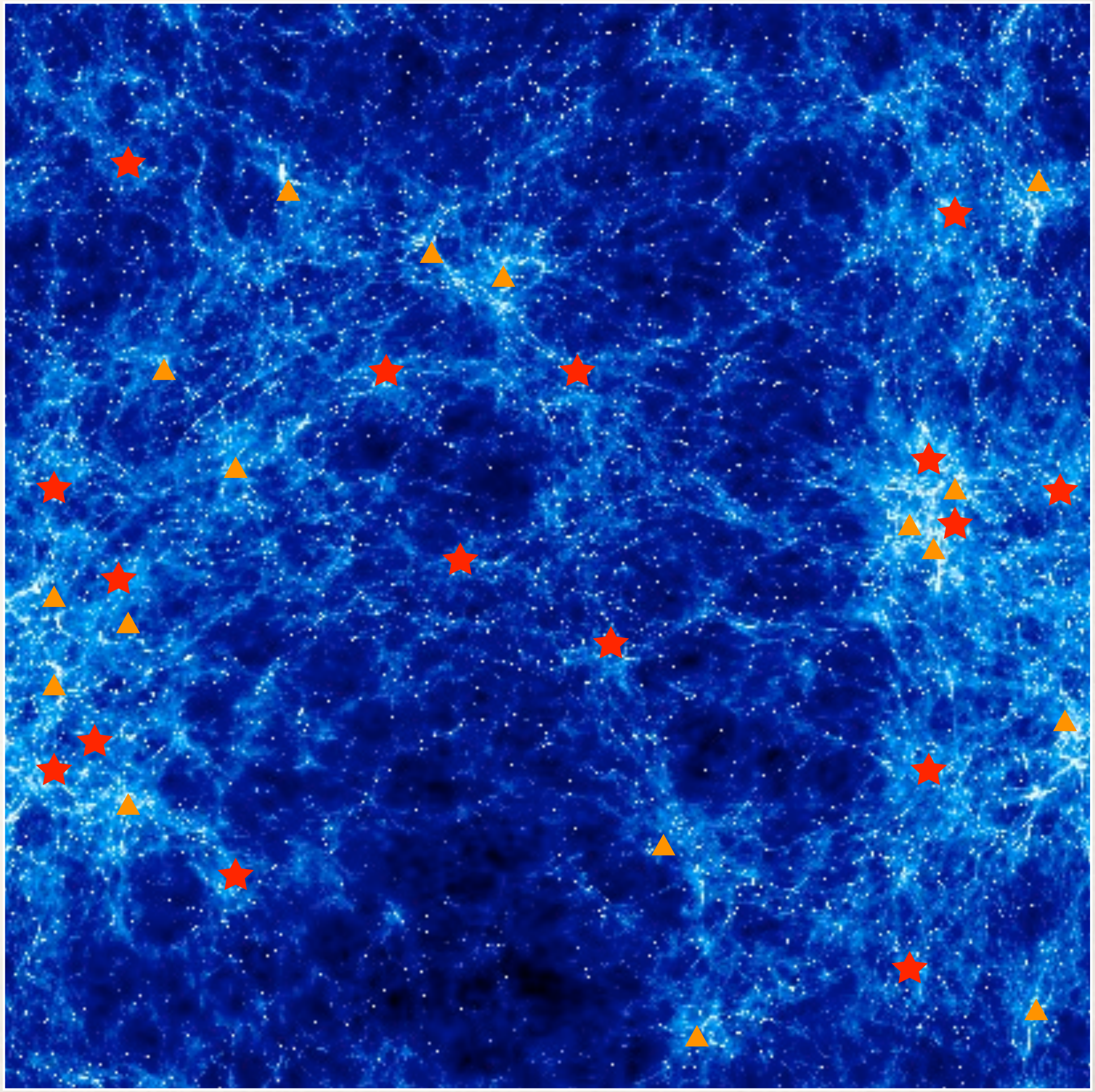
By **comparing** the **clustering of the different tracers** of large-scale structure (LRGs, ELGs, etc.), we can measure with **arbitrary accuracy*** the physical **parameters** that determine their **different clustering amplitudes**

Seljak 2008; McDonald & Seljak 2008
Gil-Marín et al. 2011
Hamaus, Seljak & Desjacques 2011,2012
Cai & Bernstein 2011
R.A. & K. Leonard, MNRAS 2013 (arXiv:1302.5444)

$$\frac{\mathcal{P}_1 = n_1\,(b_1 + f\,\mu_k^2)^2\,\cancel{P(k;z)}}{\mathcal{P}_2 = n_2\,(b_2 + f\,\mu_k^2)^2\,\cancel{P(k;z)}}$$

$$\frac{\mathcal{P}_1}{\mathcal{P}_2} = \frac{n_1\,(b_1 + f\,\mu_k^2)^2}{n_2\,(b_2 + f\,\mu_k^2)^2}$$

Cosmic Variance **does not apply!**



Multi-tracer Fisher information matrix

R.A., MNRAS 2012 (1108.5409)
R.A. & K. Leonard, MNRAS 2013 (1302.5444)

Let's say we have **several** (α = 1,2, ... N) **different types of tracers** of large-scale structure: α=1 (LRGs) , α=2 (ELGs) , α=3 (quasars) , etc.

The **multi-tracer Fisher matrix** is:

$$F_{\alpha\,\beta} = F(\log \mathcal{P}_\alpha , \log \mathcal{P}_\beta) \; = \frac{1}{4} \left[\delta_{\alpha\beta} \frac{\mathcal{P}_\alpha \, \mathcal{P}}{1+\mathcal{P}} + \frac{\mathcal{P}_\alpha \, \mathcal{P}_\beta \, (1-\mathcal{P})}{(1+\mathcal{P})^2} \right] \quad , \quad \mathcal{P} = \sum_\alpha \mathcal{P}_\alpha$$

Or, in terms of the usual parameters:

$$F_{ij} = F(\theta^i , \theta^j) = \sum_{\alpha\beta} \int \frac{d^3k \, d^3x}{(2\pi)^3} \, \frac{d \log \mathcal{P}_\alpha}{d\theta^i} F_{\alpha\beta} \frac{d \log \mathcal{P}_\beta}{d\theta^j}$$

Multi-tracer Fisher information matrix

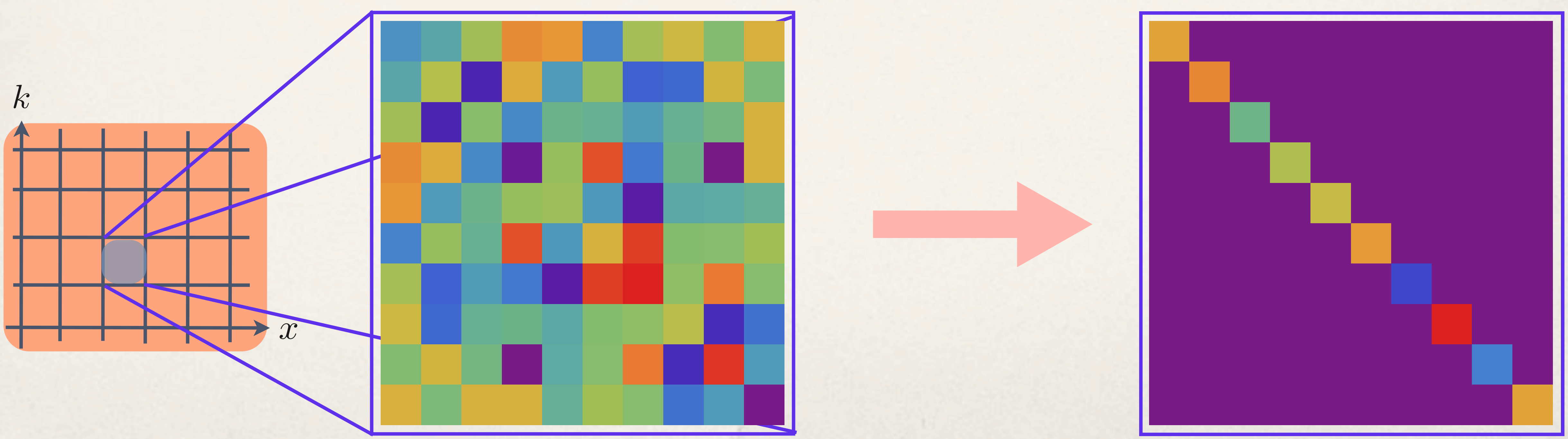
The **multi-tracer** Fisher information is **unbounded**!

$$F_{\alpha\beta} = \frac{1}{4}\left[\delta_{\alpha\beta}\frac{\mathcal{P}_\alpha \mathcal{P}}{1+\mathcal{P}} + \frac{\mathcal{P}_\alpha \mathcal{P}_\beta (1-\mathcal{P})}{(1+\mathcal{P})^2}\right]$$

But this still does not necessarily mean that the **total information** is unbounded...

In fact, there is no clear meaning to "total information" for non-diagonal Fisher matrices

But if we **diagonalize** the Fisher matrix, then **each eigenvalue** adds **positive**, **independent** amounts of information!



Multi-tracer Fisher information matrix

R.A. & K. Leonard, MNRAS 2013 (arXiv:1302.5444)

We can **diagonalize** the multi-tracer Fisher matrix by a change of variables:

$$\mathcal{P}_\alpha \to \mathcal{Q}_a$$

First, we define the **aggregate effective spectra** as: $\mathcal{S}_a = \sum_{\alpha=a}^{N} \mathcal{P}_\alpha$

The variables which diagonalize the multi-tracer Fisher matrix (eigenvectors) are:

$$\mathcal{Q}_1 = \mathcal{S}_1 = \mathcal{P}$$

Q_1 is the **total effective spectrum** of the survey. **Only it involves *P(k)***

CV-limited

$$\mathcal{Q}_a = \frac{\mathcal{S}_a}{\mathcal{P}_{a-1}} \qquad (a>1)$$

$$\vdots$$

$$\mathcal{Q}_N = \frac{\mathcal{S}_N}{\mathcal{P}_{N-1}} = \frac{\mathcal{P}_N}{\mathcal{P}_{N-1}}$$

Q_a (a>1) are **relational** variables (**ratios** of spectra between the different tracers). They **do not involve *P(k)***

NOT CV-limited

Diagonalized multi-tracer Fisher matrix

In terms of the **relational** power spectra Q_a the Fisher matrix is diagonal!

$$F_{ab} = \mathcal{F}_a \, \delta_{ab}$$

$$\mathcal{F}_1 = \frac{1}{2} \left(\frac{\mathcal{P}}{1+\mathcal{P}} \right)^2$$

< 1/2 ⇔ CV [FKP]
codifies info about $P(k)$

$$\mathcal{F}_a = \frac{1}{4} \frac{\mathcal{P}}{1+\mathcal{P}} \frac{\mathcal{S}_a \, \mathcal{P}_{a-1}}{\mathcal{S}_{a-1}}$$

unbounded
but no info about $P(k)$

$$\Rightarrow \quad F_{ij} = \int \frac{d^3k \, d^3x}{(2\pi)^3} \sum_a \frac{d \log \mathcal{Q}_a}{d\theta^i} \mathcal{F}_a \frac{d \log \mathcal{Q}_a}{d\theta^j} = \sum_a F^a_{ij}$$

Example: 2-tracer survey

Includes $P(k)$

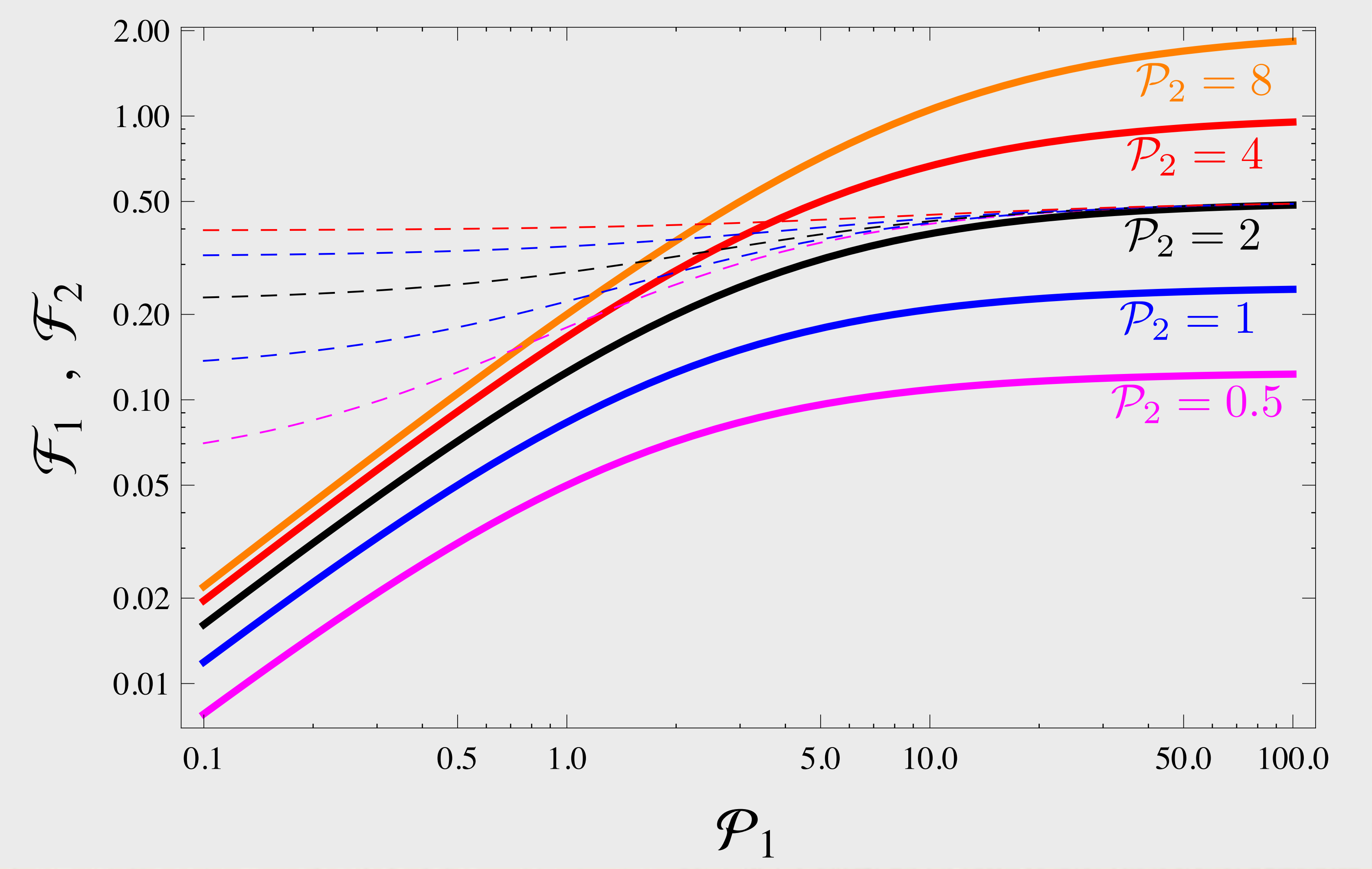
$$\mathcal{Q}_1 = \mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$$

$$\mathcal{F}_1 = \frac{1}{2}\left(\frac{\mathcal{P}_1 + \mathcal{P}_2}{1 + \mathcal{P}_1 + \mathcal{P}_2}\right)^2$$

$$\mathcal{Q}_2 = \frac{\mathcal{P}_2}{\mathcal{P}_1}$$

$$\mathcal{F}_2 = \frac{1}{4}\frac{\mathcal{P}_1 \mathcal{P}_2}{1 + \mathcal{P}_1 + \mathcal{P}_2}$$

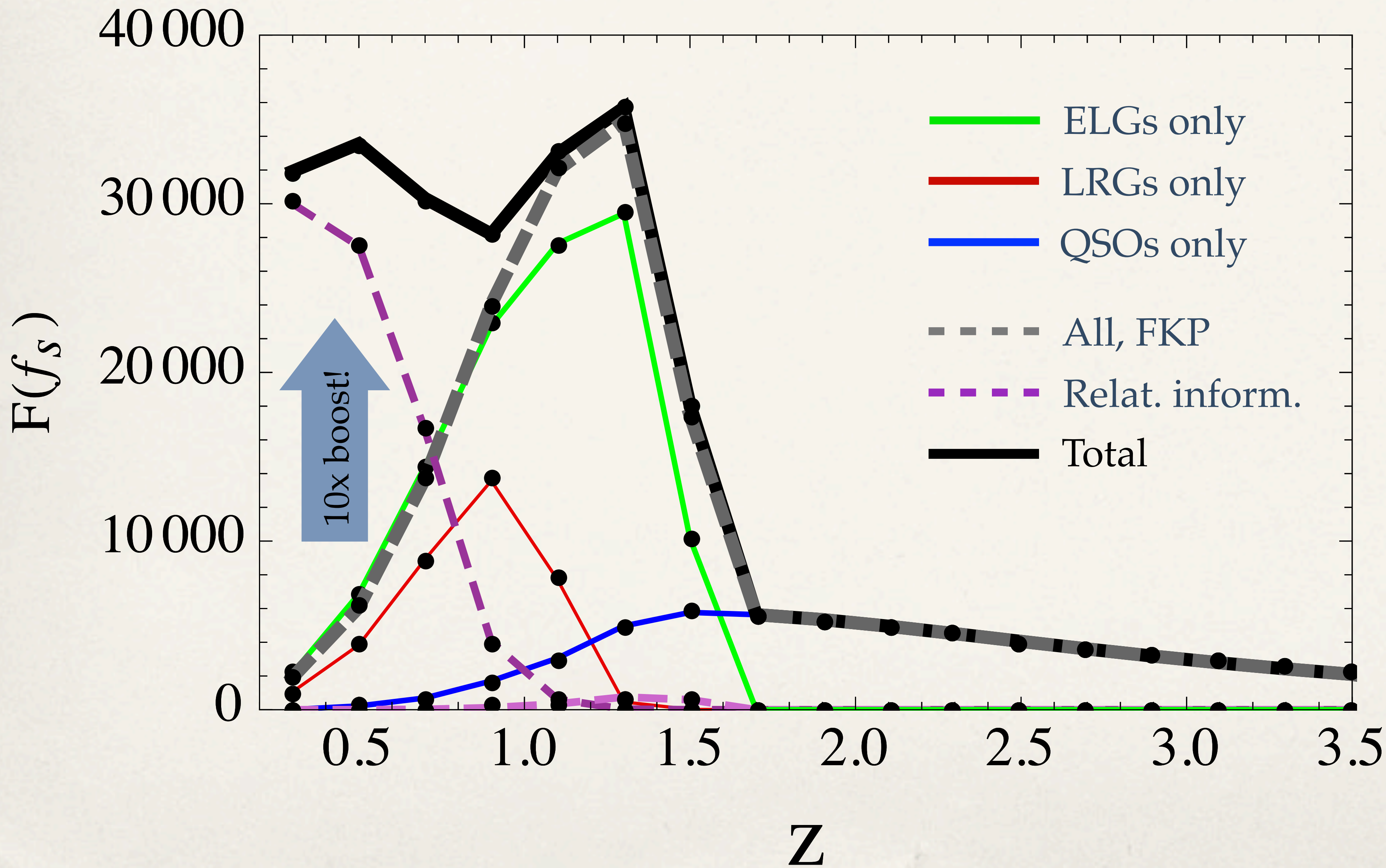
Does not include $P(k)$

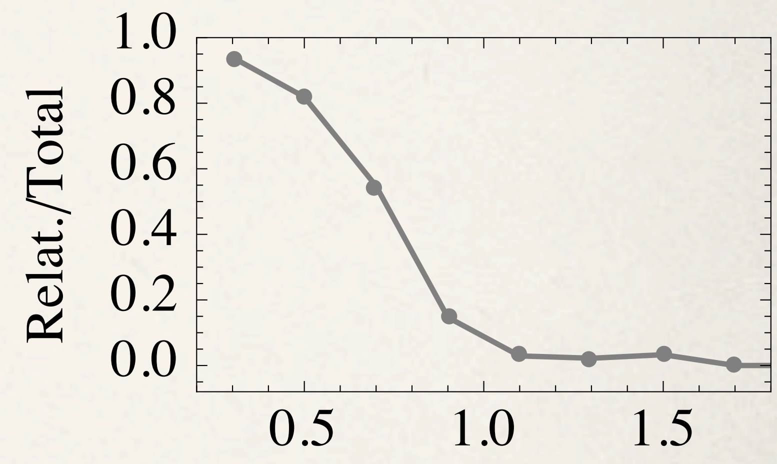


Constraints on the RSD parameter *f(z)* [J-PAS]

$$\mathcal{P}_g = n_g \, (b_g + f \, \mu_k^2)^2 \, P(k; z)$$

$$F(\theta) = \frac{1}{\sigma_c^2(\theta)}$$





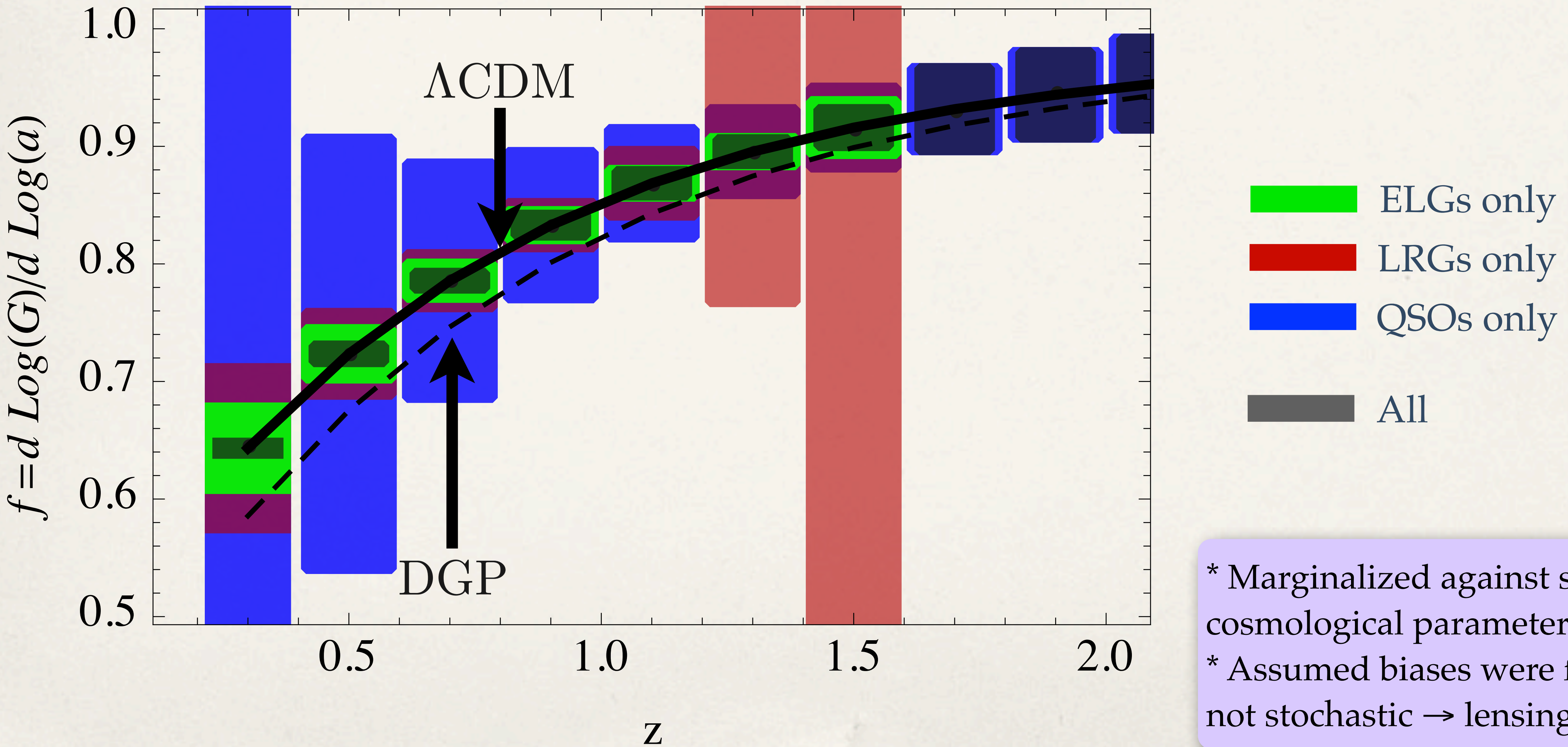
Information from the relative clusterings can improve the constraints on *f(z)* by up to ~3 at low z's!

Redshift-space distortions & modified gravity

$$\Delta G_{\mu\nu} + G_{\mu\nu} = 8\pi G\, T_{\mu\nu} + 8\pi G\, \Delta T_{\mu\nu}$$

Matter growth: $\nabla^2 \Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R(f_R) \quad \Longrightarrow \quad G(z)$

$$f(z) = \frac{d\ln G}{d\ln a} = \Omega_m^{\gamma}(z)$$



* Marginalized against shape of P(k) & cosmological parameters
* Assumed biases were fixed, linear and not stochastic → lensing would help a lot!

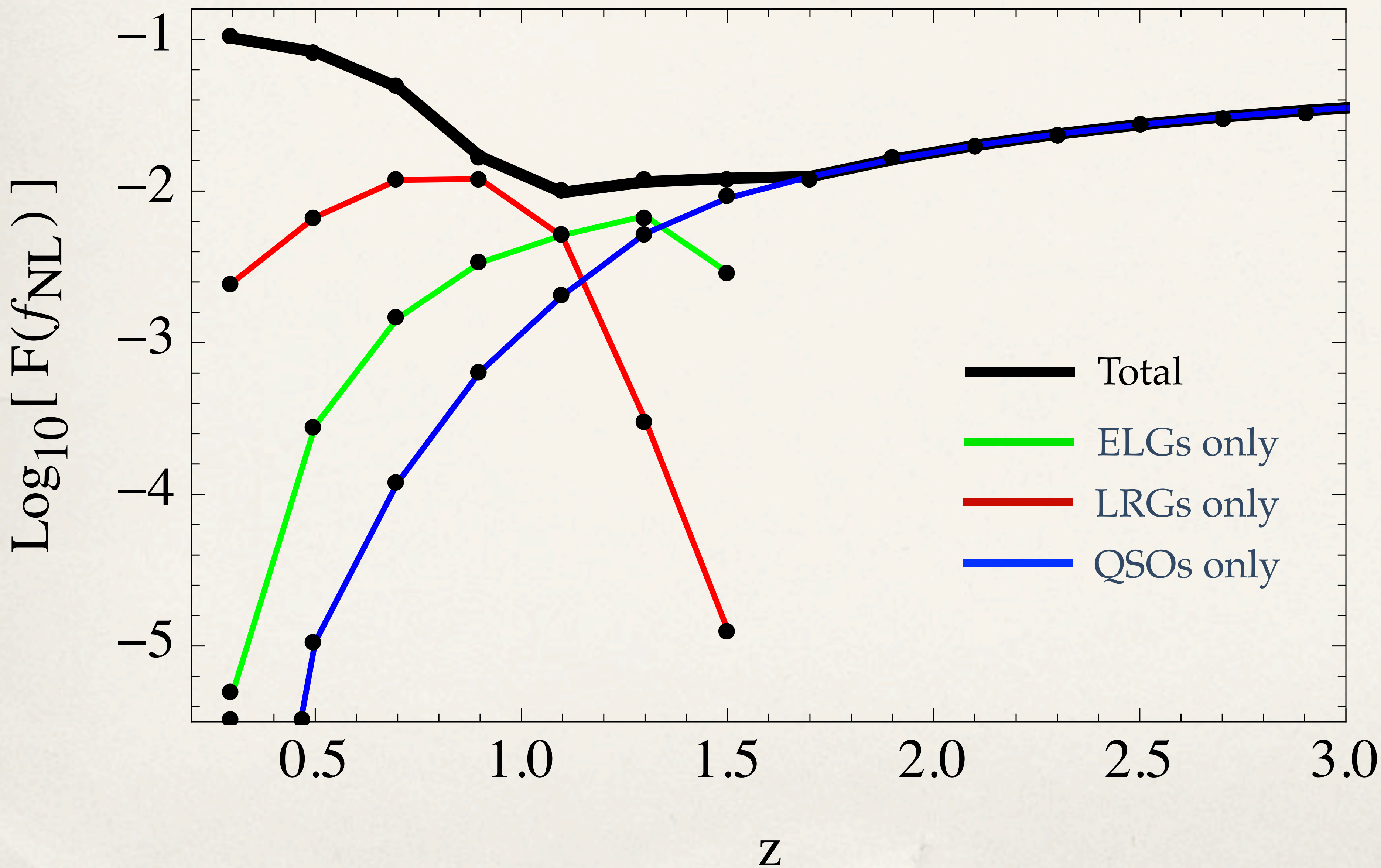
Constraints in local non-Gaussianity parameter f_{NL}

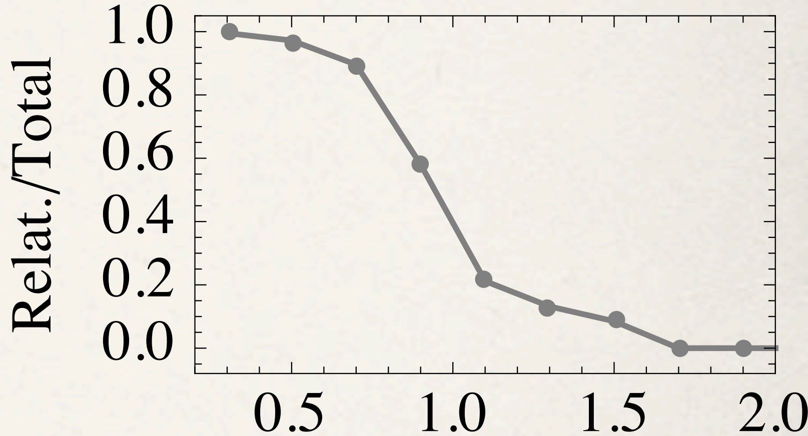
Prediction (and discriminator) of inflation models

$$\mathcal{P}_g = n_g \, (b_g + f \, \mu_k^2)^2 \, P(k; z)$$

$$b_g \rightarrow b_g + \Delta b_g(f_{NL}, k)$$

$$F(\theta) = \frac{1}{\sigma_c^2(\theta)}$$





Information from relative clustering can improve constraints on f_{NL} by ~5 at low-z!

Conclusions:

- **Cosmic variance** is a fundamental limitation **only** for measurements of the **power spectrum**

- **Multi-tracer strategies** are able to **optimally explore the new era of volume-limited surveys** of large-scale structure

- In particular, the multi-tracer approach can **enhance dramatically** the constraints on:
    - **modified gravity** (through the RSD function f)
    - **inflation** (f_{NL})

- Even BAOs can benefit: both indirectly (through marginalizations), and also directly, via enhancements of the constraints from AP tests

- Biggest challenge is covariance of biases and shot noise between tracers; we are studying those covariances with the help of N-body simulations

Forecasted distance constraints from BAOs in J-PAS

* RSDs and shape of P(k) were marginalized
* Equivalent to Seo / Eisenstein "BAOs-only" method

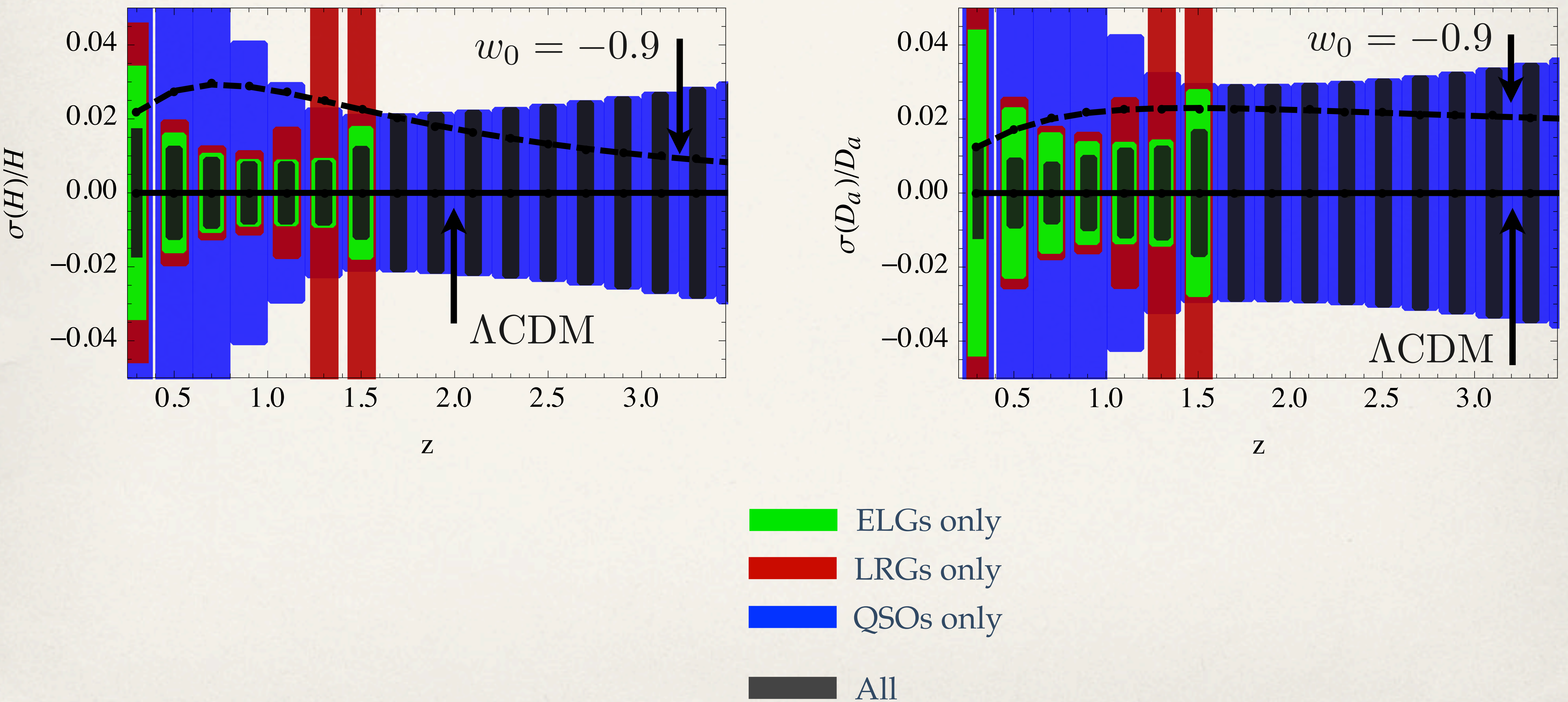
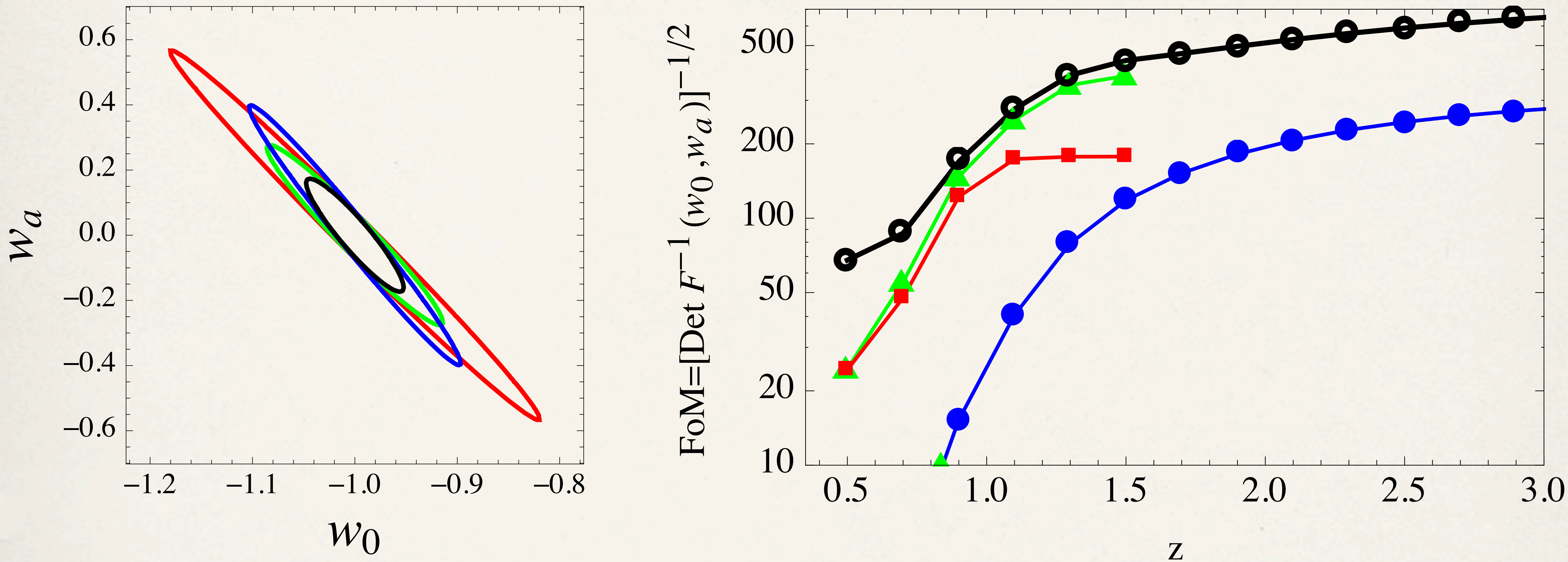


Figure of Merit for J-PAS



ELGs only

LRGs only

QSOs only

All

* With **Planck priors**
* **Marginalized** against shape of P(k) & other parameters
* RSD information was **marginalized**, but **not projected** into final set of cosmological parameters
* **Same methods/criteria as used for EUCLID papers**