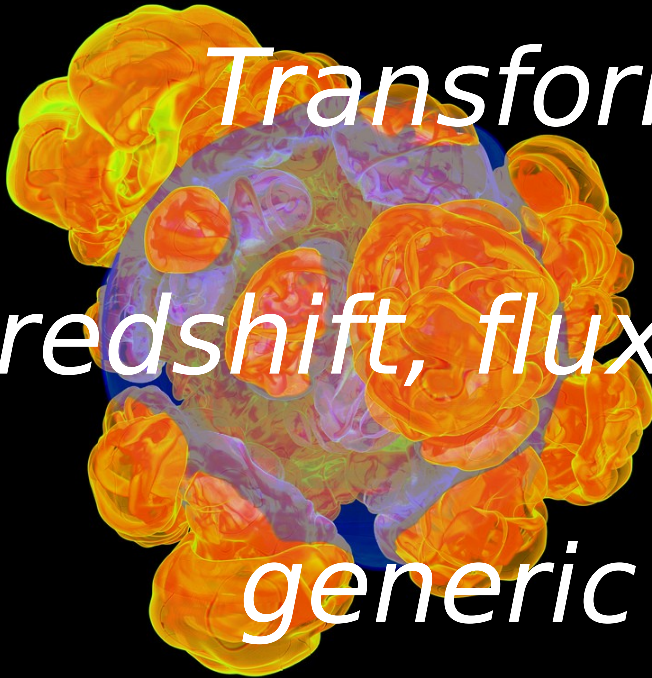
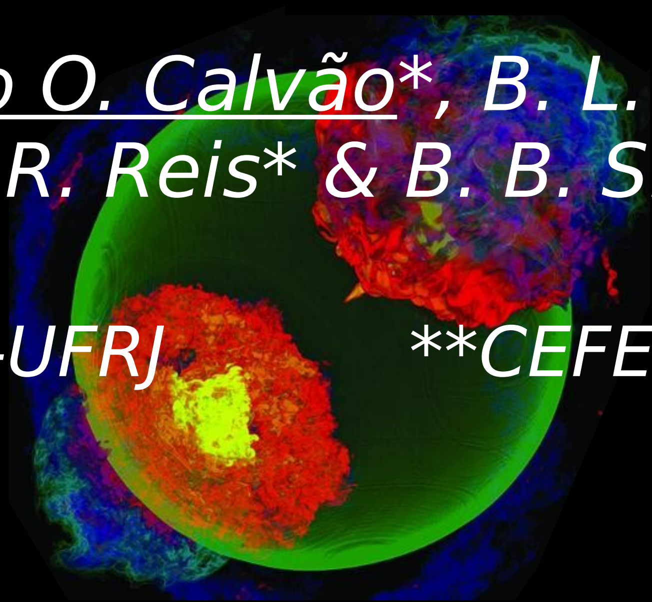


Transformation laws of redshift, flux and luminosity in generic spacetimes

Maurício O. Calvão, B. L. Lago**, R. R. R. Reis* & B. B. Siffert**

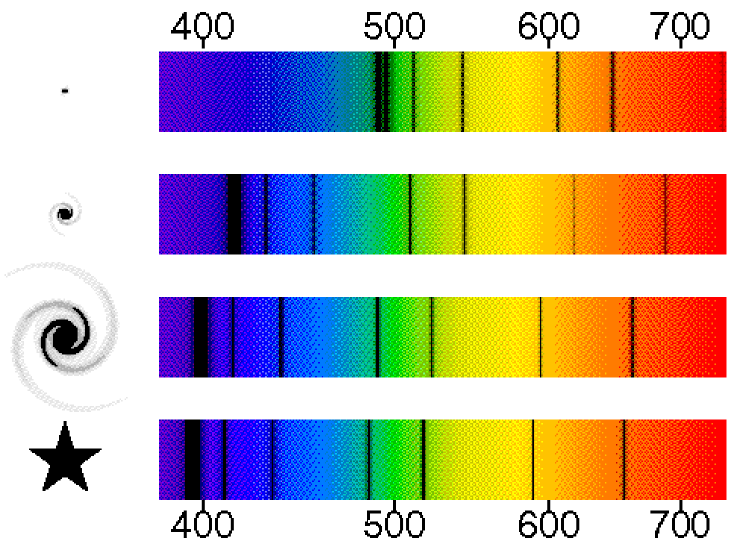
**IF-UFRJ **CEFET-RJ*



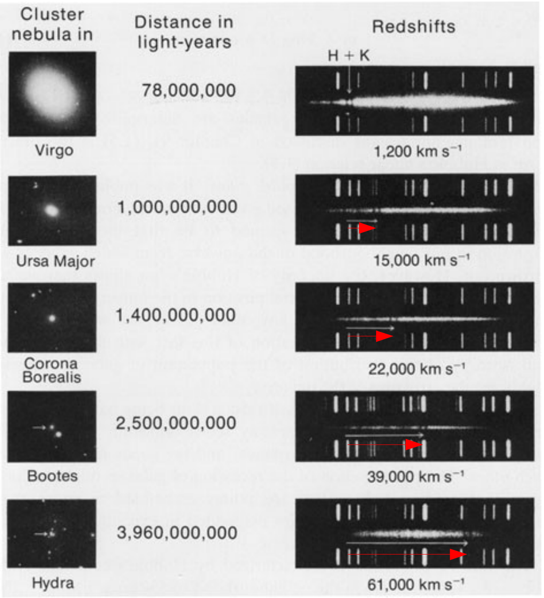
I. INTRODUCTION

I.1. MOTIVATION

redshift: Doppler, gravitational, cosmological

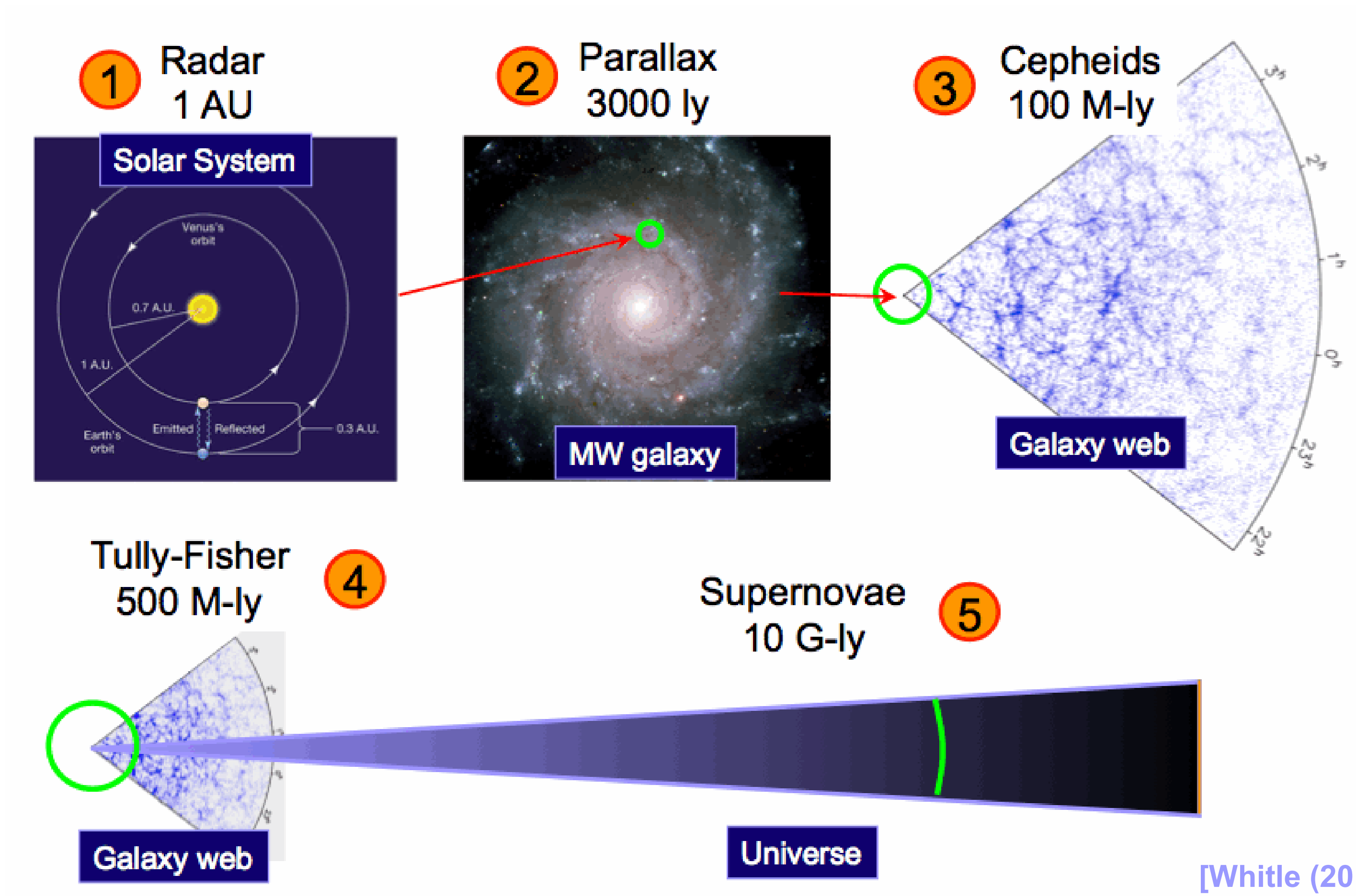


Wright (2013)]



Schombert (2013)]

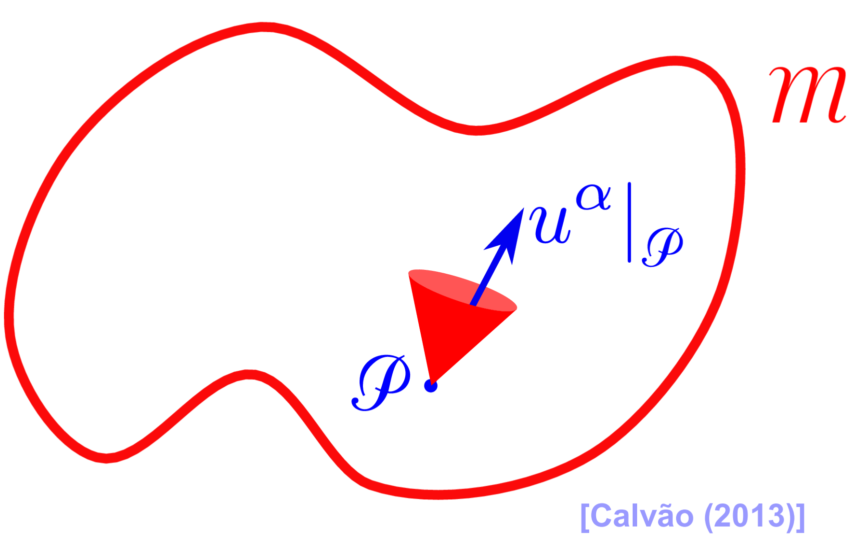
flux and luminosities



[Whitle (2013)]

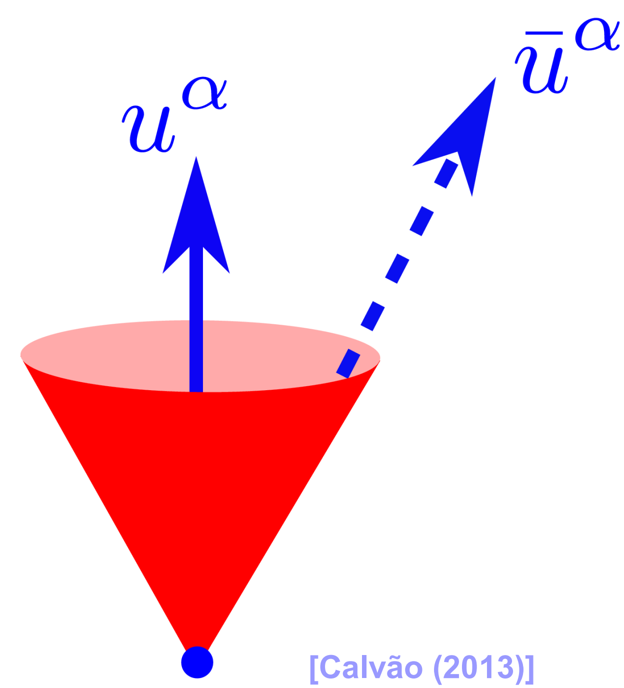
I.2. BASIC CONCEPTS

instantaneous observer



$$h^{\alpha\beta} := g^{\alpha\beta} - u^\alpha u^\beta$$

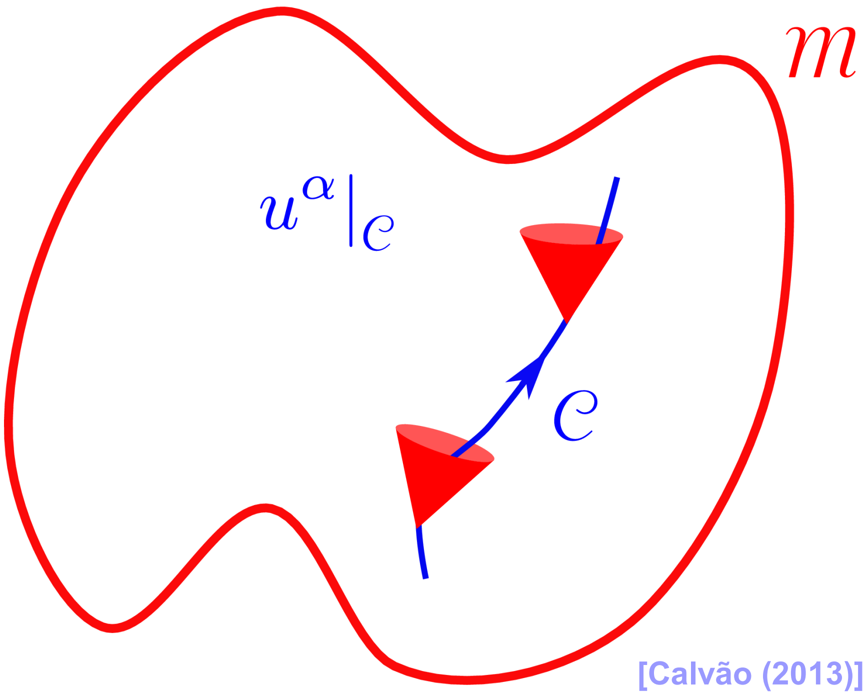
local Lorentz boost



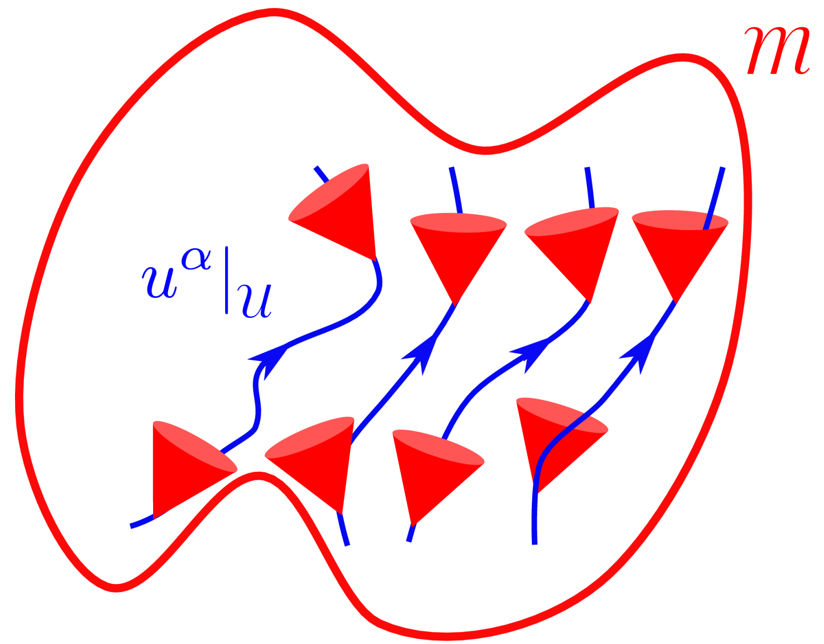
$$\bar{u}^\alpha = \gamma\,(u^\alpha + v e^\alpha)$$

observer

frame of reference



[Calvão (2013)]



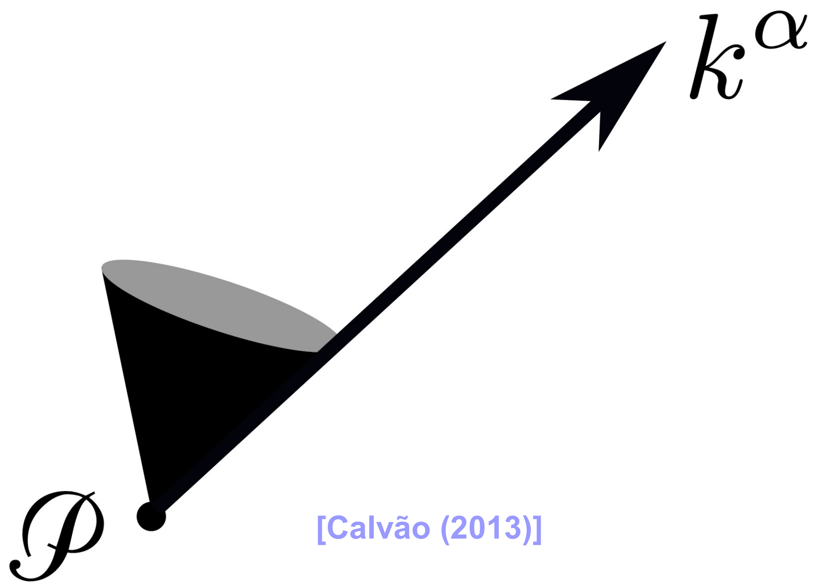
[Calvão (2013)]

$$a^\alpha := \frac{Du^\alpha}{D\tau}$$

$$u_{\alpha;\beta} = a_\alpha u_\beta + \frac{1}{3}\Theta h_{\alpha\beta} + \sigma_{\alpha\beta} + \omega_{\alpha\beta}$$

II. REDSHIFT

II.1. LOCAL (OR POINTWISE)



[Calvão (2013)]

II. REDSHIFT

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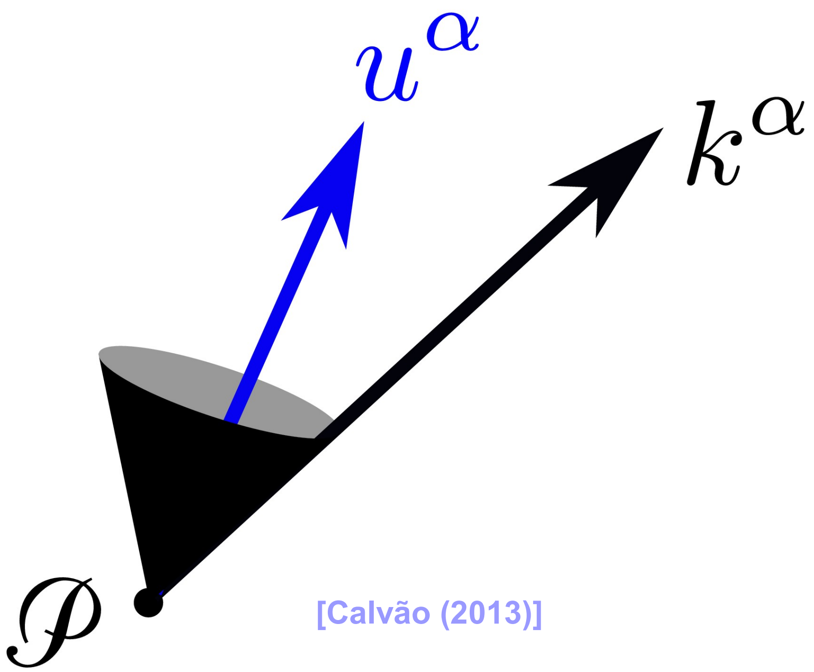
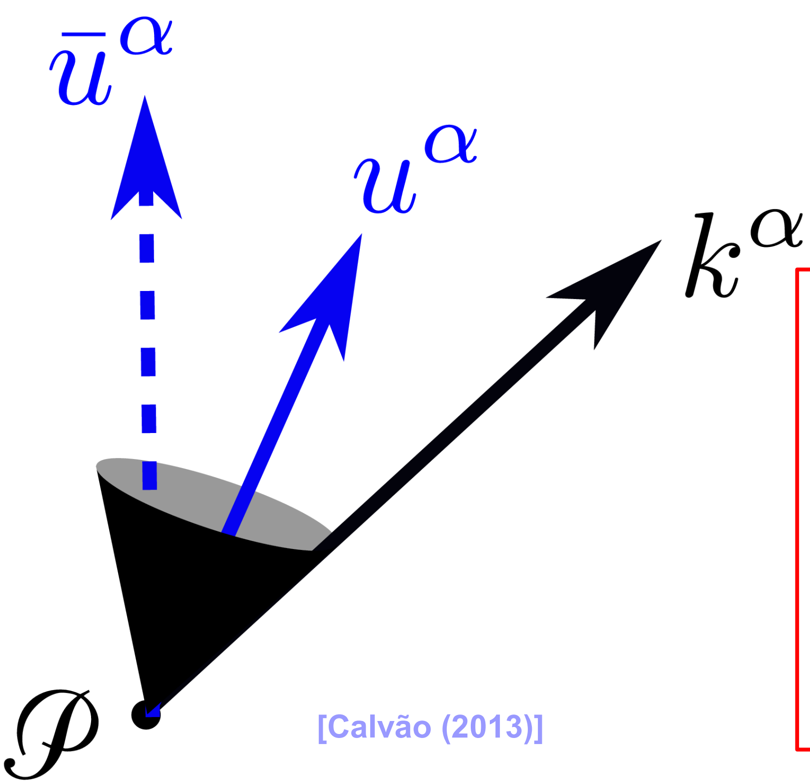


[Calvão (2013)]

$$k^\alpha = E\left(u^\alpha + n^\alpha\right)$$

II. REDSHIFT

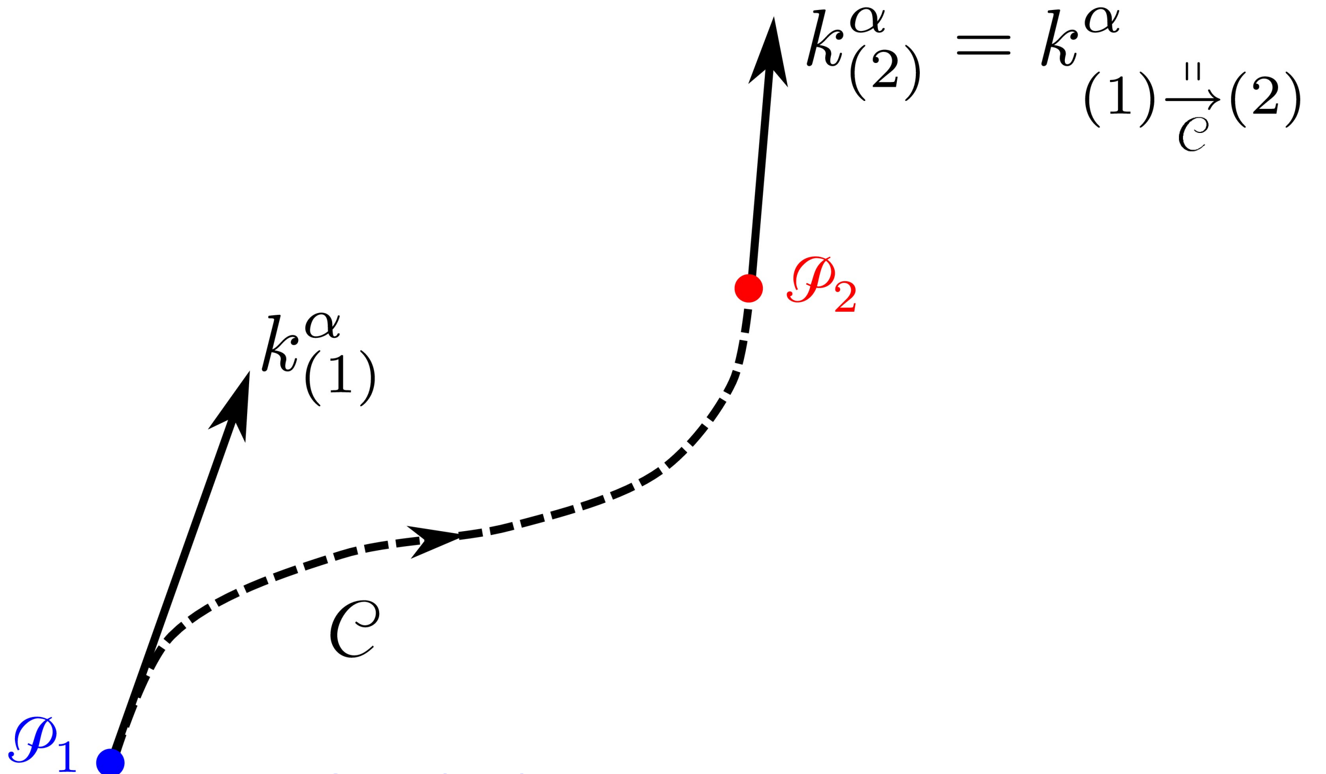
II.1. LOCAL (OR POINTWISE)



$$\bar{E} = \gamma E \left(1 + v n^\alpha e_\alpha\right)$$

$$1 + z[\mathscr{P}; k^\alpha; u^\alpha \to \bar{u}^\alpha] = \frac{\bar{\lambda}}{\lambda} = \left.\frac{k^\alpha u_\alpha}{k^\beta \bar{u}_\beta}\right|_{\mathscr{P}}$$

II.2. GLOBAL (OR AT A DISTANCE)



[Calvão (2013)]

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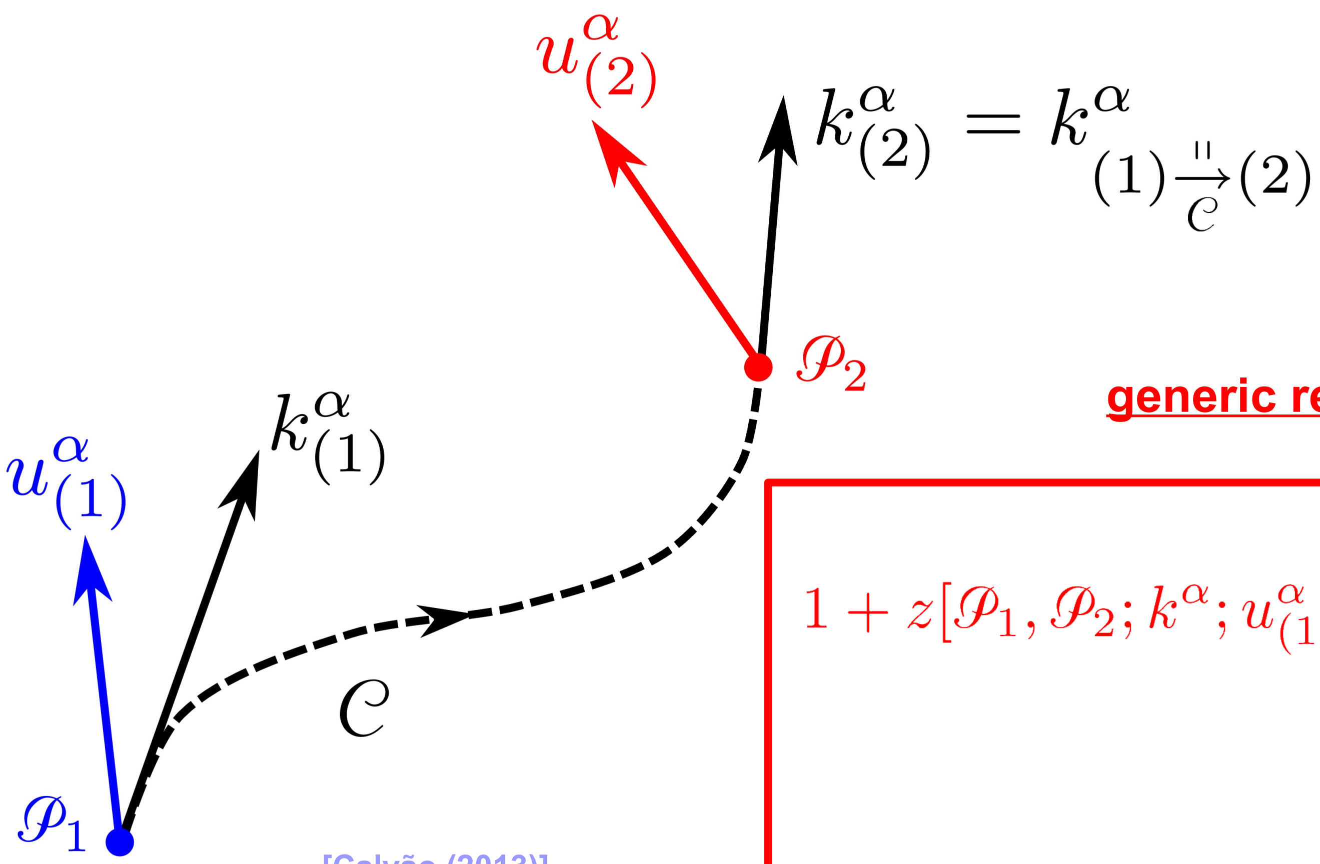


[Calvão (2013)]

generic redshift law:

$$1 + z[\mathscr{P}_1, \mathscr{P}_2; k^\alpha; u^\alpha_{(1)} \rightarrow u^\alpha_{(2)}] = \frac{\lambda_2}{\lambda_1} = \frac{(k^\alpha u_\alpha)|_{\mathscr{P}_1}}{(k^\beta u_\beta)|_{\mathscr{P}_2}}$$

II.2. GLOBAL (OR AT A DISTANCE)

in particular, for any spacetime:

$$\exists \; u^{\alpha}_{(2)} \;|\; u^{\alpha}_{(2)} = u^{\alpha}_{(1)\xrightarrow[c]{\parallel}(2)} \Rightarrow z = 0$$

$$\Downarrow$$

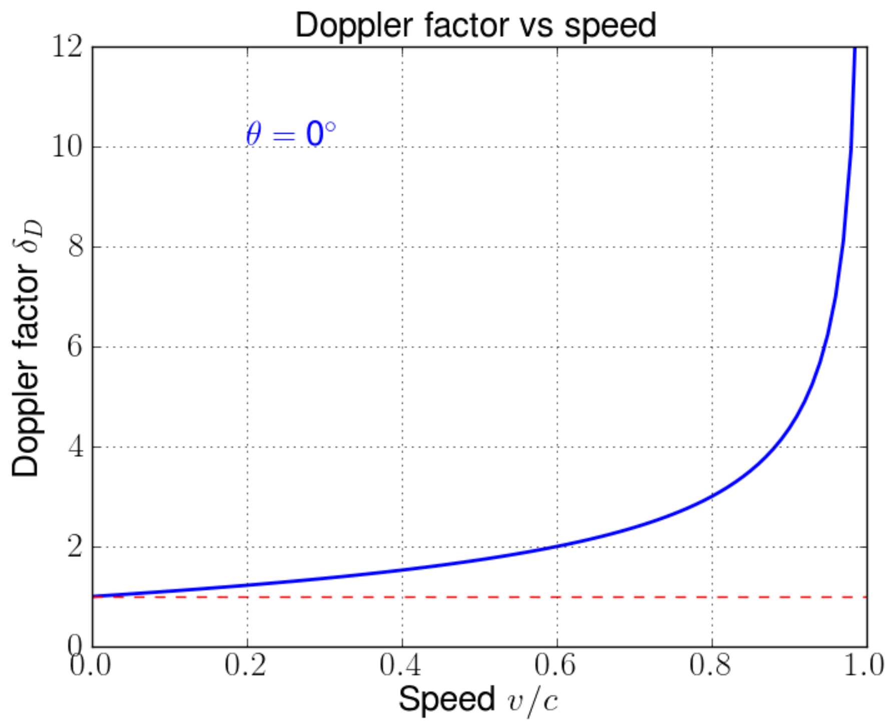
challenge to expanding space paradigm!

basis for consistent definition of absolute magnitude!

II.3. DOPPLER FACTOR

$$\delta_D(v,\theta) := [\gamma(v)\,(1 - v\cos\theta)]^{-1}$$

$$\Rightarrow 1 + z(v,\theta) = \delta_D(v,\theta)$$



[Calvão (2013)]

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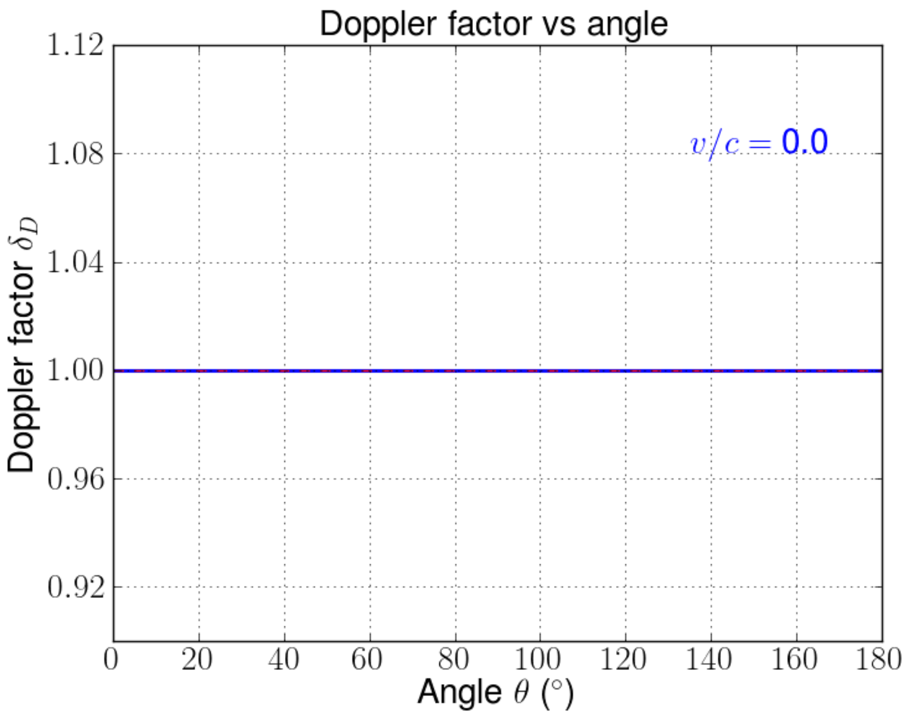

[Calvão (2013)]

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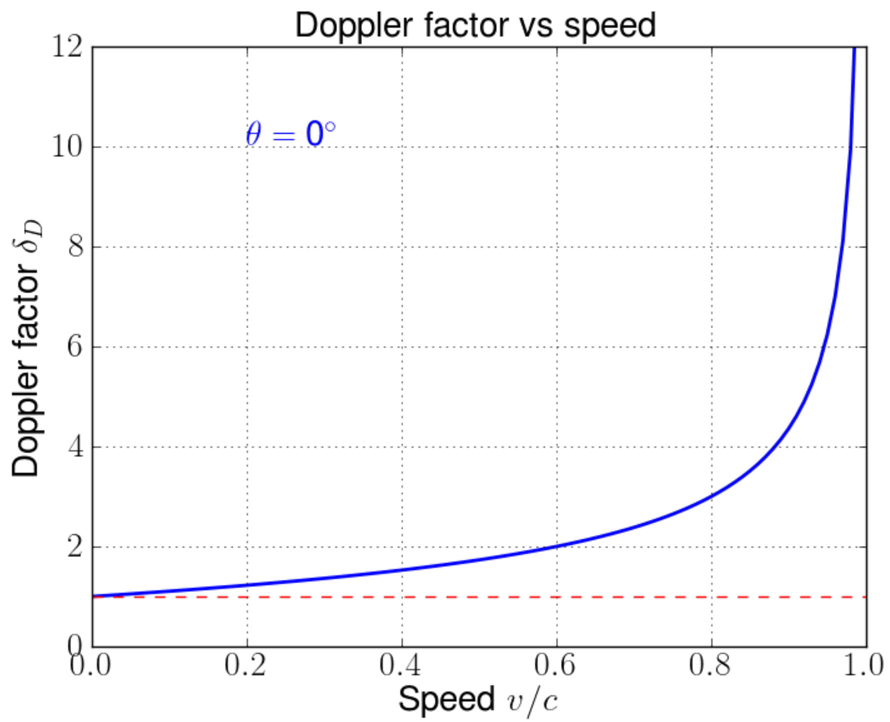
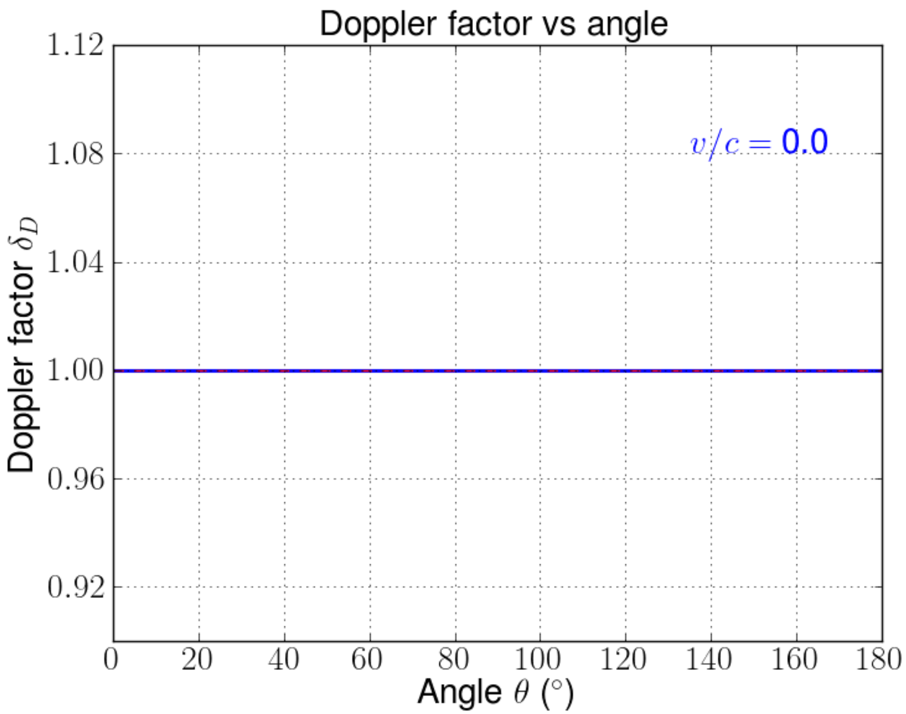
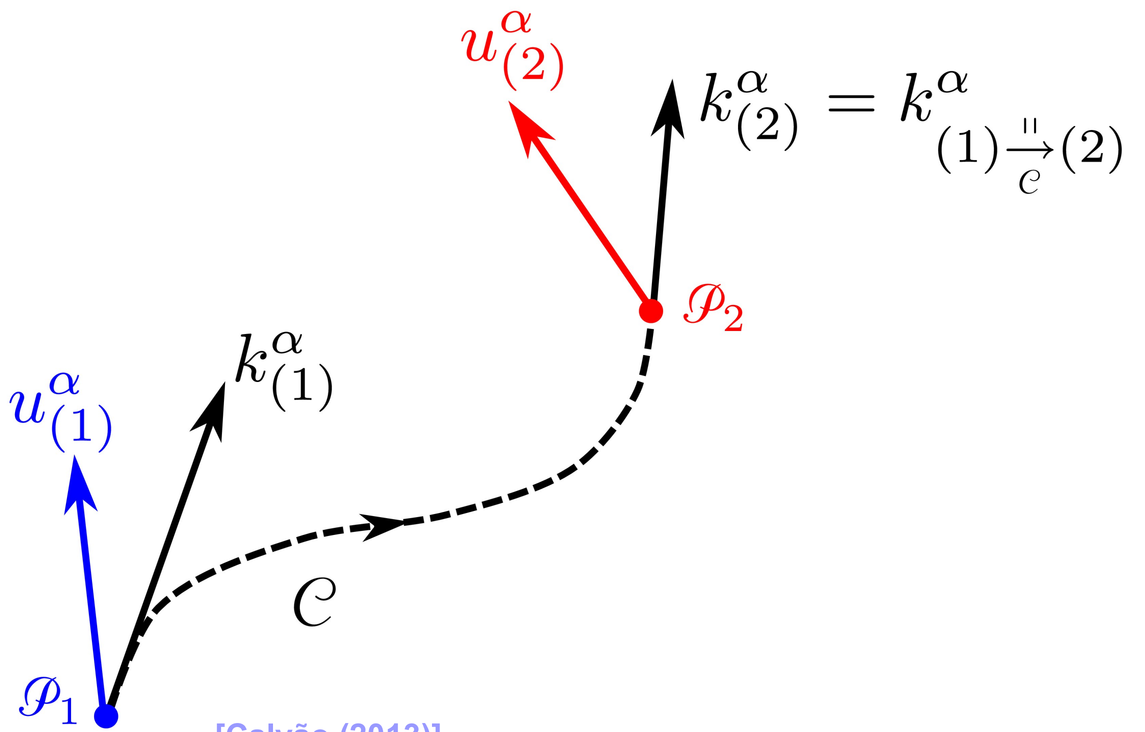


[Calvão (2013)]



[Calvão (2013)]

II.3. REDSHIFT TRANSFORMATION



[Calvão (2013)]

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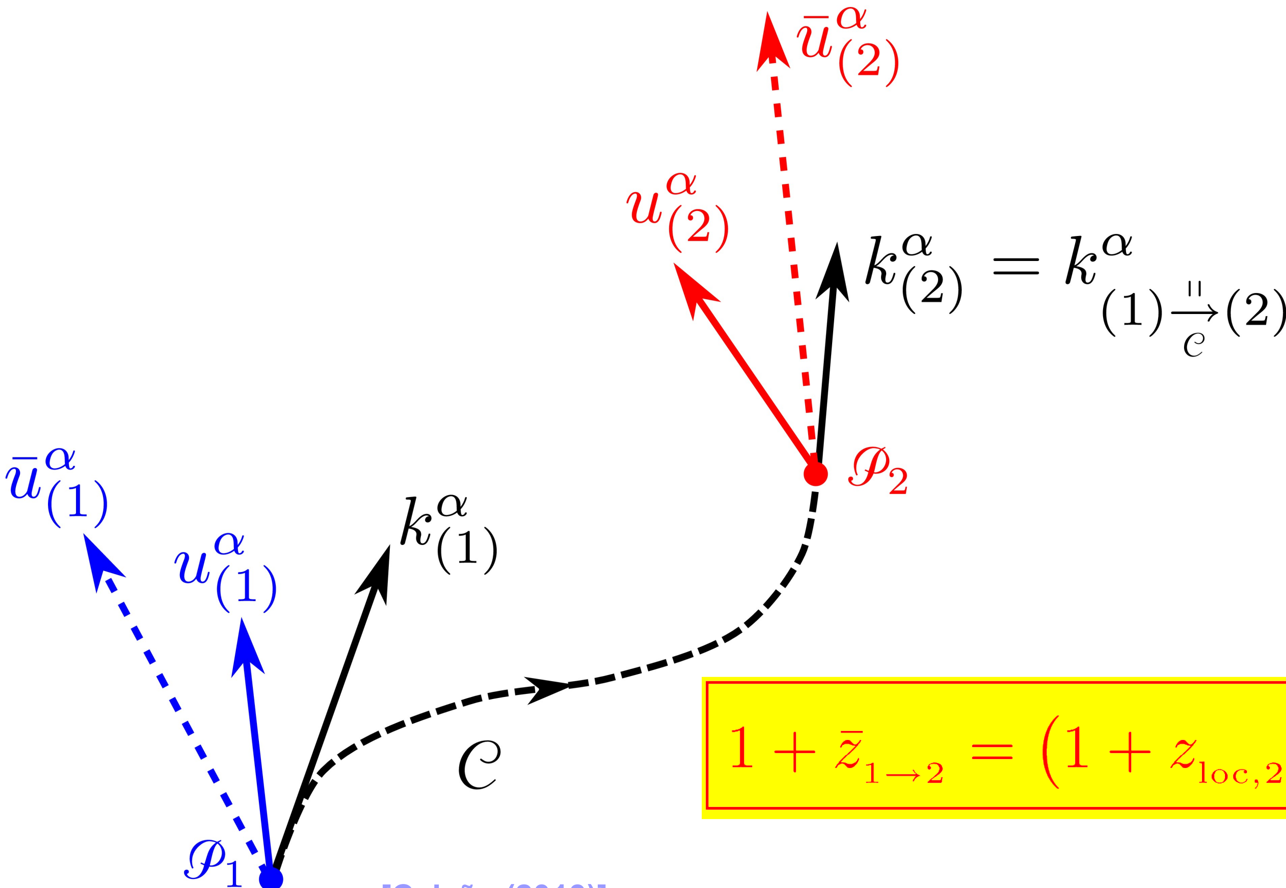


[Calvão (2013)]

$$1 + \bar{z}_{1\to 2} = \left(1 + z_{\text{loc},2}\right)\left(1 + z_{1\to 2}\right)\left(1 + z_{\text{loc},1}\right)^{-1}$$

$$1 + \bar{z}_{1\to 2} := \bar{\lambda}_2/\bar{\lambda}_1 \qquad 1 + z_{1\to 2} := \lambda_2/\lambda_1$$

$$1 + z_{\text{loc},i} := \bar{\lambda}_i/\lambda_i$$

III. FLUX

III.1. DEFINITION

$$T^{\alpha\beta} = \rho u^\alpha u^\beta - P h^{\alpha\beta} + 2q^{(\alpha}u^{\beta)} + \pi^{\alpha\beta}$$

$$f := ||q^\alpha||$$

↓ geometrical optics (eikonal) approximation

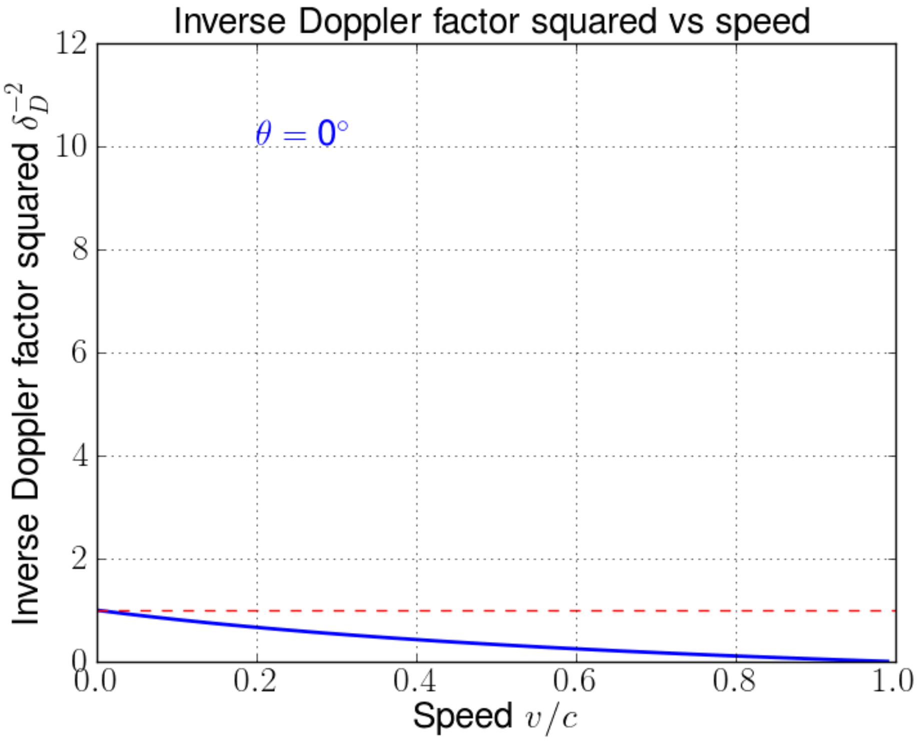
$$T^{\alpha\beta} = B^2 k^\alpha k^\beta \qquad (k_\alpha := \partial\varphi/\partial x^\alpha)$$

↓

$$\boxed{f = \rho}$$

III.2. FLUX TRANSFORMATION

$$\bar{f} = f\delta_D^{-2}$$



[Calvão (2013)]

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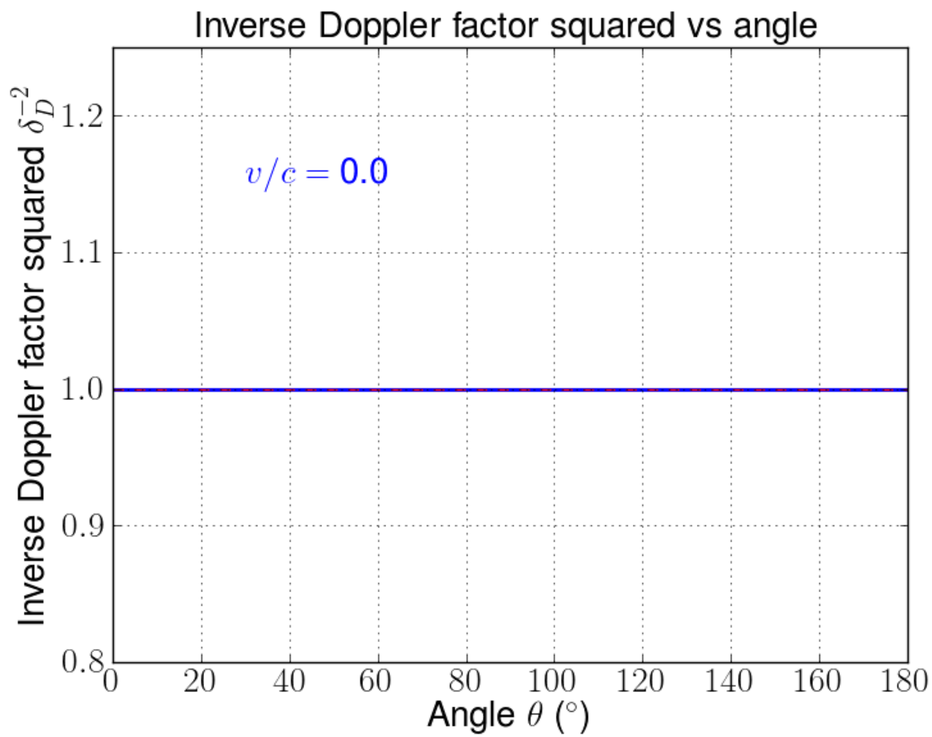

[Calvão (2013)]

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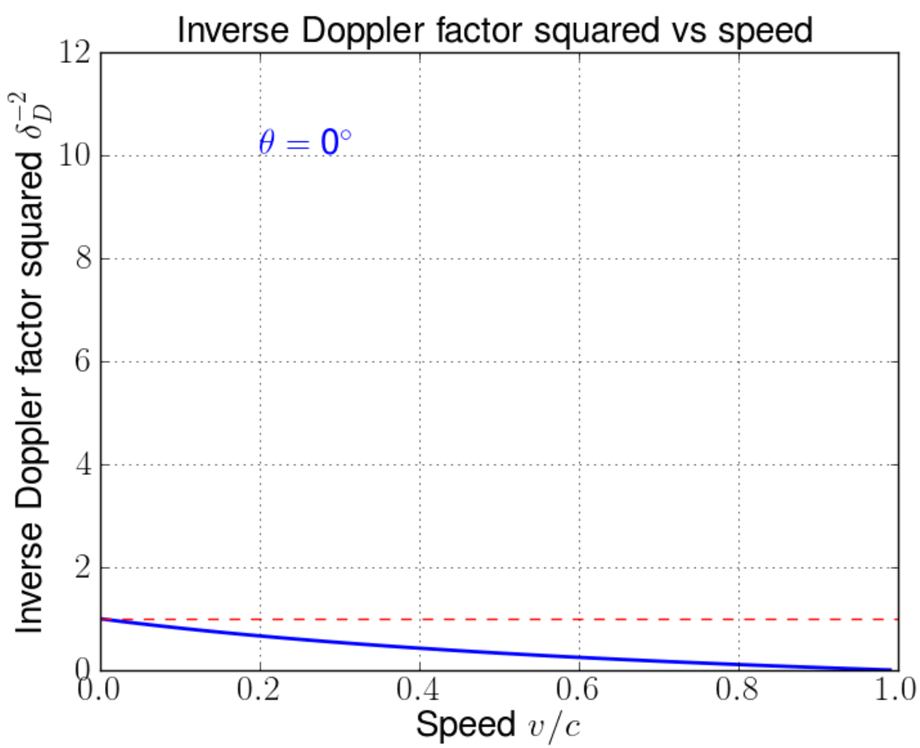
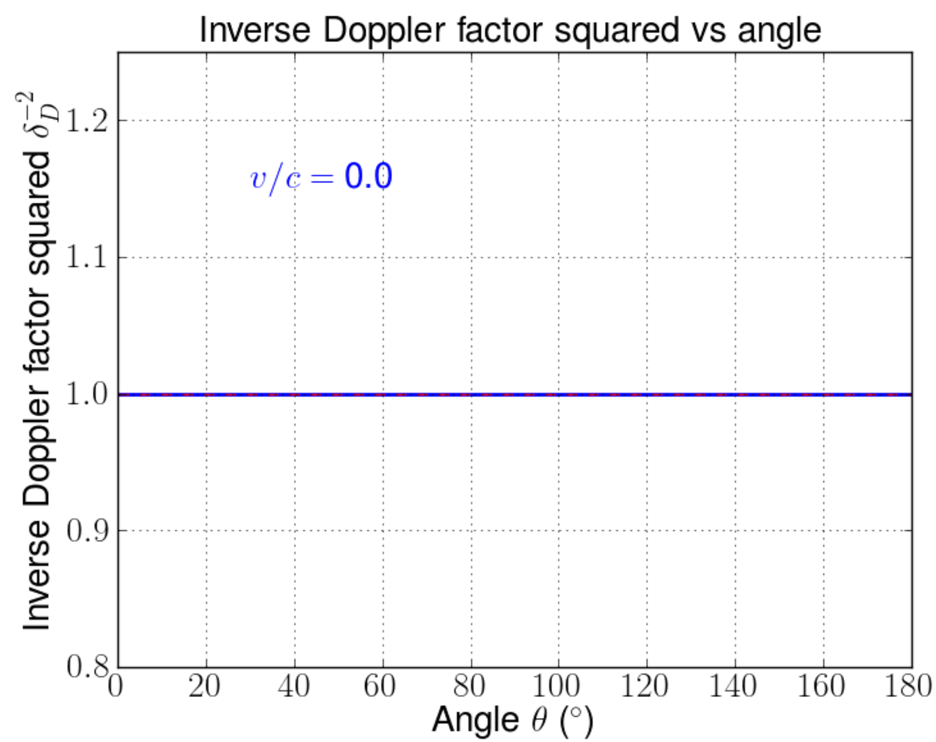


[Calvão (2013)]



[Calvão (2013)]

IV. LUMINOSITIES

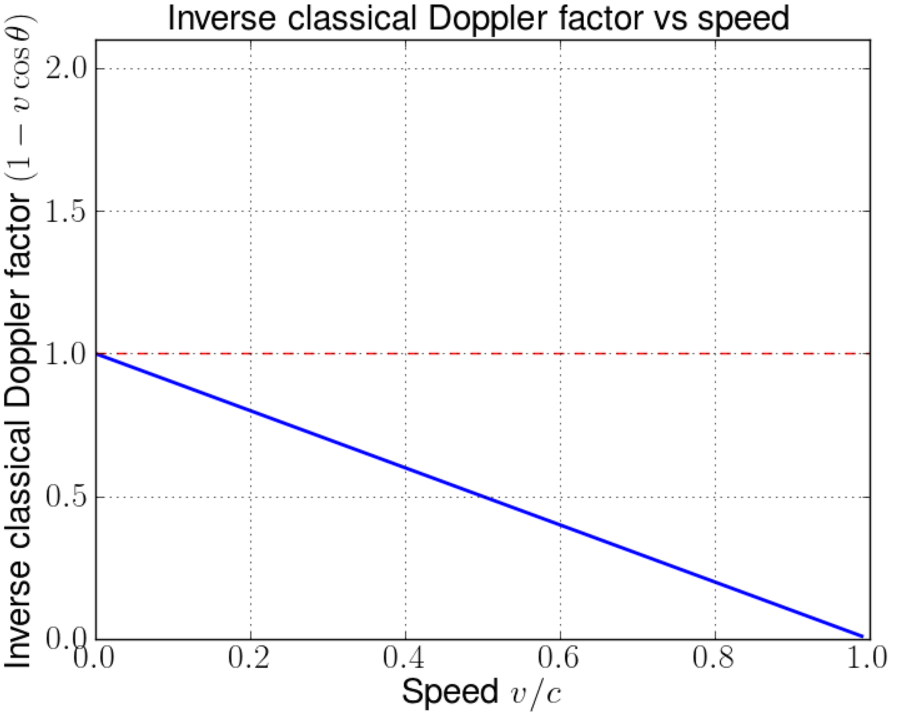
IV.1. DEFINITIONS

[Rybicki (1979)]

$L_\Omega(t, \hat{\mathbf{\Omega}})$: angular distribution of emitted (source) power, or simply angular emitted power

$\mathcal{L}_\Omega(t, \hat{\mathbf{\Omega}})$: angular distribution of received (detector) power, or simply angular received power

$$L_\Omega = \mathcal{L}_\Omega (1 - v \cos\theta)$$



[Calvão (2013)]

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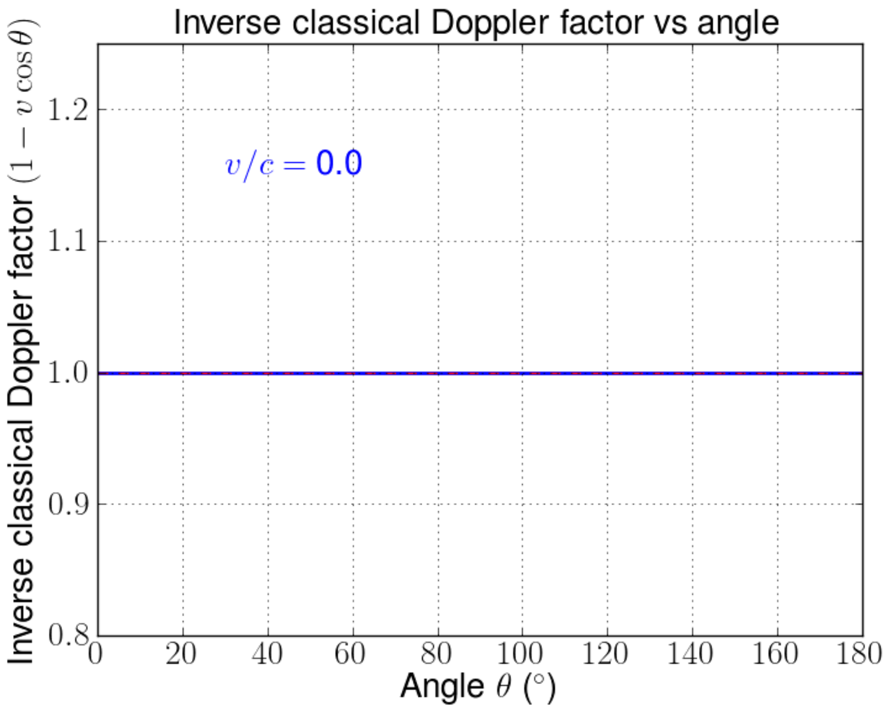

[Calvão (2013)]

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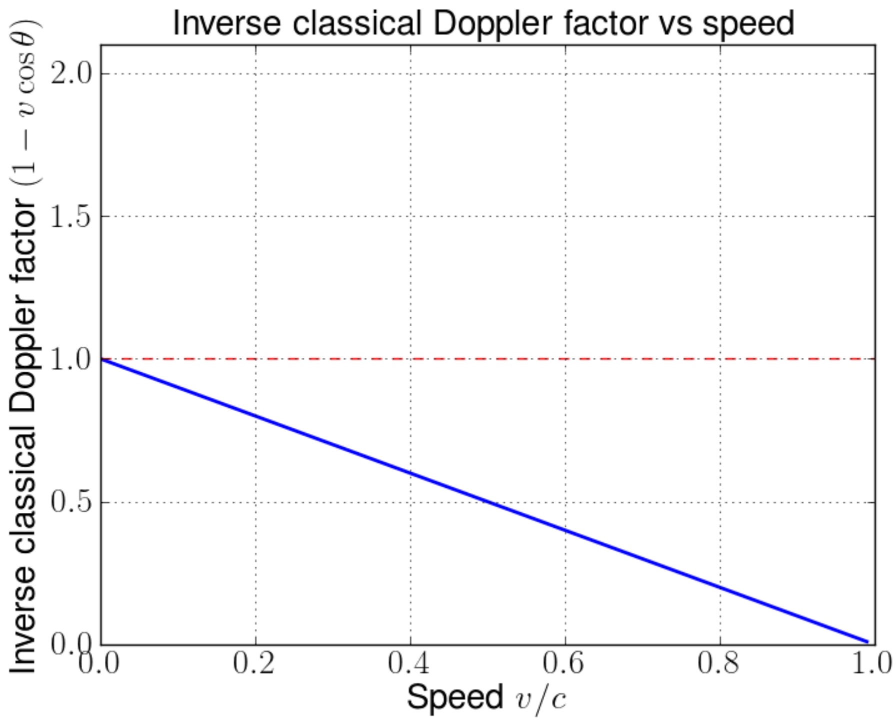
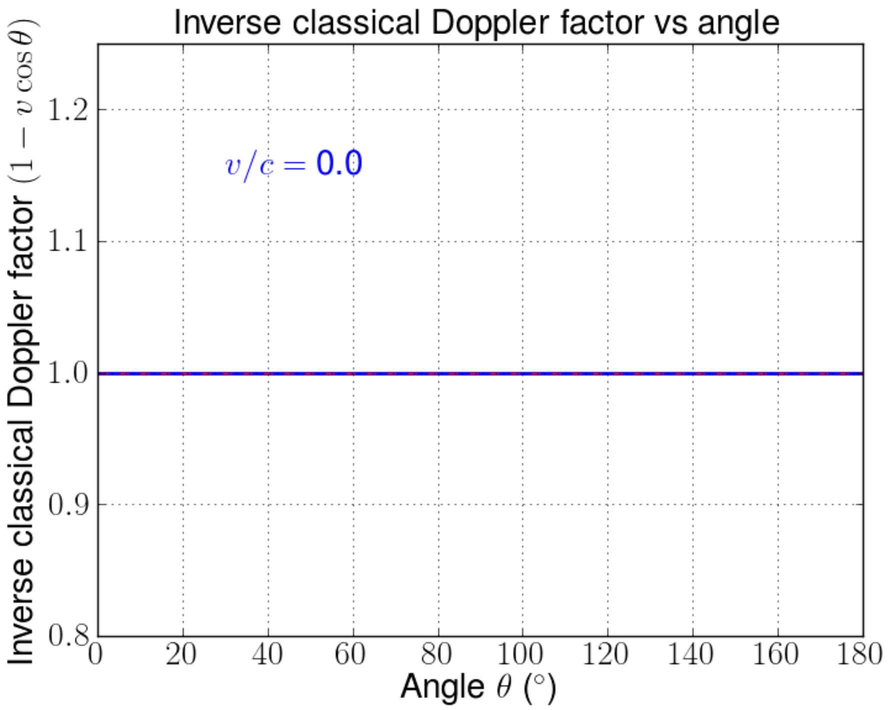


[Calvão (2013)]



[Calvão (2013)]

$L(t)$: emitted (source) power

$\mathcal{L}(t)$: received (detector) power

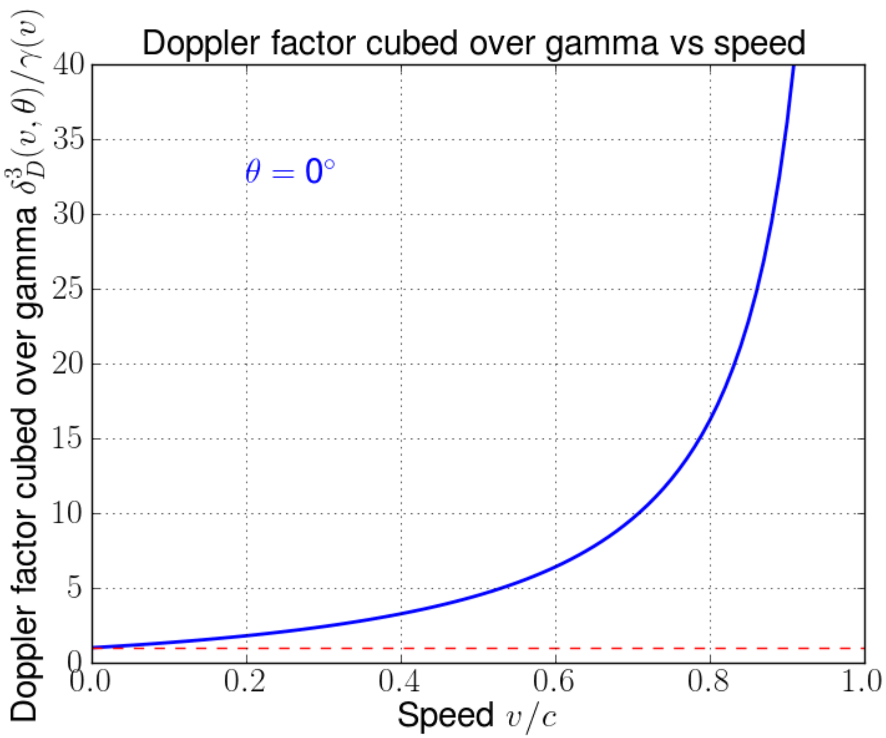
$$L = \mathcal{L} - v \oint \mathcal{L}(\hat{\mathbf{\Omega}}) \cos\theta \, d\Omega$$

isotropic case:

$$L = \mathcal{L}$$

IV.1. TRANSFORMATION LAWS

$$\bar{L}_\Omega = L_\Omega\, \gamma(v)\delta_D^3(v,\theta)$$



[Calvão (2013)]

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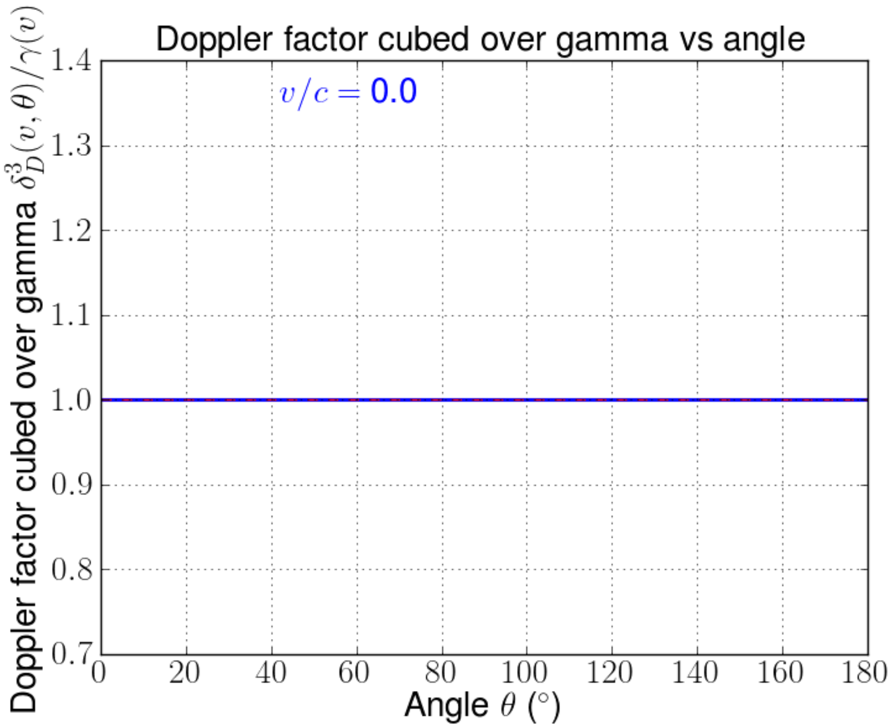

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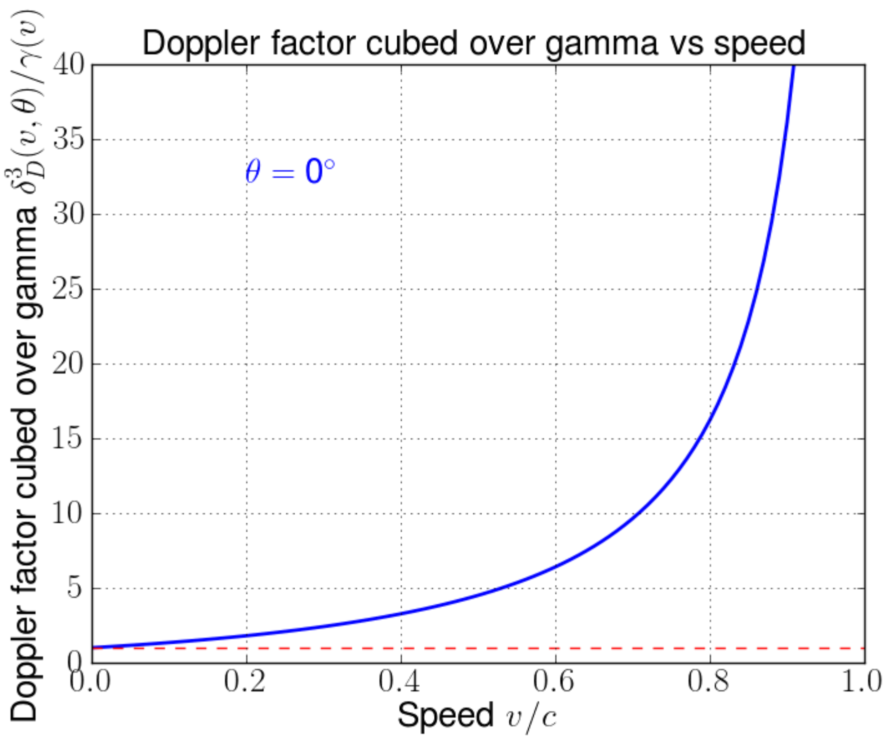
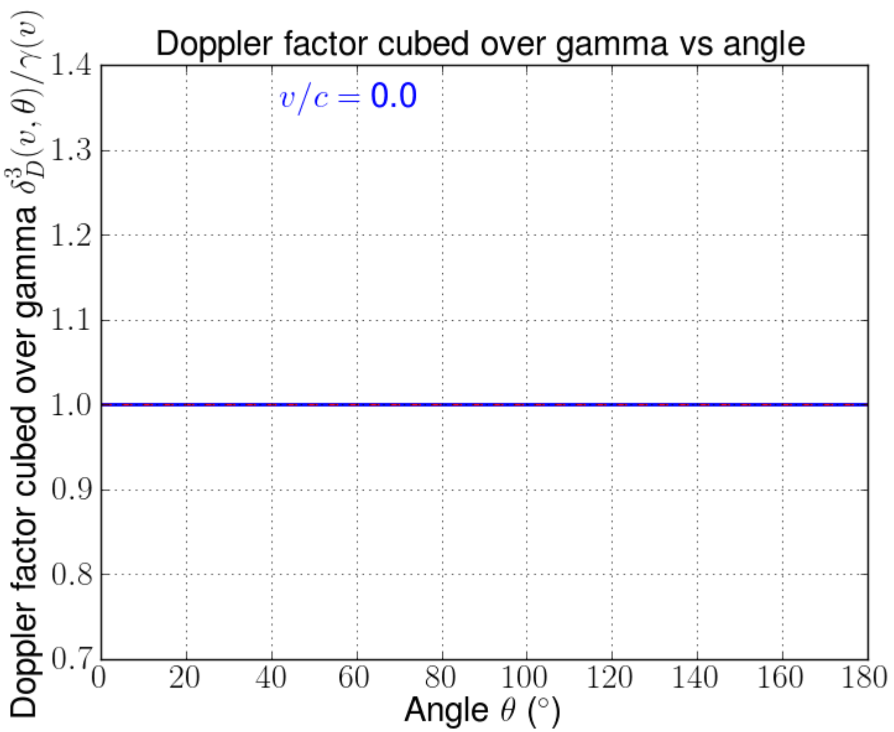


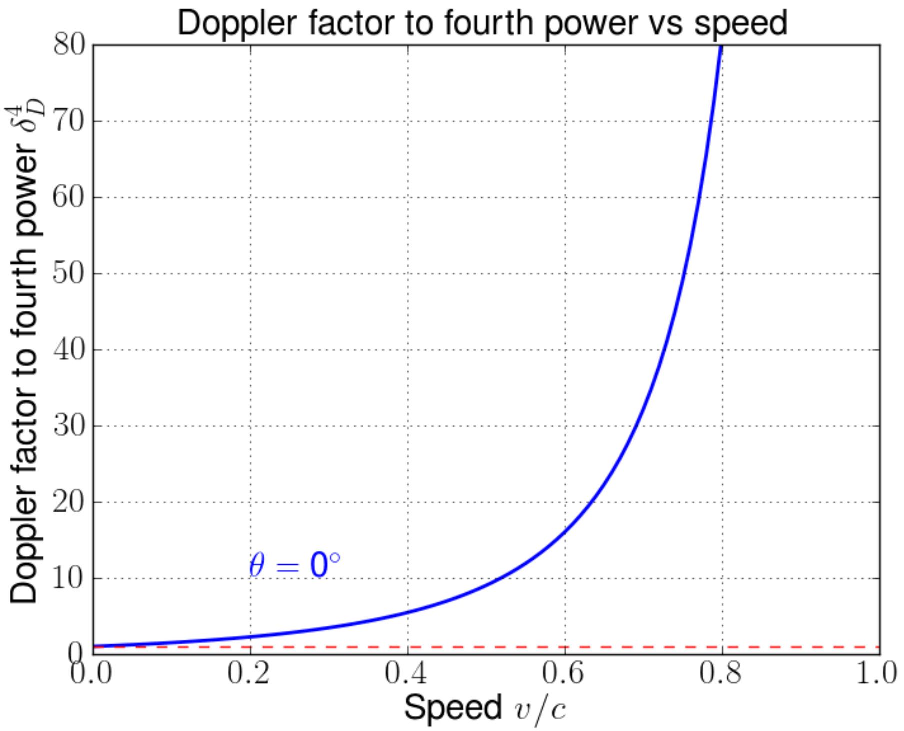
[Calvão (2013)]



[Calvão (2013)]

IV.1. TRANSFORMATION LAWS

$$\bar{\mathcal{L}}_\Omega = \mathcal{L}_\Omega\, \delta_D^4(v,\theta)$$



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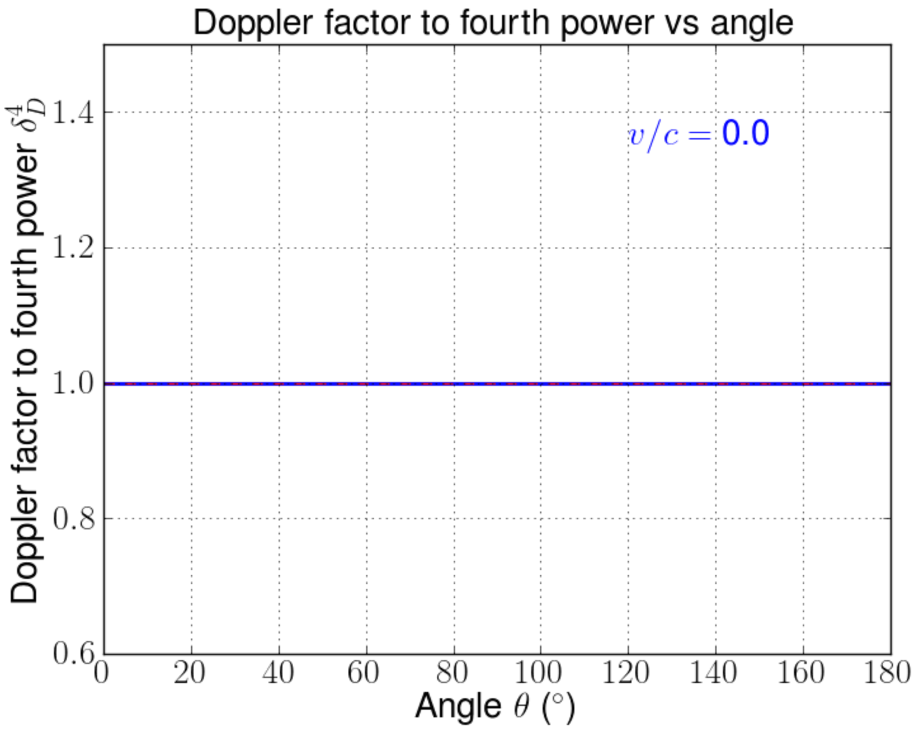

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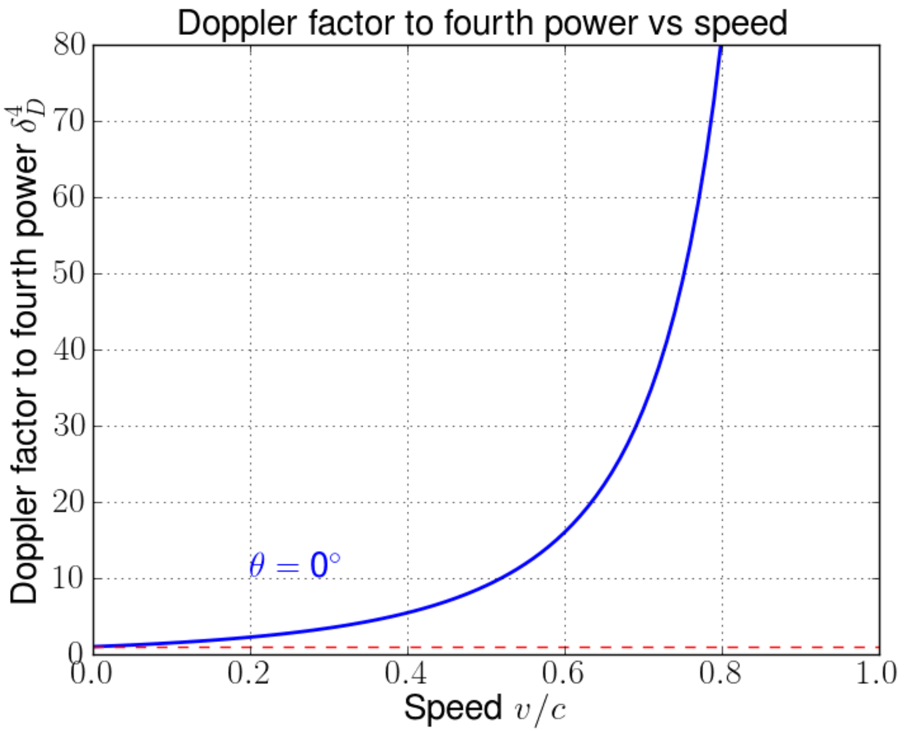

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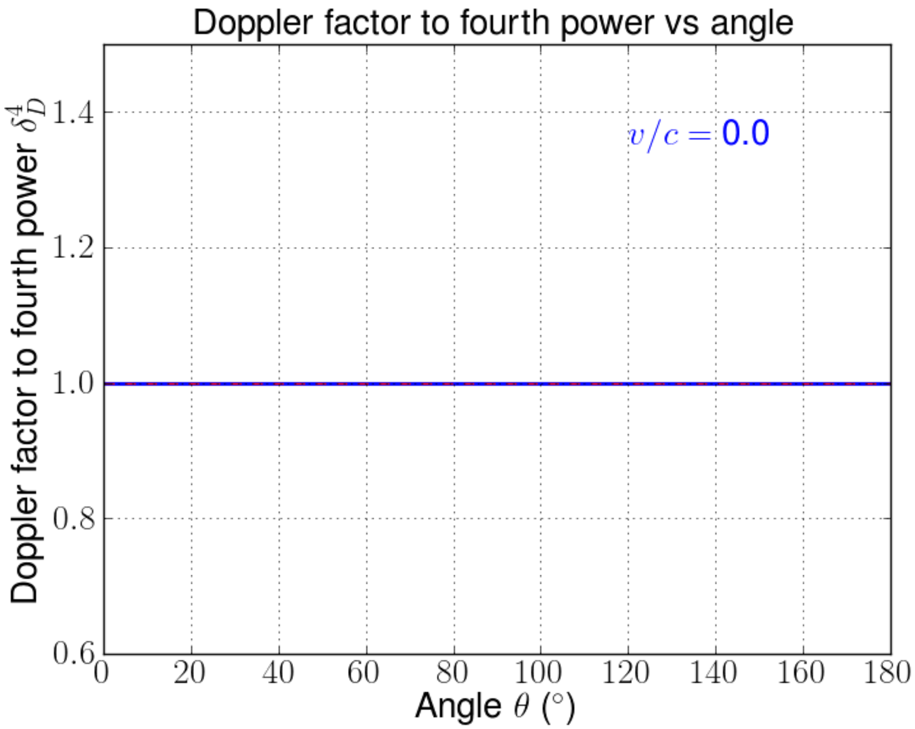

[Calvão (2013)]

IV.1. TRANSFORMATION LAWS

$$\bar{L} = L + v \oint L_\Omega(\hat{\mathbf{\Omega}}) \cos\theta \, d\Omega$$

$$\bar{\mathcal{L}} = \gamma^2 \left[\mathcal{L} + \oint \mathcal{L}_\Omega \left(2 + v\cos\theta\right) v\cos\theta \, d\Omega \right]$$

V. LUMINOSITY DISTANCES

V.1. DEFINITIONS

$$D_L := \sqrt{\frac{L}{4\pi f}}$$

$$D_{\mathcal{L}} := \sqrt{\frac{\mathcal{L}}{4\pi f}}$$

$$D_{L_\Omega} := \sqrt{\frac{L_\Omega}{f}}$$

$$D_{\mathcal{L}_\Omega} := \sqrt{\frac{\mathcal{L}_\Omega}{f}}$$

VI. CONCLUSION

MAIN RESULTS:

- We clarified the notions related to redshift, flux, luminosities and luminosity distances in a generic spacetime for arbitrary instantaneous observers, essentially without any approximation, not perturbatively!
- All Doppler effects can be interpreted as purely kinematic ones, arising from the use of non-parallely transported instantaneous observers
- In particular, in any spacetime the redshift along a given null geodesic can always be made to vanish
- Flux is essentially the norm of the energy flux vector from the energy-momentum tensor and its transformation law is trivially obtained
- There are several consistent notions of (angular) luminosity and consequent luminosity distances
- Differences due to peculiar motions and/or anisotropies do arise and are not negligible at all!!