Generalized Exponential f(R) Gravity: Steepness Control

Work in collaboration with Marcio O'Dwyer and Sérgio E. Jorás

Ioav Waga

Instituto de Física

Universidade Federal do Rio de Janeiro

Summary

We investigate a simple generalization of the metric exponential f(R) gravity theory that is cosmologically viable and compatible with solar-system tests of gravity. We show that, as compared with other viable f(R) theories, its dependence on the Ricci scalar R improves agreement with structure formation and alleviates fine-tuning.

What is causing the cosmic acceleration?

Main Possibilities

A new exotic component with negative pressure (DE) or modified gravity?

New Component

$$G_{\mu\nu} = \kappa T^{(m)}_{\mu\nu} + T_{\mu\nu}(\phi)$$

Modified Gravity

$$G_{\mu\nu} + L_{\mu\nu}(g_{\mu\nu}) = \kappa T^{(m)}_{\mu\nu}$$

Metric f(R) Gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}F(R) + \mathcal{L}_{mat}\right] \qquad F(R) = R + f(R)$$

- $f(R) \rightarrow$ simplest modification to the E-H Lagrangian ; in general $f(R, R^{\alpha\beta}R_{\alpha\beta}, R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}, \ldots)$
- $f(R)$ can be thought as a special case of a scalar-tensor theory (Brans-Dicke with $w_{BD}=0$).
- An accelerated expansion appears naturally in these models.
- Inflation can be curvature driven if $F(R) = R + \alpha R^2$. [Starobinsky (PLB 91,99,1980)]
- The same idea was explored by Capozzielo&Cardone (IJMP D12, 1963, 2003) and Carrol et al. (PRD 043528, 2004) for a late time acceleration. They considered $F(R) = R - \alpha R^{-n}$.
- The above $F(R)$ theory doesn't present a regular MDE (a $\propto$ $t^{1/2}$ and not a $\propto$ $t^{2/3}$) [Amendola et al. , PRD 75, 083504, 2007]. $\Rightarrow$ Inverse power-law $F(R)$ are incompatible with structure formation.

"Viable" f(R) theories

Starobinsky [JETPLett, 86, 157, (2007)]

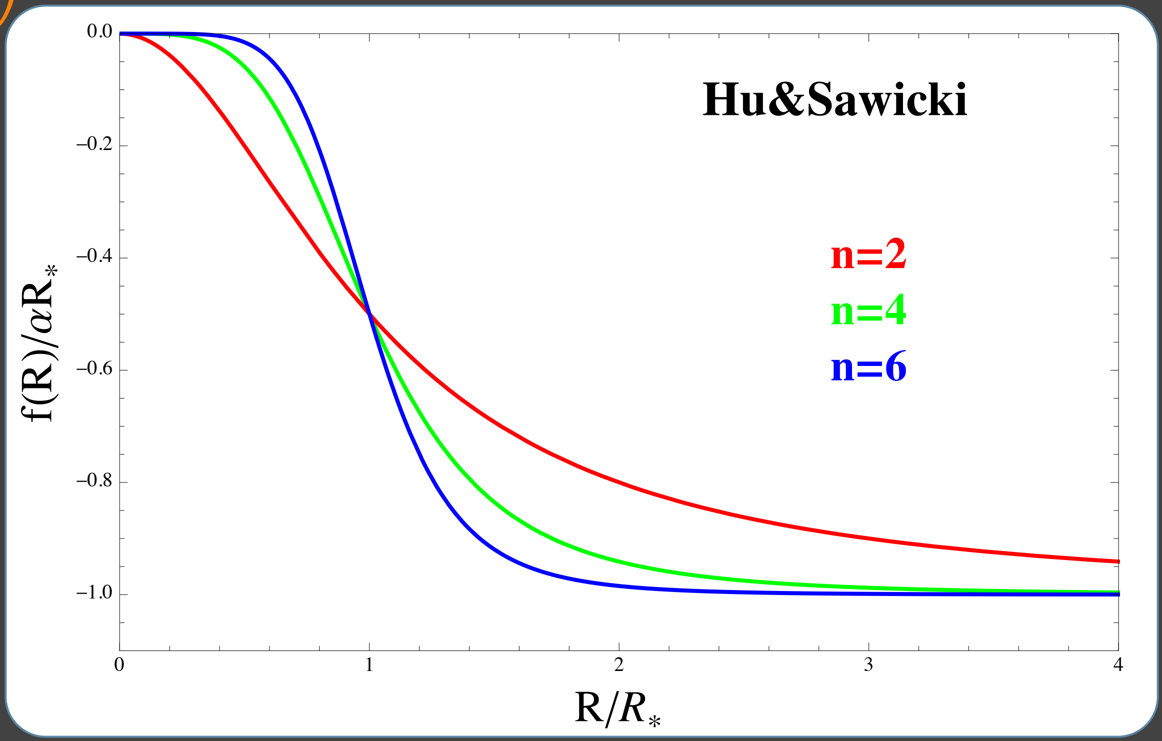
$$F(R) = R - \alpha R_* \left(1 - \frac{1}{\left[1+\left(\frac{R}{R_*}\right)^2\right]^n}\right)$$

Appleby and Battye - arXiv:0705.3199

$$F(R) = R + \frac{R_*}{2}\left(Log[Cosh[\frac{R}{R_*}] - Tanh[b]] - \frac{R}{R_*}\right)$$

Hu & Sawicki [PRD 76, 064004 (2007)]

$$F(R) = R - \alpha R_* \left(1 - \frac{1}{1+\left(\frac{R}{R_*}\right)^n}\right)$$



Exponential Gravity

Cognola et al (2008), Linder (2009)

$$F(R) = R - \alpha R_*(1 - e^{-R/R_*})$$

$F(R) \simeq R - \alpha R_*$ for $R \gg R_*$

$f(0) = 0 \Rightarrow$ desapearing cosmological constant

Generalized Exponential Gravity

$$F(R) = R - \frac{\alpha R_*}{n}\gamma\left(\frac{1}{n}, (\frac{R}{R_*})^n\right)$$

α, n and R_* are free positive parameters.

$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$ → lower incomplete gamma function

if $n = 1$ we obtain exponential gravity

$f(R) = -\alpha R_* (1 - e^{-R/R_*})$

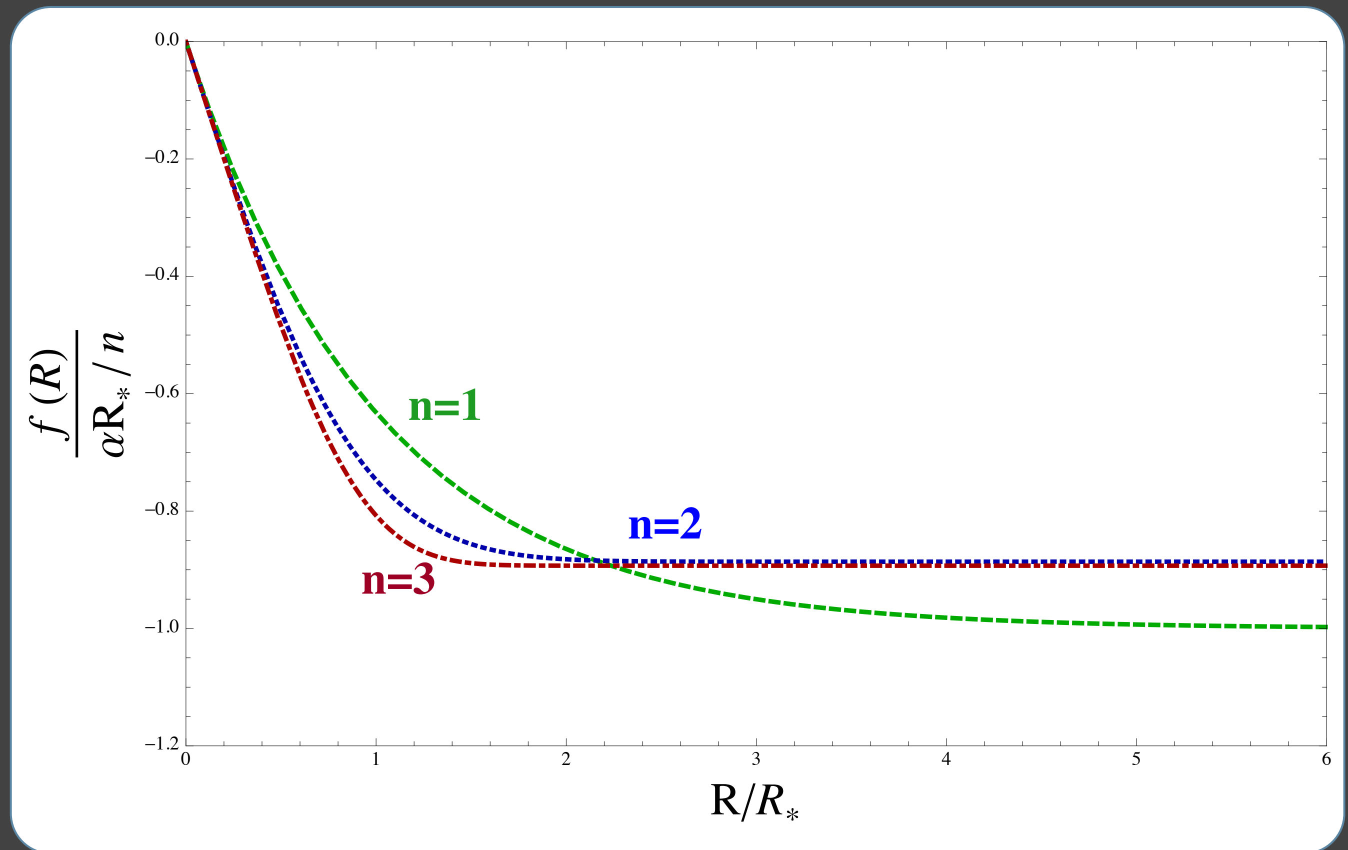
$n = 2$

$f(R) = -\alpha R_* \frac{\sqrt{\pi}}{2} \mathrm{Erf}(\frac{R}{R_*})$

$f_R \coloneqq \frac{df}{dR}$; $f_{RR} \coloneqq \frac{d^2 f}{dR^2}$

$f_R = -\alpha e^{-(R/R_*)^n}$

$f_{RR} = \frac{n\alpha}{R_*} e^{-(R/R_*)^n} (\frac{R}{R_*})^{n-1}$



Stability & Viability Conditions

Pogosian & Silvestri PRD 77, 023503 ,2008

* $f_{RR} > 0$ (no tachyons)

* $1 + f_R > 0$ ($G_{eff} = \dfrac{G}{1+f_R}$ doesn't change sign; no ghosts)

* $\lim_{R\to\infty} \dfrac{f}{R} = 0$ and $\lim_{R\to\infty} f_R = 0$ (GR is recovered at early times)

* $|f_R|$ is small at recent epochs (to satisfy solar and galactic scale constraints)

What about Cosmology?

Amendola et al PRD 75, 083504, 2007

Viable cosmology:

a)Start with a RD universe

b)Have a sadle point MD phase

c)Have a final accelerated atractor

$$m(r \approx -1) \approx 0^+ \quad \text{and} \quad \frac{dm}{dr}(r \approx -1) > -1$$

necessary condition

$$0 < m(r \approx -2) \leq 1$$

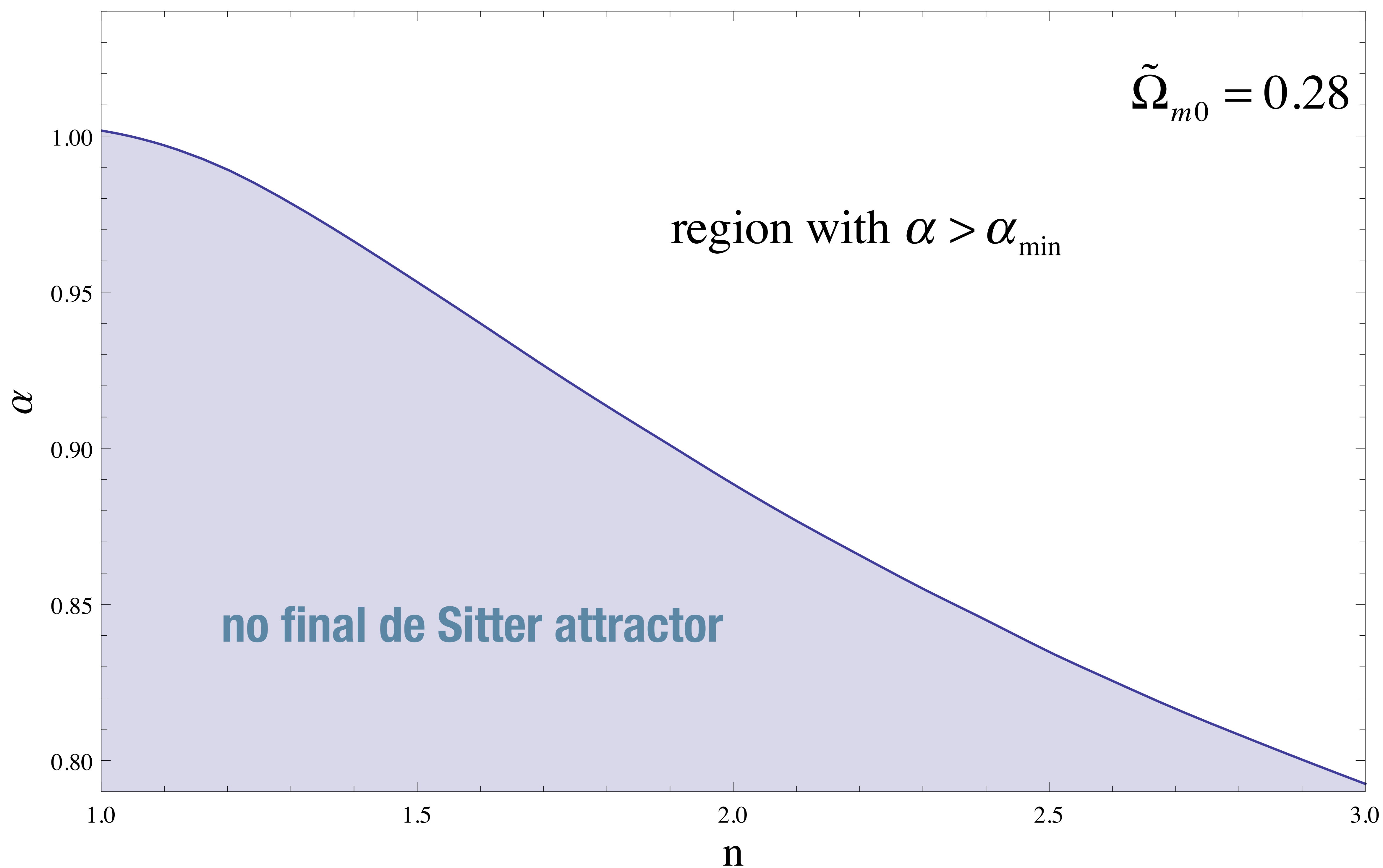
$$m := \frac{Rf_{RR}}{1+f_R} \qquad r := -\frac{R(1+f_R)}{R+f}$$

γ Gravity

$$F(R) = R - \frac{\alpha R_*}{n}\gamma\left(\frac{1}{n}, (\frac{R}{R_*})^n\right)$$

- The theory above can satisfy all the stability conditions.
- It can also satisfy the cosmological viability criteria.
- For fixed n and R_* there is a minimum value (α_{min}) of the parameter α, such that there is a final de Sitter attractor.

α_{min} x n



$\tilde{\Omega}_{m0} = 0.28$
region with $\alpha > \alpha_{min}$
no final de Sitter attractor
1.00
0.95
0.90
0.85
0.80
α
1.0
1.5
2.0
2.5
3.0
n

Modified Einstein equations

$$f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + \left(\Box f_R - \frac{1}{2} f\right) g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

For a homogeneous Universe filled with matter energy density $\bar{\rho}_m$ and radiation energy density $\bar{\rho}_r$ we use the above equation to get the modified Friedman equation:

$$H^2 + \frac{f}{6} - f_R \left(H^2 + HH'\right) + H^2 f_{RR} R' = \frac{8\pi G}{3} \bar{\rho}$$

$$' := d/dy \quad (y = ln a)$$

$$\bar{\rho} = (\bar{\rho}_m + \bar{\rho}_r)$$

$$R = 12H^2 + 6HH'$$

Introducing the following variables

$$x_1(y) = \frac{H^2}{m^2} - e^{-3y} - d - a_{eq}\, e^{-4y}$$

$$x_2(y) = \frac{R}{m^2} - 3e^{-3y} - 12\left(d + x_1(y)\right)$$

$$d := \frac{\alpha R_* \Gamma(1/n)}{6nm^2}$$

$$a_{eq} = \bar{\rho}_{r0}/\bar{\rho}_{m0} \simeq 2.9 \times 10^{-4}$$

$$m^2 := \frac{8\pi G}{3} \bar{\rho}_0 = \Omega_{m0} H_0^2$$

$$x_1'(y) = \frac{x_2(y)}{3}$$
$$x_2'(y) = \frac{R'}{m^2} + 9e^{-3y} - 4x_2(y)$$

Where

$$\frac{R'}{m^2} = \frac{e^{-3y} + a_{eq}\, e^{-4y}}{H^2 f_{RR}} - \frac{1}{m^2 f_{RR}} \left(1 + \frac{f}{6H^2}\right) + \frac{f_R}{m^2 f_{RR}} \left(\frac{R}{6H^2} - 1\right)$$

Each model is characterised by fixed values of the parameters α, n and R_*, which can be written as,

$$R_* = \frac{6nm^2}{\alpha\Gamma(1/n)} \frac{1 - \tilde{\Omega}_{m0}}{\tilde{\Omega}_{m0}} \quad \rightarrow d = (1 - \tilde{\Omega}_{m0})/\tilde{\Omega}_{m0}$$

$$\tilde{\Omega}_{m0} = 0.28$$

Here $\tilde{\Omega}_{m0}$ represents the present value of the matter density parameter that a ΛCDM model would have, if it had the same matter density $\bar{\rho}_{m0}$ as the modified gravity $f(R)$ model. As a consequence, if $\tilde{H}_0$ is the Hubble constant in the reference ΛCDM model, we should have $\tilde{\Omega}_{m0}\tilde{H}_0^2 = \Omega_{m0}H_0^2$.

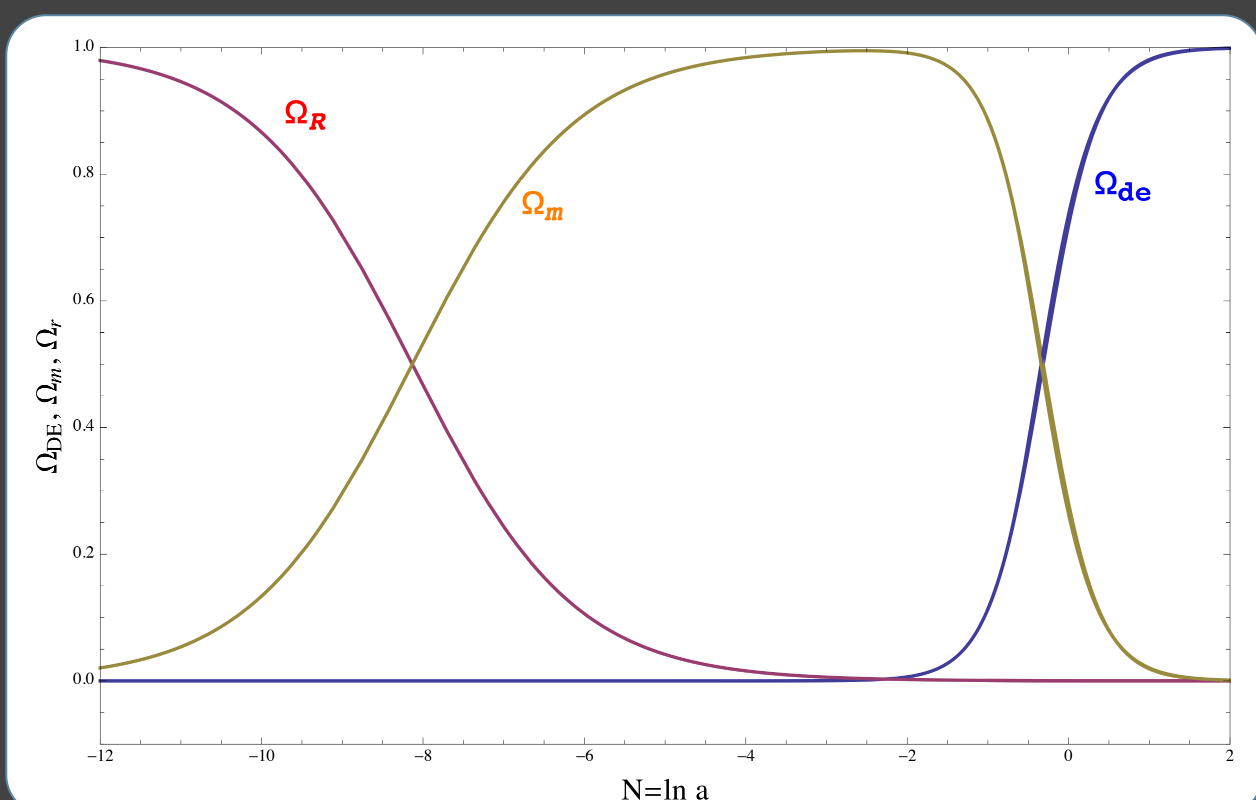
With $x_1(y)$ and $x_2(y)$ several quantities can be obtained.

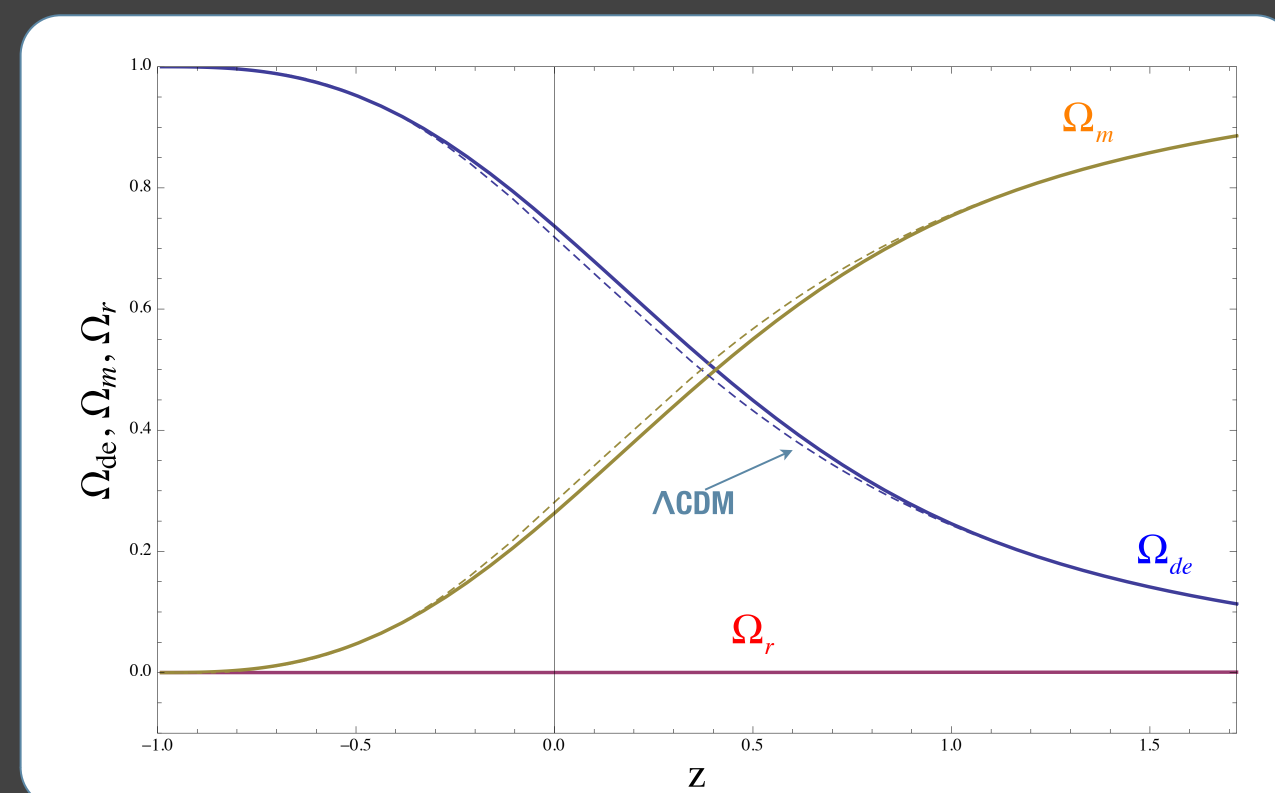
$$w_{de} = -1 - \frac{1}{9}\frac{x_2}{x_1+d}$$

$$\Omega_m = 1 - \Omega_{de} - \Omega_r$$

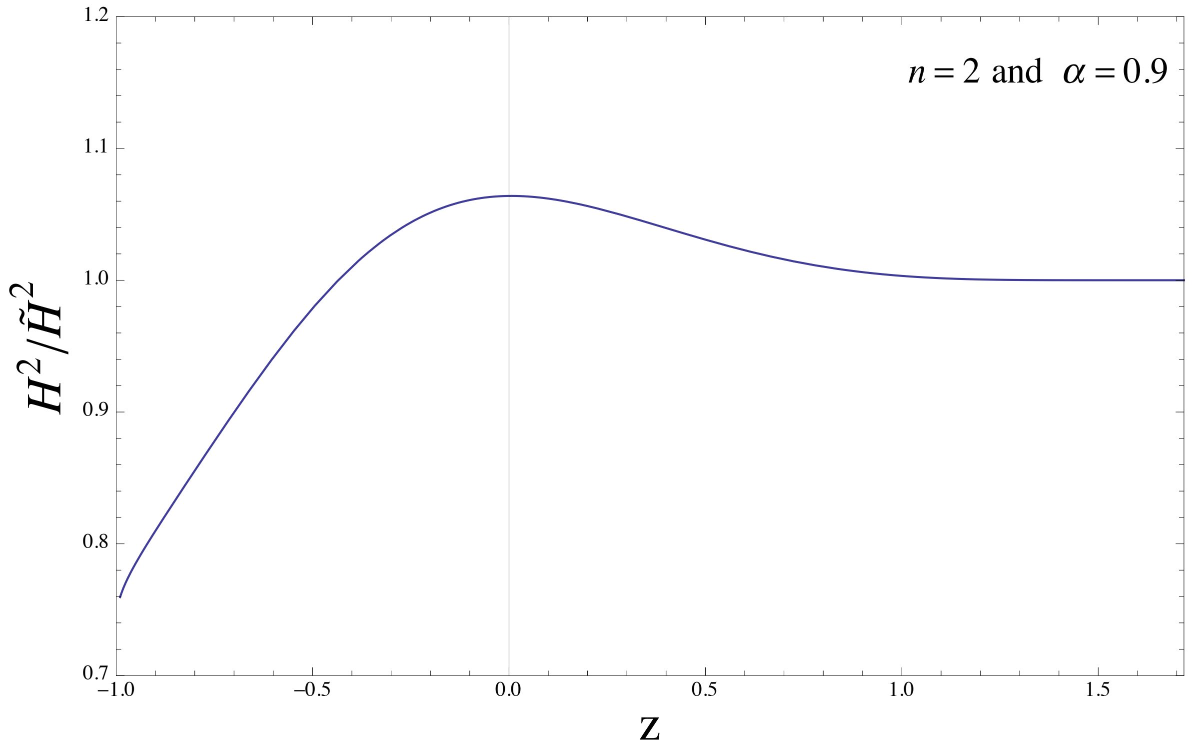
$$\Omega_{de}(y) = \frac{x_1+d}{d+x_1+e^{-3y}+a_{eq}e^{-4y}}$$

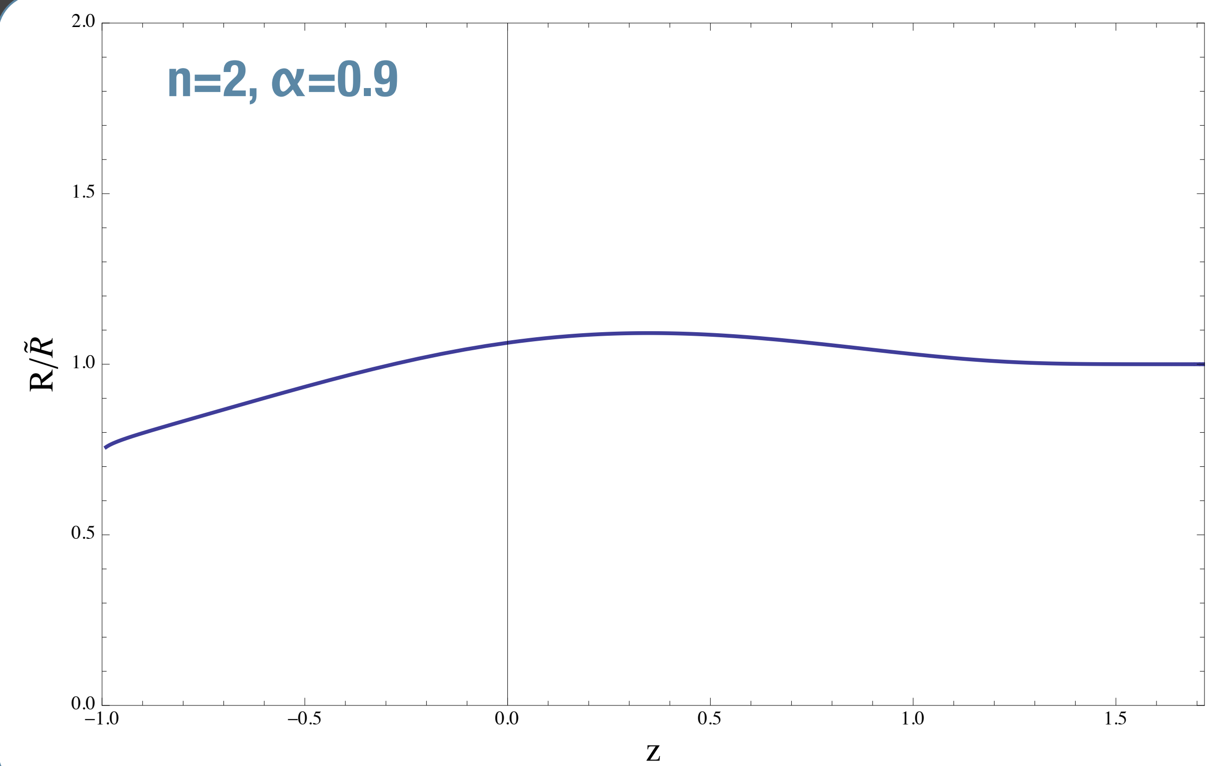
$$\Omega_r(y) = \frac{a_{eq}e^{-4y}}{d+x_1+e^{-3y}+a_{eq}e^{-4y}}$$

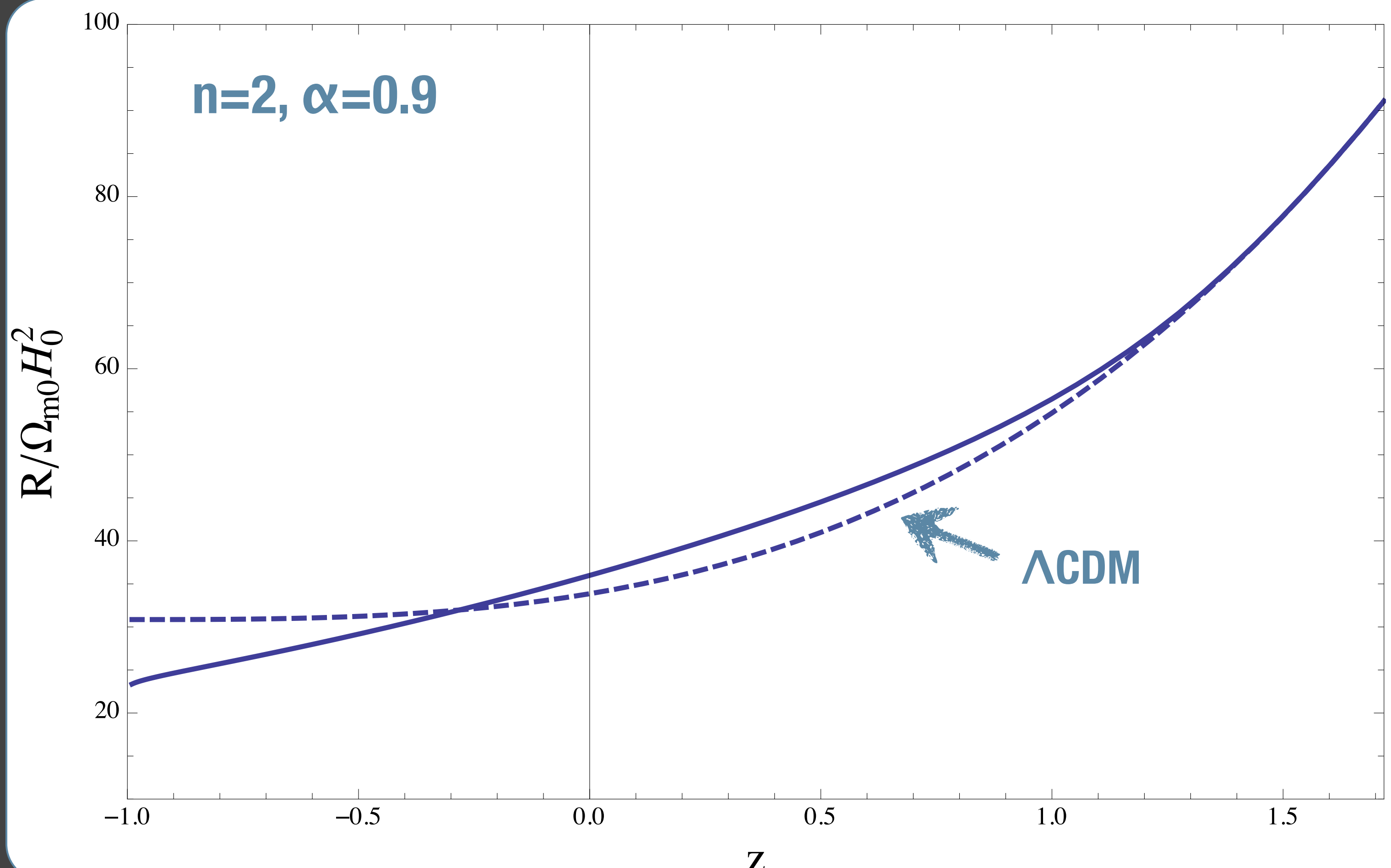




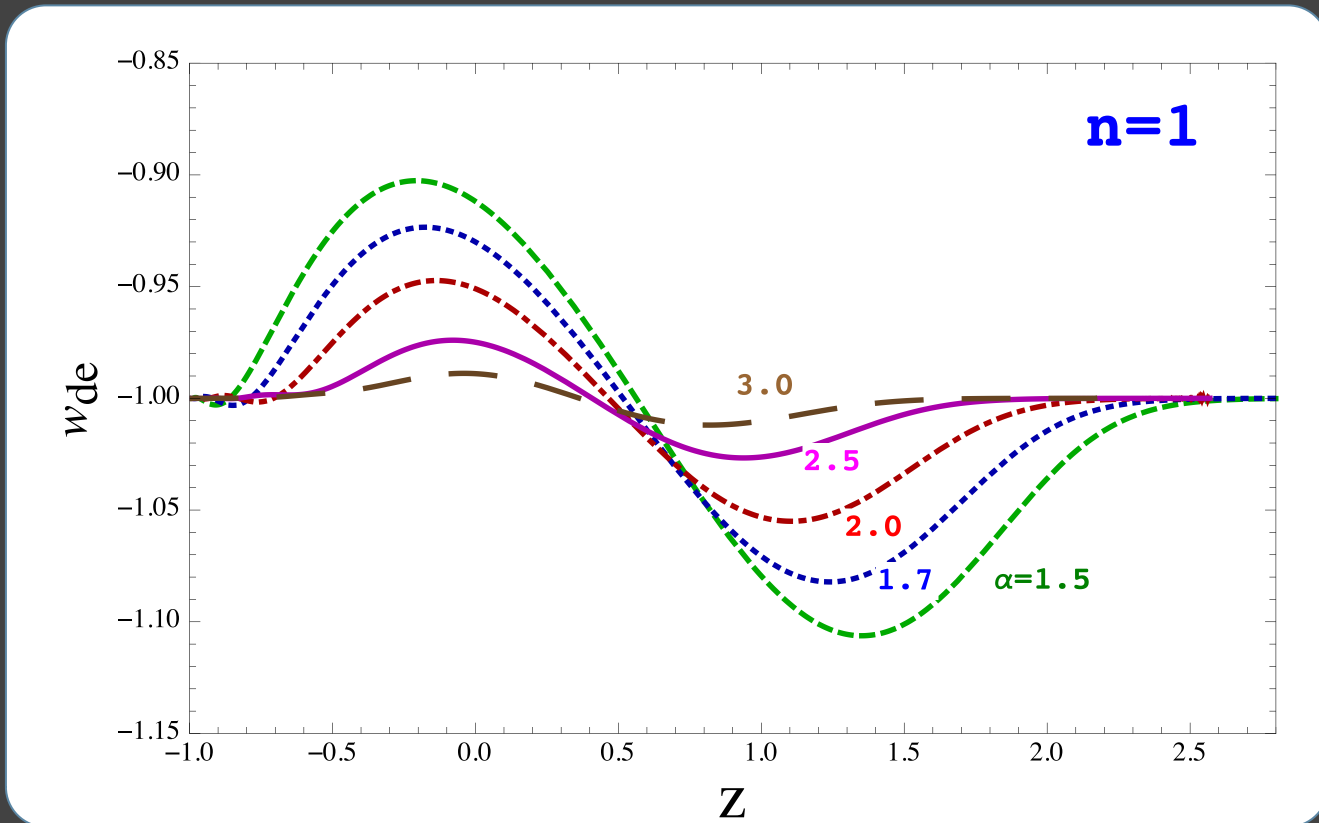
$n = 2$ and $\alpha = 0.9$

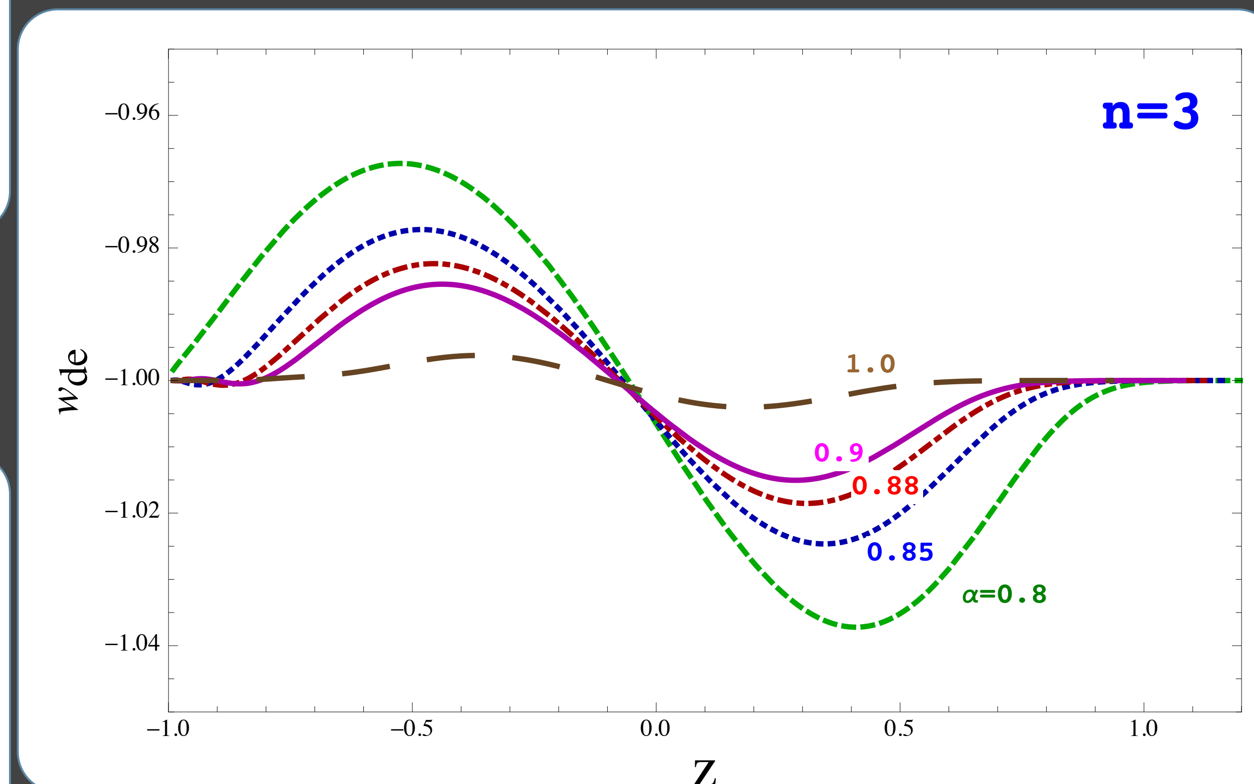

$n = 2$ and $\alpha = 0.9$
$H^2/\tilde{H}^2$
Z
1.2
1.1
1.0
0.9
0.8
0.7
−1.0
−0.5
0.0
0.5
1.0
1.5

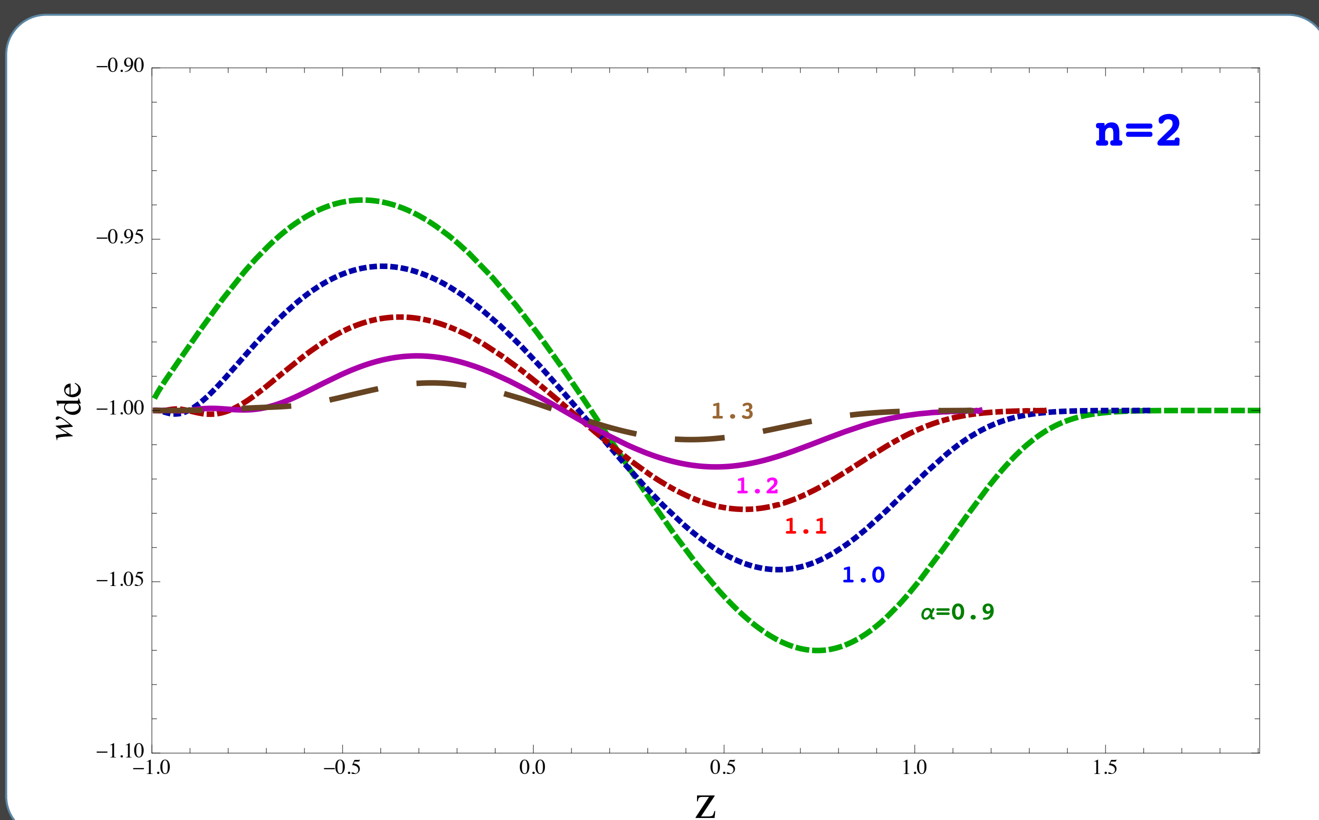

n=2, α=0.9
$R/\tilde{R}$
z
2.0
1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
1.5


n=2, α=0.9
ΛCDM
$R/\Omega_{m0}H_0^2$
Z
100
80
60
40
20
−1.0
−0.5
0.0
0.5
1.0
1.5

Equation of state parameter







Local Tests

It can be shown that solar-system constraints imply independent of the form of the f(R). (Hu&Sawicki)

$$\Rightarrow | f_{Rg} | < 4.9\times 10^{-11}$$

We have

$$| f_{R_g} | = | f_{R_0} | e^{[(\frac{R_0}{R_*})^n - (\frac{R_g}{R_*})^n]}$$

$$f_{Rg} = f_R(R = 8\pi G\rho_g)$$

$$\rho_g \sim 10^{-24} g / cm^3$$

$$R_* = \frac{1-\Omega_{m0}}{\Omega_{m0}} \frac{6n}{\alpha\Gamma(1/n)} (8\pi G\bar{\rho}_0)$$

$$\frac{R_g}{R_*} = \frac{\Omega_{m0}\alpha\Gamma(1/n)}{6n(1-\Omega_{m0})} \frac{\rho_g}{\bar{\rho}_0}$$

$$\bar{\rho}_0 = \Omega_{m0}\rho_c = \Omega_{m0}(1.9\times 10^{-29} h^2 g / cm^3)$$

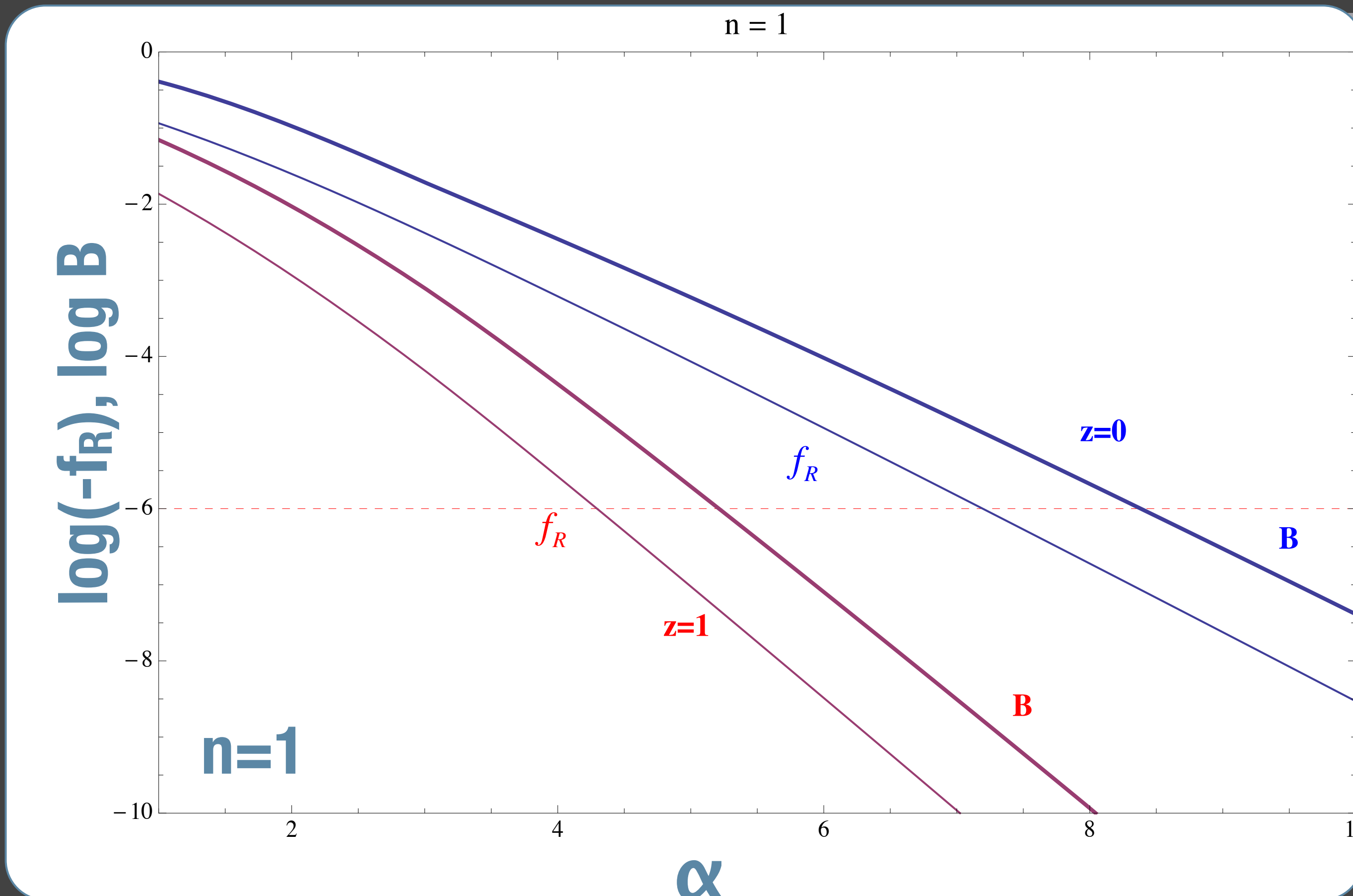
$$| f_{R_0} | < 4.9\times 10^{-11} e^{[(\frac{R_g}{R_*})^n]} > 10^{10^5} >> 1$$

Galaxy to cosmology

(Hu&Sawicki)

$$\Rightarrow | f_{R_0} | < 10^{-6}$$

Depends on when the galactic halo formed and the density profiles of the structures in which the galaxy is embedded.

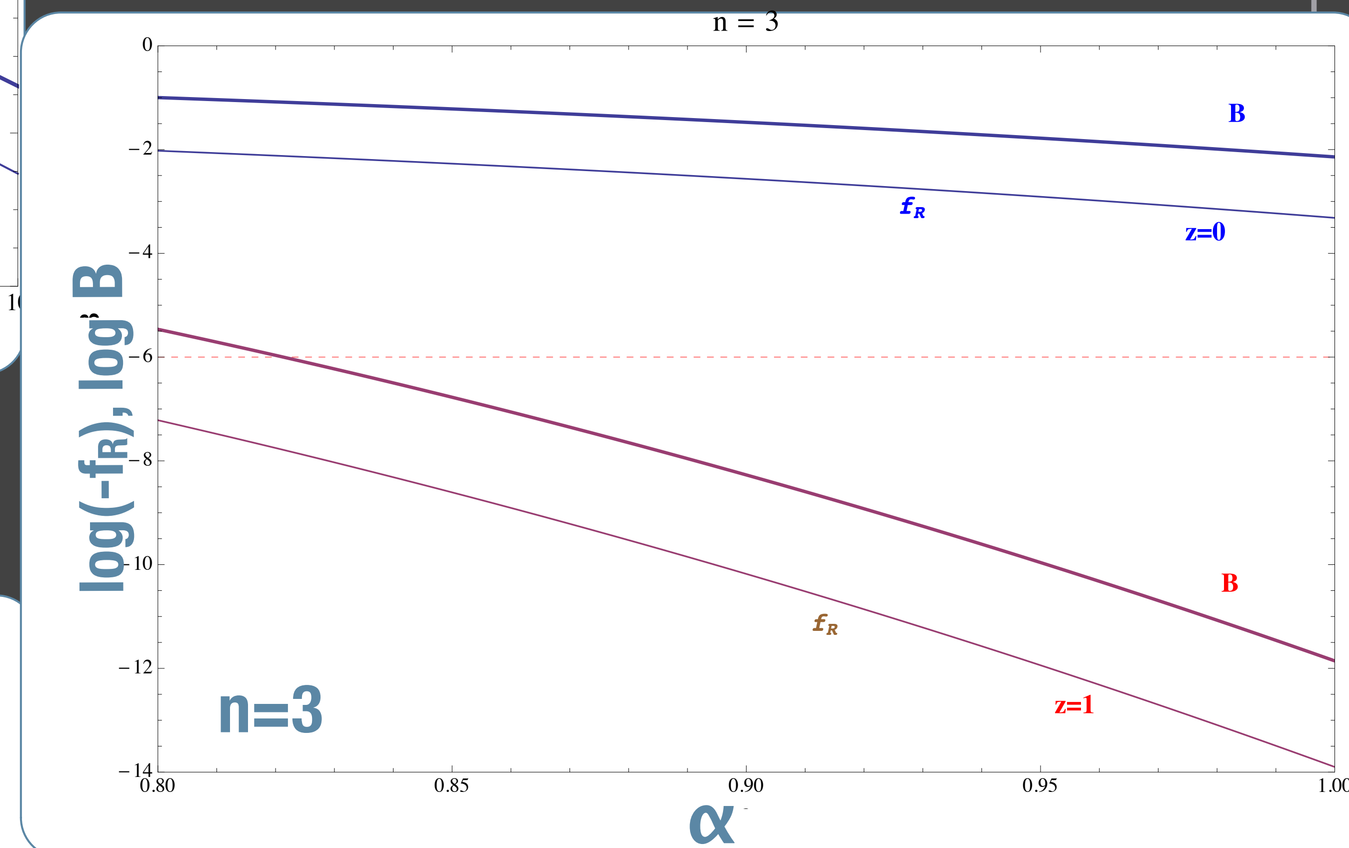


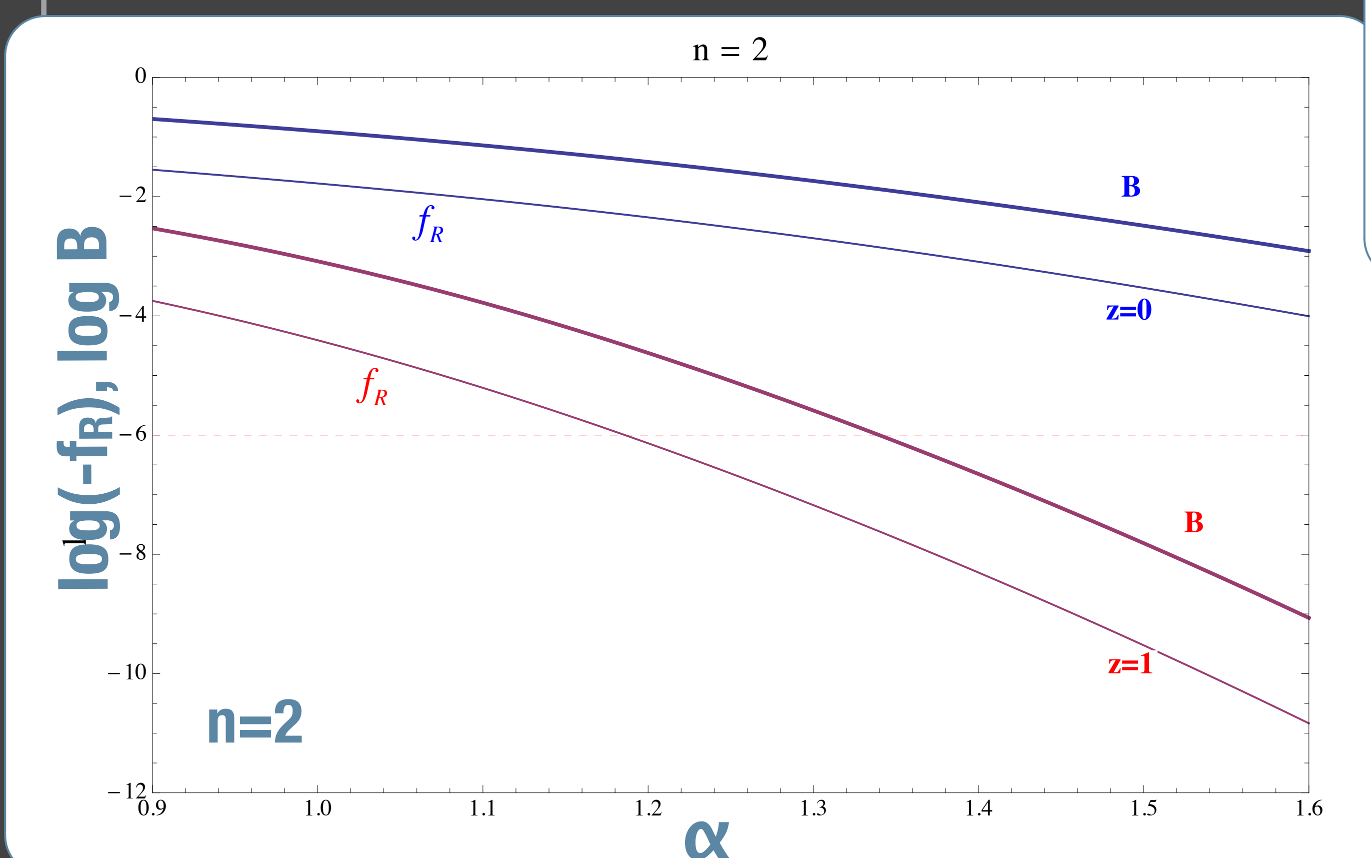
$$f_R = -\alpha e^{-(R/R_*)^n}$$

$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

$$k_c \sim aHB^{-1/2}$$

$$k>k_c \Rightarrow \gamma \simeq 1/2$$





Linear $(k < 0.1hMpc^{-1})$ perturbation growth is affected for $B \gtrsim 10^{-5}$

Evolution of Matter Density Perturbations

Here we are interested in the growth of cosmological matter density perturbations in the subhorizon regime . For $|f_R| << 1$, we have (see for instance P. Zhang (2006), Pogosian&Silvestre (2008), de la Cruz Dombriz et al. (2008))

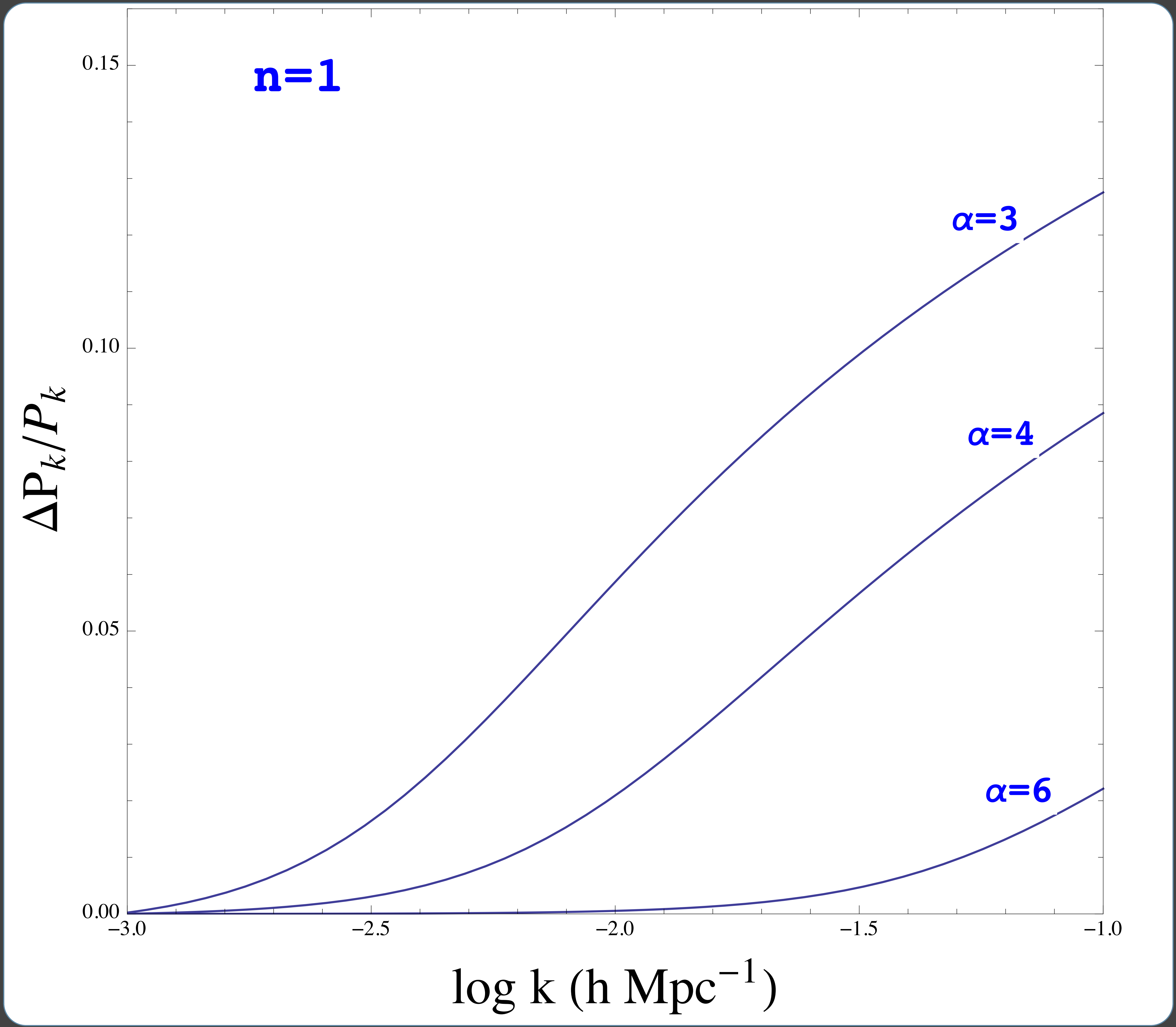
$$\delta^{''} + \delta^{'}\left(\frac{3}{a} + \frac{H^{'}}{H}\right) - \frac{\delta}{a^2}\frac{1-2Q}{2-3Q}\frac{3H_0^2\Omega_{m0}}{a^3H^2(1+f_R)} = 0$$

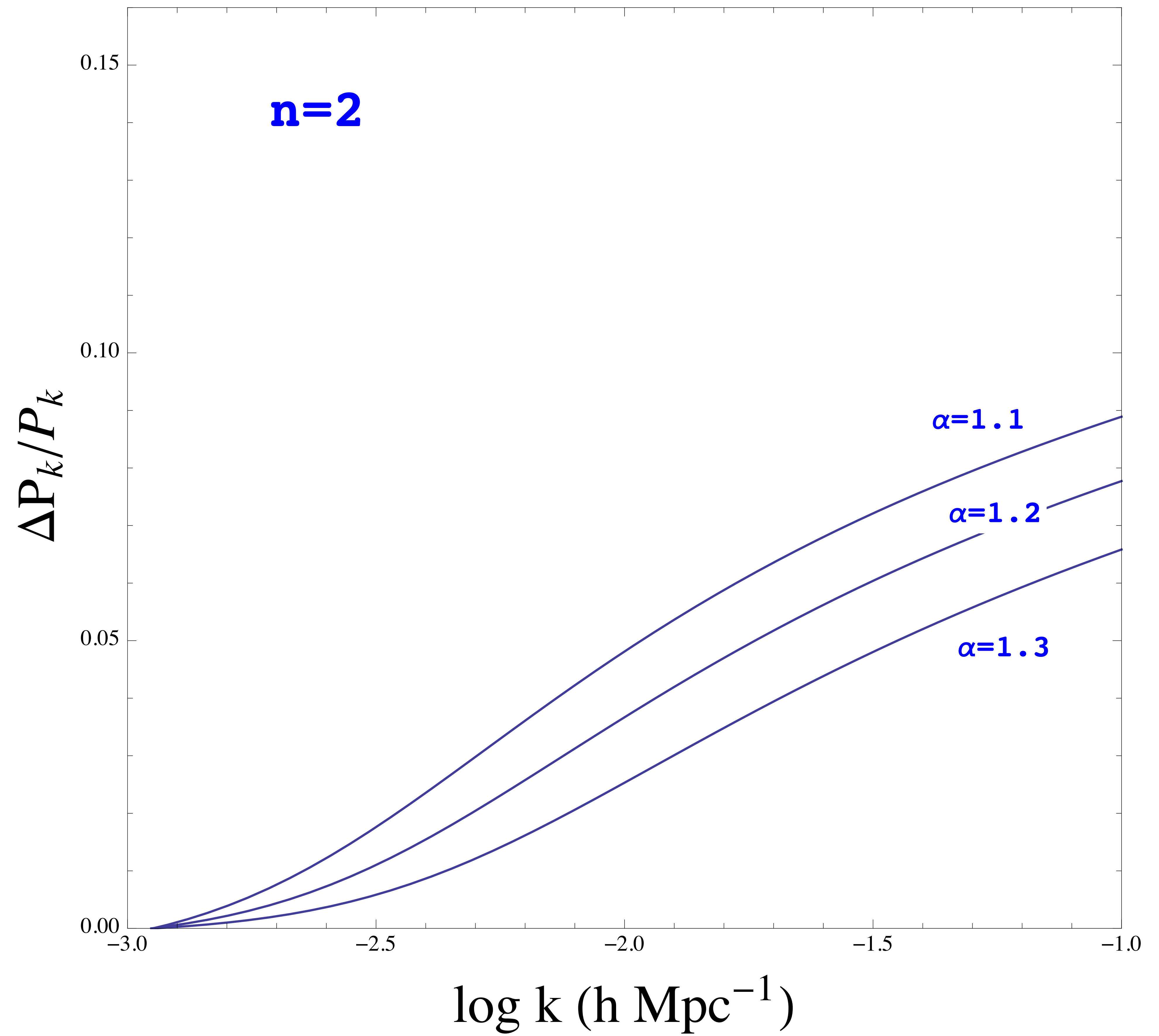
$$Q(k,a) := -\frac{2f_{RR}c^2k^2}{(1+f_R)a^2} \qquad {}' := \frac{d}{da}$$

In GR f_R=0, Q=0, there is no scale dependence in the linear regime.
For wCDM the growing mode is given by (Silveira&Waga (1994))

$$\delta(z) = \frac{1}{1+z}\,{}_2F_1\left[-\frac{1}{3w}, \frac{w-1}{2w}, 1-\frac{5}{6w}, -(1+z)^{3w}\frac{1-\Omega_{m0}}{\Omega_{m0}}\right]$$

We calculated numerically the growing mode for the f(R) theory and obtained the fractional change in the matter power spectrum P(k) relative to ΛCDM.


n=1
α=3
α=4
α=6
0.15
0.10
0.05
0.00
−3.0
−2.5
−2.0
−1.5
−1.0
ΔP_k/P_k
log k (h Mpc^−1)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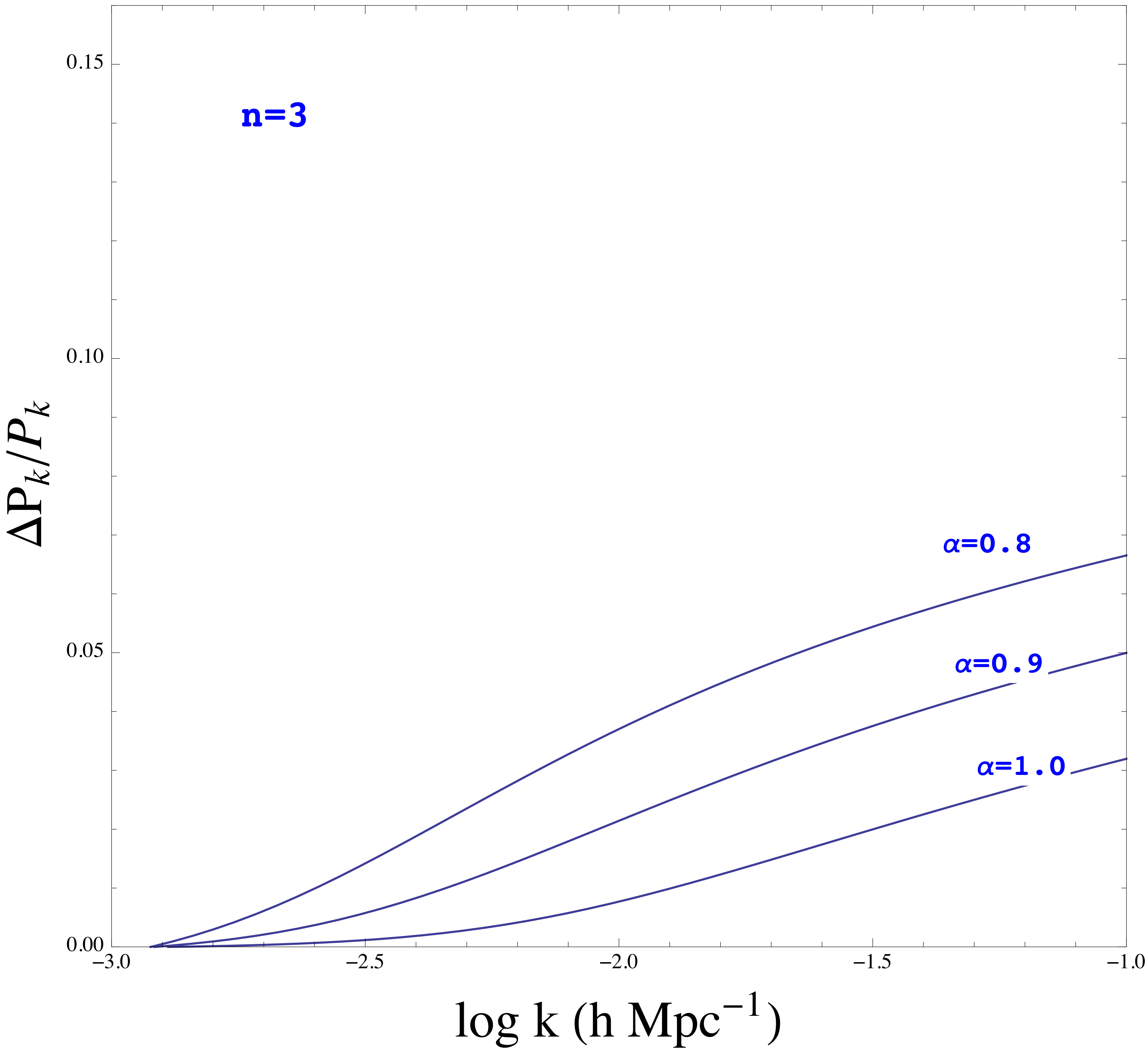
α=1.3
0.15
0.10
0.05
0.00
−3.0
−2.5
−2.0
−1.5
−1.0
ΔP_k/P_k
log k (h Mpc^−1)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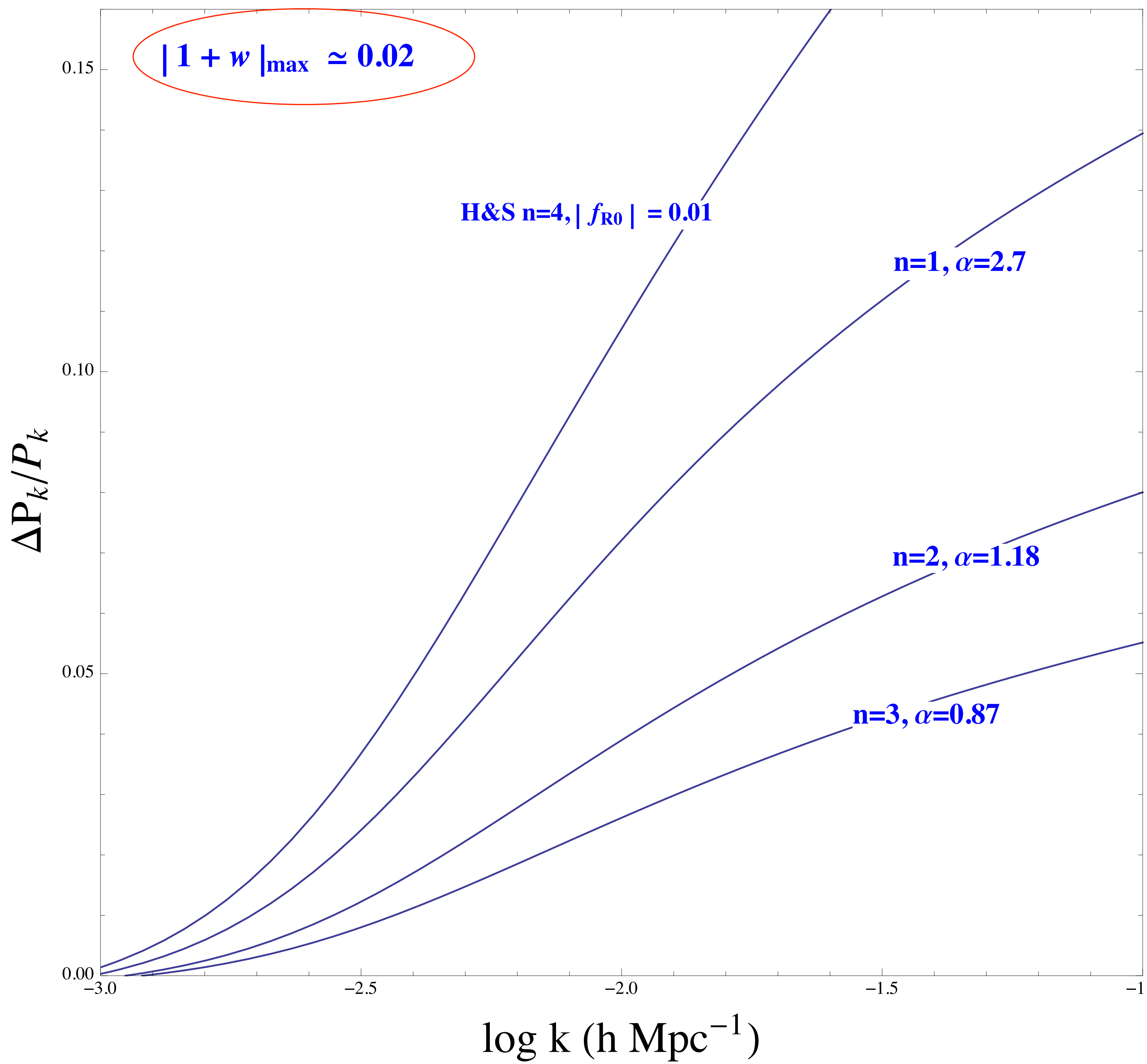
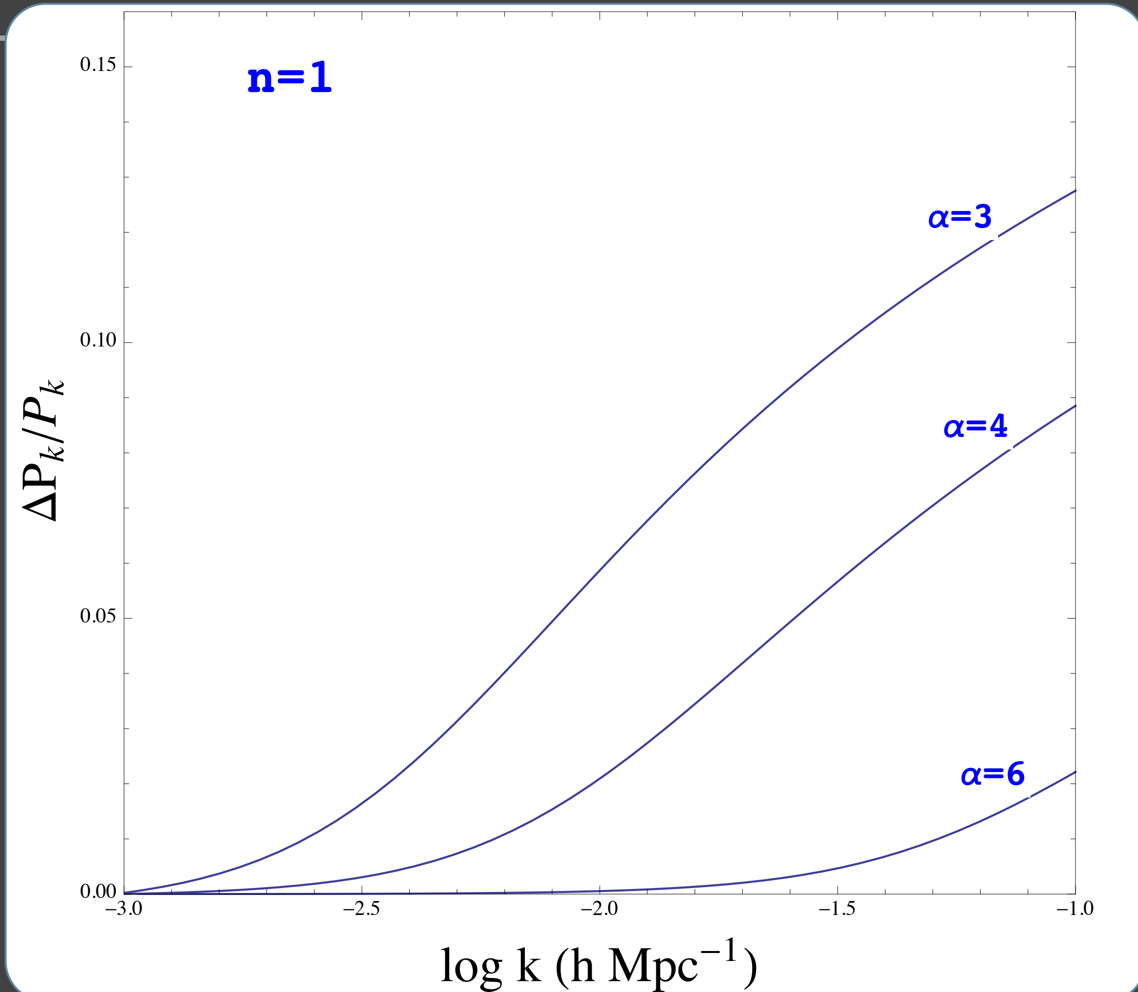
α=0.9
α=1.0
0.15
0.10
0.05
0.00
−3.0
−2.5
−2.0
−1.5
−1.0
$\Delta P_k / P_k$
log k (h Mpc^{-1})



Wednesday, May 15, 13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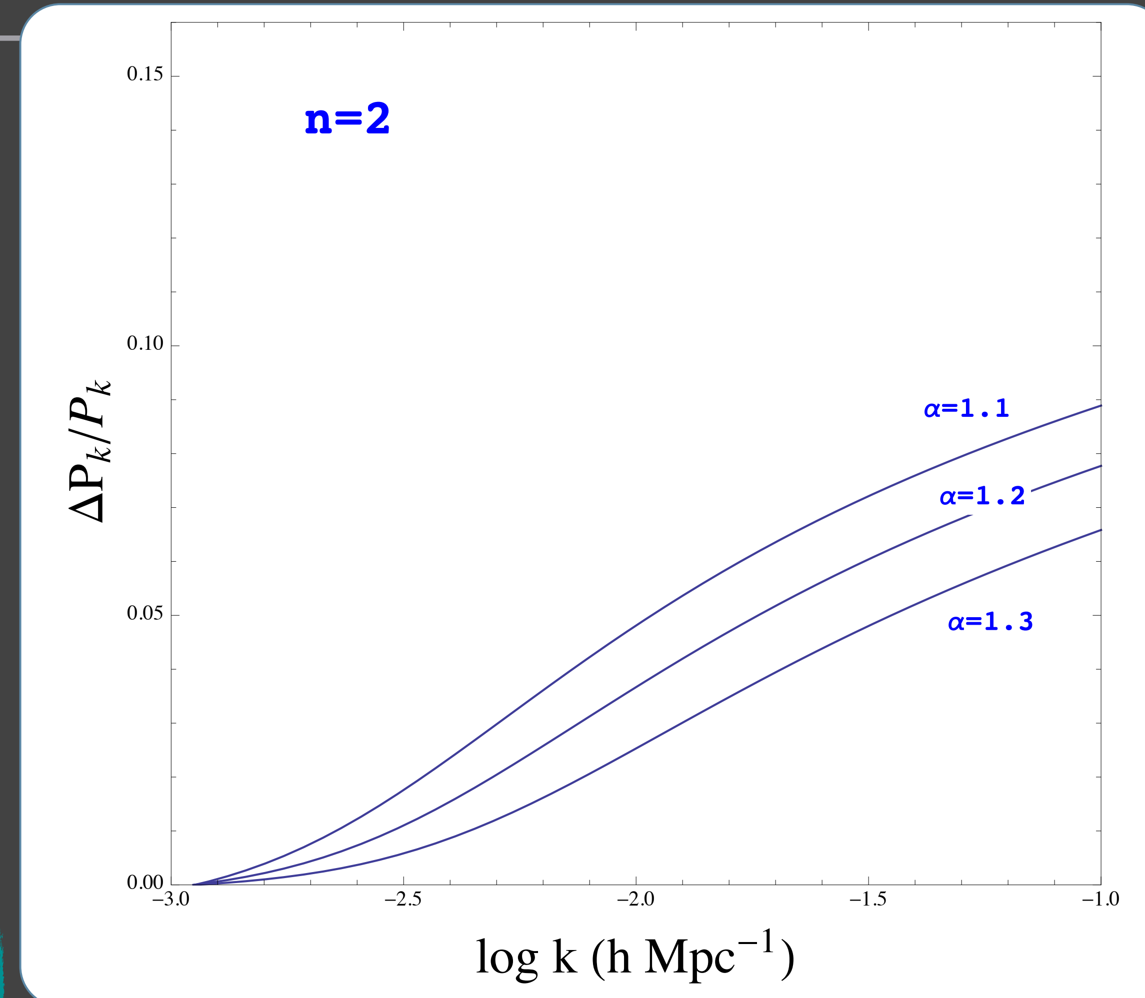
α=3
α=4
α=6
ΔP_k/P_k
0.00
0.05
0.10
0.15
−3.0
−2.5
−2.0
−1.5
−1.0
log k (h Mpc⁻¹)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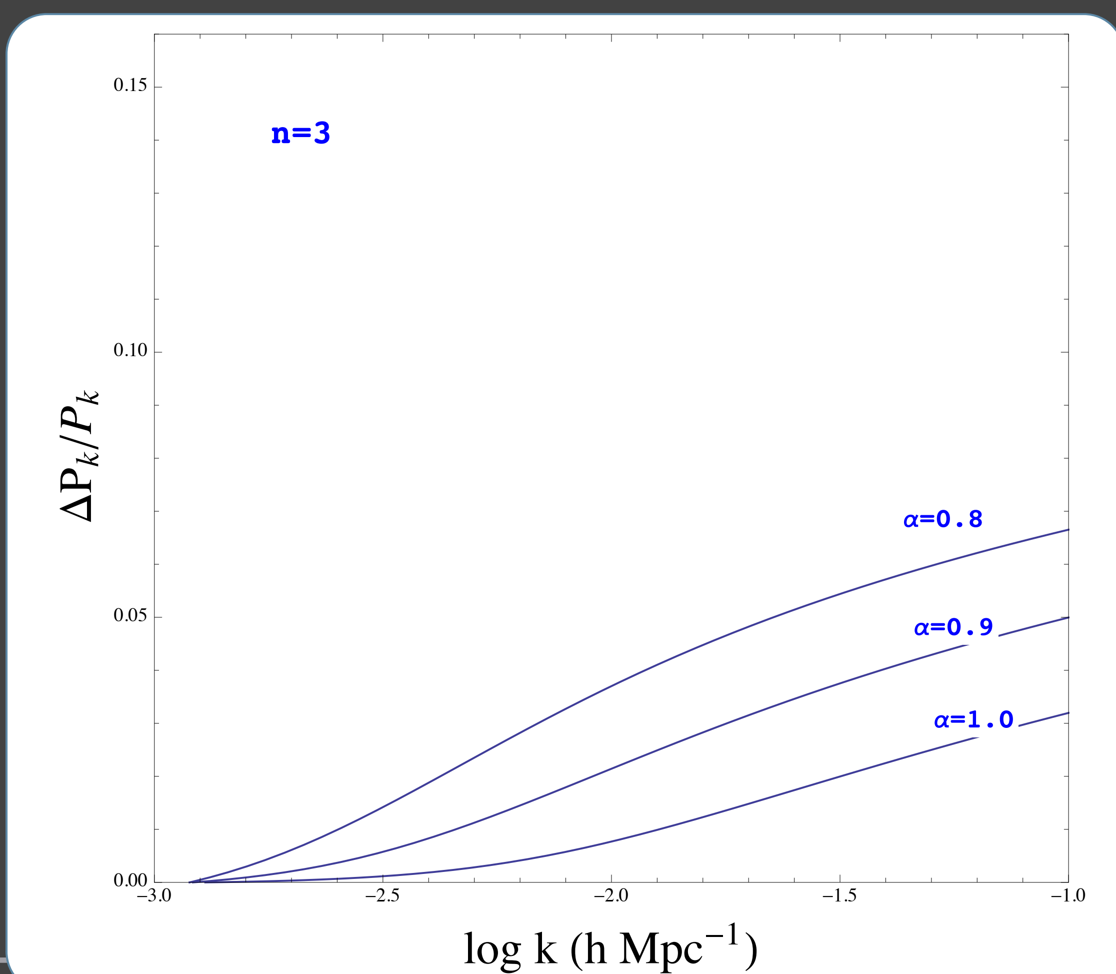
α=1.1
α=1.2
α=1.3
ΔP_k/P_k
0.00
0.05
0.10
0.15
−3.0
−2.5
−2.0
−1.5
−1.0
log k (h Mpc⁻¹)

z=0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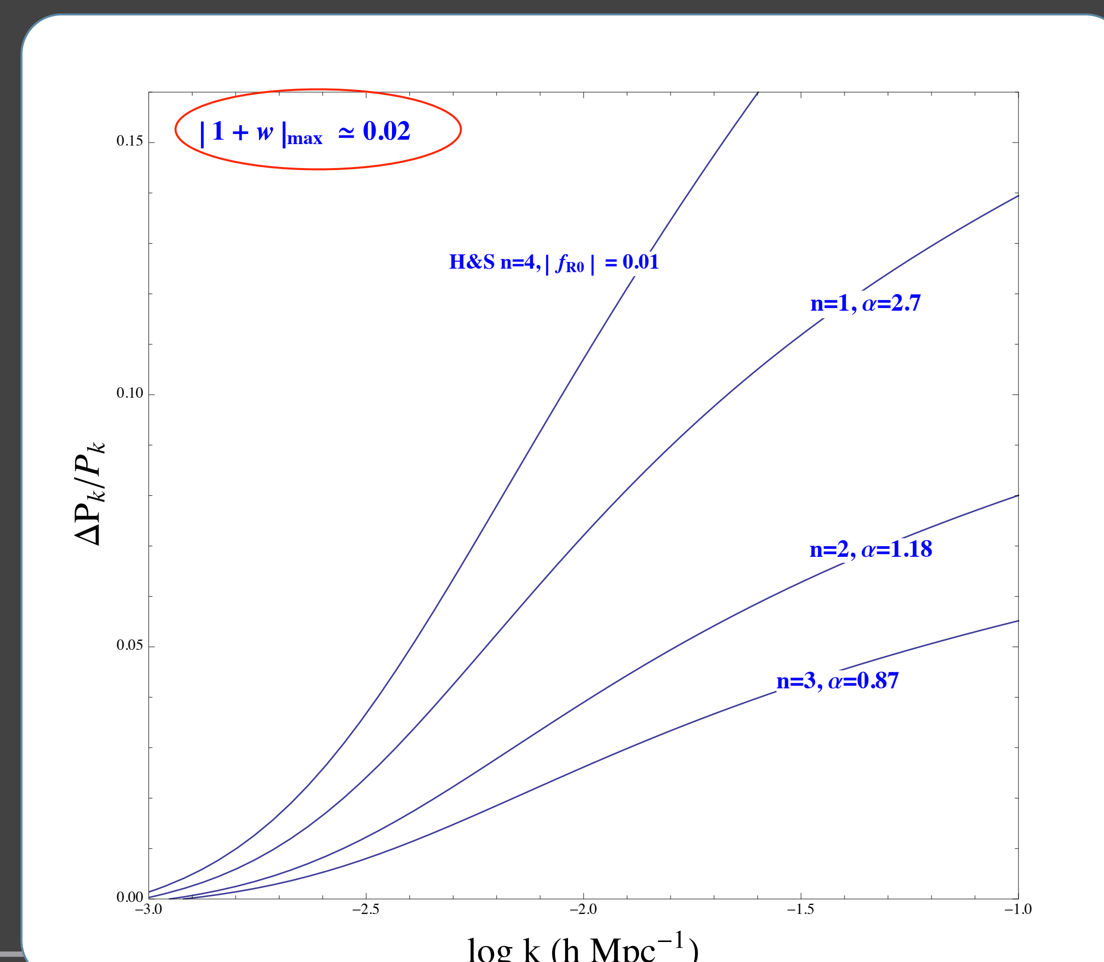
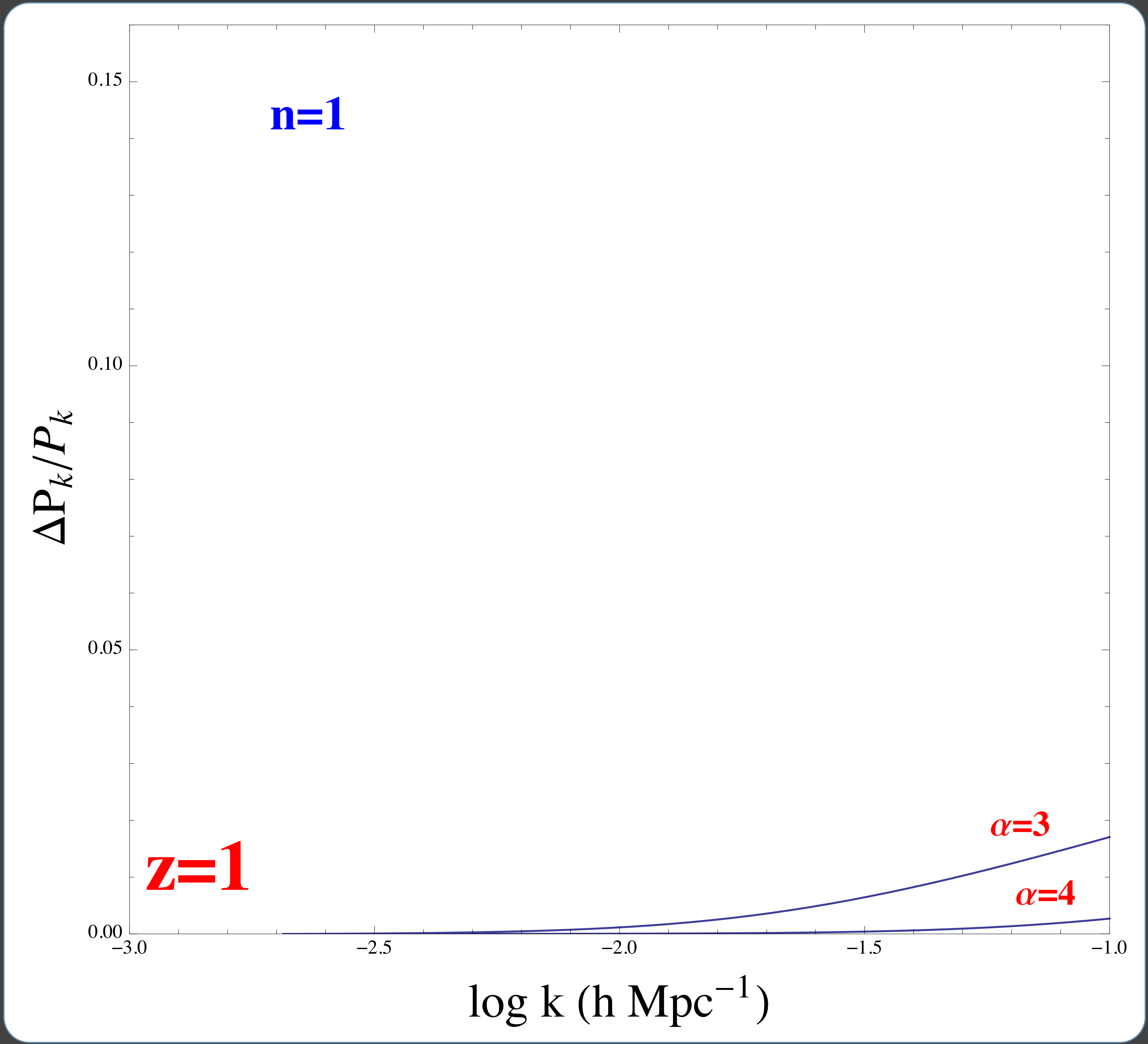
α=0.8
α=0.9
α=1.0
ΔP_k/P_k
0.00
0.05
0.10
0.15
−3.0
−2.5
−2.0
−1.5
−1.0
log k (h Mpc⁻¹)


|1 + w|max ≃ 0.02
H&S n=4, | fR0 | = 0.01
n=1, α=2.7
n=2, α=1.18
n=3, α=0.87
ΔP_k/P_k
0.00
0.05
0.10
0.15
−3.0
−2.5
−2.0
−1.5
−1.0
log k (h Mpc⁻¹)


n=1
z=1
α=3
α=4
0.15
0.10
0.05
0.00
−3.0
−2.5
−2.0
−1.5
−1.0
ΔP_k/P_k
log k (h Mpc^−1)

Conclusions

- It is very difficult to have modified f(R)-gravity models that satisfy all the viability and stability criteria, with a cosmic expansion history distinct from ΛCDM and being, at the same time, in accordance with large scale structure formation and local tests of gravity.

- We have presented a class of generalized exponential f(R) gravity theory, with a parameter controlling the steepness, that facilitates agreement with observations and can give rise to viable f(R) models distinct from ΛCDM.

- Further investigations are necessary, in particular cosmological simulations should be performed, trying to constrain even more the parameter space of the γ-gravity theory, checking if it will remain a viable and interesting modified gravity theory.