Gravitational Waves and New Perspectives for Quantum Gravity

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- **Gravitational waves and stability of classical solutions.**

General Relativity

(GR) is a complete theory of classical gravitational phenomena.

The most important solutions of GR have specific symmetries.

1) Spherically-symmetric solution. Stars ... Black holes.
2) Isotropic and homogeneous metric. Universe.

In both cases there are singularities, curvature and density of matter become infinite, hence GR is not valid at all scales.

Dimensional consideration: Planck scale,

length $l_P = G^{1/2}\,\hbar^{1/2}\,c^{-3/2} \approx 1.4 \cdot 10^{-33}\,cm;$

time $t_P = G^{1/2}\,\hbar^{1/2}\,c^{-5/2} \approx 0.7 \cdot 10^{-43}\,sec;$

mass $M_P = G^{-1/2}\,\hbar^{1/2}\,c^{1/2} \approx 0.2 \cdot 10^{-5} g \approx 10^{19}\,GeV\,.$

Three choice for Quantum Gravity (QG)

One may suppose that the fundamental units indicate to the presence of fundamental physics at the Planck scale.

We can classify the possible approaches into three distinct groups. Namely, we can

● Quantize both gravity and matter fields. This is the most fundamental approach and the main subject of this talk.

●● Quantize only matter fields on classical curved background (semiclassical approach).
QFT and Curved space-time are well-established notions, which passed many experimental/observational tests.

●●● Quantize something else. E.g., in case of (super)string theory both matter and gravity are induced.

●● **The renormalizable QFT in curved space requires introducing a generalized action of gravity (external field).**

$$S_{vac} = S_{EH} + S_{HD}\,,$$

where $$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}\,.$$

is the Einstein-Hilbert action with the cosmological constant.

S_{HD} **includes higher derivative terms.**

$$S_{HD} = \int d^4x\sqrt{-g}\,\{a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\}\,,$$

where $$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2$$

is the square of the Weyl tensor in $d = 4$ **and**

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

is integrand of the Gauss-Bonnet term (topological term in d=4**).**

General considerations about higher derivatives:

- **One should definitely quantize both matter and gravity, for otherwise the QG theory would not be complete.**

- **The diagrams with matter internal lines in a complete QG are be exactly the same as in a semiclassical theory.**

- **This means one can not quantize metric without higher derivative terms in a consistent manner, since these terms are produced already in the semiclassical theory.**

- **Indeed, most of the achievements in curved-space QFT are related to the renormalization of higher derivative vacuum terms, including Hawking radiation, Starobinsky inflation and others.**

Quantum Gravity (QG)

starts from some covariant action of gravity,

$$S = \int d^4x\sqrt{-g}\ \mathcal{L}(g_{\mu\nu})\,.$$

$\mathcal{L}(g_{\mu\nu})$ **can be of GR,** $\mathcal{L}(g_{\mu\nu}) = -\kappa^{-2}(R + 2\Lambda)$ **or some other.**

Gauge transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}$. **The metric transforms as**

$$\delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\nabla_{\mu}\xi_{\nu} - \nabla_{\nu}\xi_{\mu}\,.$$

In the case of $\quad g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\,,$

$$\delta h_{\mu\nu} = -\frac{1}{\kappa}\,(\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}) - h_{\mu\alpha}\partial_{\nu}\xi^{\alpha} - h_{\nu\alpha}\partial_{\mu}\xi^{\alpha} - \xi^{\alpha}\partial_{\alpha}h_{\mu\nu} = R_{\mu\nu\,,\,\alpha}\xi^{\alpha}\,.$$

The gauge invariance of the action means

$$\frac{\delta S}{\delta h_{\mu\nu}}\,\cdot\,R_{\mu\nu\,,\,\alpha}\,\cdot\,\xi^{\alpha} = 0\,.$$

One can prove that the same is true for the Effective Action.

Let us use power counting.

As the first example consider quantum GR.

$$S \,=\, -\,\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,.$$

For the sake of simplicity we consider only vertices with maximal K_ν. **Then we have** $r_l = K_\nu = 2$ **and, combining**

$$D+d \,=\, \sum_{l_{int}}(4-r_l)\,-\,4n\,+\,4\,+\,\sum_\nu K_\nu$$

with

$$l_{int} = p + n - 1$$

we arrive at the estimate ($D = 0$ **means log. divergences)**

$$D+d \,=\, 2+2p\,.$$

This output means that quantum GR is not renormalizable and we can look for some other starting point.

Perhaps, the most natural is HDQG.

Reason: we need HD's anyway for quantum matter field.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}$$

and S_{HD} **include higher derivative terms**

$$S_{HD} = -\int d^4x\sqrt{-g}\left\{\frac{1}{2\lambda}\,C^2 + \frac{1}{\rho}\,E + \tau\Box R + \frac{\omega}{3\lambda}\,R^2\right\}\,,$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2\,,$$

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2\,.$$

K. Stelle, Phys. Rev. D (1977).

Buchbinder, Odintsov, I.Sh., Effective Action in Quantum Gravity. (1992 - IOPP).

Propagators and vertices in HDQG are not like in quantum GR.
Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ and we have K_4, K_2, K_0 vertices. The superficial degree of divergence

$$D + d = 4 - 2K_2 - 2K_0.$$

This theory is definitely renormalizable. Dimensions of counterterms are 4, 2, 0.

Well, there is a price to pay: Massive ghosts

$$G_{\text{spin}-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2 + k^2}\right), \quad m \propto M_P\,.$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

The main point of this talk is a new proposal concerning ghosts and related difficulty of QG.

Including even more derivatives was initially thought to move massive pole to even higher mass scale,

$$S = S_{EH} + \int d^4x\sqrt{-g}\,\Big\{a_1 R^2_{\mu\nu\alpha\beta} + a_2 R^2_{\mu\nu} + a_3 R^2 + \,...$$

$$+ c_1 R_{\mu\nu\alpha\beta}\Box^k R^{\mu\nu\alpha\beta} + c_2 R_{\mu\nu}\Box^k R^{\mu\nu} + c_3 R\Box^k R + \, b_{1,2,..} R^{k+1}_{...}\Big\}\,.$$

Simple analysis shows this theory is superrenormalizable, but the massive ghosts are still here. For the case of real poles:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2+m_1^2} + \frac{A_2}{k^2+m_2^2} + \cdots + \frac{A_{N+1}}{k^2+m_{N+1}^2}$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, **the signs of the corresponding terms alternate,** $A_j \cdot A_{j+1} < 0.$

Asorey, Lopez & I. Sh., hep-th/9610006; IJMPhA (1997).

$$D + d \,=\, 4 \,+\, k(1-p)\,.$$

Once again: what is bad in the higher-derivative gravity?

For the linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad (1)$$

we meet

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

Tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(M.J.G. Veltman, 1963).*

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(M.V. Ostrogradsky, 1850).*

- **Without ghost one violates unitarity of the S -matrix.**

There were several attempts to solve the HD ghost problem.

Stelle, Salam & Strathdee, Tomboulis,
Antonidis & Tomboulis, Johnston, Hawking, ...

In what follows we suggest a new approach which is much simpler and is probably working.

Assumption we made to condemn higher derivative theory:

● One can draw conclusions using linearized gravity approximation. S-matrix of gravitons is the main object.

●● Ostrogradsky instabilities or Veltman scattering are relevant independent on the energy scale, in all cases they produce run-away solutions and "Universe explodes".

There is a simple way to check all these assumptions at once.

Take higher derivative theory of gravity and verify the stability with respect to the linear perturbations on some, physically interesting, dynamical background.

For the sake of generality, consider not only classical HD terms, but also take into account the semiclassical corrections, derived by integrating conformal anomaly.

$$\langle T^\mu_\mu \rangle = \{\beta_1 C^2 + \beta_2 E + a' \Box R\} \,, \quad \textbf{where}$$

$$\begin{pmatrix} \omega \\ -b \\ c \end{pmatrix} = \begin{pmatrix} \beta_1 \\ -\beta_2 \\ \beta_3 \end{pmatrix} = \frac{1}{360(4\pi)^2} \begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

N_0 **conformal real scalars**, $N_{1/2}$ **Dirac spinors**, N_1 **vectors**.

One can use $\langle T^\mu_\mu \rangle$ to find the finite part of 1-loop EA

$$\frac{2}{\sqrt{-g}}\, g_{\mu\nu} \frac{\delta\, \Gamma_{ind}}{\delta g_{\mu\nu}} = \langle T^\mu_\mu \rangle = \frac{1}{(4\pi)^2} \left(\omega C^2 + bE + c\Box R \right) .$$

Solution *(Riegert, Fradkin & Tseytlin, PLB-1984).*

Useful notation:

$$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\, R\Box + \frac{1}{3}\, (\nabla^\mu R)\nabla_\mu \,.$$

Anomaly-induced effective action of vacuum

$$\Gamma_{ind} = S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2} \int d^4x\sqrt{-g(x)}\, R^2(x)$$

$$+ \frac{\omega}{4} \int_x \int_y C^2(x)\, G(x,y)\, \big(E - \frac{2}{3}\Box R\big)_y$$

$$+ \frac{b}{8} \int_x \int_y \big(E - \frac{2}{3}\Box R\big)_x\, G(x,y)\, \big(E - \frac{2}{3}\Box R\big)_y\,,$$

where $\int_x = \int d^4x\sqrt{-g}\,, \qquad \Delta_4\, G(x,y) = \delta(x,y)$.

One can rewrite this expression using auxiliary scalars,

$$\Gamma_{ind} \,=\, S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2} \int_x R^2(x) + \int_x \Big\{ \frac{1}{2}\,\varphi\Delta_4\varphi - \frac{1}{2}\,\psi\Delta_4\psi$$

$$+ \frac{a}{8\pi\sqrt{-b}}\,\psi\, C^2 + \varphi\, \Big[\frac{\sqrt{-b}}{8\pi}\,\big(E - \frac{2}{3}\,\Box R\big) - \frac{a}{8\pi\sqrt{-b}}\, C^2 \Big] \Big\}\,.$$

where $S_c[\bar{g}_{\mu\nu}] = S_c[g_{\mu\nu}]$ **is an integration constant.**

Consider a Cosmological Model based on the action

$$S_{total} \,=\, -\,\frac{M_P^2}{16\pi}\,\int d^4x\sqrt{-g}\,(R+2\Lambda) + S_{matter} + S_{vac} + \bar{\Gamma}_{ind}\,.$$

Equation of motion in physical time $dt = a(\eta)d\eta\,,$ k=0

$$\frac{\ddddot{a}}{a} + \frac{3\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3} - \frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{2\Lambda}{3}\right) \,=\, 0\,,$$

where $k = 0, \pm 1,\ \Lambda$ **is the cosmological constant.**

Particular solutions *(Starobinsky, PLB-1980)* **:** $a(t) \,=\, a_o\,\exp(Ht).$

Hubble parameter $$H \,=\, \frac{M_P}{\sqrt{-32\pi b}}\,\left(1 \pm \sqrt{1+\frac{64\pi b}{3}\,\frac{\Lambda}{M_P^2}}\right)^{1/2}\,.$$

Consider unstable inflation, matter (or radiation) dominated Universe and assume that the Universe is close to the classical FRW solution. The equation is

$$\frac{\ddddot{a}}{a} + \frac{3\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5 + \frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}$$

$$-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{2\Lambda}{3}\right) = -\frac{1}{3c}\,\rho_{matter}\,,$$

The terms of the first line, of quantum origin, behave like $1/t^4$.

The second line terms, of classical origin, behave like $1/t^2$.

After certain time the "quantum" terms become negligible.

Conclusion: For the dynamics of conformal factor, classical solutions are very good low-energy approximations in the theory with quantum corrections and/or higher derivatives.

Stability & Gravitational Waves

As far as classical action and quantum, anomaly-induced term, both have higher derivatives, an important question is whether the stability of classical solutions in cosmology holds or not.

Consider small perturbation

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}\,, \quad h_{\mu\nu} = \delta\, g_{\mu\nu}\,,$$

where $g^0_{\mu\nu} = \{1,\ -\delta_{ij}\, a^2(t)\}$, $\mu = 0,1,2,3$ **and** $i = 1,2,3$.

$$h_{\mu\nu}(t,\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{k}}\, h_{\mu\nu}(t,\vec{k})\,.$$

Using the conditions $\partial_i\, h^{ij} = 0$ **and** $h_{ii} = 0$**, together with the synchronous coordinate condition** $h_{\mu 0} = 0$, **we arrive at the equation for the tensor mode**

Fabris, Pelinson and I.Sh., (2001);
Fabris, Pelinson, Salles and I.Sh., (2011).

$$
\Big(2f_1+\frac{f_2}{2}\Big)\ \ddddot{h}+\Big[3H\Big(4f_1+f_2\Big)+4\dot{f}_1+\dot{f}_2\Big]\ \dddot{h}+\Big[3H^2\Big(6f_1+\frac{f_2}{2}-4f_3\Big)
$$

$$
+H\Big(16\dot{f}_1+\frac{9}{2}\dot{f}_2\Big)+6\dot{H}\Big(f_1-f_3\Big)+2\ddot{f}_1+\frac{1}{2}\big(\ddot{f}_2+f_0+f_4\ddot{\varphi}\big)+\frac{3}{2}f_4H\dot{\varphi}-\frac{2}{3}f_5\dot{\varphi}^2\Big]\ \ddot{h}
$$

$$
-\Big(4f_1+f_2\Big)\ \frac{\nabla^2\ddot{h}}{a^2}+\Big[\dot{H}\Big(4\dot{f}_1-6\dot{f}_3\Big)-21H\dot{H}\Big(\frac{1}{2}f_2+2f_3\Big)-\ddot{H}\Big(\frac{3}{2}f_2+6f_3\Big)
$$

$$
+3H^2\Big(4\dot{f}_1+\frac{1}{2}\dot{f}_2-4\dot{f}_3\Big)-9H^3\big(f_2+4f_3\big)+H\Big(4\ddot{f}_1+\frac{3}{2}\ddot{f}_2\Big)+\frac{3}{2}f_4\dot{\varphi}\Big(3H^2+\dot{H}\Big)
$$

$$
+H\big(3f_4\ddot{\varphi}+\frac{3}{2}f_0-2f_5\dot{\varphi}^2\big)+\frac{1}{2}f_4\ \dddot{\varphi}\ -\frac{4}{3}f_5\dot{\varphi}\ddot{\varphi}\Big]\ \dot{h}-\Big[H\Big(4f_1+f_2\Big)+4\dot{f}_1+\dot{f}_2\Big]\ \frac{\nabla^2\dot{h}}{a^2}
$$

$$
+\Big[5f_4H\ \dddot{\varphi}\ +f_4\ \ddddot{\varphi}\ -\big(36\dot{H}H^2+18\dot{H}^2+24H\ddot{H}+4\ \dddot{H}\ \big)\Big(f_1+f_2+3f_3\Big)
$$

$$
-H\dot{H}\Big(32\dot{f}_1+36\dot{f}_2+120\dot{f}_3\Big)-8\ddot{H}\Big(\dot{f}_1+\dot{f}_2+3\dot{f}_3\Big)-H^2\Big(4\ddot{f}_1+6\ddot{f}_2+24\ddot{f}_3\Big)
$$

$$
-4\dot{H}\Big(\ddot{f}_1+\ddot{f}_2+3\ddot{f}_3\Big)-9f_4\dot{\varphi}\Big(H^3+H\dot{H}\Big)+f_4\ddot{\varphi}\Big(3H^2+5\dot{H}\Big)-H^3\Big(8\dot{f}_1+12\dot{f}_2+48\dot{f}_3\Big)
$$

$$
+f_5\dot{\varphi}^2\Big(\frac{1}{2}H^2+\frac{1}{3}\dot{H}\Big)+\frac{2}{3}f_5H\dot{\varphi}\ddot{\varphi}-\frac{1}{6}f_5\ddot{\varphi}^2+\frac{1}{3}f_5\dot{\varphi}\,\dddot{\varphi}\,\Big]\,h+f_0\Big[2\dot{H}+3H^2\Big]\,h
$$

$$
+\Big[H\Big(2\dot{f}_1+\frac{1}{2}\dot{f}_2\Big)+2\dot{H}\Big(f_1+f_2+3f_3\Big)
$$

$$
-\frac{1}{2}\left(\ddot{f}_2+f_4\ddot{\varphi}+f_0+3f_4H\dot{\varphi}\right)-\frac{1}{3}f_5\dot{\varphi}^2\Big]\,\frac{\nabla^2h}{a^2}+\Big[2f_1+\frac{1}{2}f_2\Big]\,\frac{\nabla^4h}{a^4}\;=\;0\,,
$$

where the f - terms are defined as

$$
f_0\;=\;-\frac{{M_P}^2}{16\pi}\,;\qquad f_1\;=\;a_1+a_2-\frac{b+\omega}{2\sqrt{-b}}\,\varphi+\frac{\omega}{2\sqrt{-b}}\,\psi\,;
$$

$$
f_2\;=\;-2a_1-4a_2+\frac{\omega+2b}{\sqrt{-b}}\,\varphi-\frac{\omega}{\sqrt{-b}}\,\psi;
$$

$$
f_3\;=\;\frac{a_1}{3}+a_2-\frac{3c+2b}{36}-\frac{3b+\omega}{6\sqrt{-b}}\,\varphi+\frac{\omega}{6\sqrt{-b}}\,\psi;
$$

$$
f_4\;=\;-\frac{4\pi\sqrt{-b}}{3}\,;\qquad f_5\;=\;\frac{1}{2}\,.
$$

Qualitative results were achieved by using

I. Analytical methods. We can approximately treat all coefficients as constants. There is a mathematically consistent way to check when (and whether) it works. With the Wolfram's Mathematica software, manipulating our equation is not so difficult, in fact.

II. Numerical methods. Modern methods to study the CMB spectrum (CMBEasy software) can be pretty well applied to gravitational waves and gives the results which are perfectly consistent with the output of method I.

Net Result: The stability does not actually depend on quantum corrections. It is completely defined by the sign of the classical coefficient a_1 **of the Weyl-squared term. The sign of this term defines whether graviton or ghost has positive kinetic energy!**

We can distinguish the following two cases:

- **The coefficient of the Weyl-squared term is** $a_1 < 0$ **Then**

$$G_{\mathrm{spin}-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,,$$

there are no growing modes up to the Planck scale, $\vec{k}^2 \approx M_P^2$**.**

For the dS background this is in a perfect agreement with
Starobinsky, Let.Astr.Journ. (in Russian) (1983);
Hawking, Hertog and Real, PRD (2001).

- **The classical coefficient of the Weyl-squared term** $a_1 > 0$**.**

$$G_{\mathrm{spin}-2}(k) \sim \frac{1}{m^2}\left(-\frac{1}{k^2} + \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

and there are rapidly growing modes at any scale.

So, where is the ghost??

In fact, the result is natural. The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$ and $\mathcal{O}(R^4_{....})$, Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2_{....})$ - terms.

For $a_1 < 0$ there are no growing tensor modes in the higher derivative gravity on cosmological backgrounds.

Massive ghosts are present only in the vacuum state. We just do not observe them "alive" until the typical energy scale is below the Planck mass.

- **All in all, massive ghosts do not pose real danger below the Planck scale. Above M_P we need new ideas to fight ghosts.**

Conclusions

- For QG with higher derivatives (HDQG) the propagator includes massive nonphysical mode(s) called ghosts.

- These massive ghosts are capable to produce terrible instabilities, but ... for this end there should be at least one such ghost excitation in the initial spectrum.

- At least in the cosmological case, the ghost is not actually generated below Planck scale.

- The final conclusion is that the HDQG looks as a perfect candidate to be an effective QG below the Planck scale.