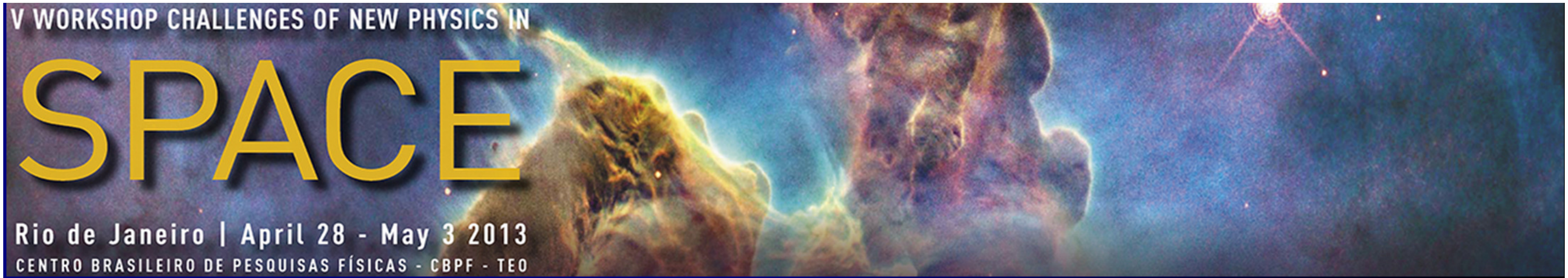

The search for a cosmic topology: Where to look for it, and how to interpret results

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Outline

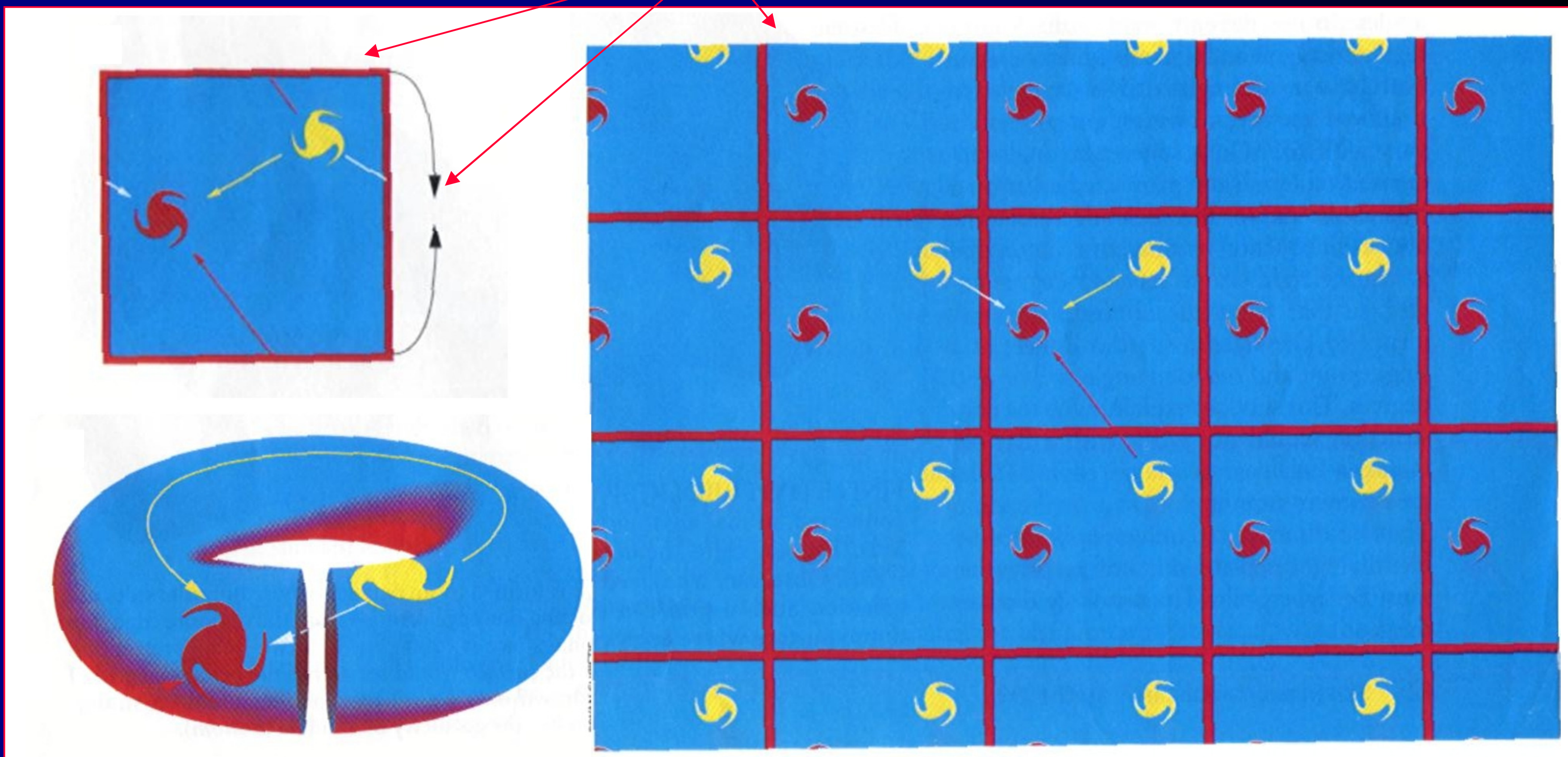
1. Cosmic topology and detectability

2. Circles in the sky - Detecting cosmic topology

3. Constraining observable parameters

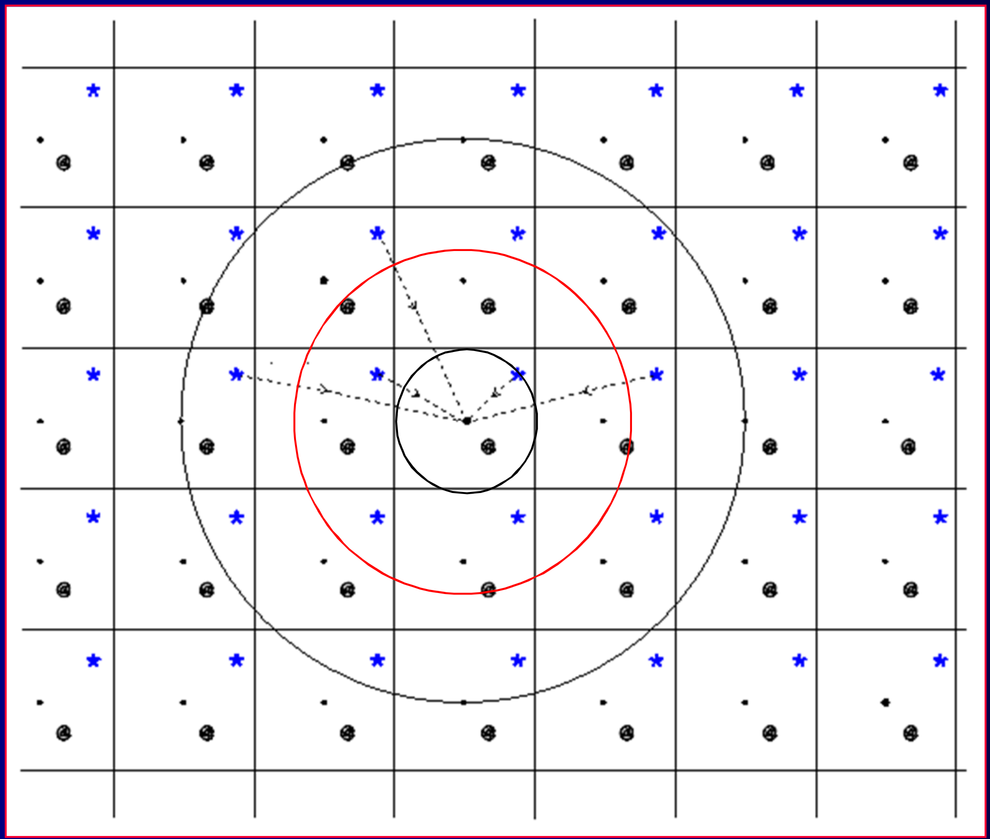
4. Where to look for topology, and what it can tell us

Detecting topology: multiple images



Non-trivial topology ➡ Multiple images in covering space

Detectability: one holonomy at a time



$\mathcal{M} = \frac{M}{\Gamma}, \qquad M = R^3,\ S^2\ ou\ H^3$

- Each element in Γ generates a different image
- However, only images within the horizon are detectable
- γ is a Clifford Translation (CT) if $d(x, \gamma x) = cte.$ for any x

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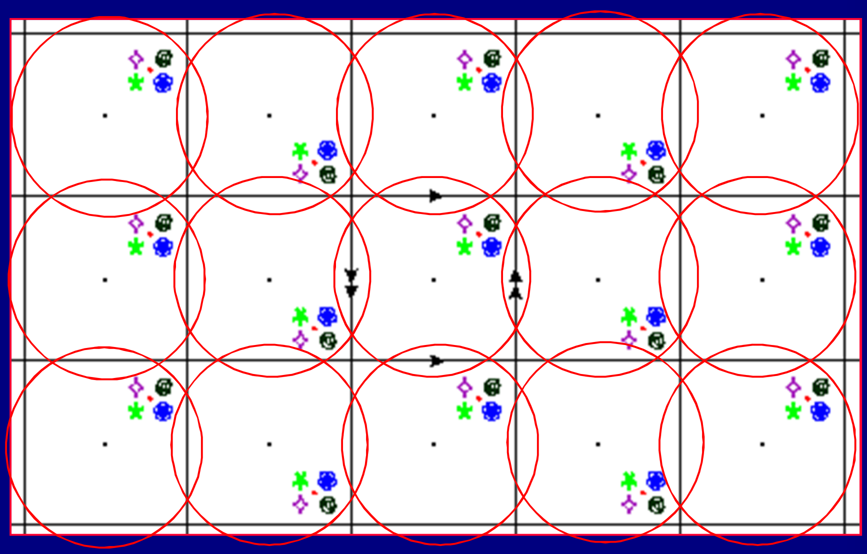
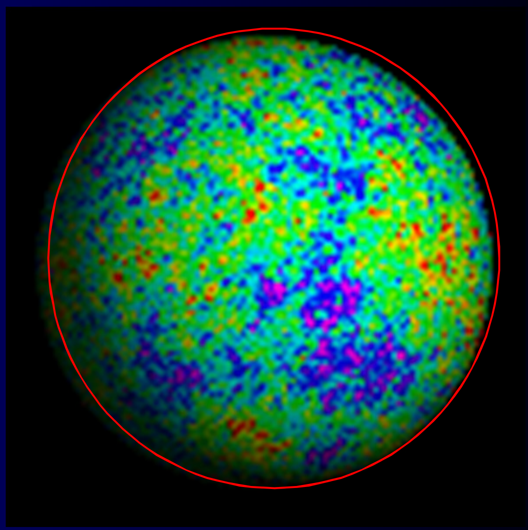
Circles in the sky

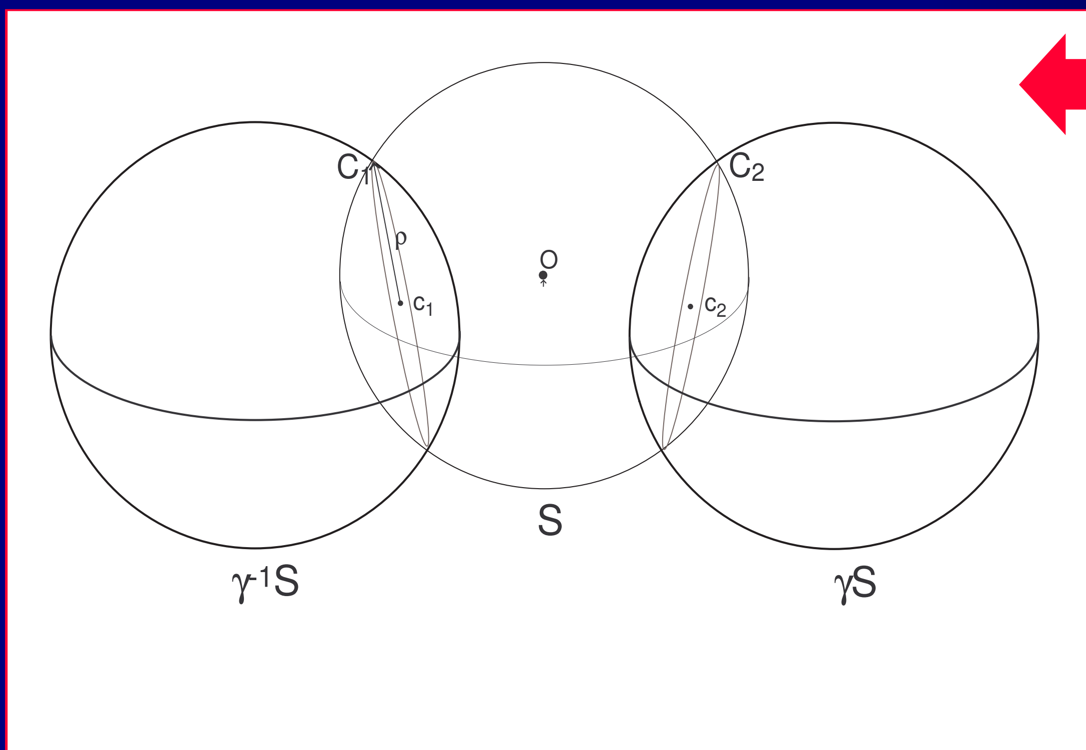
- In a universe with non-trivial topology, copies of the fundamental domain will tesselate the covering space
- If the topology is detectable, copies of the LSS will "spill over" and intersect along circles
- Along such intersecting circles temperature fluctuations will match

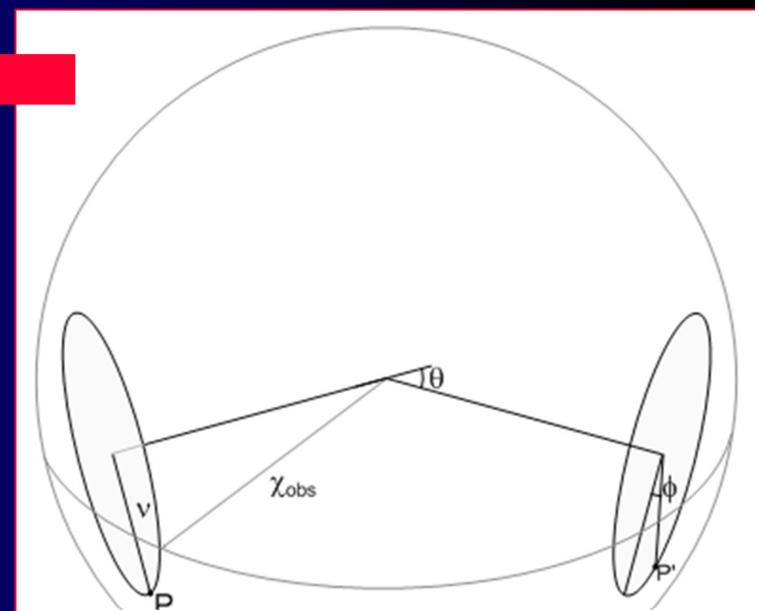




Circles in the sky

- A Pair of circles in the CMB map with idential patterns of temperature fluctuations should be observable any detectable holonomy γ
- **If γ is a CT, the corresponding pair of circles will be antipodal**





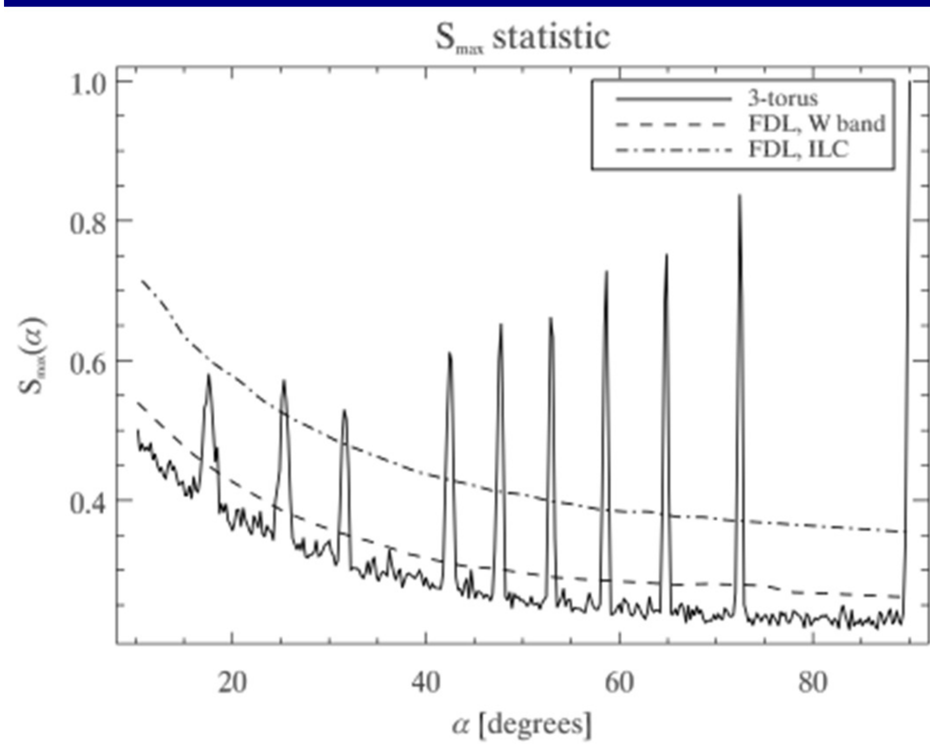
$$S(\varphi, \rho, \hat{c}_1, \hat{c}_2, \epsilon) = \frac{\langle 2T_1(\epsilon\phi)T_2(\phi+\varphi)\rangle}{\langle T_1(\phi)^2 + T_2(\phi+\varphi)^2\rangle}$$

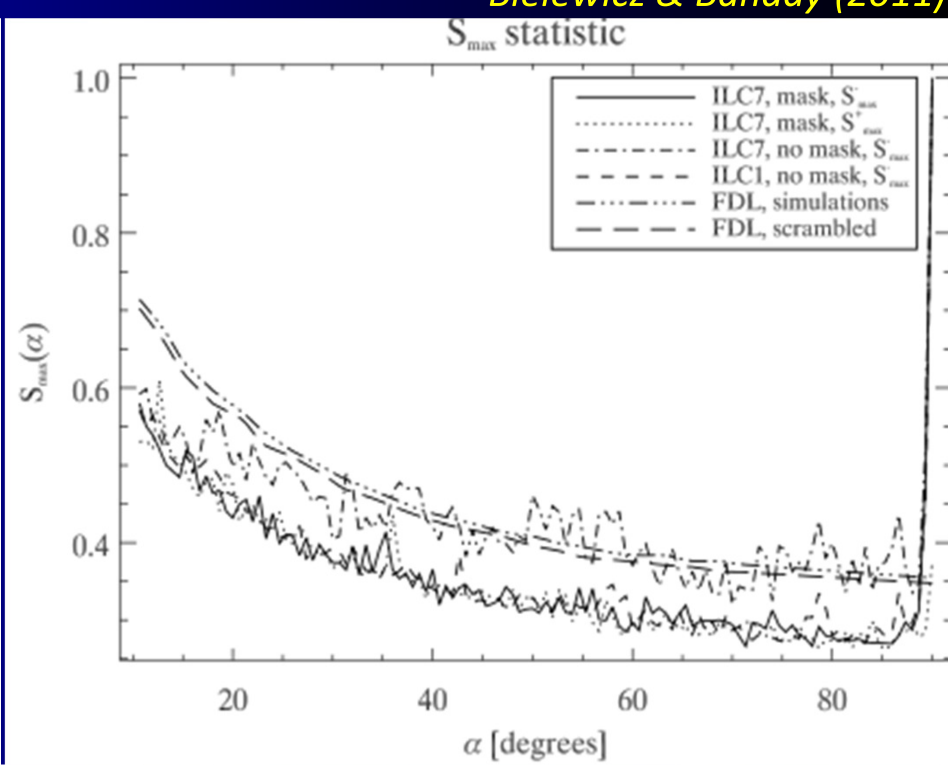
The S_{max} statistic

Proposed by Cornish, Spergel & Starkman in 1998 and modified (m-weigthing) in 2004

$$S_{ij}(\alpha,\beta) = \frac{2\sum_m m T_{im}(\alpha) T^*_{jm}(\alpha) e^{-im\beta}}{\sum_n n\left[|T_{in}(\alpha)|^2 + |T_{jn}(\alpha)|^2\right]}$$

Bielewicz & Banday (2011)





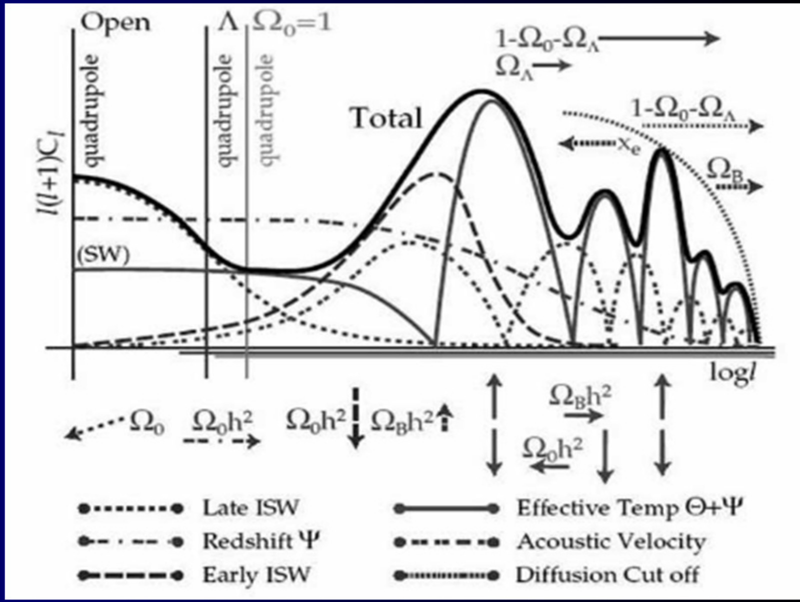
… but there are complications

Dominant:

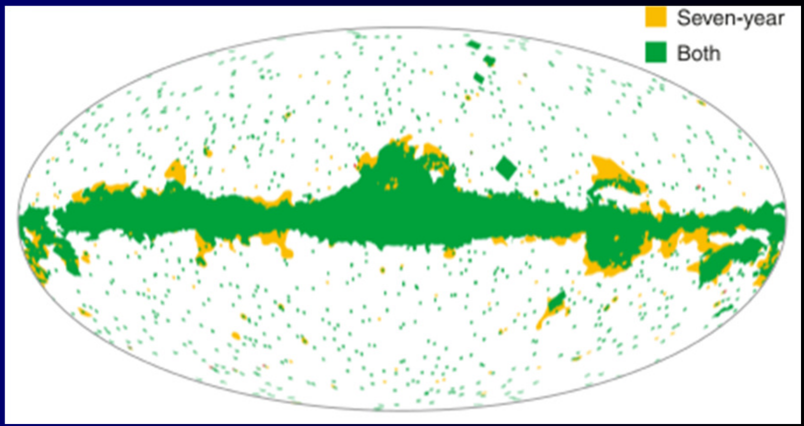
- ISW
- Doppler
- Foreground contamination

Sub-dominant:

- Last scattering shell thickness
- Beam profile
- Detector noise
- Proper movement
- Etc...



Hu, Sugiyama & Silk (1997)



KQ85, *Gold et. al. (2010)*

The searches: nothing (definite) so far II

- Cornish et al (2004), Key et al (2007)

  Nearly antipodal, WMAP 1y, ILC full sky
  no signal

- Bielewicz & Banday, 2011

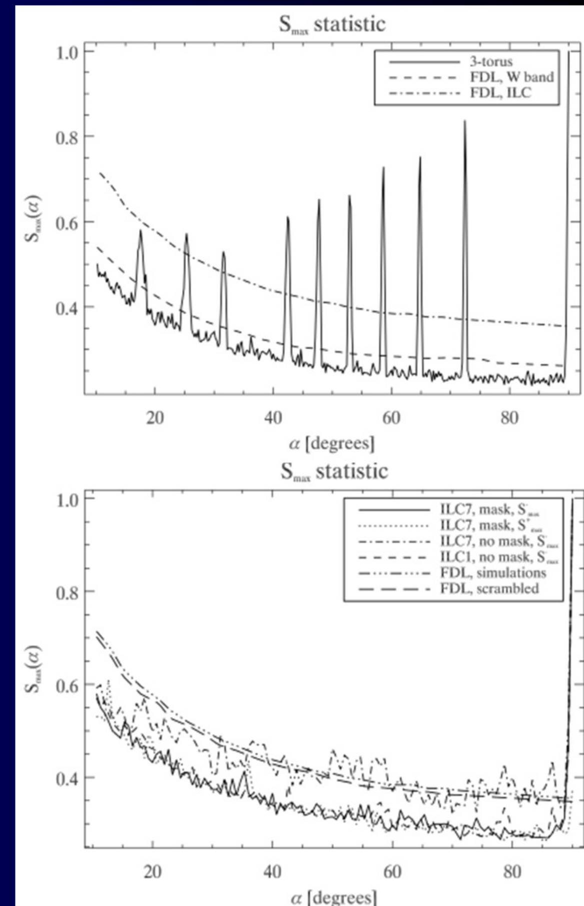
  Antipodal only, WMAP 7y W band, KQ85y7 mask
  no signal

- Vaudrevange et al, 2012

  All pairs, WMAP 7y W band + ILC inside mask
  weird signal, very likely foreground contamination

- Aurich & Lustig, 2013

  Antipodal only, WMAP 7y W + V bands, KQ75 & KQ85 masks
  marginal signal, likely foreground contamination



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(nearly) Excluding a detectable cosmic topology in the inflationary limit

- We can show that for small χ_{obs}

$$\theta \le \frac{2\cos\nu}{|z_2||z_1|}\chi_{obs} \qquad (spherical)$$

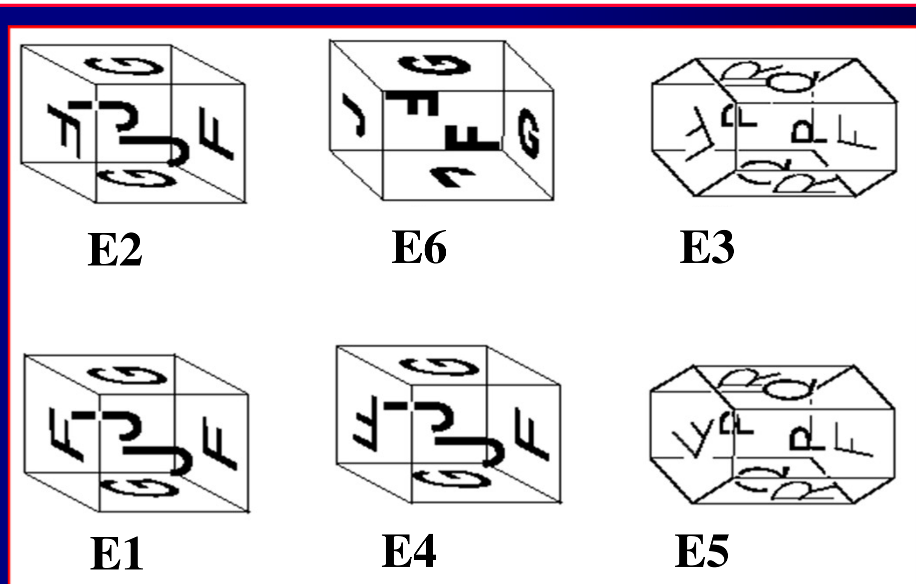
$$\theta \le 2\cos\nu\,\chi_{obs} \qquad (hyperbolical)$$

- **Yes, in the inflationary limit**

  - For observers in hyperbolical universes, $\theta \le 1.1°$
  - For 99% of observers in spherical universes, $\theta \le 10°$

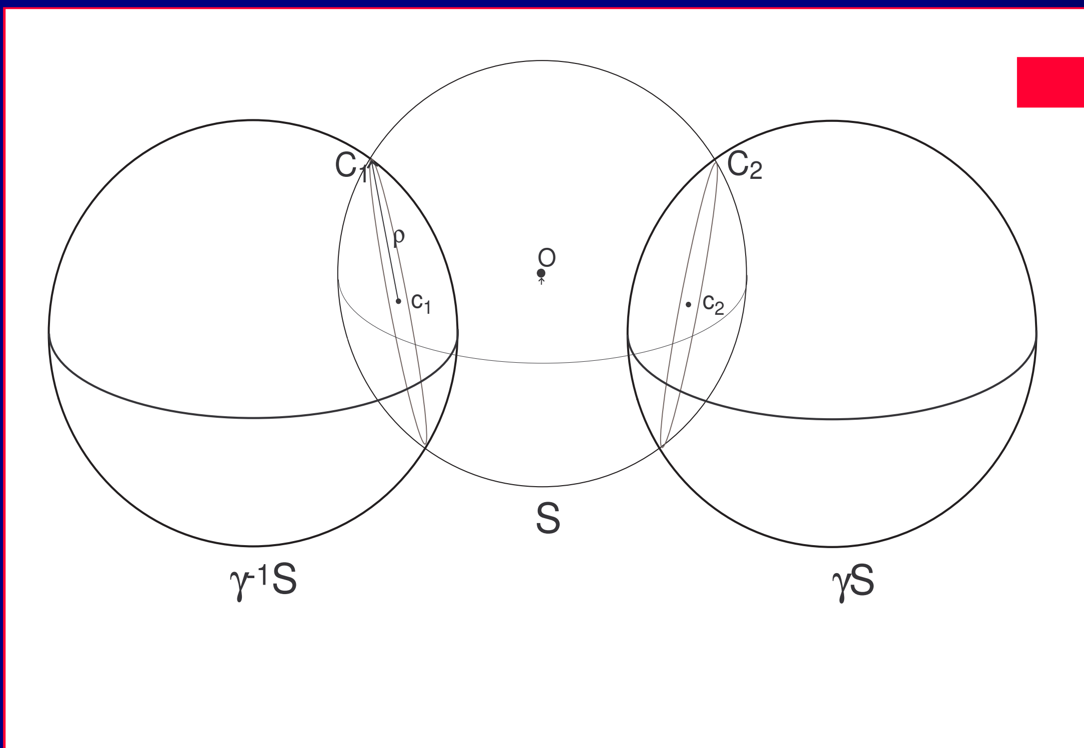
θ_{max} for the orientable flat manifolds

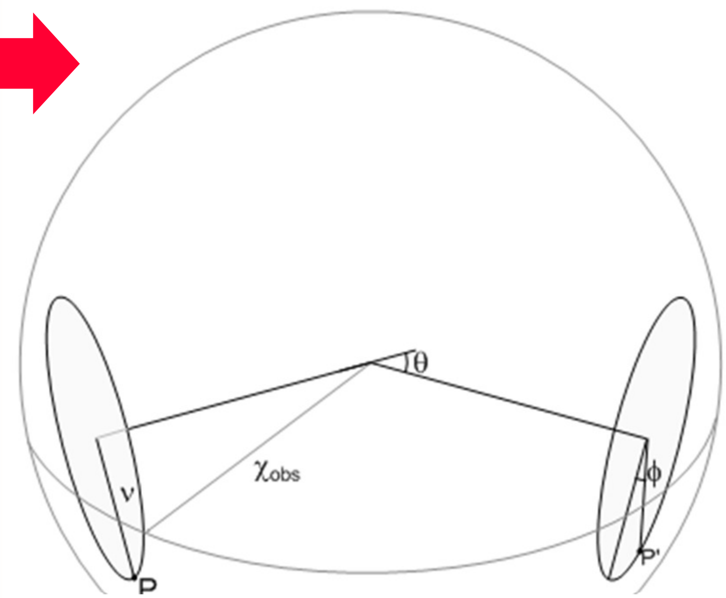
Symbol	Manifold	n	θ_{max}
E_1	three-torus	1,1,1	0°
E_2	half turn space	1,1,2	120°
E_3	quarter turn space	1,1,4	86°
E_4	third turn space	1,1,3	109°
E_5	sixth turn space	1,1,6	59°
E_6	Hantzsche-Wendt space	2,2,2	120°



Determining the holonomy in flat spaces

- One should be able to obtain holonomy parameters from the CitS parameters
- But is determining one holonomy enough to determine all the topology?
- **For non-translational flat holonomies, there is!**





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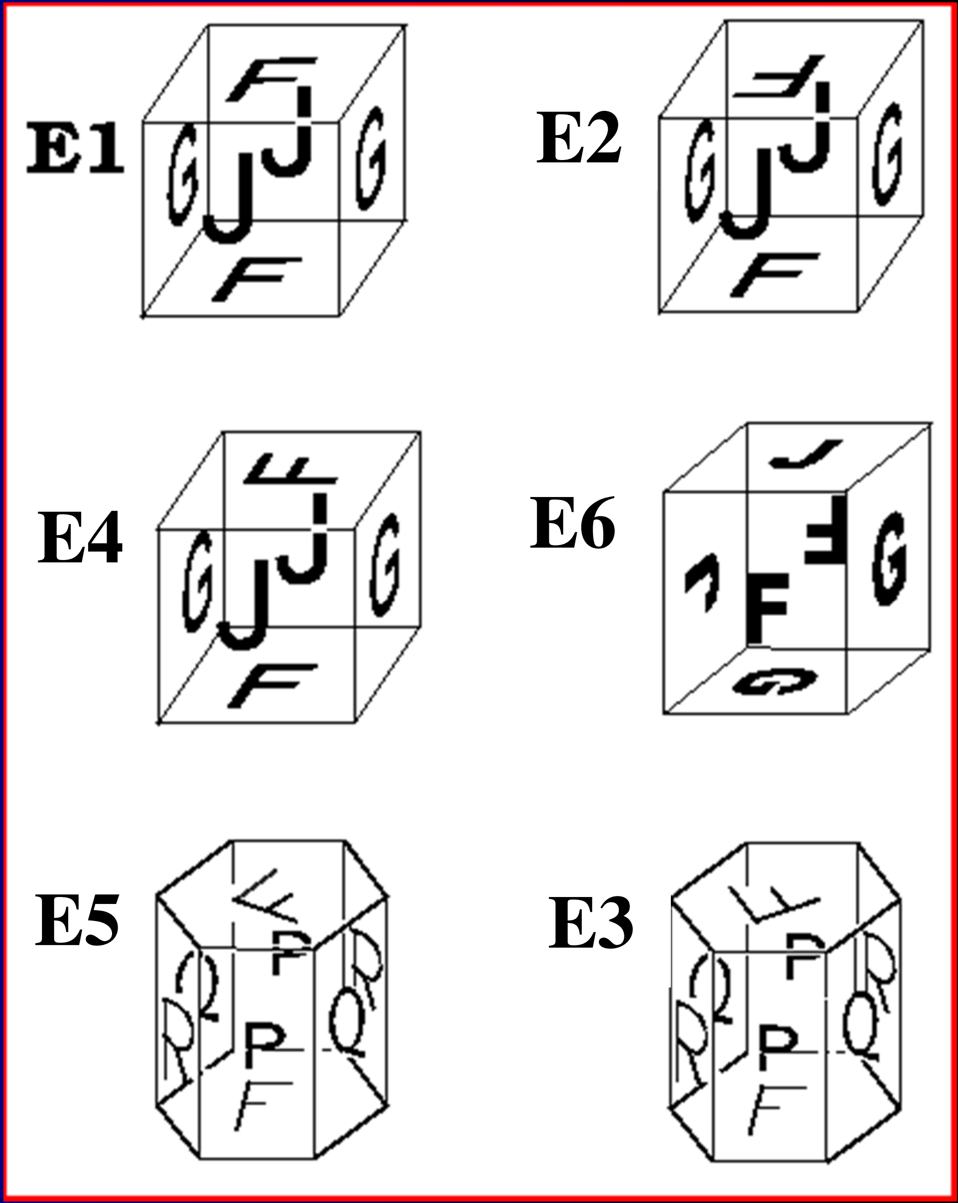
The (classes of) orientable flat 3-manifolds

- No characteristic scale
- Some parameters are free
- All but E_6 have as generators of Γ
  - 2 translations
  - 1 'screw motion'
  - α = 0, π, $\pi/2$, $\pi/3$, $\pi/6$
- E_6 ('Hantzche-Wendt') is generated by 3 screw motions with $\alpha_i = \pi/2$, $i=1,2,3$



Determining the holonomy II

- The twist parameter of the holonomy can be expressed in termos of CitS parameters θ and ϕ

$$\cos\alpha = \frac{(\cos\phi + 1)(\cos\theta + 1)}{2} - 1$$

- The compactifiation length can also be expressed

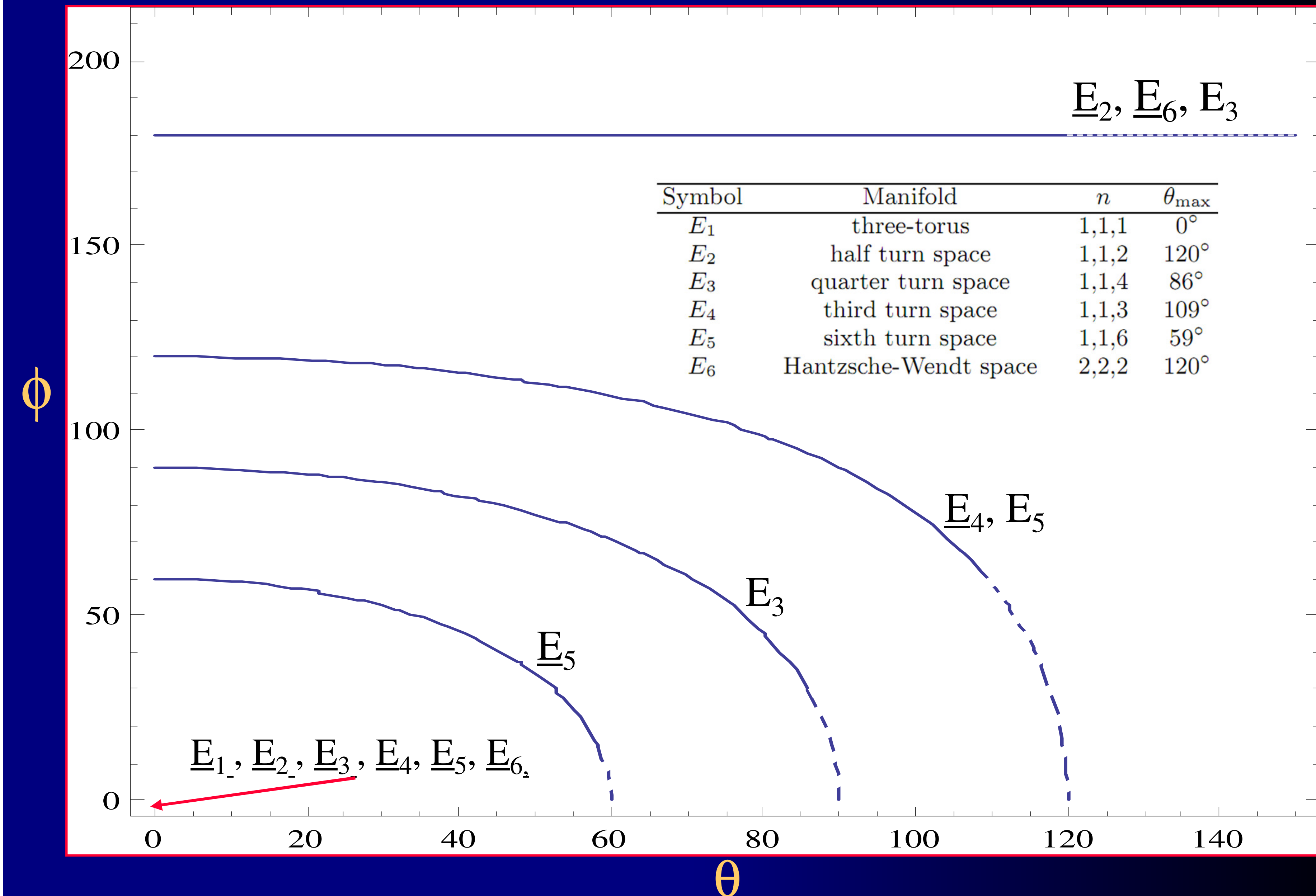
$$L = \chi_{obs} \cos\nu \sqrt{\frac{\cos\theta - \cos\alpha}{1 - \cos\alpha}}$$

L can take any value (non-rigidity), but α is constrained:

$$\alpha = 2\pi/n, \quad n = \{1,2,3,4,6\}$$

- In general more than one topology will include each given holonomy. However, only one of them will generate the shortest geodesic

Determining the topology from a single pair detection



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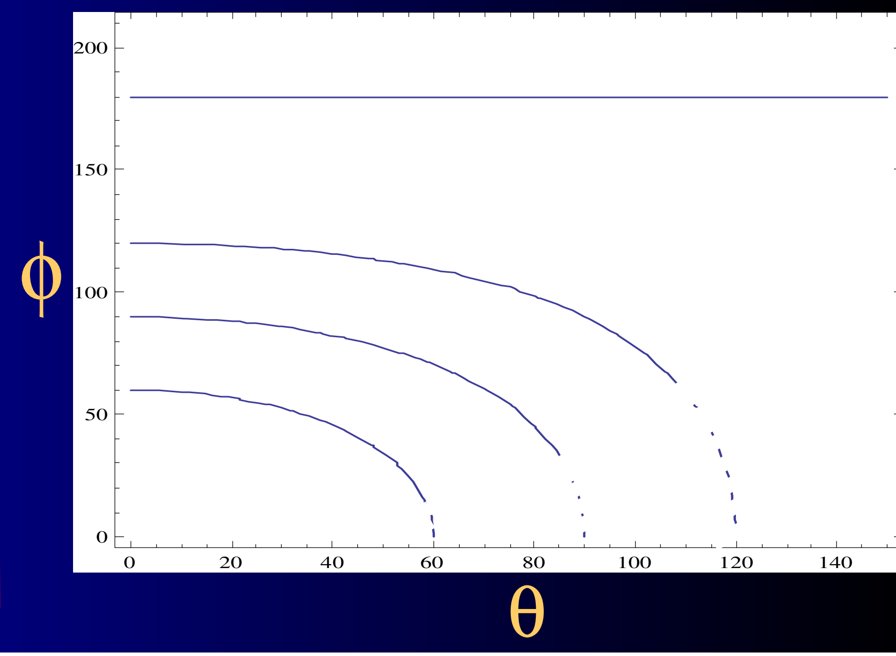
Where to look?

	Homogeneous	Non-homogeneous
Flat	• Antipodal circle pairs • No phase shift	• Non-antipodal pairs • Phase shift specified by deviation from antipodicity
Spherical	• Antipodal pairs • Phase shift $< 2.7^\circ$ (Planck) $< 0.004^\circ$ (Inflation)	• Nearly antipodal pairs • Small phase shift $< 10^\circ$ for most observers (inflation)

What can a pair detection it tell us?

	Antipodal	**Non-antipodal**
Phase = 0	• It's a flat Universe!	• Universe is not globally homogeneous • Either flat or positively-curved, depending on values of ϕ and θ • In many cases, can fully specify topology
Phase ≠ 0	• It's a positively-curved Universe! • Universe is globally homogeneous • With sufficient precision, value of Ω_k	



The way forward

- The issue of a detectable cosmic topology is <u>not</u> settled!

- Focused searches: Look only for e.g. flat holonomies, or translational holonomies:
  - Computationally more efficient
  - Reduces false-positive threshold

- Systematically look for matching partial circle (arcs)

- Look for topology in polarization maps? (cross-validation)