Time evolution of non-symmetric Robinson-Trautman spacetimes

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Robinson-Trautman (RT) spacetimes

- Solution of the vacuum Einstein equations

$$R_{ab} = 0$$

describing a compact object surrounded by gravitational waves.

Non-symmetric RT spacetimes

- The metric

$$ds^2 = -\left(K - 2\frac{m_0}{r} - r(\ln Q^2)_u\right) du^2 - 2dudr + \frac{r^2}{Q^2} d\Omega^2,$$

with $Q = Q(u, \theta, \phi)$ and

$$K = Q^2 \left(1 + \frac{1}{2}\nabla^2_\Omega \ln Q^2\right)$$

is the Gaussian curvature of the surface

$$r = 1, \quad u = u_0 \quad \text{constant}$$

Non-symmetric RT spacetimes

- The metric

$$ds^2 = -\left(K - 2\frac{m_0}{r} - r(\ln Q^2)_u\right) du^2 - 2dudr + \frac{r^2}{Q^2} d\Omega^2,$$

for $Q(u,\theta,\phi) = Q_0$

$$K = Q_0^2$$

Schwarzschild spacetime is recovered.

Non-symmetric RT spacetimes

- Einstein equations for the RT spacetime

$$6m_0 \frac{\partial}{\partial u}\left(\frac{1}{Q^2}\right) = \nabla^2_\Omega K$$

Non-symmetric RT spacetimes

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integration over S^2

$$\frac{d}{du}\int_{S^2} \frac{dS}{Q^2(u,\theta,\phi)} = 0$$

Non-symmetric RT spacetimes

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$$6m_0 \frac{\partial}{\partial u}\left(\frac{1}{Q^2}\right) = \nabla^2_\Omega K$$

integration over S^2

$$\int_{S^2} \frac{dS}{Q^2(0,\theta,\phi)} = 4\pi$$

Non-symmetric RT spacetimes

- Asymptotic evolution

$$\nabla^2_{\Omega} K = 0$$

Non-symmetric RT spacetimes

- Asymptotic evolution

$$\nabla^2_\Omega K = 0 \quad \Rightarrow \quad K \text{ constant}$$

Non-symmetric RT spacetimes

- Asymptotic evolution

$$\nabla^2_\Omega K = 0 \quad \Longrightarrow \quad K \text{ constant}$$

- Symmetric case

$$Q = Q(u, \theta)$$

Non-symmetric RT spacetimes

- Galerkin spectral method

Non-symmetric RT spacetimes

- Galerkin spectral method

$$Q(u,\theta,\phi) = \sum_{\ell=0}^{N} \sum_{m=-\ell}^{\ell} b_m^\ell(u) Y_\ell^m(\theta,\phi)$$

$$K(u,\theta,\phi) = \sum_{\ell=0}^{N} \sum_{m=-\ell}^{\ell} a_m^\ell(u) Y_\ell^m(\theta,\phi)$$

Non-symmetric RT spacetimes

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Orthogonal modes

$$\langle Y_\ell^m, Y_{\ell'}^{m'} \rangle = \int_{S^2} Y_\ell^m Y_{\ell'}^{m' *} \, dS = \frac{4\pi}{(2\ell+1)} \delta_{\ell\ell'} \delta_{mm'},$$

Non-symmetric RT spacetimes

- Galerkin spectral method

$$6m_0 \frac{\partial}{\partial u}\left(\frac{1}{Q^2}\right) = \nabla^2_\Omega K$$

Non-symmetric RT spacetimes

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$$6m_0 \frac{\partial}{\partial u}\left(\frac{1}{Q^2}\right) = \nabla^2_\Omega K \quad \Rightarrow \quad -12m_0 \frac{\partial}{\partial u} Q = Q^3 \nabla^2_\Omega K$$

Non-symmetric RT spacetimes

- Galerkin spectral method

$$6m_0 \frac{\partial}{\partial u}\left(\frac{1}{Q^2}\right) = \nabla^2_\Omega K \quad \Longrightarrow \quad -12m_0 \frac{\partial}{\partial u} Q = Q^3 \nabla^2_\Omega K$$

$$-12m_0 \frac{\partial}{\partial u} \langle Q, Y_\ell^m \rangle = \langle Q^3 \nabla^2_\Omega K, Y_\ell^m \rangle$$

Non-symmetric RT spacetimes

- Galerkin spectral method

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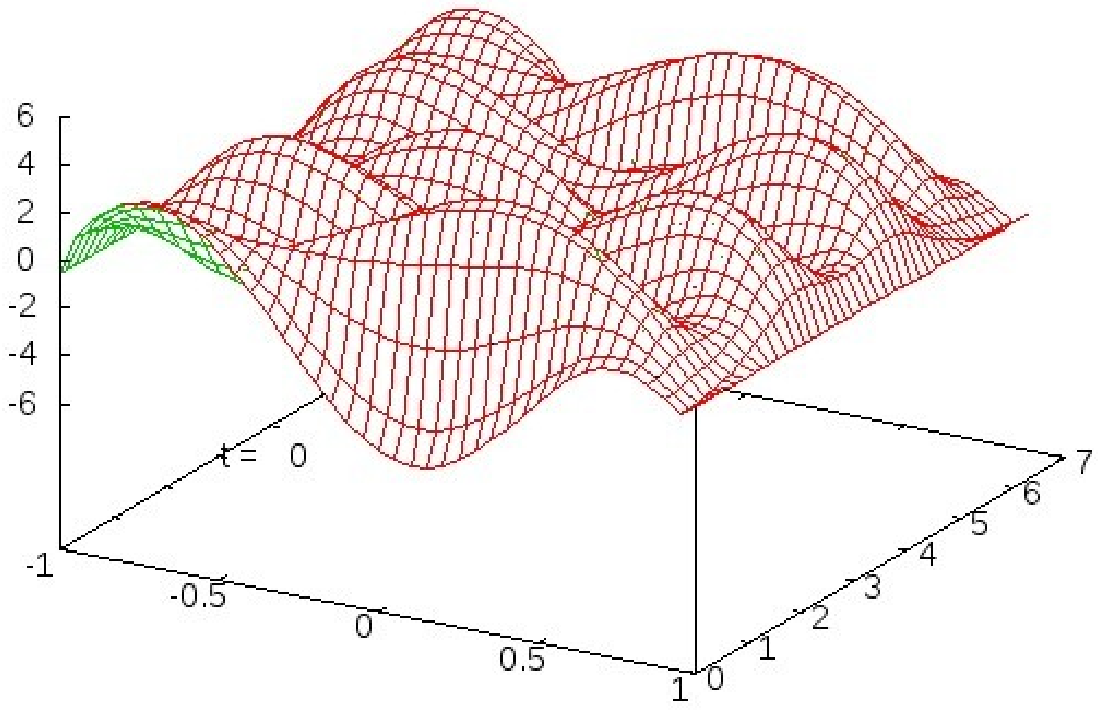
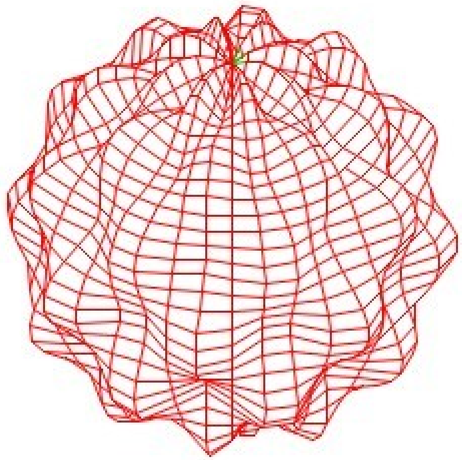
$$-12m_0 \frac{\partial}{\partial u} \langle Q, Y_\ell^m \rangle = \langle Q^3 \nabla^2_\Omega K, Y_\ell^m \rangle$$

$$\frac{48\pi m_0}{(2\ell+1)} \frac{d}{du} b_\ell^m(u) = -\langle Q^3 \nabla^2_\Omega K, Y_\ell^m \rangle$$

Non-symmetric RT spacetimes

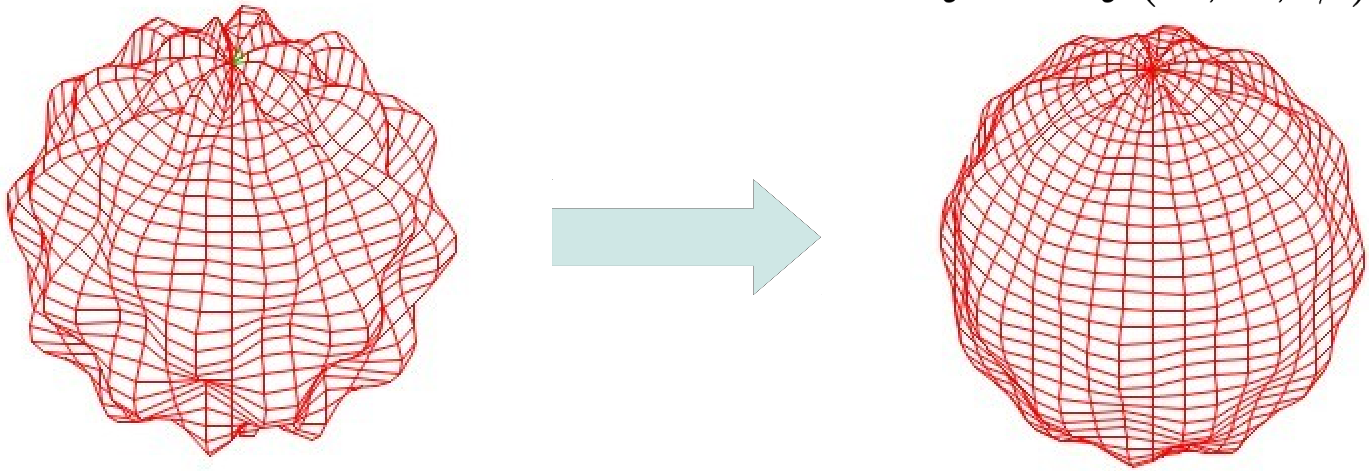


Non-symmetric RT spacetimes



$$Q = Q(0, \theta, \phi)$$

$$Q = Q(u, \theta, \phi)$$



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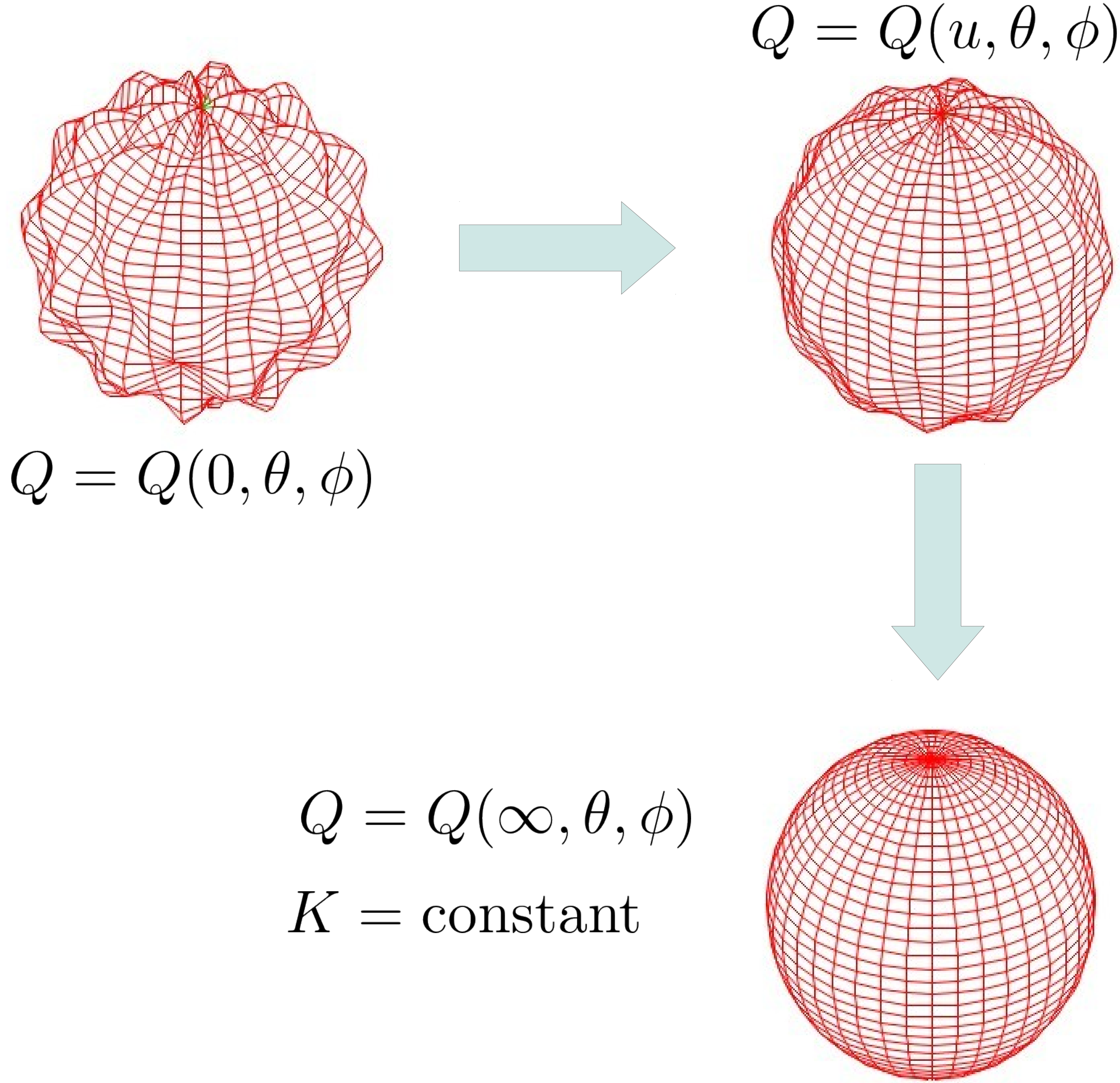
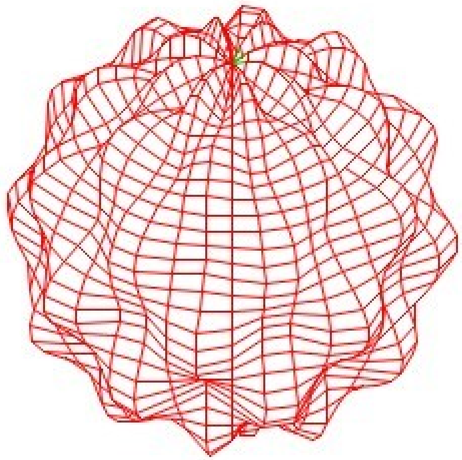

$Q = Q(u, \theta, \phi)$
$Q = Q(0, \theta, \phi)$
$Q = Q(\infty, \theta, \phi)$
$K = \text{constant}$



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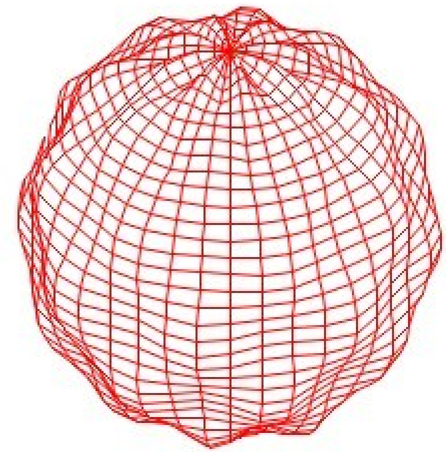
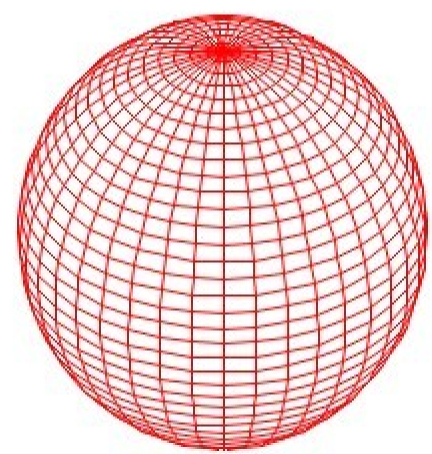


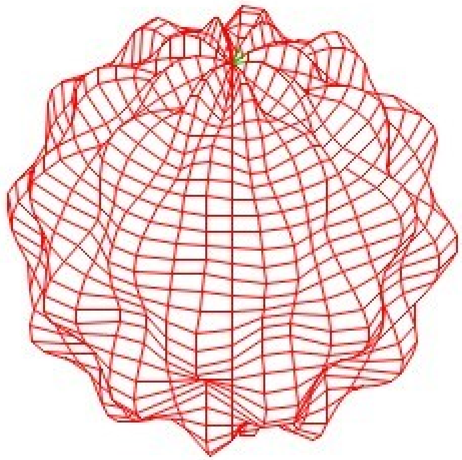


$$Q = Q(\infty, \theta, \phi)$$

$$K = \text{constant}$$

$$Q(\infty, \theta, \phi) = Q_0 + Q_1 \cos\theta$$

$$Q_0^2 - Q_1^2 = 1$$



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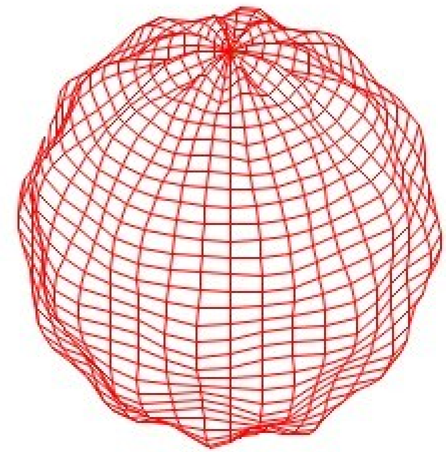

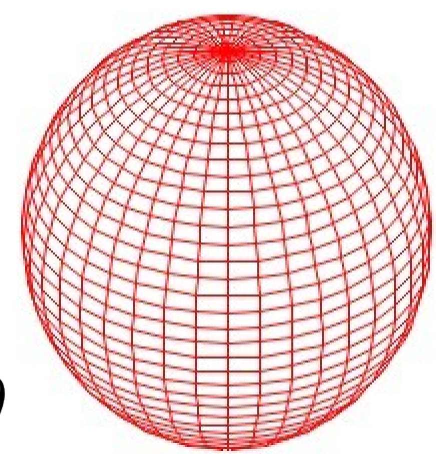

$$Q = Q(\infty, \theta, \phi)$$

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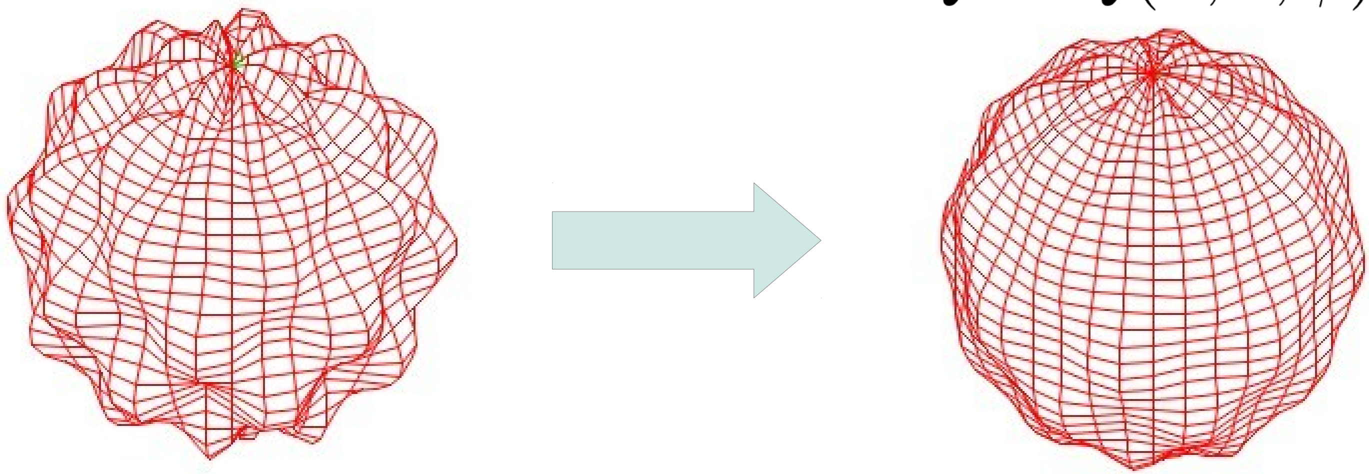
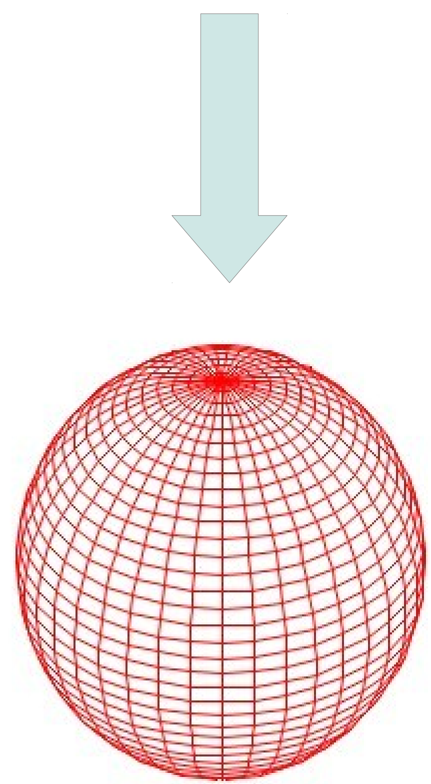
$$Q(\infty, \theta, \phi) = \cosh\alpha + \sinh\alpha\cos\theta$$

$$Q = Q(u, \theta, \phi)$$



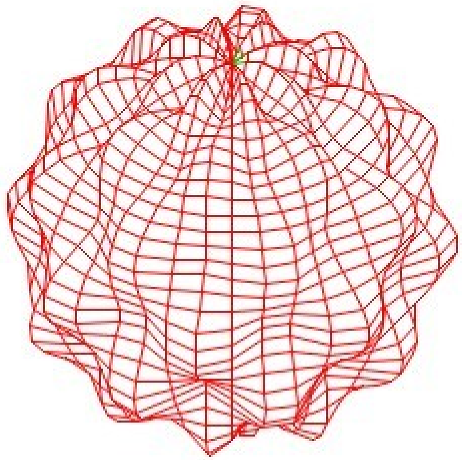
$$Q = Q(0, \theta, \phi)$$



$$Q(\infty, \theta, \phi) = \cosh\alpha + \sinh\alpha\cos\theta$$

$$M(u) = m_0 \int_{S^2} \frac{dS}{Q^3(u, \theta, \phi)} = \frac{m_0}{\sqrt{1 - v^2}}$$

$$v = \tanh\alpha$$



$$Q = Q(0, \theta, \phi)$$

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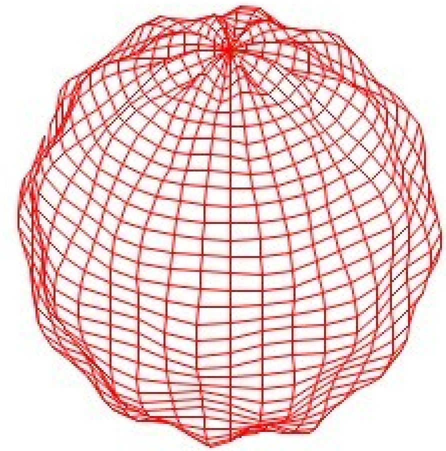
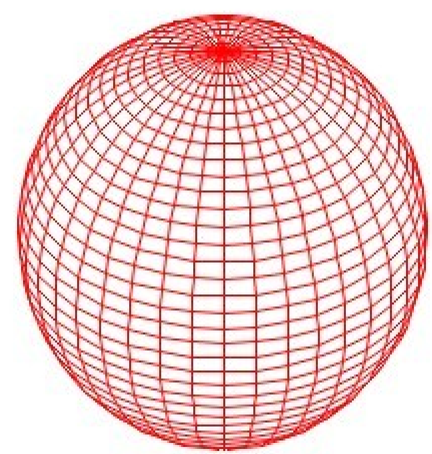
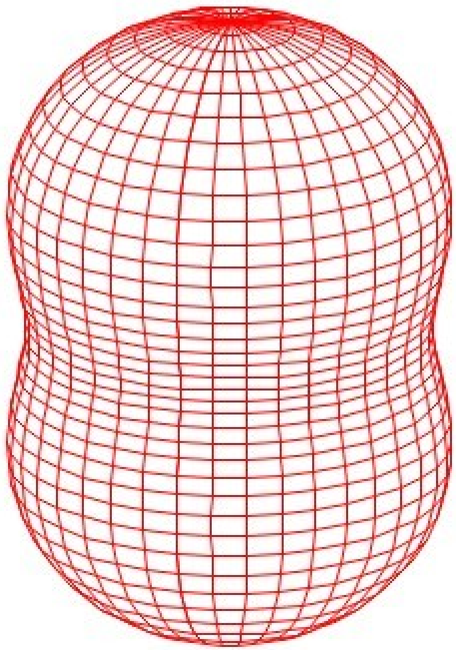




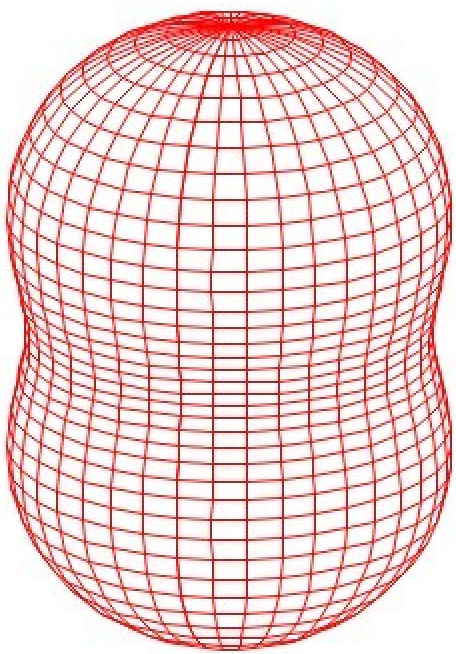
$$Q(\infty, \theta, \phi) = \cosh \alpha + \sinh \alpha \cos \theta$$

$$P(u) = \frac{m_0}{\sqrt{1 - v^2}}(1, 0, 0, v)$$

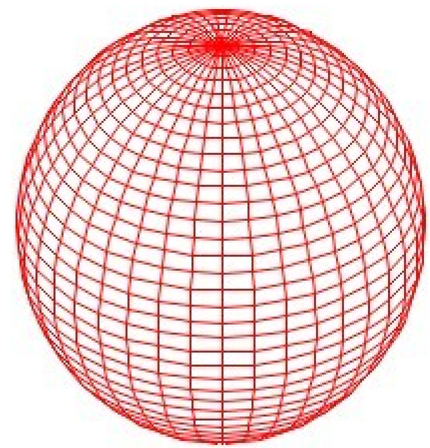
$$v = \tanh \alpha$$



$$Q = Q(0, \theta)$$



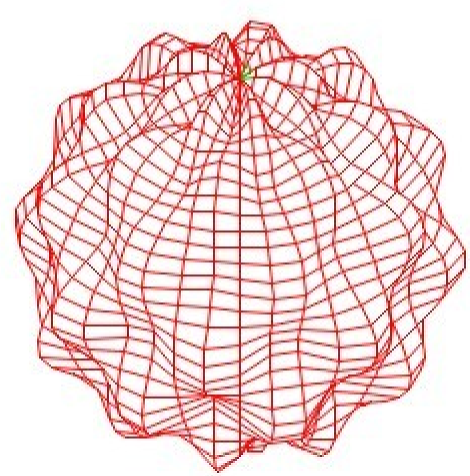
$$Q = Q(0,\theta)$$



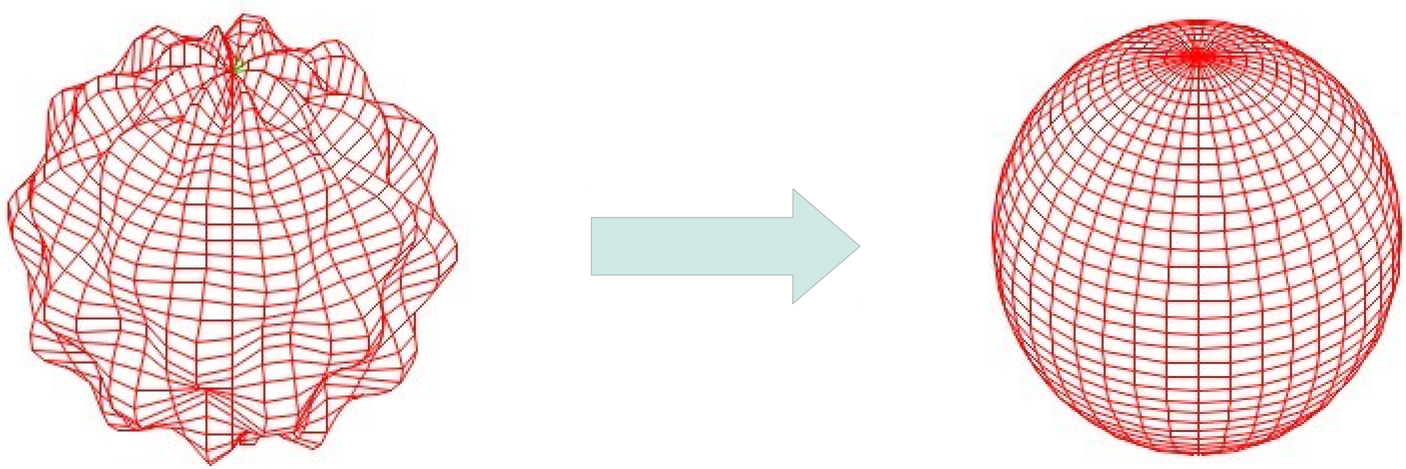
$$Q(\infty,\theta) = Q_0$$

$$v = 0$$

$$\frac{M(\infty)}{M(0)}$$



$$Q = Q(0, \theta, \phi)$$



$$Q = Q(0, \theta, \phi)$$

$$P(\infty) = \frac{m_0}{\sqrt{1 - \mathbf{v}^2}}(1, v_x, v_y, v_z)$$

- Thanks!