On the quantum-classical correspondence for the scattering dwell time

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C. H. Lewenkopf & ROV,
quant-ph/0406056; JPA **37**, 131 (2004)

also with
A. M. Ozorio de Almeida

Campinas, June 22, 2004

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Motivation

Universal fluctuations in mesoscopic transport

Conduction can be described as single particle scattering via Landauer-Büttiker formalism

Semiclassical approach to universal fluctuations of scattering observables: cross-sections, transmission coefficients (conductance), Wigner time delay ...

$$t_W(E) = -\frac{i\hbar}{N}\sum_{a,b=1}^{N} S_{ab}^{*}\frac{\partial S_{ab}}{\partial E}$$

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Summary

Statement of the dwell time problem

Classical scattering
average classical time delay NEW!

Dwell time in quantum mechanics
first attempts: Eisenbud & Wigner, Smith
Wigner time delay
connection with density of states
classical limit

Conclusions
quantum-classical correspondence ?

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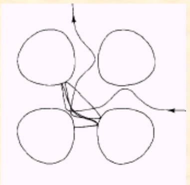
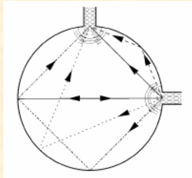
The problem

One particle scattered by a potential in 2D, 3D



Asymptotically free particle scattered by a smooth potential.

Electronic scattering through a mesoscopic cavity.



Idealization of mesoscopic transport problem. Cavity plus waveguide geometry.

What is the duration of the scattering process?
(meaningful question in classical mechanics)

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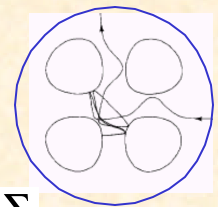
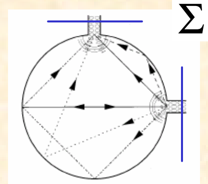
Classical scattering

Control surfaces allow to define dwell time with precision



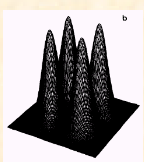


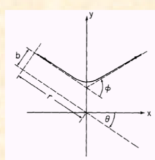
Σ Σ

dwell time : $t = \int_{t_{in}}^{t_{out}} dt$

Dependence on initial conditions: reaction functions

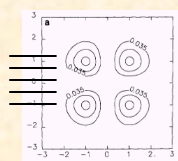
Ott's chaos book
Bleher, Grebogi & Ott, 1990



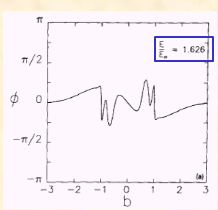


r, b, q local coordinates

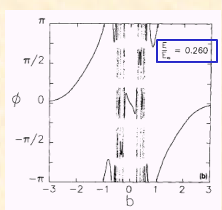
E energy



$q = p$
$b = \cdots$



regular



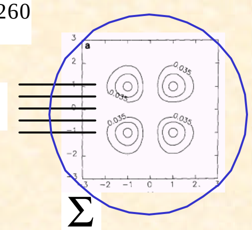
chaotic

Fractality is caused by stretching and folding.

Dependence on initial conditions: dwell time

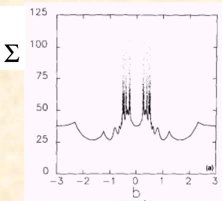
$\frac{E}{E_m} = 0.260$

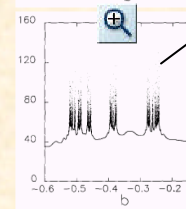
b



Σ

t_Σ



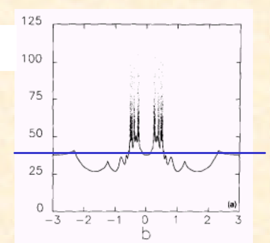


∞

Initial states on stable manifolds of prisoner trajectories have infinite dwell times.

Classical dwell time: statistical approach

t



$P(t) = ?$ distribution of dwell times

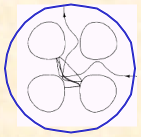
$\langle t \rangle = ?$

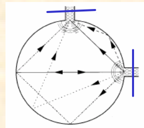
Does the average over initial states exist?

It depends on how initial states are weighted: determined by experimental setting or theoretical considerations.

Scattering as a Poincaré section mapping

The scattering map can be seen as the first return map to Σ,
Σ is equipped with coordinates (q,p)





E fixed

$$\Sigma = \{(q, p_q)\} \qquad \Sigma' = \{(y, p_y)\} \cup \cdots$$

- The scattering map is area preserving.
- The "natural" weight of a set of incoming (outgoing) states is its area in Σ (Liouville measure).

Average dwell time

The average dwell time

$$\langle t \rangle_\Sigma = \frac{1}{A(\Sigma)} \int_\Sigma t(q,p)\, dq dp \qquad A(\Sigma) = \int_\Sigma dq dp$$

is finite and can be expressed in terms of simple geometric properties of the scattering system.

Proof in three steps

- *Abstract scattering system*
- *Billiard + waveguides, dwell time measured in bounces*
- *General case*

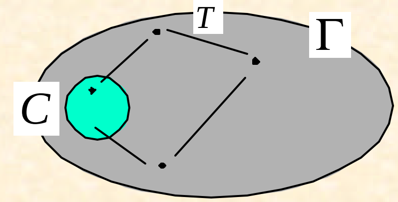
1. Recurrence map as a scattering system

$T : \Gamma \to \Gamma$

area preserving

$C \subset \Gamma$

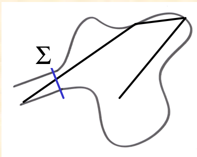
entrance/exit

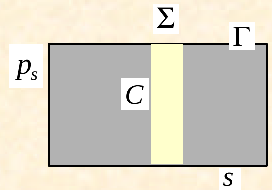


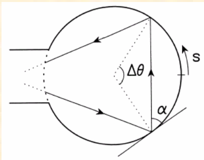
Poincaré's recurrence theorem: "All" points in C return to C

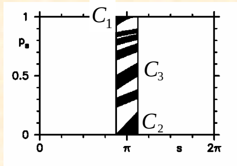
$$\langle n \rangle_C = \frac{\int_C dx\, n(x)}{A(C)} = ?$$

2. Billiard + waveguide, E fixed



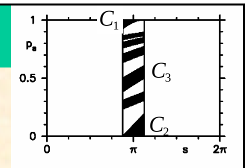






If time is measured in bounces, scattering by a billiard is a Poincaré recurrence map.

Computing the average



$$\langle n\rangle_C = \frac{\int_C dx\, n(x)}{m(C)} = \frac{\sum_{n=1}^{\infty} n\, m(C_n)}{m(C)}$$

But

$$\sum_{n=1}^{\infty} n\, m(C_n) = m\left(\bigcup_{n=1}^{\infty} T^n C\right)$$

Ergodic theory,
Cornfeld-Fomin-Sinai, p. 20

Then

$$\langle n\rangle_C = \frac{m\left(\bigcup_{n=1}^{\infty} T^n C\right)}{m(C)}$$

$\bigcup_{n=1}^{\infty} T^n C$

inner phase space explored by scattering trajectories

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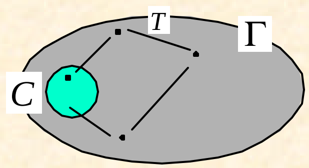
Special cases

If T is ergodic then

$$\bigcup_{n=1}^{\infty} T^n C = \Gamma \quad \forall C$$



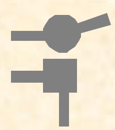
$$\Rightarrow \langle n\rangle_C = \frac{m(\Gamma)}{m(C)}$$ Kac's lemma (1947)

Even for non-ergodic T we may have $\bigcup_{n=1}^{\infty} T^n C = \Gamma$, e.g.,

The general result is

$$\langle n\rangle_C = \frac{m(\Gamma')}{m(C)}$$

Does not depend on the dynamics!



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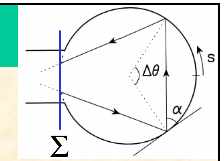
Continuous time



Observation:

$$\langle t\rangle_\Sigma < \langle n\rangle_\Sigma \frac{D}{v} < \infty$$

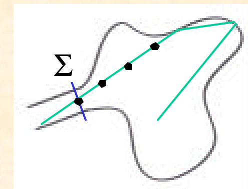
D diameter

v velocity

Can the previous scheme be adapted to the continuous case?

Consider the stroboscopic map !

$$\langle t\rangle_\Sigma = \lim_{\Delta t\to 0} \Delta t \frac{m(\overline{\Gamma}')}{m(\overline{C})}$$



$\overline{C}, \overline{\Gamma}'$ are 4 dimensional

$\overline{C}$ set of incoming states

$\overline{\Gamma}'$ inner phase space explored by scattering trajectories

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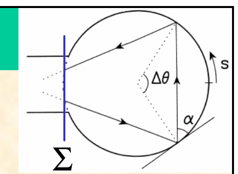
Constructing $\overline{C}$ from C



Let's C grow in two directions:

(1) normal to the energy surface ΔE

(2) along the flow Δt

$$\overline{C} = \{(y, p_y, E, t)\}$$

$$\langle t\rangle_\Sigma = \lim_{\Delta t\to 0} \Delta t \frac{m(\overline{\Gamma}')}{m(\overline{C})}$$

$\overline{\Gamma}'$ energy shell of width ΔE inside the scatterer

With this choice:

$$m(\overline{C}) = m(C)\Delta E \Delta t \qquad m(\overline{\Gamma}') = \frac{\partial \Omega'}{\partial E}\Delta E$$

Ω'

volume inside the energy shell to the right of Σ

$$m(C) = \Omega_\Sigma$$

volume in Σ inside the energy shell of Σ

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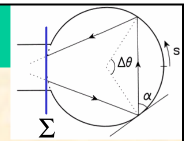
Final result



$$\langle t\rangle_\Sigma = \frac{1}{\Omega_\Sigma}\frac{\partial \Omega'}{\partial E}$$

density of states

number of channels

$$\Omega = \int_{\substack{H\le E\\ x>0}} dx\,dy\,dp_x\,dp_y \qquad \Omega_\Sigma = \int_{H\,(x=0,\,p_x=0)\le E} dy\,dp_y$$

These formulas are also valid for smooth systems !

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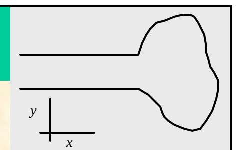
Multichannel scattering



in the waveguide $H = H_y + \frac{p_x^2}{2m_*}$

$$H_y f_n(y) = e_n f_n(y)$$

$e_n < E$ open channels, propagating

$e_n > E$ closed channels, evanescent

$$e_n + \frac{\hbar^2 k_n^2}{2m_*} = E$$

$$\psi(x,y;E) = \sum_{n=1}^{N} a_n \frac{e^{+ik_n x}}{\sqrt{k_n}} f_n(y) + \sum_{n=1}^{\infty} b_n \frac{e^{-ik_n x}}{\sqrt{k_n}} f_n(y)$$

$$b_m = \sum_{n=1}^{N} S_{mn} a_n$$ unitary scattering S-matrix

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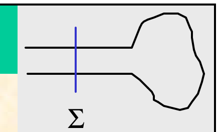
The S-matrix as a QPM



For fixed x, *e.g.*, x=0

$$S:\ \underbrace{\sum_{n=1}^{N} a_n \frac{f_n(y)}{\sqrt{k_n}}}_{\psi_{in}} \to \underbrace{\sum_{n=1}^{\infty} b_n \frac{f_n(y)}{\sqrt{k_n}}}_{\psi_{out}}$$

- The classical limit of the quantum scattering map (S-matrix) is a Poincaré section map
- Unitarity of S corresponds to classical area preservation
- Averages over (quantum) channels correspond to averages over Σ with the Liouville weight

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Simplest version of quantum time delay

s-wave scattering, spherical symmetry

stationary phase peak

incoming wavepacket, $r \to \infty$

$$r\psi_{in} \approx \int_0^\infty dE\,|A(E)|\,\exp\left\{i\left[-\frac{pr}{\hbar} - \frac{Et}{\hbar} + \delta(E)\right]\right\}$$

$r_* = v(t_0 - t)$

$t_0 = \hbar(d\delta/dE)$

outgoing wavepacket, $r \to \infty$

$$r\psi_{out} \approx \int_0^\infty dE\,|A(E)|\,\exp\left\{i\left[+\frac{pr}{\hbar} - \frac{Et}{\hbar} + \delta(E) + 2\eta(E)\right]\right\}$$

$r_* = v(t - t_0 - \Delta t)$

$$S(E) = e^{2i\eta(E)}$$

S “matrix”

delay time $\Delta t = 2\hbar \frac{d\eta}{dE}$

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Eisenbud-Wigner time delay

$$\Delta t = 2\hbar \frac{d\eta}{dE} = -i\hbar S^* \frac{dS}{dE}$$

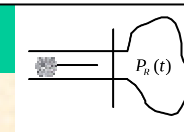
Eisenbud (1948)
Wigner (1955)

Example: hard core of radius a

$$S(E) = e^{-2ika} \qquad k = p/\hbar = mv/\hbar$$

$$\Delta t = -\frac{2a}{v}$$ time advance

Time delay matrix



$$t_d = \int_0^\infty dt\, P_R(t) = \sum_{a,b} A_a^* A_b \langle \hat{t}_{ab}(E) \rangle$$

$$\hat{t}(E) = -i\hbar S^+ \frac{dS}{dE}$$ time-delay matrix
Smith (1960)

$$\Delta t = -i\hbar S^* \frac{dS}{dE}$$ Wigner-Eisenbud

$\langle \hat{t}_{aa} \rangle$ time delay for a wavepacket inciding via channel a

Averaging over channels (equal weights)

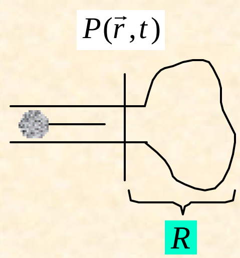
$$t_W(E) = -\frac{i\hbar}{N} \mathrm{Tr}\left(S^+ \frac{dS}{dE} \right)$$ Wigner time-delay

A more general definition

$$P_R(t) = \int_R P(\vec{r},t)\, d\vec{r}$$

$$t_d = \int_0^\infty dt\, P_R(t)$$

classical dwell time in R



$$P(\vec{r},t) = |\psi(\vec{r},t)|^2 \quad \Longrightarrow$$ quantum dwell time definition

$$t_d = \int_0^\infty dt \int_R |\psi(\vec{r},t)|^2\, d\vec{r}$$

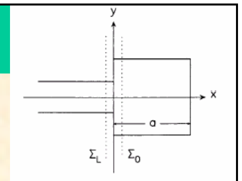
Connection with density of states

S-matrix pole decomposition

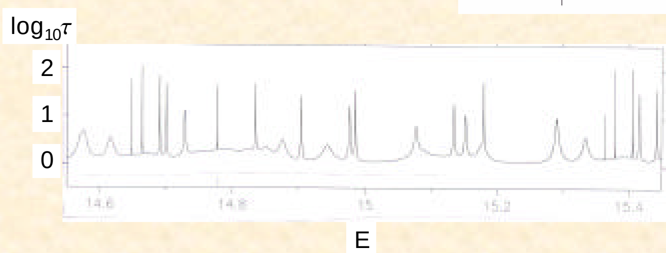
$$t_W(E) \approx -\frac{2\hbar}{N} \mathrm{Im}\,\mathrm{Tr} \frac{1}{E - H_{\mathrm{eff}}} = \frac{\hbar}{N} \sum_n \frac{\Gamma_n}{(E - E_n)^2 + \Gamma_n^2/4}$$

$$t_W(E) = \frac{h}{N} \rho(E) + \cdots$$ Krein-Friedel-Lloyd

Examples



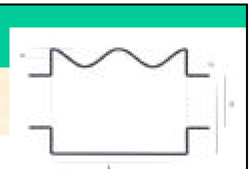
Wigner time delay as indicator of resonant structure

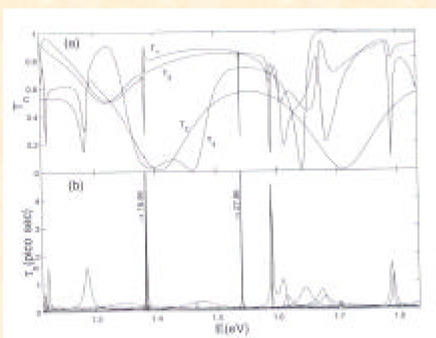


15 open channels, overlapping resonances
A. M. Ozorio de Almeida & ROV (1999)

Examples II

transmission *vs* Wigner time delay





transmission coefficients

partial time delays

4 channels in each guide, Akguc & Reichl (2000)

Energy average

$$t_W(E) \approx \frac{h}{N} r(E)$$

Averaging over a window containing many resonances ...

$$\langle t_W \rangle \approx \frac{h}{N} \langle r \rangle = \frac{h}{N\Delta}$$

In the semiclassical limit

$$\underbrace{N \approx \frac{1}{h} \int_{H_y<E} dy dp_y}_{\Omega_\Sigma} \qquad \underbrace{\langle r \rangle \approx \frac{1}{h^2} \frac{\partial}{\partial E} \int_{\substack{H \le E \\ x>0}} dx dy dp_x dp_y}_{\Omega}$$

$$\Rightarrow \quad \langle t_W \rangle \approx \frac{1}{\Omega_\Sigma} \frac{\partial \Omega}{\partial E}$$

independent of $\hbar$
purely geometric

Quantum-classical correspondence ?

$$\langle\langle t_W \rangle\rangle \approx \frac{1}{\Omega_\Sigma} \frac{\partial \Omega}{\partial E}$$

energy averaged Wigner time delay

$$\langle t \rangle_\Sigma = \frac{1}{\Omega_\Sigma} \frac{\partial \Omega'}{\partial E}$$

classical time delay

- There is no correspondence in general, *i.e.*, for systems with mixed phase space
- Differences are due to tunneling
- Very thin resonances also contribute to the quantum average time delay

Final remark

$$\langle\langle t_W \rangle\rangle \approx \frac{1}{\Omega_\Sigma}\frac{\partial \Omega}{\partial E}$$

energy averaged
Wigner time delay

under certain
conditions

$$= \langle t \rangle_\Sigma$$

classical
time delay

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