

# Conductance fluctuations in ballistic quantum dots with mixed dynamics – Fano lineshapes

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*The 12th Brazilian Workshop on Semiconductor Physics (BWSP-12) will be held during the week of April 4-8, 2005 in [São José dos Campos](#), São Paulo. The scientific [program](#) and content of the BWSP-12 will continue the tradition of this [series](#) of biannual conferences and will cover all aspects of the frontiers of the semiconductor physics. This is the first time the conference is being held in São José dos Campos, a well known scientific and technological center, pioneer in the semiconductor physics in Brazil. The BWSP-12, with the Brazilian Physical Society (SBF), celebrates the [2005 World Year of Physics](#).*

# Abstract

**Ballistic quantum dots with ergodic classical dynamics exhibit universal conductance fluctuations well described by random matrix theories. However, when the dot contains classical structures (“islands”) that cannot be accessed from outside, deviations from the universal behavior are expected.**

**If islands are large enough as compared with the area quantum, they can support quantum states weakly coupled to the reservoirs, which manifest themselves as thin resonances.**

**The fact that islands are classically isolated implies that a standard semiclassical approach in terms of open orbits is not appropriate. Instead, it is preferable to decompose the dot into parts which do not contain islands. Each part is then treated with the standard semiclassical theory. Eventually, the original scattering matrix is obtained using the exact quantum rules of composition of  $S$ -matrices. We apply this scheme to describe non-universal conductance patterns observed in a recent experiments of mesoscopic transport.**

**The predictions of the theory are compared with the results of numerical simulations obtained with the recursive Green’s function method.**

# Experiments

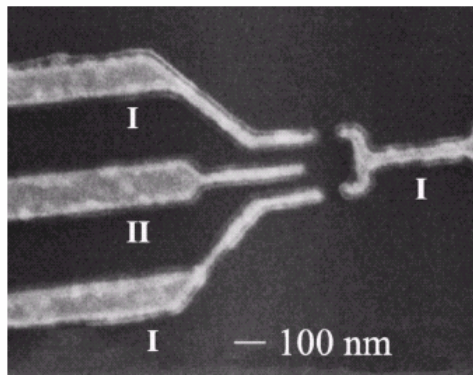
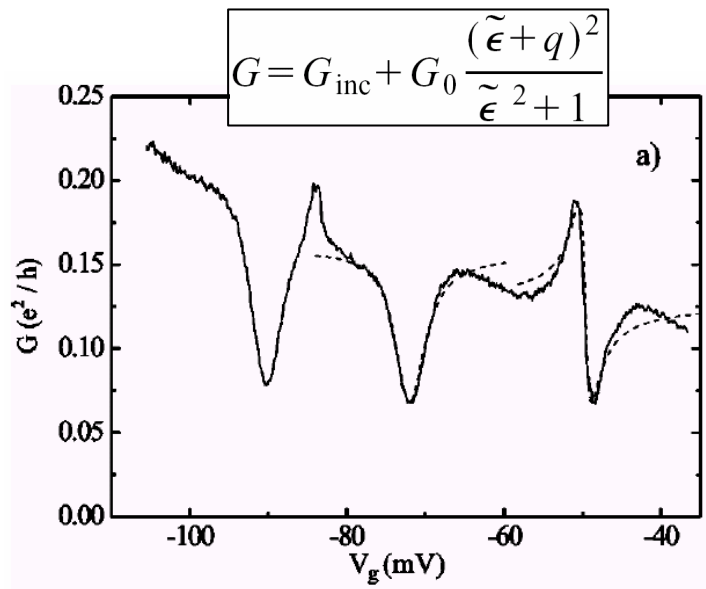
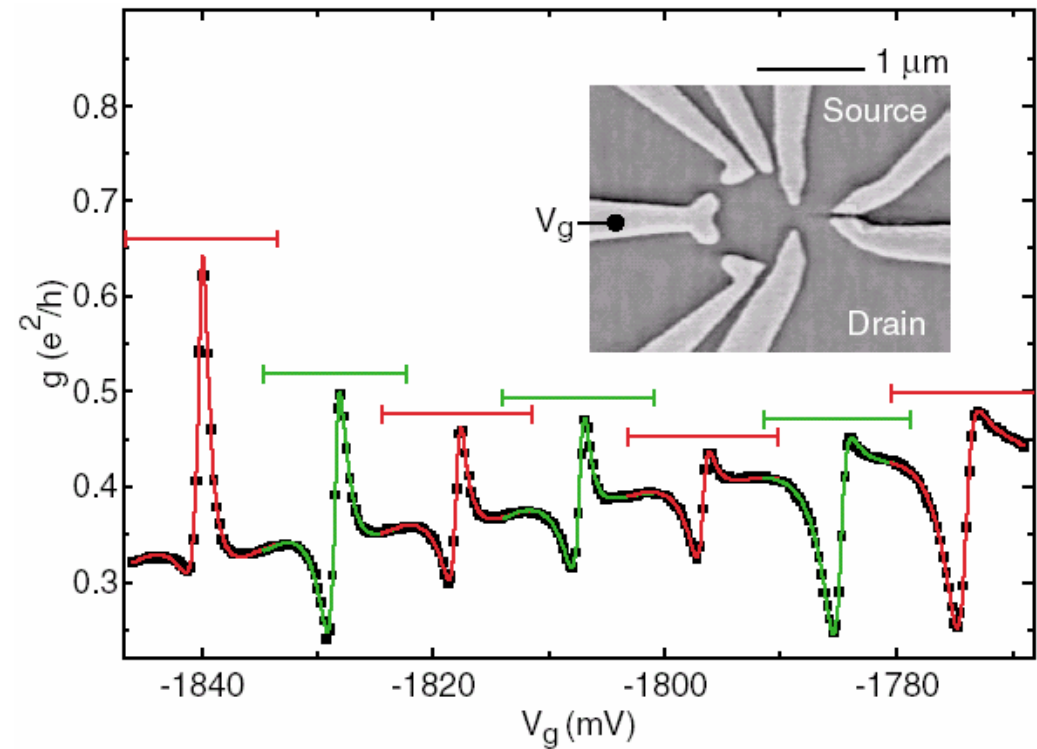


FIG. 1. Electron micrograph of a SET showing the split gates (*I*) that define the tunnel barriers and the additional gate electrode (*II*) that adjusts the potential energy on the droplet.

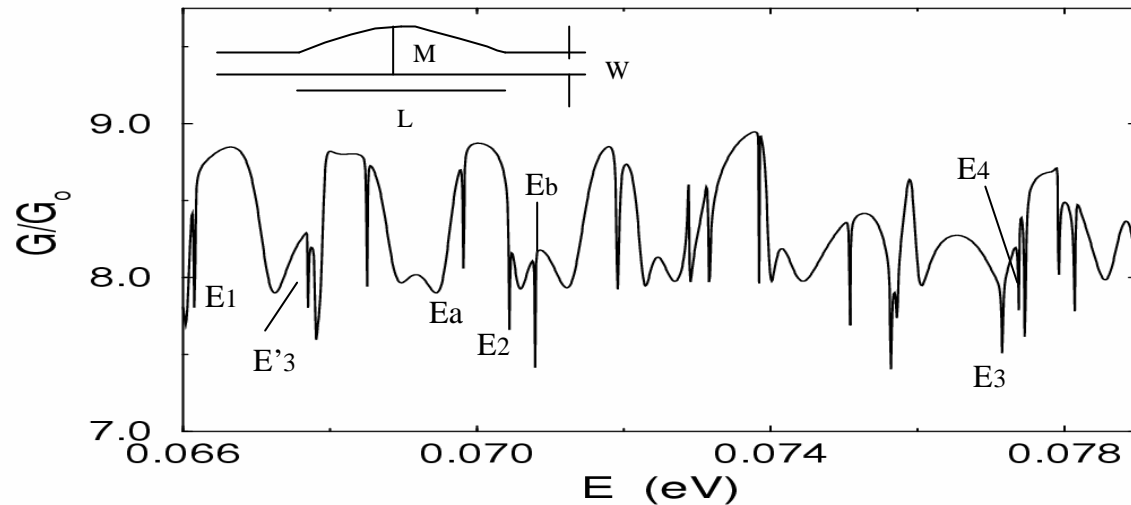
Johnson, Marcus, Hanson, Gossard  
PRL 2004



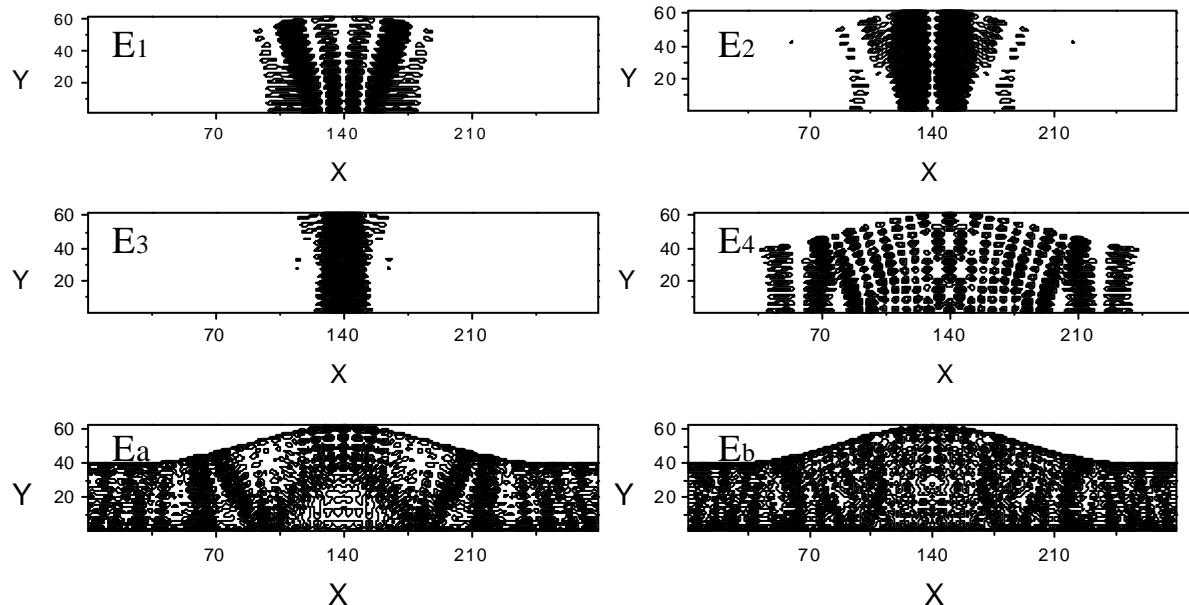
Göres, Goldhaber-Gordon, Heemeyer, Kastner,  
Shtrikman, Mahalu, Meirav  
PRB 2000

# Our system: cosine shaped ballistic cavity

Numerical method:  
recursive Green's function  
M. Mendoza  
PhD thesis, IFGW/UNICAMP, 2003



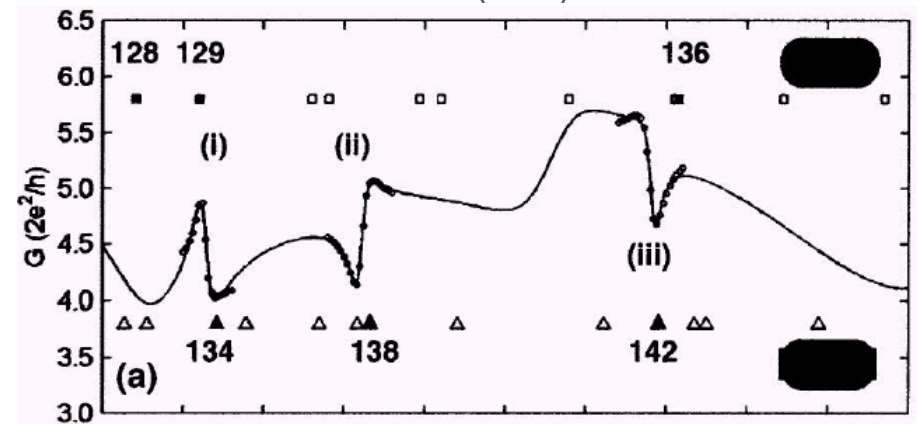
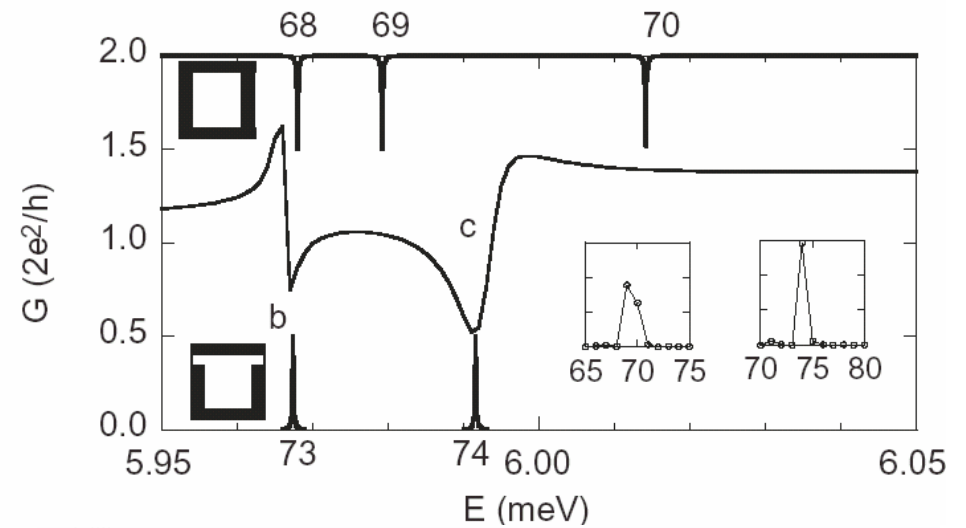
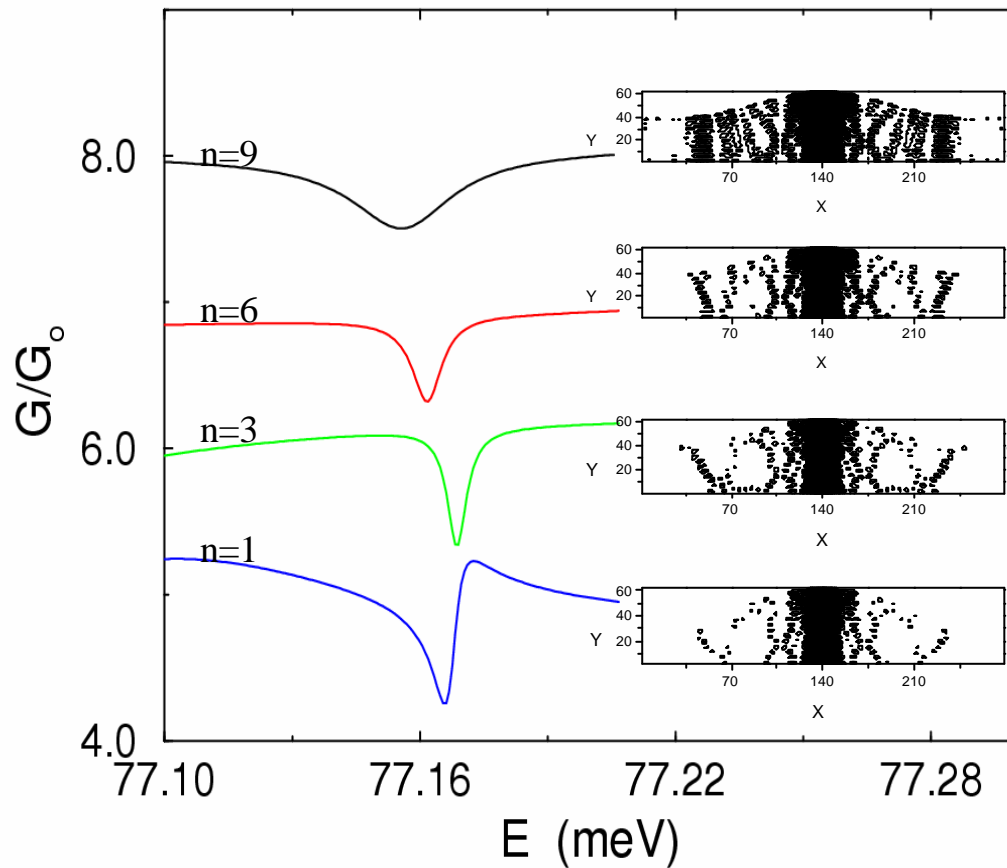
conductance  
( = transmission )  
vs energy



parameters:  
 $L = 442$  nm  
 $M = 124$  nm  
 $W = 80$  nm  
 $m = 0.067 m_0$

local density of states

# Lineshape



Akis, Bird, Vasileska, Ferry, Moura, Lai  
2003

# Strategy A: Poincaré section decomposition

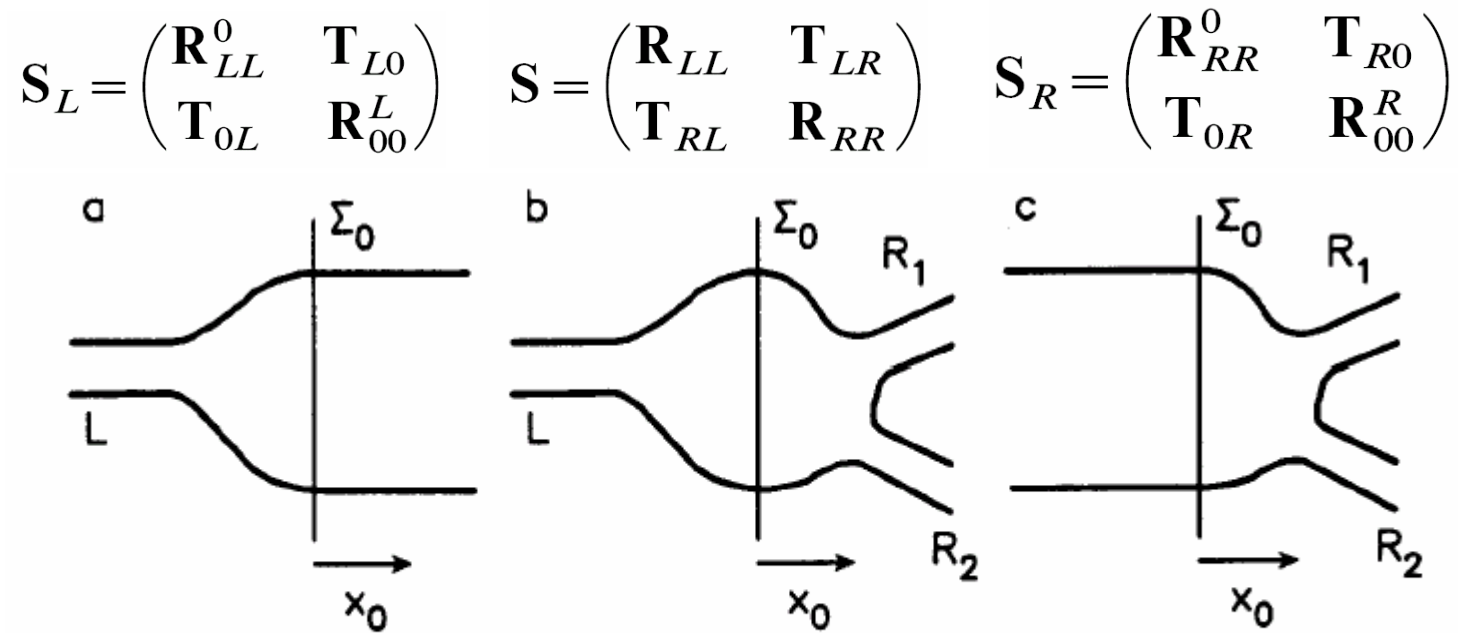


FIG. 2. The multilead scattering system. The section  $\Sigma_0$  decomposes the full scatterer (b) into the nonresonant systems (a) and (c). Thick curves represent a single contour of the potential energy.

Ozorio de Almeida & Vallejos  
AOP 1999, PE 2001

$$\mathbf{R}_{LL} = \mathbf{R}_{LL}^0 + \mathbf{T}_{L0} [\mathbb{1} - \mathbf{R}_{00}^R \mathbf{R}_{00}^L]^{-1} \mathbf{R}_{00}^R \mathbf{T}_{0L},$$

$$\mathbf{R}_{RR} = \mathbf{R}_{RR}^0 + \mathbf{T}_{R0} [\mathbb{1} - \mathbf{R}_{00}^L \mathbf{R}_{00}^R]^{-1} \mathbf{R}_{00}^L \mathbf{T}_{0R},$$

$$\mathbf{T}_{LR} = \mathbf{T}_{L0} [\mathbb{1} - \mathbf{R}_{00}^R \mathbf{R}_{00}^L]^{-1} \mathbf{T}_{0R},$$

$$\mathbf{T}_{RL} = \mathbf{T}_{R0} [\mathbb{1} - \mathbf{R}_{00}^L \mathbf{R}_{00}^R]^{-1} \mathbf{T}_{0L}.$$

# Position of resonances

A thin resonance occurs for an energy such that

$$\det(\mathbb{1} - \mathbf{R}_{00}^R \mathbf{R}_{00}^L) = \det(\mathbb{1} - \mathbf{R}_{00}^L \mathbf{R}_{00}^R) \rightarrow 0.$$

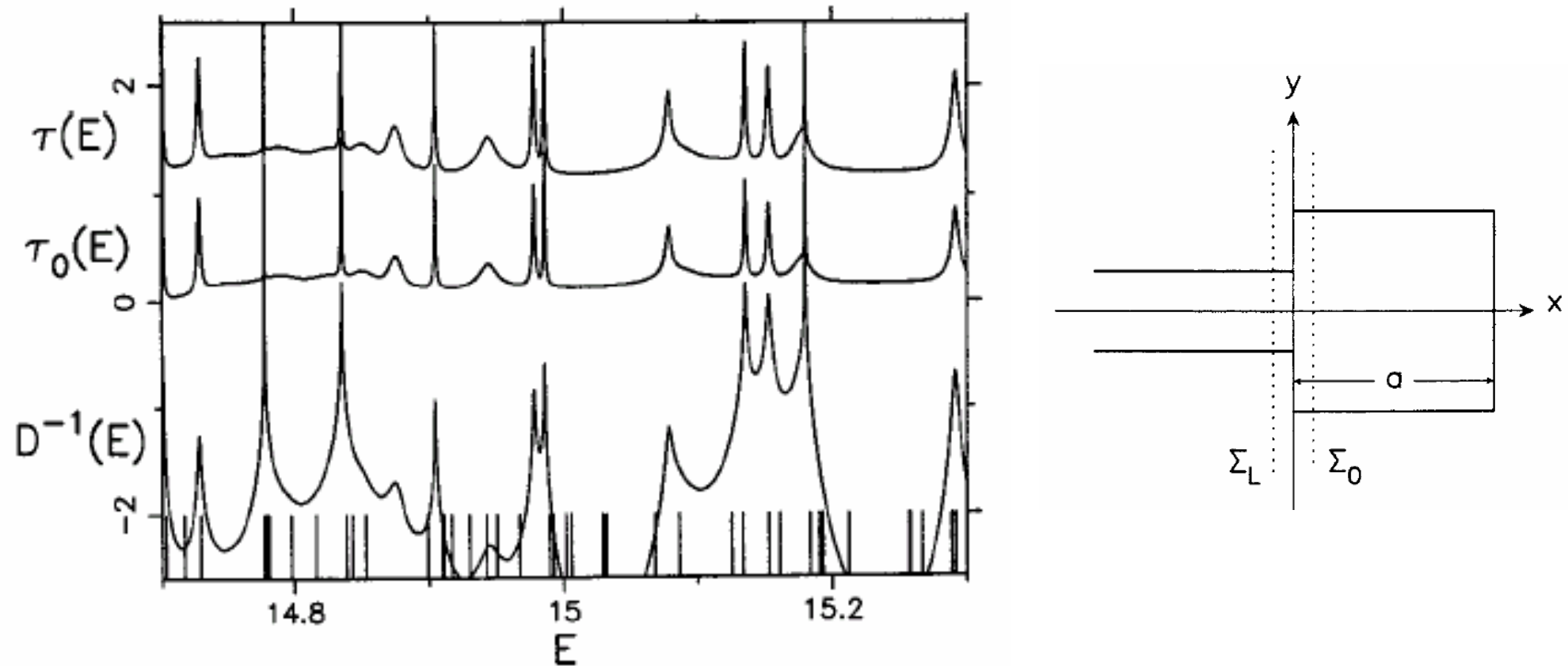
Time delay (similar to density of states)

$$\tau(E) = -\frac{i\hbar}{\Lambda} \frac{d}{dE} \log \det \mathbf{S}_{\text{open}}$$

Resonant part of the density of states !?

$$\tau_0(E) = -i\hbar \frac{d}{dE} \log \det(1 - \mathbf{R}_0)^{-1}$$

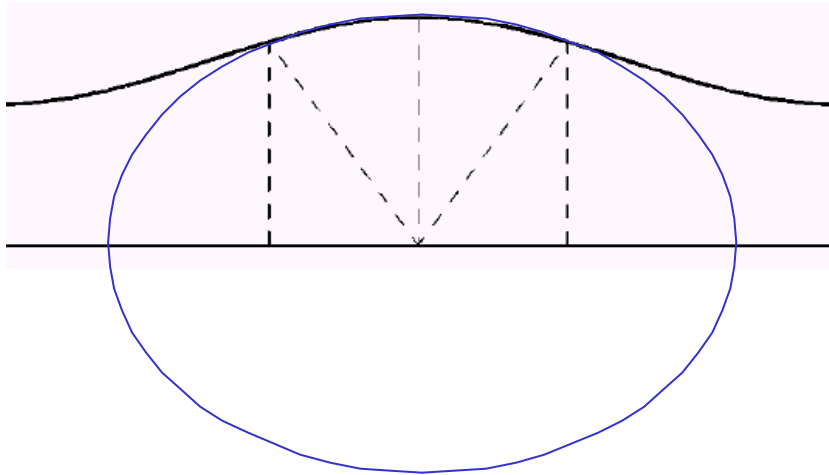
# Resonant density of states



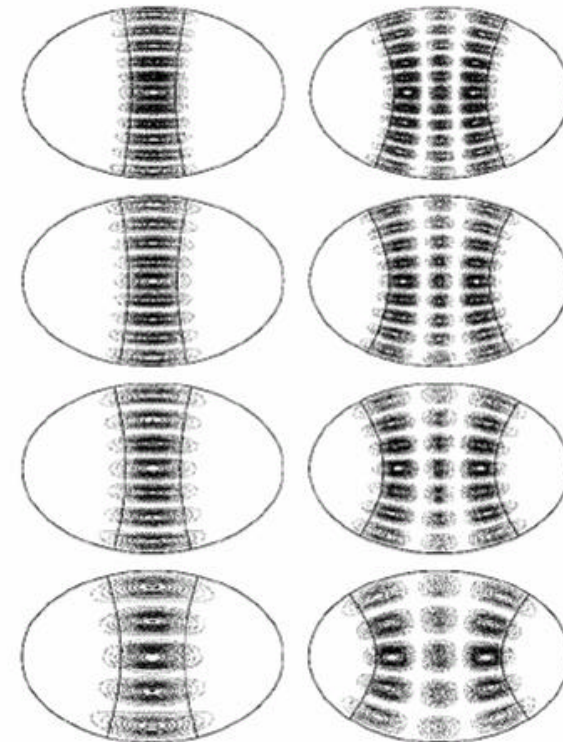
**FIG. 11.** Log<sub>10</sub>-linear plot of the exact time delay  $\tau(E)$ , the section time delay  $\tau_0(E)$ , and the spectral determinant  $D(E)$ . The spikes at the bottom represent the energy levels of the bound system obtained by closing the cavity with a hard wall at  $x = 0$ .



# A subset of resonances -- Elliptical billiard

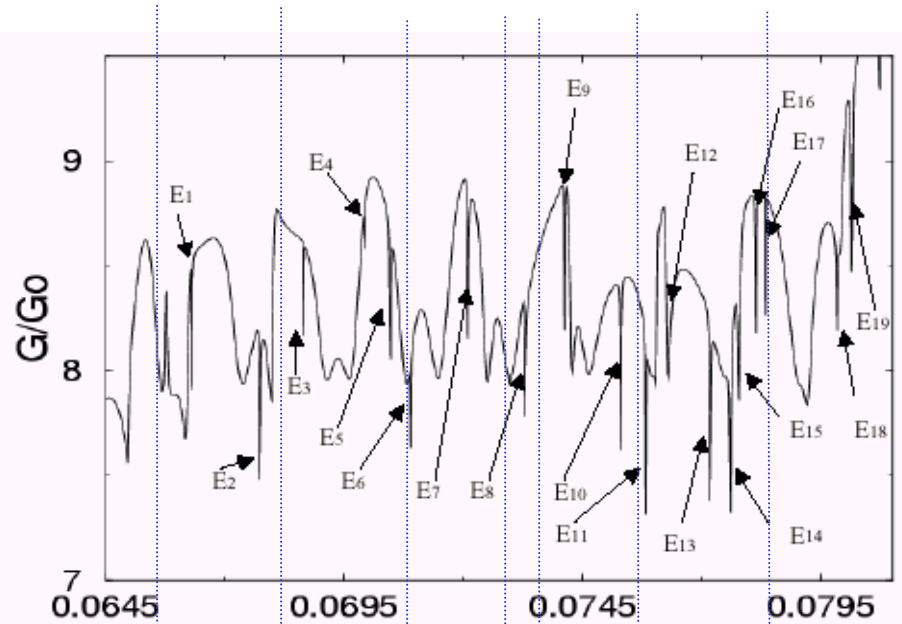


**Cosine billiard and osculating ellipse.**

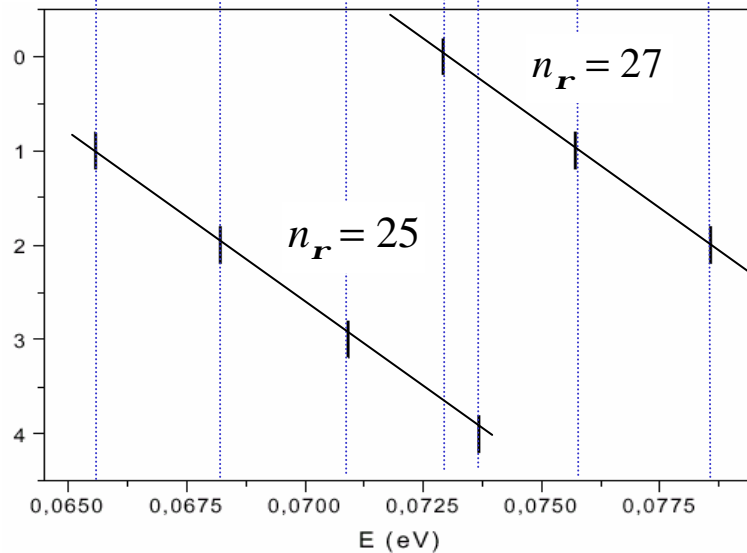


**Some eigenfunctions of the elliptical billiard with  $n_\phi=0$  (left) and  $n_\phi=2$  (right). All eigenfunctions are even in  $x$  and  $y$ . As  $n_\rho$  increases while keeping  $n_\phi$  fixed, these families localize in smaller neighbourhoods of  $x=0$ .**

# Quantization rule



Energy levels of quasi-bound states (bottom) versus conductance (top).



$$E \approx \frac{1}{32m_*} \left( \frac{2p\hbar}{b} \right)^2 \left[ \left( n_F + \frac{1}{2} \right) \frac{2}{p} \arccos \left( \frac{c}{a} \right) + (n_r + 1) \right]^2$$

# Work in progress

1. Study of lineshape for an isolated S-matrix pole coupled to one or many continua.

Analysis of background phase shift associated to direct transmission.

2. Linewidth:

Semiclassical calculation of widths for the decay of island states immersed in a chaotic sea.