I) Wigner function correlations & II) Semiclassical propagation of wavepackets in open systems

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Instituto do Milênio
Informação Quântica

II Encontro do Instituto do Milênio de Informação Quântica
Departamento de Física – UFMG, Belo Horizonte, Brazil
September 29 / 2004

I) Wigner function correlations

with
AM Ozorio de Almeida (CBPF)
M Saraceno (Buenos Aires)

Motivations (1)

$$\int dx W_r(x) W_r(x-x) = tr\, r T_x\, r T_x^+$$

displacement D(α)

density matrix

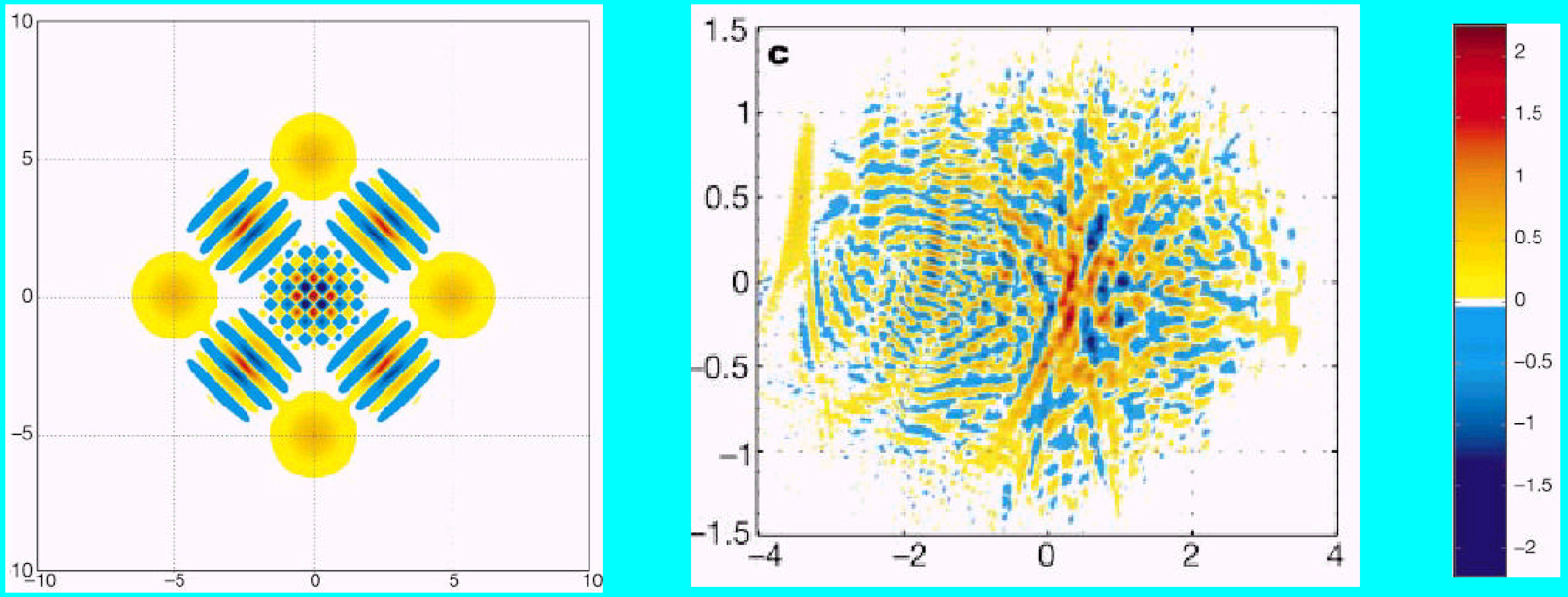
Averaging over Gaussian ξ gives the fidelity of a Gaussian displacement channel,

Fidelity of Gaussian channels,

CM Caves & K Wódkiewicz, quant-ph/0409063

Wigner function correlations – Motivations (2)

Sub-Planck structures in phase space and its relevance for quantum decoherence, WH Zurek, Nature 2001



Zurek: chaotic states hipersensitive to perturbations (shifts), i.e.,

$\int dxW(x)W(x-x)$ decays very fast, *i.e.,* $x_* \approx \hbar / D$

Question

What is the shape of the Wigner correlation for typical states?

Sub-Planck structure, decoherence, and many-body environments,
A Jordan & M Srednicki, quant-ph/0112139
Action scales for quantum decoherence and their relation to structures in phase space,
D Alonso, S Brouard, JP Palao, RS Mayato, PRA 2004

Chaotic states, small displacements → Berry-Voros conjecture

$$W(x) \propto d\left(H(x) - E\right)$$

Conclusion → power-law decay of correlations for few freedoms

Our results – 1) general properties

$$\left(2p\hbar\right)^{f} c\left(x\right) = \int dx e^{ix\wedge x/\hbar} W\left(x\right)$$

characteristic function or chord function

$$\int dx W(x) W(x - x) = \int dh e^{ih\wedge x/\hbar} \left|c\left(h\right)\right|^{2}$$

Wiener--Khinchin

Easy to prove: for a pure state the Wigner correlation is Fourier invariant

$$\int dh e^{ih\wedge x/\hbar} \left|c\left(h\right)\right|^{2} = \left(2p\hbar\right)^{f} \left|c\left(x\right)\right|^{2}$$

Then

$$\int dx W(x)W(x-\boldsymbol{x}) = (2p\hbar)^f \left|\boldsymbol{c}(\boldsymbol{h})\right|^2$$

for a pure state it suffices to study χ

Example: parity eigenstate $\Pi y = \pm y$

$$\boldsymbol{c}(\boldsymbol{x}) = \pm 2^{-f} W(-\boldsymbol{x}/2)$$

➡

$$\int dx W(x)W(x-\boldsymbol{x}) \propto \left|W(\boldsymbol{x}/2)\right|^2$$

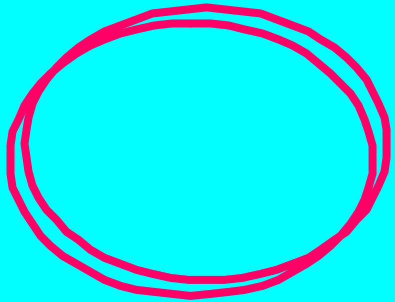
Correlations on all scales!

If parity symmetry is broken ...

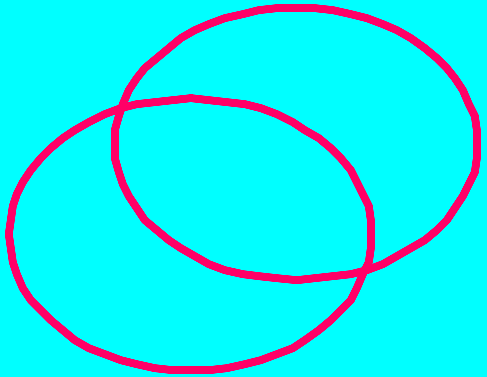
Characteristic function -- semiclassical analysis

$$c(x) \propto \langle y | T_{-x} | y \rangle$$

WKB → torus overlap

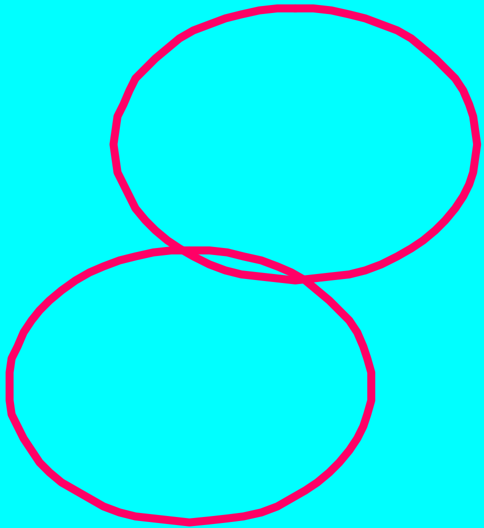


$$\int_{torus} dq\, e^{ix(q)\wedge x}$$



standard
stationary phase

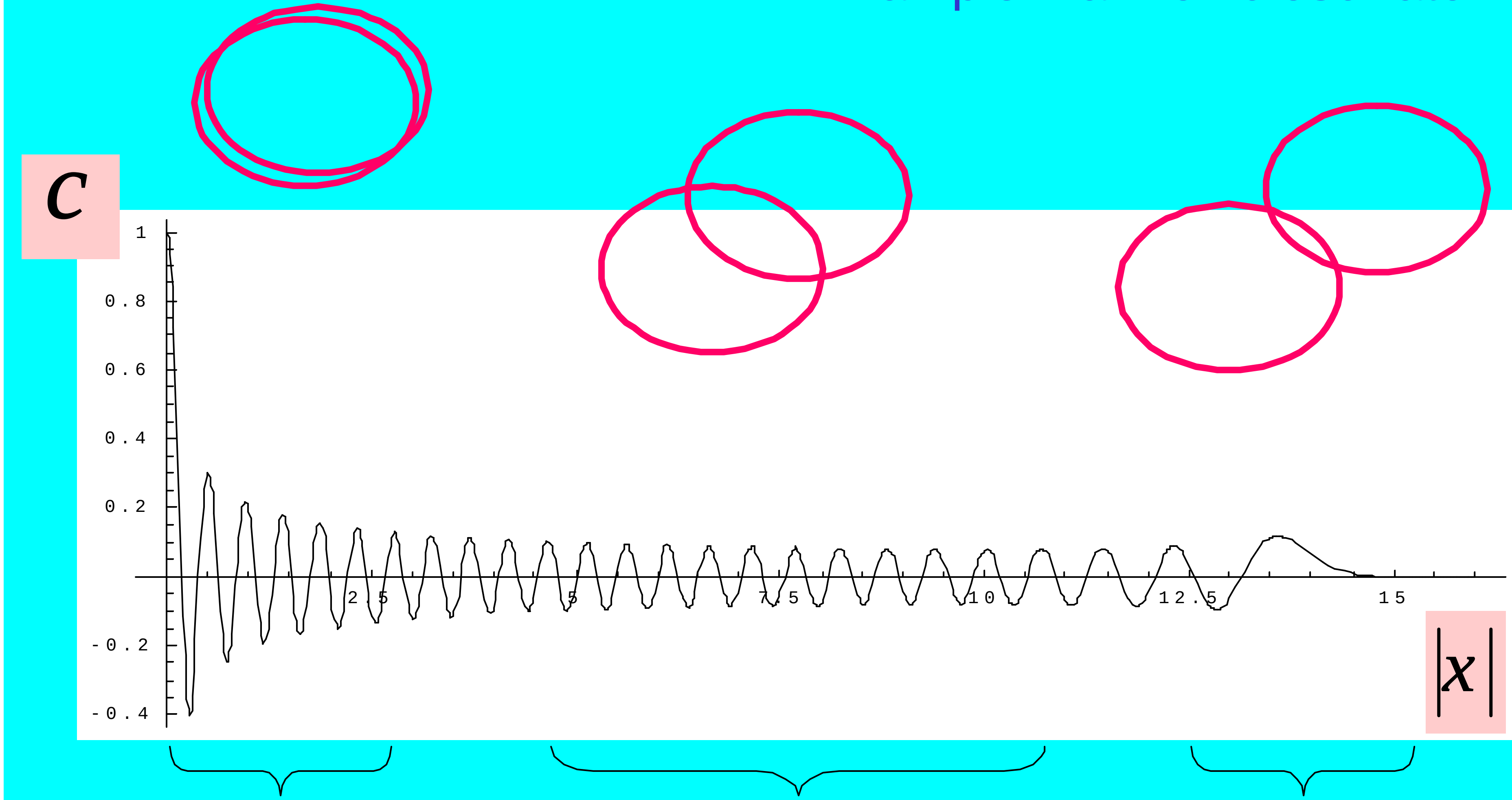
$$A_1 e^{iS_1/\hbar} + A_2 e^{iS_2/\hbar}$$



coalescing points:
Airy function

WKB characteristic function

Example: harmonic oscillator



Bessel

cosine

Airy

II) Semiclassical propagation of wavepackets in open systems

with AM Ozorio de Almeida & F Toscano (UFRJ)

Goal

Describe propagation of wavepackets with Lindblad equation in semiclassical regimes

Plan

- Unitary nonlinear shear of squeezed wavepackets
- Unitary arbitrary stretching and folding
- Linearization of Lindblad dynamics around center of mass trajectory (F Nicacio's thesis)
- General case of nonlinear Hamiltonian and arbitrary Lindbladians
- Classification of time scales

Quantum-classical transition (UFRJ)

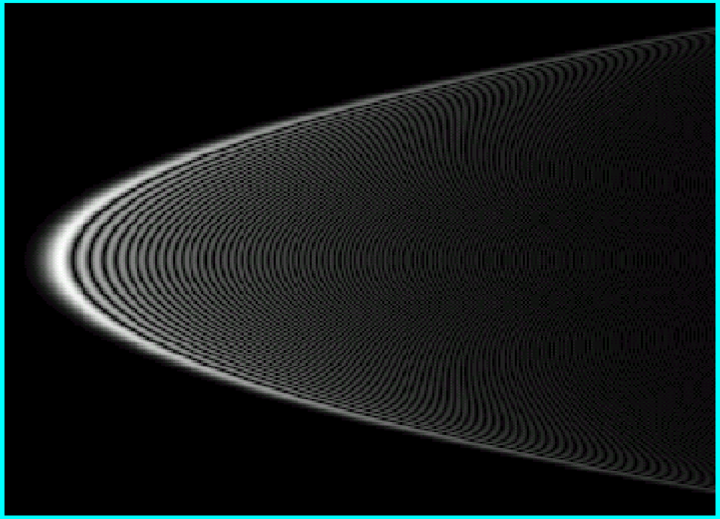
Chaos, decoherence, state protection and the quantum-classical transition for trapped ions
ARR Carvalho, PhD thesis, UFRJ 2002

Dissipation, diffusion, and the quantum-classical limit in phase space
ARR Carvalho, L Davidovich, RL de Matos Filho & F Toscano, 2003

Environmental effects in the quantum-classical transition for the delta-kicked harmonic oscillator
ARR Carvalho, RL de Matos Filho & L Davidovich, PRE 2004

Achievements

- ✓ Partial understanding of development of quantum structures in a nonlinear shear



Wigner function of a squeezed Gaussian state after a transverse nonlinear (cubic) shear

- ✓ Defined basic ingredients for a semiclassical theory of nonlinear unitary wavepacket evolution (Wigner propagator)

Students

From left to right: Raphael NP Maia (MSc, PhD), Raúl O Vallejos, Rômulo F Abreu (PhD), Maria JB Moura (former SI), Fernando AN Nicacio (MSc)

Theses in progress

- ✓ Fernando AN Nicacio (MSc)
  Semiclassical Lindblad propagation

- ✓ Rômulo F Abreu (PhD)
  i) Construction of a test bench for semiclassical theories, i.e., Lindblad evolution of density matrix, Wigner & characteristic functions, etc.
  ii) ...

- ✓ Raphael NP Maia (PhD)
  i) Cumulant expansion of density propagators
  ii) ...