Generalized Cat States

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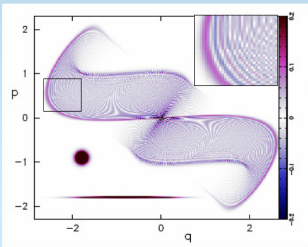
Motivation

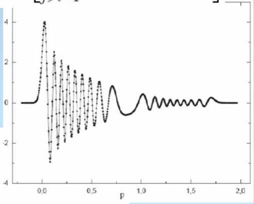
$$W^{\Psi}(x) \propto \left\langle \Psi \middle| \hat{R}_x \middle| \Psi \right\rangle$$

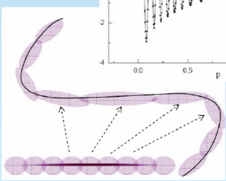
Pure superposition: $|\Psi\rangle = \sum_{i=1}^{M} C_i |\psi_i\rangle,$

$$W^{\Psi}(x) := \frac{1}{(\pi\hbar)^n}\left[\sum_{i=1}^{M} |C_i|^2 \langle\psi_i|\hat{R}_x|\psi_i\rangle\right] + \frac{2}{(\pi\hbar)^n}\mathrm{Re}\left[\sum_{j>i=1}^{M} C_i C_j^* \langle\psi_j|\hat{R}_x|\psi_i\rangle\right]$$

Nearby orbit approximation:







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Wigner Function

$$|\mathsf{S};\xi\rangle := \hat{T}_\xi \hat{M}_{\mathsf{S}}|0\rangle$$

2-Superposition: $|\psi\rangle = \frac{1}{\sqrt{1+2\mathrm{Re}\left[a^* b \langle \mathsf{U};u|\mathsf{V};v\rangle\right]}}\left(a|\mathsf{U};u\rangle + b|\mathsf{V};v\rangle\right)$

Wigner:

$$W_\psi(x) = \frac{1}{1+2\mathrm{Re}[a^*b\langle \mathsf{U};u|\mathsf{V};v\rangle]}\left\{|a|^2\mathcal{G}_{2n}^{\mathbb{R}}\left[x;\left(\mathsf{U}\mathsf{U}^\top\right)^{-1};u\right] + |b|^2\mathcal{G}_{2n}^{\mathbb{R}}\left[x;\left(\mathsf{V}\mathsf{V}^\top\right)^{-1};v\right]\right.$$

interference→ $\left. + \, 2\,\mathrm{Re}\left[a^* b \langle \mathsf{U};u|\hat{R}_x|\mathsf{V};v\rangle\right]\right\}$

$$\langle 0|\hat{M}_{\mathsf{U}}^\dagger \hat{T}_u^\dagger \hat{R}_x \hat{T}_v \hat{M}_{\mathsf{V}}|0\rangle = \frac{2^n\, i^{\nu-\frac{1}{2}\mathrm{sng}\,\mathsf{C}_{\mathsf{U}}^{-1}\mathsf{V}}}{\sqrt{\det[(\mathsf{U}+\mathsf{V})+i(\mathsf{U}-\mathsf{V})\mathsf{J}]}}\, e^{\frac{i}{\hbar}x\wedge\zeta+\frac{i}{2\hbar}\zeta\wedge\bar{\eta}}\, \mathcal{G}_{2n}^{\mathbb{C}}[x;\mathsf{G};\bar{\eta}]$$

$$\mathsf{G} = 2\left(\mathsf{U}\mathsf{U}^\top + \mathsf{V}\mathsf{V}^\top\right)^{-1} - i\left(\mathsf{U}\mathsf{U}^\top + \mathsf{V}\mathsf{V}^\top\right)^{-1}\left(\mathsf{U}\mathsf{U}^\top - \mathsf{V}\mathsf{V}^\top\right)\mathsf{J}$$

$\bar{\eta} = \frac{1}{2}(u+v)$ e $\zeta := v - u$ *Complex, Symmetric, Symplectic*

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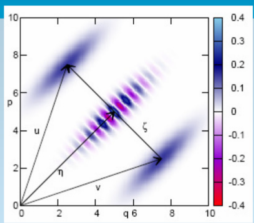
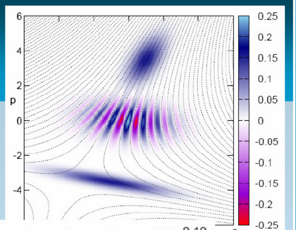
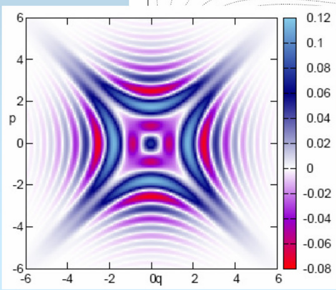
2D Gallery



standard cat state
$\mathsf{U} = \mathsf{V} = \mathsf{S}$





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2N Dimensions: Euler Decomposition & Sylvester Law of Inertia

Euler decomposition for $\mathsf{G}' = 2\left(\mathbb{I}_2 + \mathsf{V}\mathsf{V}^\top\right)^{-1} - i\left(\mathbb{I}_2 + \mathsf{V}\mathsf{V}^\top\right)^{-1}\left(\mathbb{I}_2 - \mathsf{V}\mathsf{V}^\top\right)\mathsf{J}$

$$\mathsf{S} = \mathsf{O}\mathsf{Z}\mathsf{O}', \quad \text{com} \quad \begin{cases} \mathsf{Z} = \mathrm{Diag}\left(z_1,\dots,z_n,1/z_1,\dots,1/z_n\right), & z_i \geq 1\ \forall i \\ \mathsf{O}',\mathsf{O} \in \mathrm{Sp}(2n,\mathbb{R}) \cap \mathrm{SO}(2n) \end{cases}$$

$$\mathsf{S}\mathsf{S}^\top = \mathsf{O}\mathsf{Z}\mathsf{O}'\mathsf{O}'^\top\mathsf{Z}\mathsf{O}^\top = \mathsf{O}\mathsf{Z}^2\mathsf{O}^\top$$

law of Inertia for $\mathsf{G} = 2\left(\mathsf{U}\mathsf{U}^\top + \mathsf{V}\mathsf{V}^\top\right)^{-1} - i\left(\mathsf{U}\mathsf{U}^\top + \mathsf{V}\mathsf{V}^\top\right)^{-1}\left(\mathsf{U}\mathsf{U}^\top - \mathsf{V}\mathsf{V}^\top\right)\mathsf{J}$

$$(n_+, n_-, n_0)\left[\mathrm{BAB}^\top\right] = (n_+, n_-, n_0)\left[\mathrm{A}\right]$$

In 2N-D the intereference pattern is also hyperbolic !!

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Decoherence of Generalized Cats

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \frac{1}{\hbar}\sum_k\left(\hat{L}_k\hat{\rho}\hat{L}_k^\dagger - \frac{1}{2}\hat{L}_k^\dagger\hat{L}_k\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{L}_k^\dagger\hat{L}_k\right),$$

linear Lindblads: $\hat{L}_j(\hat{x}) := l_j' \cdot \hat{x} + i l_j'' \cdot \hat{x}$ *+ quadratic H*

$$\frac{\partial W}{\partial t}(x,t) = \mathcal{L}_W^H\left[W(x,t)\right] + \mathcal{L}_W^\flat\left[W(x,t)\right] + \mathcal{L}_W^\sharp\left[W(x,t)\right]$$

$\mathcal{L}_W^H\left[W(x,t)\right] = \{H(x), W(x,t)\}$ ⟶ *Hamiltonian*

$\mathcal{L}_W^\flat\left[W(x,t)\right] = \frac{\hbar}{2}\sum_{j=1}^{M}\left[\mathsf{J}l_j' \cdot \frac{\partial^2 W}{\partial x \partial x}\mathsf{J}l_j' + \mathsf{J}l_j'' \cdot \frac{\partial^2 W}{\partial x \partial x}\mathsf{J}l_j''\right]$ ⟶ *difussive*

$\mathcal{L}_W^\sharp\left[W(x,t)\right] = \alpha\left[x \cdot \frac{\partial W}{\partial x} + 2nW(x,t)\right]$ $\quad \alpha := \sum_{j=1}^{M} l_j' \wedge l_j''$ ⟶ *contractive*

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Decoherence II

Lie-Trotter

Gaussian case

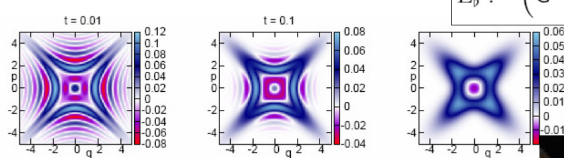
$$W(x,t) = \mathcal{G}_{2n}^{\mathbb{R}}\left[x; e^{2\gamma_t}\left(\mathsf{S}\Delta_0\mathsf{S}^\top + \Gamma\right)^{-1}; e^{-\gamma_t}\mathsf{S}\eta_0\right]$$

S → Hamiltonian !
γ → contraction !
Γ → difussion !?

$$\gamma_t := \int_0^t \alpha\, dt' \qquad \Gamma = 2\int_0^t \sum_{j=1}^{M} \mathsf{J}^\top\left(l_j' l_j'^\top + l_j'' l_j''^\top\right)\mathsf{J}\, d\tau$$

$$\mathcal{I}_\flat(x) = \frac{2^n\, i^{\nu-\frac{1}{2}\mathrm{sng}\,\mathsf{C}_{\mathsf{U}}^{-1}\mathsf{V}}}{\sqrt{\det\left[(\mathsf{U}+\mathsf{V}) + i(\mathsf{U}-\mathsf{V})\mathsf{J}\right]}}\, e^{-\frac{1}{4\hbar}\zeta\wedge\Gamma\Omega_\flat\mathsf{J}\zeta - \frac{i}{2\hbar}\bar{\eta}\cdot[\mathsf{G}\Gamma - \mathbb{I}_{2n}]\Omega_\flat\mathsf{J}\zeta}$$
$$\times\ \mathcal{G}_{2n}^{\mathbb{C}}\left[x;\mathrm{E}_\flat;\bar{\eta}_\flat\right]\exp\left(-\tfrac{i}{\hbar}x\cdot\Omega_\flat\mathsf{J}\zeta\right),$$

$$\mathrm{E}_\flat := \left(\mathsf{G}^{-1} + \Gamma\right)^{-1}$$



Intereference pattern survives decoherence !!

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