

Caos Quântico

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Ministério da Ciência e Tecnologia

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Resumo

- Caos clássico
- A velha teoria quântica
- Caos quântico

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Caos Clássico

História



Henri Poincaré (1895)
 problema gravitacional de N corpos (Oskar II)
 [estabilidade do sistema solar]
 > o problema de três corpos não tem solução analítica



Jacques Hadamard (1898)
 partícula livre numa superfície de curvatura negativa constante
 > sensibilidade exponencial às condições iniciais

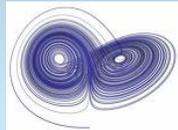
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Lorenz

Edward Lorenz (1961) usando um computador para simular um modelo de clima "descobre" a sensibilidade às condições iniciais (efeito borboleta).

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$


sistema dissipativo, não hamiltoniano

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Dinâmica regular vs. irregular

Exemplo de dinâmica regular: o pêndulo simples



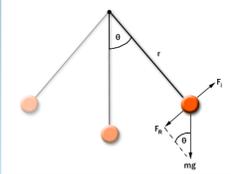
Galileo 1602



C. Huygens 1656

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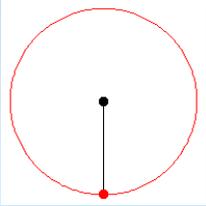
O pêndulo ideal

$$H(p_\theta, \theta) = \frac{p_\theta^2}{2ml^2} - mgl \cos \theta$$


$$\dot{p}_\theta = -\frac{\partial H}{\partial \theta} = -mgl \sin \theta, \dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{ml^2}$$

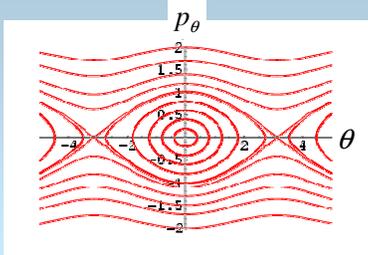
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O pêndulo ideal



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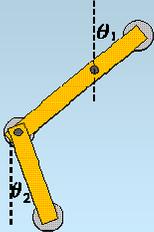
Espaco de fases



Similar a qualquer sistema de um grau de liberdade

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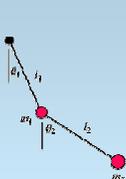
Dinâmica irregular: o pêndulo duplo




chaoticpendulums
www.chaoticpendulums.com

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O pêndulo duplo ideal



$$\dot{\theta}_1 = \frac{\partial H}{\partial p_{\theta_1}} = \frac{l_2 p_{\theta_1} - l_1 p_{\theta_2} \cos(\theta_1 - \theta_2)}{l_1^2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]}$$

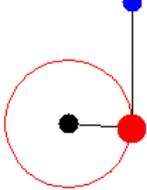
$$\dot{\theta}_2 = \frac{\partial H}{\partial p_{\theta_2}} = \frac{l_2 (m_1 + m_2) p_{\theta_2} - l_2 m_2 p_{\theta_1} \cos(\theta_1 - \theta_2)}{l_1^2 l_2^2 m_2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]}$$

$$\dot{p}_{\theta_1} = -\frac{\partial H}{\partial \theta_1} = -(m_1 + m_2) g l_1 \sin \theta_1 - C_1 + C_2$$

$$\dot{p}_{\theta_2} = -\frac{\partial H}{\partial \theta_2} = -m_2 g l_2 \sin \theta_2 + C_1 - C_2$$

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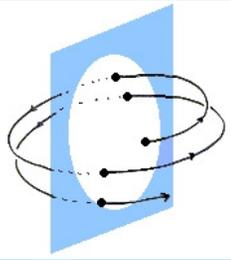
Uma trajetória (xy)



como é a dinâmica no espaço de fases (quatro dimensões)?

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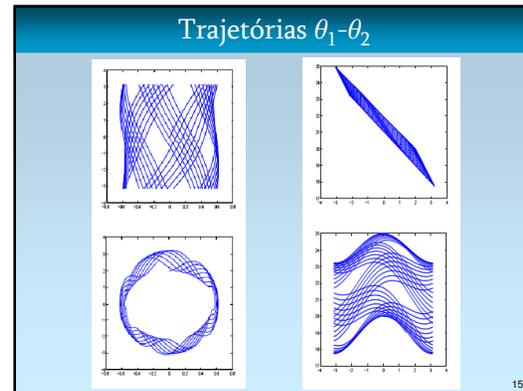
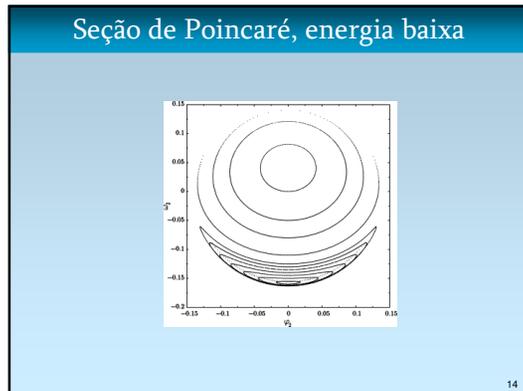
Espaço de fases: seção de Poincaré



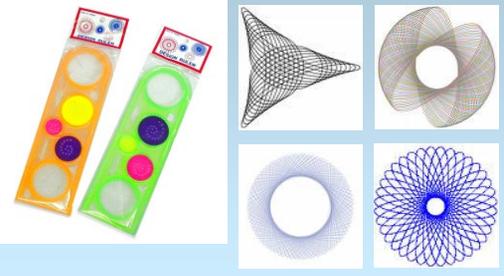
$\theta_1, \theta_2, p_1, p_2$

exemplo:
 $\theta_1 = 0$
 θ_2, p_2

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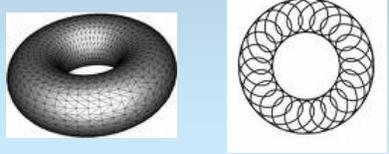
Dinâmica quase-periódica



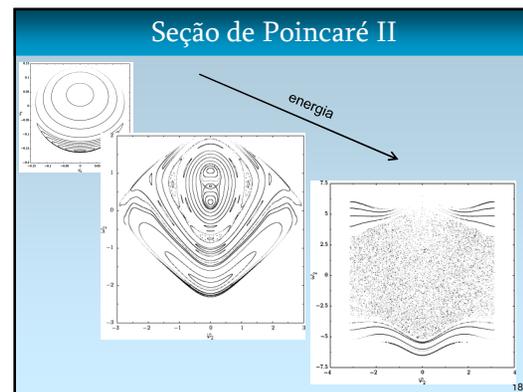
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Quase-periodicidade – Espaço de fases

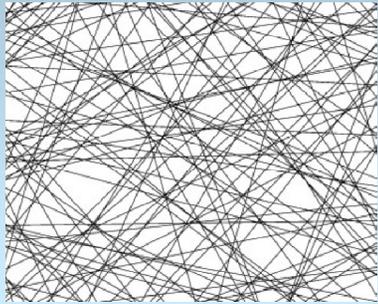
Espaço de fases folheado por toros



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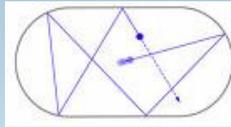


Trajétoria caótica no plano $\theta_1-\theta_2$

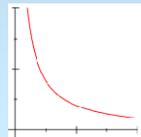


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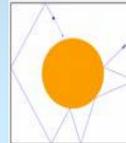
Bilhares



Bunimovich (estádio)



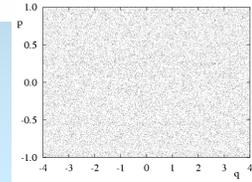
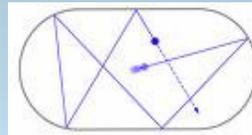
hipérbole



Sinai

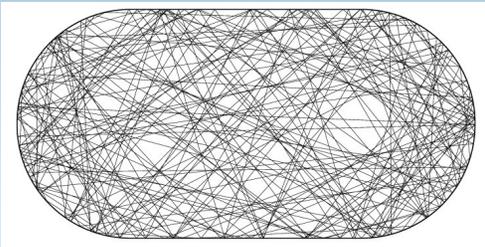
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Bilhares – seção de Poincaré-Birkhoff



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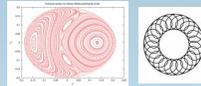
Uma trajetória



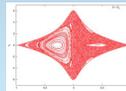
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Dinâmica clássica – Resumo

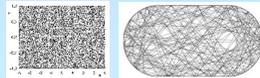
sistemas integráveis



sistemas mistos



sistemas completamente caóticos



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Mecânica Quântica

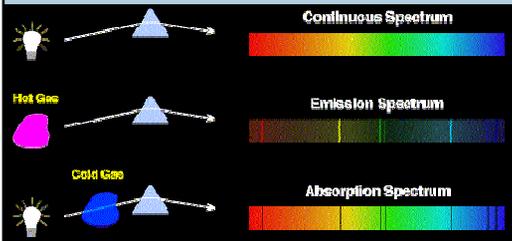
A velha mecânica quântica

Max Planck (1900) > hipótese quântica:
a energia é emitida ou absorvida em quantidades discretas

Albert Einstein (1905), efeito fotoelétrico (1839),
a luz consiste de fótons

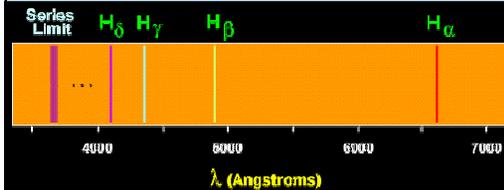
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Séries espectrais



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Hidrogênio



$$\frac{1}{\lambda_{\text{vac}}} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$n_1 = 2$$

Balmer
1885

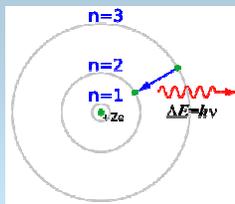
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Séries

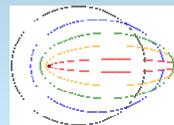
n_1	n_2	Name	Converge toward
1	$2 \rightarrow \infty$	Lyman series	91.13 nm
2	$3 \rightarrow \infty$	Balmer series	364.51 nm
3	$4 \rightarrow \infty$	Paschen series	820.14 nm
4	$5 \rightarrow \infty$	Brackett series	1458.03 nm
5	$6 \rightarrow \infty$	Pfund series	2278.17 nm
6	$7 \rightarrow \infty$	Humphreys series	3280.56 nm

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O átomo de Bohr



Niels Bohr 1913



$$\oint p_i dq_i = 2\pi n_i \hbar$$

regra de quantização
Sommerfeld 1915

integral em um periodo

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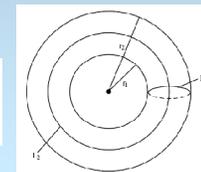
Einstein 1917

$$\oint p_i dq_i = 2\pi n_i \hbar$$

$$1 \leq i \leq \ell$$

depende das coordenadas
de separação

$$\oint \sum_k p_k dq_k = 2\pi n_k \hbar$$



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Pergunta

Como se generalizam as fórmulas de quantização quando a dinâmica clássica não é integrável?

Einstein 1917
[RBEF 2005]

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Caos Quântico

Sistemas caóticos - Fórmula do traço

$$\hat{H}, \{\varepsilon_k\}, \{\psi_k\}$$

estados ligados

$$\hat{U} = e^{-i\hat{H}t/\hbar}$$

operador de evolução

$$\hat{G}_E^+ = -\frac{i}{\hbar} \int_0^\infty dt e^{-i\hat{H}t/\hbar} e^{iEt/\hbar}$$

operador de Green



Martin Gutzwiller
>1966

$$\text{tr} \hat{G}_E^+ = -\frac{i}{\hbar} \sum_k \int_0^\infty dt e^{-i(E-\varepsilon_k)t/\hbar}$$

$$-\text{Im} \text{tr} \hat{G}_E^+ = \pi \sum_k \delta(E - \varepsilon_k)$$

densidade de níveis

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Fórmula do traço II

$$\hat{G}_E^+ = -\frac{i}{\hbar} \int_0^\infty dt e^{iEt/\hbar} e^{-i\hat{H}t/\hbar}$$

$$\text{tr} \hat{G}_E^+ \propto \int_0^\infty dt e^{iEt/\hbar} \text{tr} e^{-i\hat{H}t/\hbar}$$

$$\text{tr} e^{-i\hat{H}t/\hbar} = \int dq \langle q | e^{-i\hat{H}t/\hbar} | q \rangle$$

Propagador:
soma sobre caminhos
fechados (Feynman)
 $q \rightarrow q$

Limite semiclássico:

- 1) caminhos fechados \rightarrow trajetórias fechadas de tempo t
- 2) trajetórias fechadas \rightarrow trajetórias periódicas de período t
- 3) Fourier: tempo \rightarrow energia

$$-\text{Im} \text{tr} \hat{G}_E^+ = \pi \sum_k \delta(E - \varepsilon_k) \approx C(E) + \text{Re} \sum_\nu A_\nu(E) e^{iS_\nu(E)/\hbar}$$

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Fórmula do traço III

$$\rho(E) = \sum_k \delta(E - \varepsilon_k) \approx \rho_0(E) + \text{Re} \sum_\nu A_\nu(E) e^{iS_\nu(E)/\hbar}$$

ν trajetórias periódicas de energia E

$S_\nu(E)$ ação

$A_\nu(E)$ amplitude, estabilidade

sistemas caóticos: Gutzwiller
bilhares: Balian & Bloch
sistemas integráveis: Berry & Tabor

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Dificuldades

$$\rho(E) \approx \rho_0(E) + \text{Re} \sum_\nu A_\nu(E) e^{iS_\nu(E)/\hbar}$$

> Convergência (proliferação exponencial de órbitas)

> Esquemas alternativos (regularização)



Michael Berry



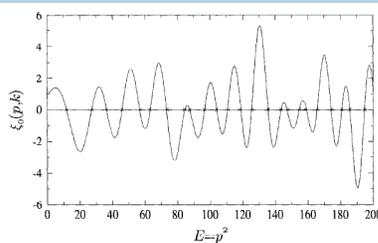
Jonathan Keating

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Verificações

1. Problema de Kepler anisotrópico, Gutzwiller 1982

2. Bilhar hipérbolo, Keating & Sieber 1994



n	E _n ⁰	E _n ¹	ΔE _n (%)
1	11.74	11.80	0.4
2	27.33	27.10	-1.5
3	36.78	36.95	0.7
4	48.79	48.83	0.1
5	58.63	58.71	-0.5
6	62.89	63.10	0.0
7	73.67	73.11	-6.7
8	84.21	84.45	3.0
9	87.34	87.42	1.1
10	90.55	90.48	-0.9
11	103.83	104.02	2.5
12	111.19	110.51	-6.0
13	118.29	118.81	7.1
14	125.63	125.36	-2.8
15	130.55	135.48	-1.0
16	143.36	143.66	1.4
17	147.86	147.15	-7.4
18	154.84	154.04	-5.2
19	158.17	158.16	14.6
20	165.79	165.02	-2.8
21	173.64	174.24	-6.0
22	180.73	180.79	4.9
23	185.77	185.73	-16.6
24	194.85	194.74	-1.6

Densidade de níveis para bilhares

Bilhares $\rho(E) \approx \rho_0(E) + \text{Re} \sum_{\nu} A_{\nu}(E) e^{iS(E)/\hbar}$

$$E = \frac{p^2}{2m} = \frac{(\hbar k)^2}{2m}$$

$$\rho_{\mu}(k) = \frac{1}{2\pi} \text{Re} \sum_{p,m} \frac{l_p}{\sinh \frac{m\alpha_p}{2}} \exp \left[im \left(kl_p - \frac{\mu_p \pi}{2} \right) \right]$$

$\nu \rightarrow p, m$

Função Z dinâmica – Determinante espectral

$$\rho_{\mu}(k) = -\frac{1}{\pi} \text{Im} \frac{d}{dk} \ln Z(k)$$

André Voros 1988

$$Z(k) = \prod_{p,m} \left\{ 1 - \exp \left[i \left(kl_p - \frac{\mu_p \pi}{2} \right) - \left(m + \frac{1}{2} \right) \alpha_p \right] \right\}$$

p: órbitas periódicas primitivas
m: repetições

Ainda exige regularização !

Conclusões

Êxito para alguns sistemas simples, muito difícil em geral

Domínio de validade?

Outras questões interessantes ...

Outra pergunta

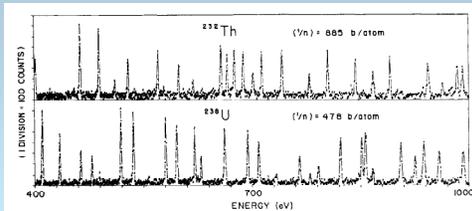
Que particularidades exhibe a mecânica quântica de sistemas classicamente caóticos?

Resposta:

- (1) flutuações espectrais universais, para um sistema fechado
- (2) flutuações universais das seções de choque, para sistemas de espalhamento

Flutuações universais \iff Teoria das matrizes aleatórias

Espectro nuclear



espectroscopia de nêutrons lentos (tempo de voo)

Quantidade estatística de interesse: distribuição de espaçamentos entre níveis consecutivos.

Teoria das matrizes aleatórias

Descrição de mínima informação: **Wigner 1950's**

hamiltoniano nuclear ↔ ensemble de matrizes aleatórias

simetria de reversão temporal?

SIM:
matrizes reais simétricas

NÃO:
matrizes complexas hermitianas

elementos independentes e gaussianamente distribuídos

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Teste I

VOLUME 48, NUMBER 16 PHYSICAL REVIEW LETTERS 19 APRIL 1982

Fluctuation Properties of Nuclear Energy Levels: Do Theory and Experiment Agree?

R. U. Haq
*Physique Théorique et Mathématique, Université Libre de Bruxelles, B-1050 Brussels, Belgium, and
Department of Physics, University of Toronto, Toronto M5S 1A7, Canada*

and

A. Pandey
*Division de Physique Théorique, Institut de Physique Nucléaire, F-91190 Orsay, Cédex, France, and
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627*

and

O. Bohigas
*Division de Physique Théorique, Institut de Physique Nucléaire, F-91190 Orsay, Cédex, France
(Received 11 December 1981)*

The fluctuation properties of nuclear energy levels are analyzed with new spectrally averaged measures. A remarkably close agreement between the predictions of random-matrix theories and experiment is found.

PACS numbers: 21.10.Ma, 24.60.+m

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Teste II

1730 níveis, diferentes núcleos

Orthogonal ($\beta = 1$) $p(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right)$,

Unitary ($\beta = 2$) $p(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right)$,

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Hipótese RMT

VOLUME 52 2 JANUARY 1984 NUMBER 1

Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws

O. Bohigas, M. J. Giannoni, and C. Schmit
*Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France
(Received 2 August 1983)*

It is found that the level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

PACS numbers: 05.40.+j, 03.65.lz

Oriol Bohigas

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Hipótese RMT II

740 níveis, diferentes valores de R

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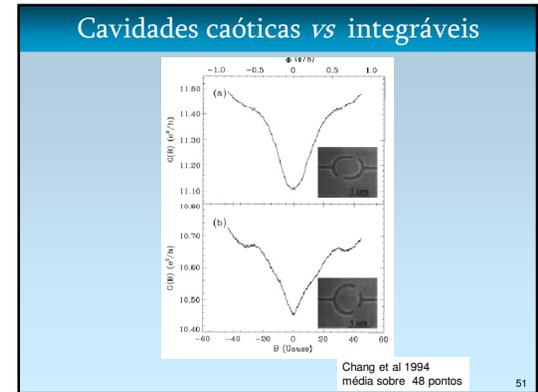
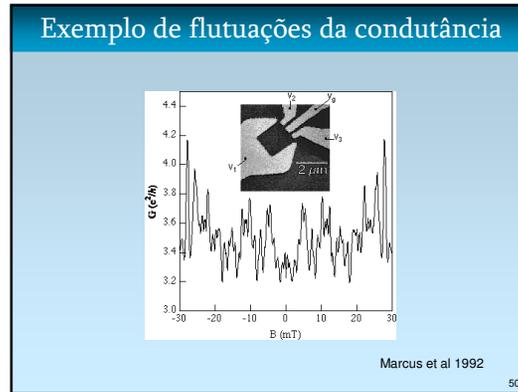
Sistemas abertos

Flutuações universais da condutância

dispositivos submicrométricos + baixas temperaturas (<100mK) → transporte coerente quântico

GaAs Al_xGa_{1-x}As 1 μm

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Flutuações mesoscópicas - Teoria

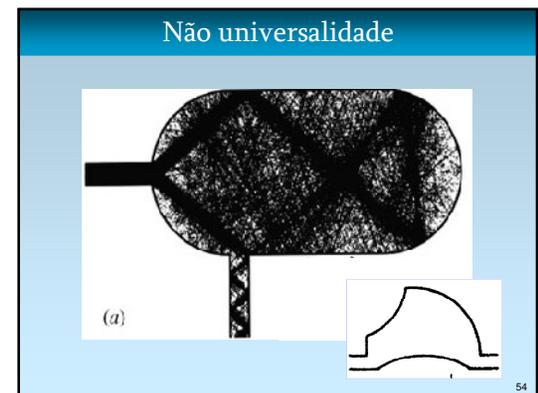
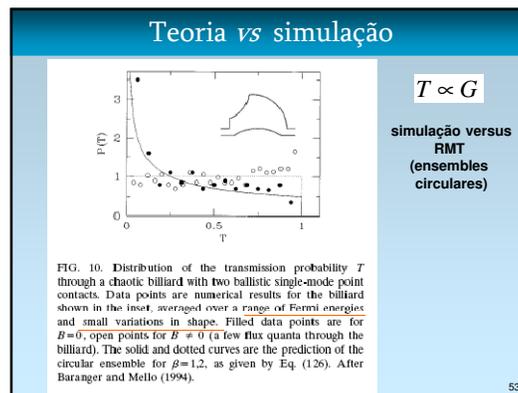
$$G = \frac{2e^2}{h} \sum_{\substack{a \in L \\ b \in R}} |S_{ab}|^2$$

Fórmula de Landauer
freqüências, temperatura, voltagem baixas, interação e-e desprezível

coeficiente de transmissão S matriz de espalhamento (unitária)

caos! → ensemble de matrizes unitárias COE, CUE

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Tópico especial

A função ζ de Riemann

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \text{Re}(s) > 1 \quad + \text{extensão analítica}$$

$$\zeta(s) = \prod_{\text{primos}} \frac{1}{1-p^{-s}} \quad \text{fórmula de Euler (1737)}$$

 existem infinitos primos (Euclides, -300)

$$\sum_p \frac{1}{p} = \infty$$

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O teorema dos números primos (PNT)

$$\pi(x) = \#\{p < x\}$$

$$\pi(x) \sim \frac{x}{\ln x}, \quad x \rightarrow \infty$$

conjecturas: Gauss, Legendre ~1800
provas: Hadamard, de la Vallée Pousin 1896

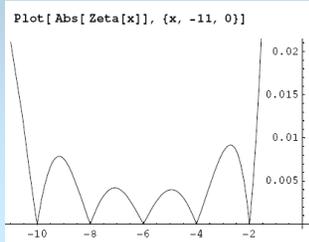
provas baseadas em propriedades dos zeros da função Z de Riemann

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Os zeros de $\zeta(s)$

$$s = -2, -4, -6, \dots$$

zeros triviais



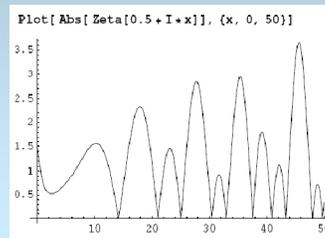
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Zeros de $\zeta(s)$. II

$$0 \leq \text{Re}(s) \leq 1 \quad \text{faixa crítica} \\ \text{zeros não triviais}$$

Conjectura de Riemann:
os zeros não triviais ficam na linha crítica

$$\text{Re}(s) = \frac{1}{2}$$



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Primos & zeros de $\zeta(s)$

Se a conjectura de Riemann for verdadeira então:

$$\left| \pi(x) - \int_2^x \frac{dv}{\ln v} \right| \leq c \sqrt{x} \ln x$$

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Prêmio



Clay Mathematics Institute
Dedicated to increasing and disseminating mathematical knowledge

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Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a \$7 million prize fund for the solution to these problems, with \$1 million allocated to each. During the *Millennium Meeting* held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled *The Importance of Mathematics*, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

Birch and Swinnerton-Dyer Conjecture
Hodge Conjecture
Navier-Stokes Equations P vs NP
Poincaré Conjecture
Riemann Hypothesis
Yang-Mills Theory

Rules
Millennium Meeting Videos

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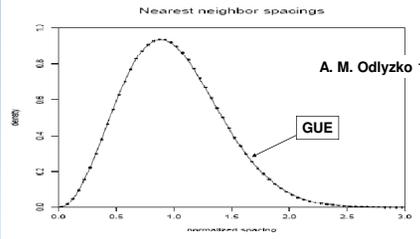
Conjectura de Hilbert-Pólya

Existe um operador hermitiano tal que seus autovalores correspondem às partes imaginárias dos zeros da função zeta de Riemann

$$\varepsilon_k : \zeta\left(\frac{1}{2} + i\varepsilon_k\right) = 0$$

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Zeta function & RMT



A. M. Odlyzko 1987

FIGURE 1. Probability density of the normalized spacings δ_n . Solid line: Gue prediction. Scatterplot: empirical data based on a billion zeros near zero # $1.3 \cdot 10^{16}$.

Analogia

$$Z(k) = \prod_{p,m} \left\{ 1 - \exp \left[i \left(kl_p - \frac{\mu_p \pi}{2} \right) - \left(m + \frac{1}{2} \right) \alpha_p \right] \right\}$$

Bilhães

$$\tilde{Z}\left(\frac{1}{2} + ik\right) = \prod_{p,m} \left\{ 1 - \exp \left[-ik\tilde{l}_p - \left(m + \frac{1}{2} \right) \tilde{\alpha}_p \right] \right\}^{-1}$$

Zeta de Riemann

p : primo

$\tilde{l}_p = \ln p$

$\tilde{\alpha}_p = \ln p$

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Conjectura

My purpose in this speculative paper is to extend the old suggestion that the E_j are real because they are eigenvalues of some Hermitian operator \hat{H} . The extensions are that if \hat{H} is regarded as the Hamiltonian of a quantum-mechanical system then

- (i) \hat{H} has a classical limit
- (ii) the classical orbits are all chaotic (unstable)
- (iii) the classical orbits do not possess time-reversal symmetry.

Berry 1986

Fim

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Fim