A model of the nucleon structure

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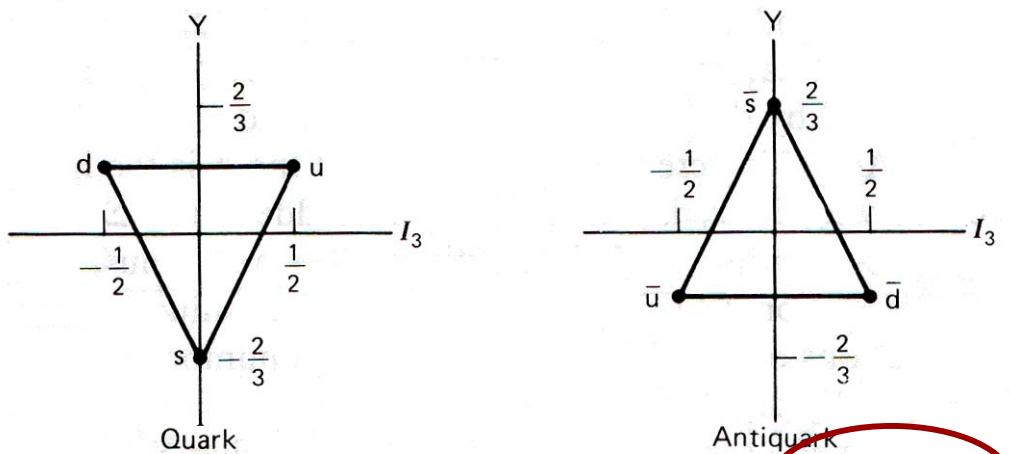
Outline

- Introduction
- The structure of the proton (experiment)
  - Deep Inelastic Scattering
  - Drell-Yan dilepton production
  - Open questions
- The structure of the proton (theory)
  - The strange sea of the proton
- Conclusions

Introduction

- According to the quark model, all hadrons are made up of a small variety of basic entities called quarks
- Baryons are a three quarks bound state
- Mesons are a quark – antiquark bound state
- In $SU(3)_f$

$$Q = I_3 + Y/2$$



$SU(3)$ quark and antiquark multiplets; $Y \equiv B + S$

Hypercharge Y

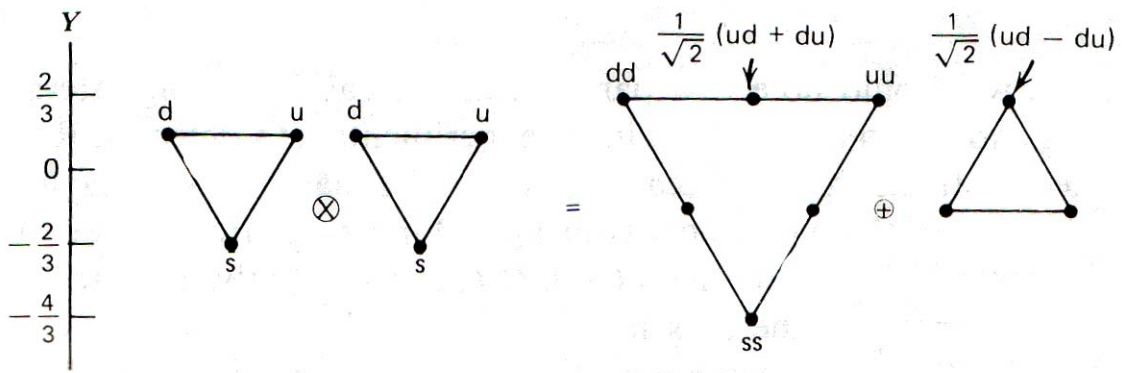
B = baryon number

S = strangeness

- Baryons in $SU(3)_f$ are in the $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

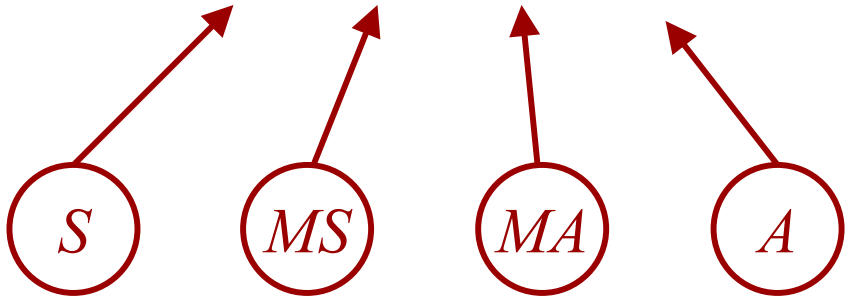
Step by step:

1) $3 \otimes 3 = 6 \oplus \bar{3}$

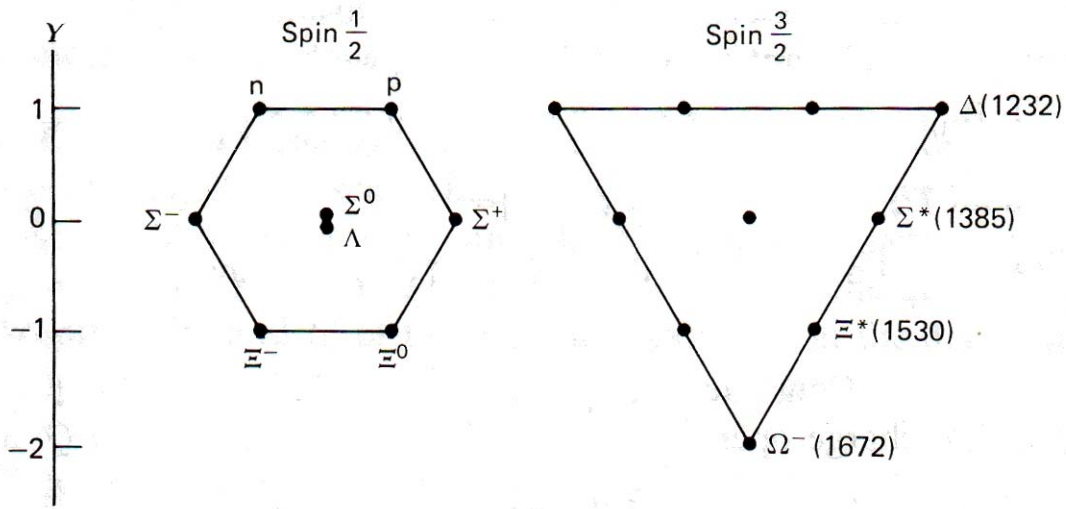


The qq $SU(3)$ multiplets; $3 \otimes 3 = 6 \oplus \bar{3}$.

2) $(6 \oplus \bar{3}) \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

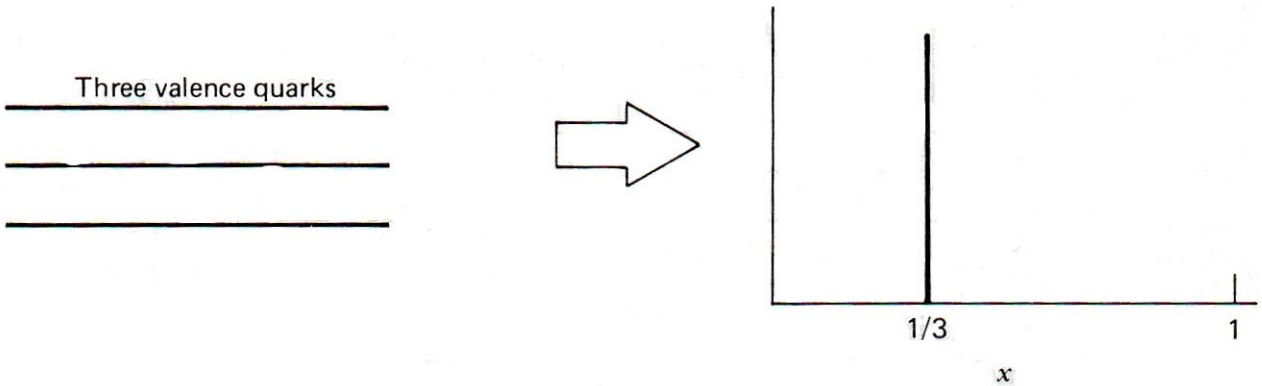


3) impose that the total baryon wave function has to be antisymmetric

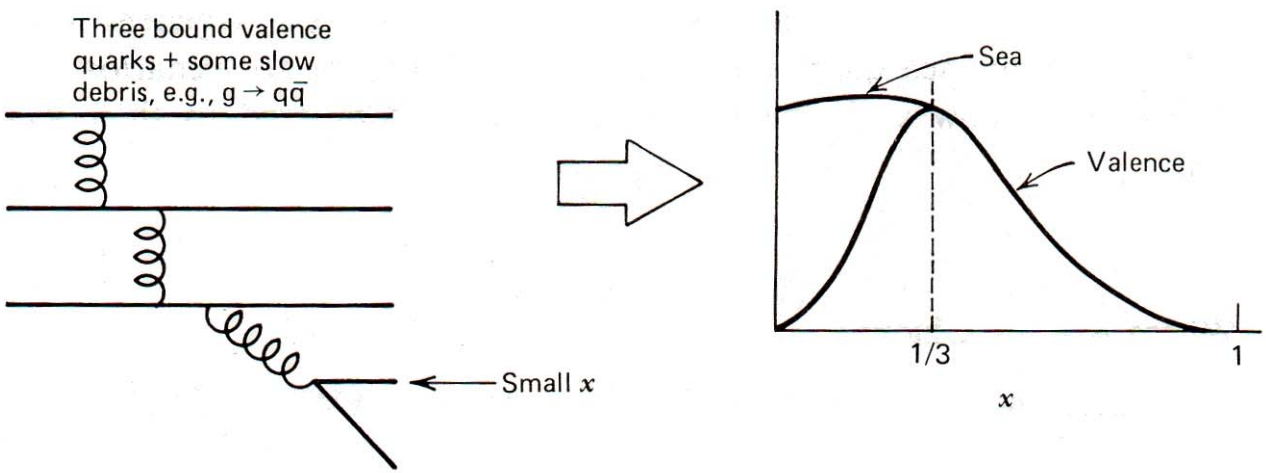


Ground-state baryons: (8, 2) + (10, 4).

Then the nucleon seems to be



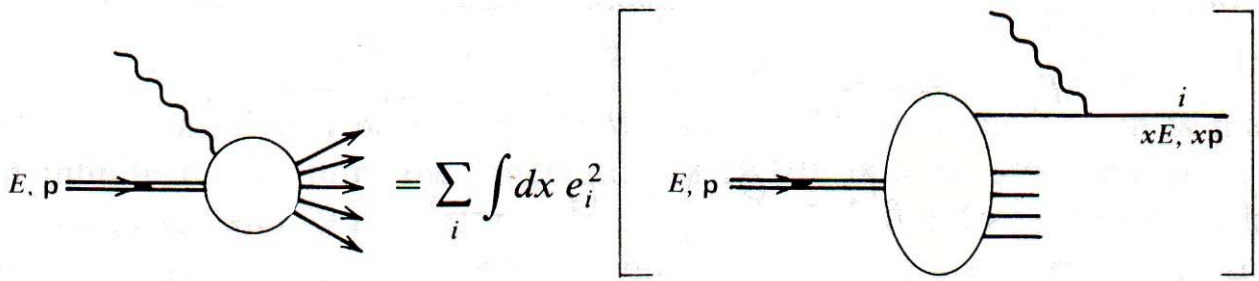
However, what experiments show is more likely



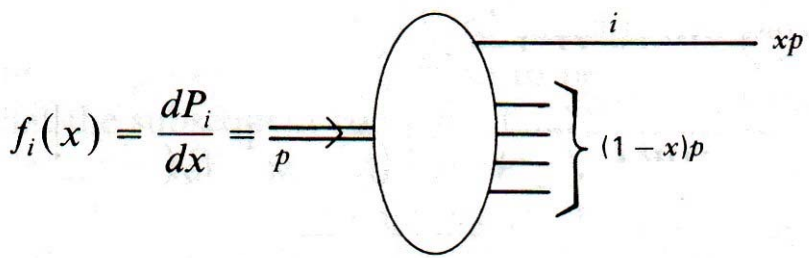
The structure of the proton

(experimental)

Parton Model



where partons have a momentum distribution



The kinematics of the parton model can be summarized as

$$\sum_i \int dx\ x f_i(x) = 1$$

probability density of finding the parton i with momentum fraction x

	Proton	Parton
Energy	E	xE
Momentum	p_L $p_T = 0$	xp_L $p_T = 0$
Mass	M	$m=(x^2E^2-x^2p_L^2)^{1/2} = xM$

but:

• if the parton momentum is xp, its energy can only be xE if $m = M = 0$!

• Equivalently, *a proton can only emit a parton with $p_T = 0$ if both have zero mass* (if a massive particle decays, there is a non-zero angle between the decay products)

• if we define a Lorentz frame in which

$$|\boldsymbol{p}| \gg m, M$$

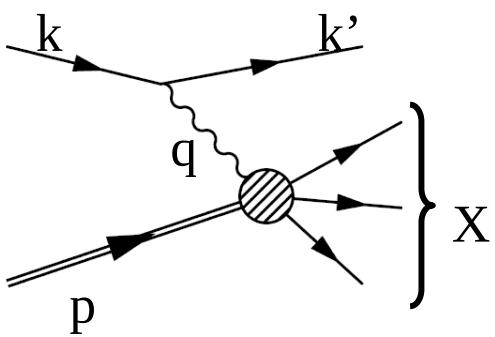
all the masses can be neglected and the kinematics of the parton model becomes exact

In practice, **p** ~ few GeV is enough

The Parton Model is valid in the

INFINITE MOMENTUM FRAME

Deep Inelastic Scattering (DIS)

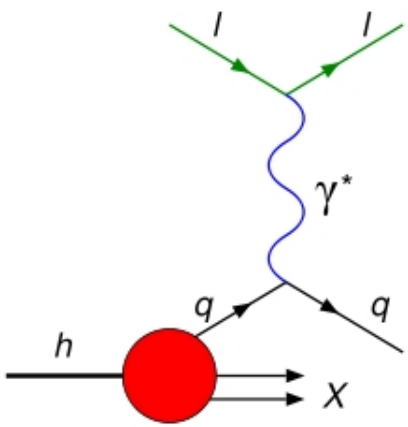


$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 sin^4\frac{\theta}{2}}\left[W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right]$$

$$q^2 \; ; \; \nu = \frac{p\cdot q}{M} = E - E'$$

$Q^2 \rightarrow \infty$

$$\begin{cases} \nu W_2(\nu,Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x) \\ MW_1(\nu,Q^2) \rightarrow F_1(x) = \dfrac{1}{2x}F_2(x) \end{cases}$$



$$x = \frac{Q^2}{2M\nu} \; ; \; Q^2 = -q^2$$ Bjorken scaling

$$\left.\frac{d\sigma}{dtdu}\right|_{ep\rightarrow eX} = \sum_i \int dx f_i(x)\left(\frac{d\hat{\sigma}}{d\hat{t}d\hat{u}}\right)_{eq_i\rightarrow eq_i}$$

alternatively, defining $y = \frac{\nu}{E}$

$$M\nu_{max}\frac{d\sigma}{dxdy} = \frac{2\pi\alpha^2}{x^2y^2}\left\{xy^2F_1(x) + \left[(1-x) - \frac{Mxy}{2\nu_{max}}\right]F_2(x)\right\}$$

Structure functions

proton

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2\left[u^p(x) + \bar{u}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[d^p(x) + \bar{d}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[s^p(x) + \bar{s}^p(x)\right]$$

---

neutron

$$\frac{1}{x}F_2^{en}(x) = \left(\frac{2}{3}\right)^2\left[d^p(x) + \bar{d}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[u^p(x) + \bar{u}^p(x)\right] + \left(\frac{1}{3}\right)^2\left[s^p(x) + \bar{s}^p(x)\right]$$

Flavor sum rules

$q(x) = q_v(x) + q_s(x)$

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx = 2$$ → two u-quarks in the proton

$$\int_0^1 [d(x) - \bar{d}(x)]\, dx = 1$$ → one d-quark in the proton

$$\int_0^1 [s(x) - \bar{s}(x)]\, dx = 0$$ → zero net strangeness in the proton

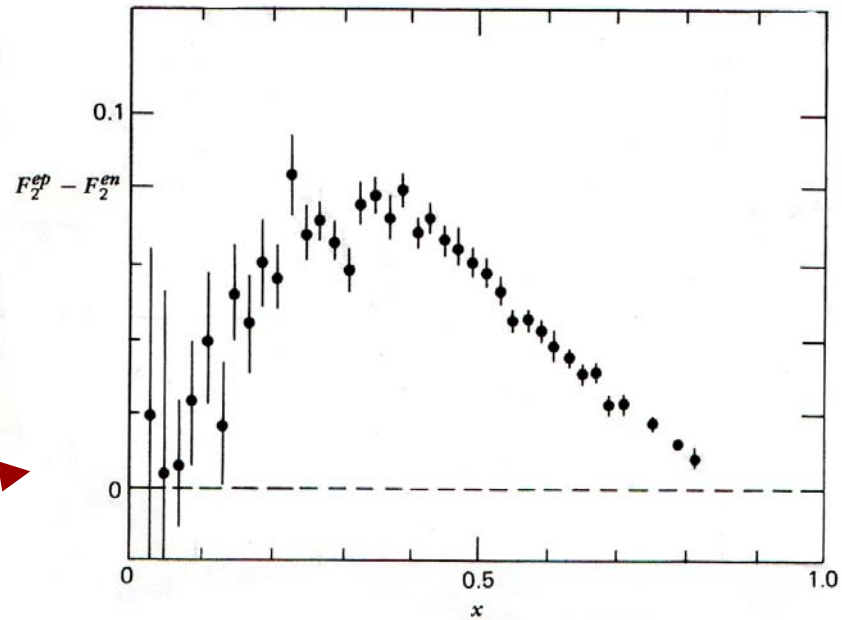
and gluons

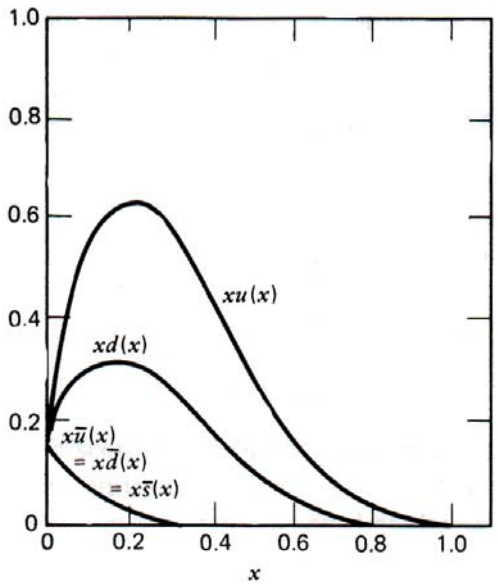
$$\int_0^1 x(u + \bar{u} + d + \bar{d} + s + \bar{s})dx = 1 - \epsilon_g$$

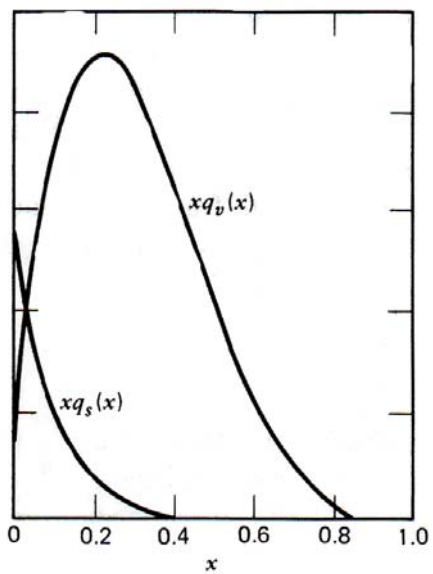
$$\epsilon_g \sim 0.46$$

Quarks and anti-quarks inside the proton

$$\frac{1}{x}\left[F_2^{ep}(x) - F_2^{en}(x)\right] = \frac{1}{3}\left[u_v(x) - d_v(x)\right]$$







The role of gluons and the "Bjorken scaling violation"

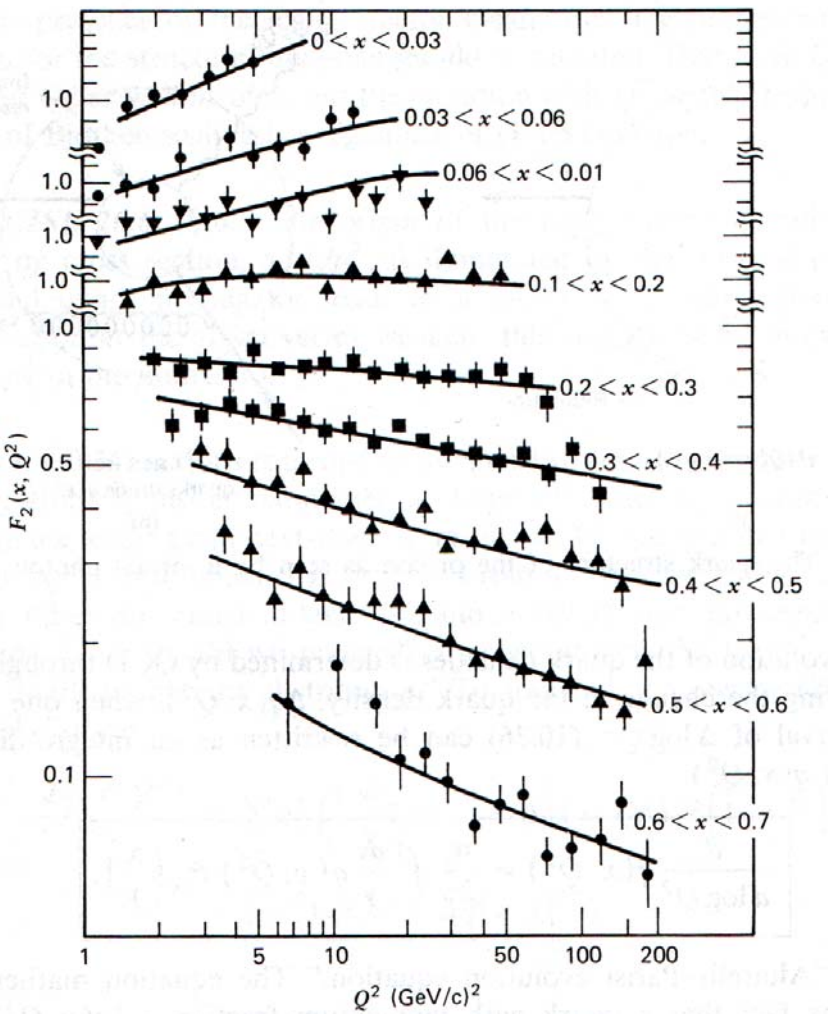
A more convincing picture of the proton in e + p scattering:

- if $Q^2 < Q_0^2 \sim 1\ GeV^2$ → the proton behaves like a point particle: $(e + p \rightarrow e + p) \rightarrow G_E(Q^2), G_M(Q^2)$ (Elastic Scattering regime)

- if $Q^2 \sim$ few GeV^2 → the proton is smashed in the collision: $(e + p \rightarrow e + X) \rightarrow W_1(\nu,Q^2), W_2(\nu,Q^2)$ (Inelastic Scattering regime)

- if $Q^2 > Q_0^2$ → lepton interacts with point constituents inside the proton: $(e + p \rightarrow e + X) \rightarrow F_1(x), F_2(x)$ – Bjorken scaling (Deep Inelastic Scattering regime)

But… if Q^2 is too large → deviations from Bjorken scaling !

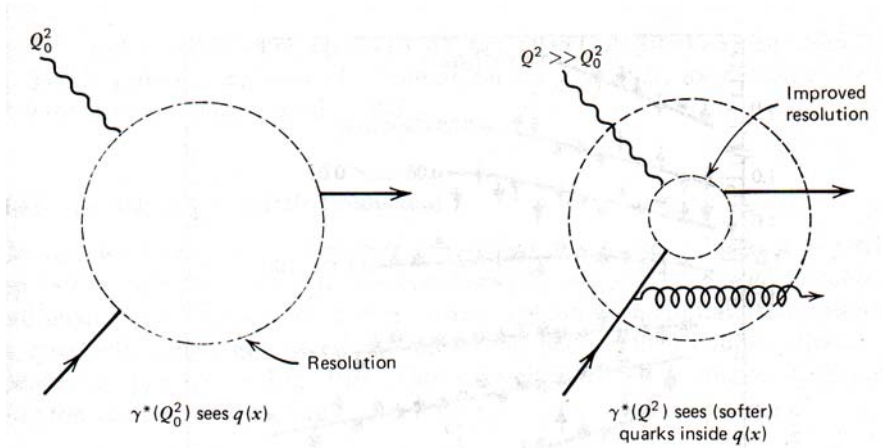


As Q^2 increases, the structure function $F_2(x,Q^2)$ increases at small x and decreases at large x

The origin of this behavior is gluon emission:
$\gamma^* + q \rightarrow q + g$

QCD dixit:

if $Q^2 \gg Q_0^2$ ➔ each parton appears as surrounded by its own cloud of partons

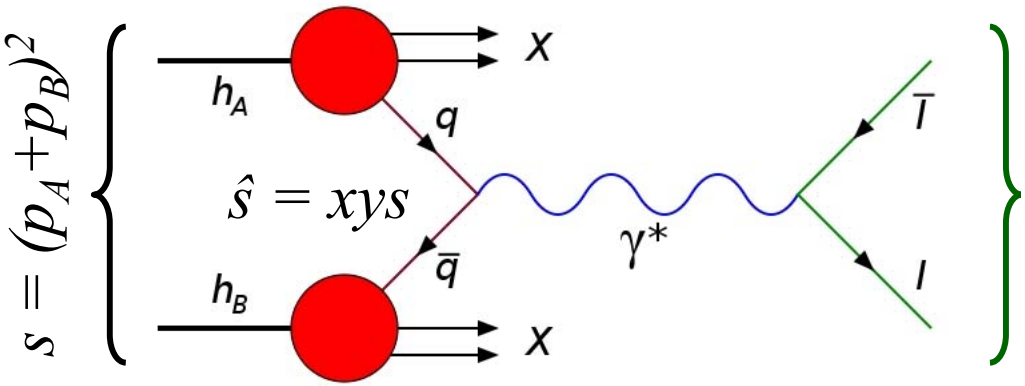


DGLAP evolution

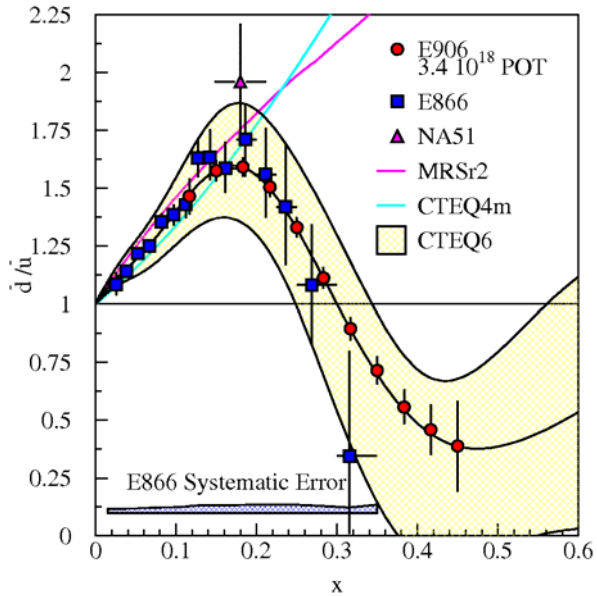
$$\frac{dq_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$

$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_i q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]$$

Drell-Yan dilepton production



invariant mass of the l^+l^- pair



$$\left.\frac{d\sigma}{dQ^2}\right|_{pp\to l^+l^-X} = \frac{4\pi\alpha^2}{9Q^4}\sum_q e_q^2 \int dx \int dy f_q(x,Q^2) f_{\bar{q}}(x,Q^2)\delta\left(1 - xy\frac{s}{Q^2}\right)$$

if you know the valence quark distributions,
you can measure the sea quark distributions

Open questions

- What is the structure of the proton at the scale where QCD evolution starts ?

The forms

$$
\begin{aligned}
q_i(x,Q_0^2) &= N_q x^{a_q}(1-x)^{b_q}(1+e_q x^{d_q}) \\
\bar{q}_i(x,Q_0^2) &= N_{\bar{q}} x^{a_{\bar{q}}}(1-x)^{b_{\bar{q}}}(1+e_{\bar{q}} x^{d_{\bar{q}}}) \\
g(x,Q_0^2) &= N_g x^{a_g}(1-x)^{b_g}(1+e_g x^{d_g})
\end{aligned}
$$

are used for the sole reason that they fit better experimental data

- Is the quark – antiquark sea symmetric ?
  For instance, the strange sea has to satisfy

$$\int_0^1 dx\,[s(x) - \bar{s}(x)] = 0$$

  but there is still room for

$$s(x) \neq \bar{s}(x)$$

- How pdfs are (evolve) at low x ?
  Color Glass Condensate, etc.

- etc.

The structure of the proton
(theory)

- Assume that at some low Q_0^2 scale the proton is made of valence quarks, $v_u(x)$ and $v_d(x)$

$$v_{u/d}(x) = \frac{1}{\beta(a_{u/d}+1, b_{u/d}+1)} x^{a_{u/d}} (1-x)^{b_{u/d}}$$

- Valence quarks interact by the exchange of gluons (needed, they have to form a bound state)

- You can describe the emission of gluons from valence quarks using

$$P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

Probability of gluon emission with momentum fraction z from a parent quark

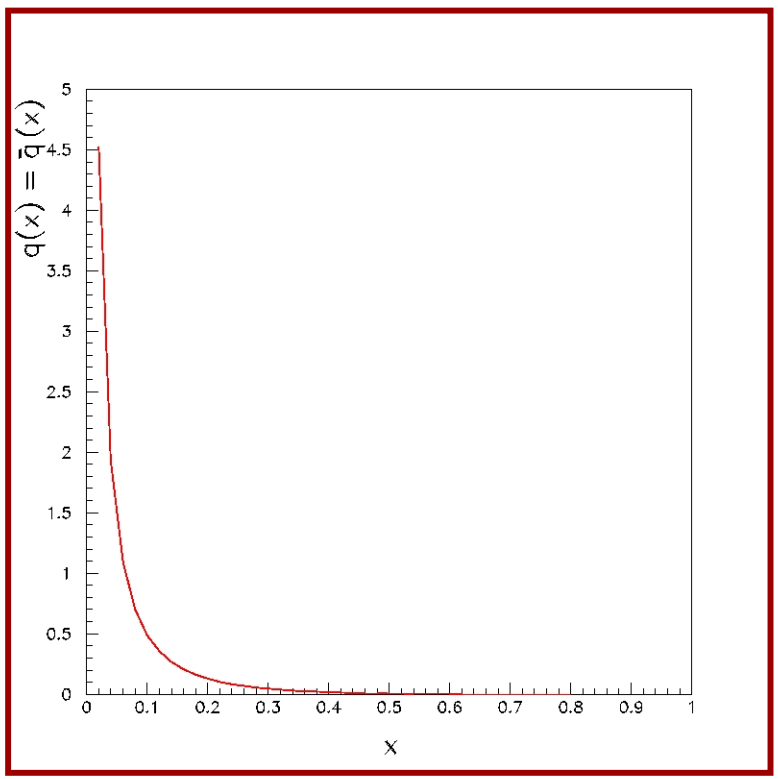
- Assume now that the gluon, before interacting with another valence quark, produces a q-$\bar{q}$ pair. This gluon splitting can be described by

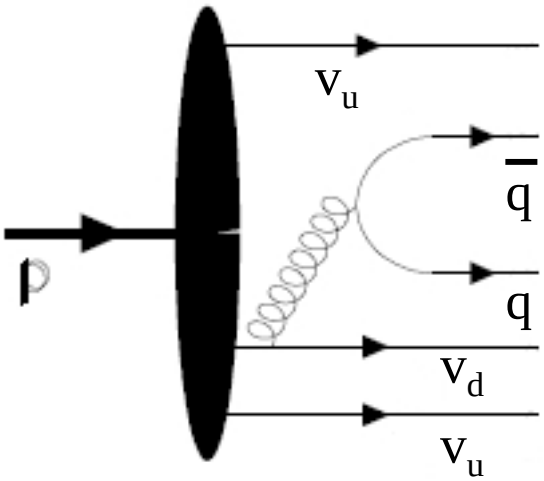
$$P_{qg}(z) = \frac{1}{2}\left(z^2+(1-z)^2\right)$$

Probability of q-$\bar{q}$ creation with momentum fraction z from a parent gluon

- Then the joint probability density of having a q or a $\bar{q}$ coming from the subsequent decays v → v + g → v + q + $\bar{q}$, is

$$q(x,Q^2) = \bar{q}(x,Q^2) = N\frac{\alpha_{st}^2(Q^2)}{(2\pi)^2}\int_x^1 \frac{dy}{y}P_{qg}\left(\frac{x}{y}\right)\int_y^1 \frac{dz}{z}P_{gq}\left(\frac{y}{z}\right)v(z)$$





QCD scale: q-$\bar{q}$ pair creation should occur at some value of the order of $Q_0 \leq 1$ GeV. We use $Q_0 = 0.7$ GeV (then $\alpha_{st}(Q_0^2)/2\pi \sim 0.3$)

N = normalization constant depending on the flavor being created

$$q(x,Q^2) = \bar{q}(x,Q^2) = N \frac{\alpha_{st}^2(Q^2)}{(2\pi)^2} \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) \int_y^1 \frac{dz}{z} P_{gq}\left(\frac{y}{z}\right) v(z)$$

- The next step is to let the q – $\bar{q}$ pair to interact with the valence quarks.
- Using the ideas of recombination models we get

$$P_M(x) = \int_0^x \frac{dy}{y} \int_0^{x-y} \frac{dz}{z} F(y,z) R(x,y,z)$$

$$F(y,z) = \beta y v(y) z \bar{q}(z) \rho(y,z)$$

$$R(y,z) = \alpha \frac{yz}{x^2} \delta\left(1 - \frac{y+z}{x}\right)$$

- And similar for the Baryon inside the proton

with the constraints

$$\int_0^1 dx\,[P_B(x) - P_M(x)] = 0,$$

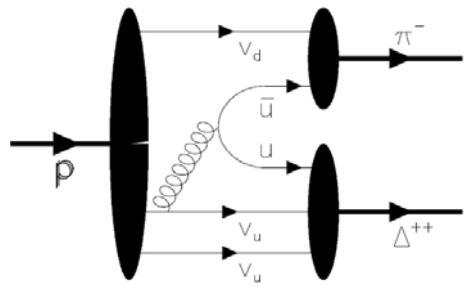
Flavor sum rule

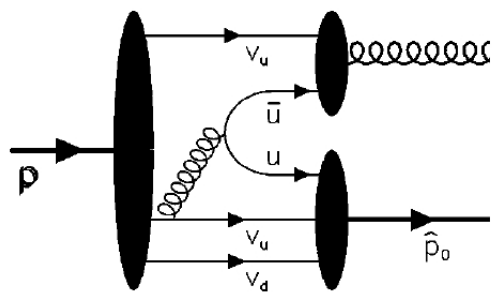
$$\int_0^1 dx\,[xP_B(x) + xP_M(x)] = 1,$$

Momentum sum rule

The proton wave function at Q_0^2 can be thought as

$$|p\rangle = a_0\,|p_0\rangle + a_1\,|pg\rangle + \sum_{i=2}^{n} a_i\,|M_i B_i\rangle$$





then, e. g., the gluon distribution is

$$g(x,Q_0^2) = |a_1|^2 \frac{(1-x)^{a_g}}{x} \int_0^x dy y\bar{q}(y)(y-x)v_q(y-x)$$

where we have used

$$\rho(y,z) = (1-y-z)^{a_g}$$

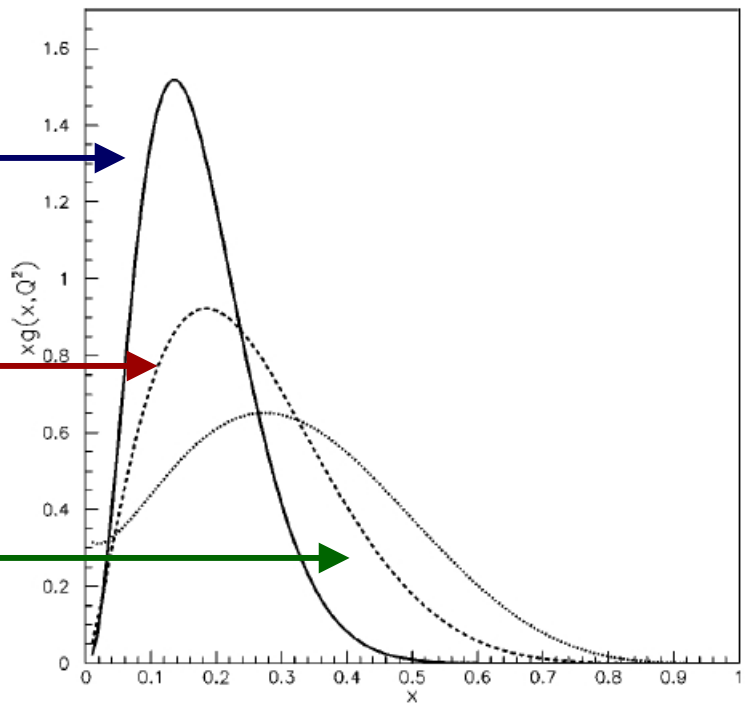
Our $g(x,Q_0^2)$

(J.M; H.R.C.; PRD63 (2000) 014001)

Model with Gaussian pdfs

(PLB **432** (1998) 402)

GRV(94)-HO

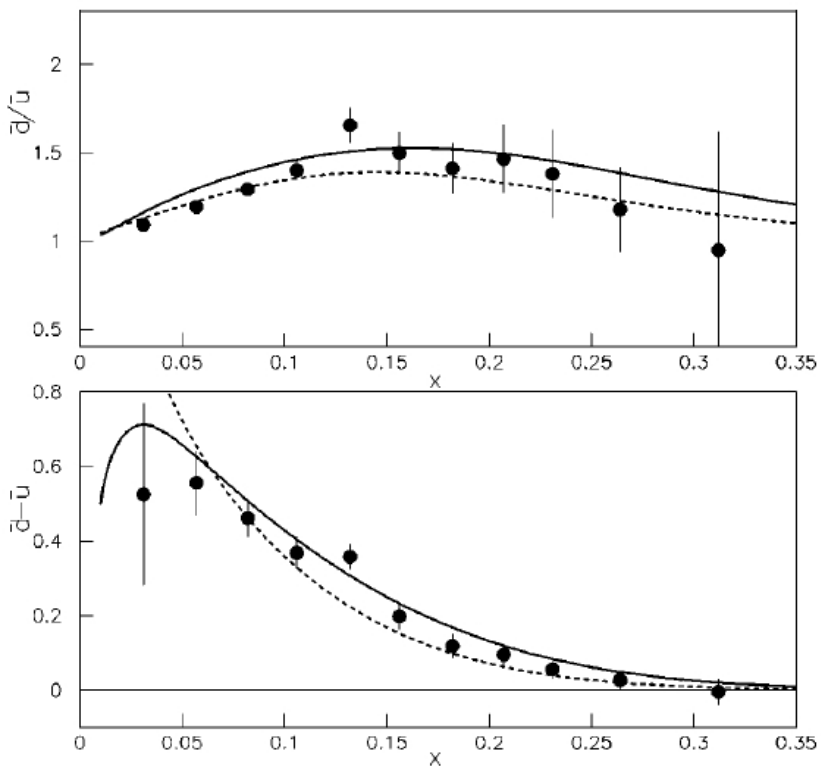


the $\bar{u}$ and $\bar{d}$ sea,

$$\bar{d}^{NP}(x,Q_V^2) = \int_x^1 \frac{dy}{y}\left[P_{\pi N}(y) + \frac{1}{6}P_{\pi\Delta}(y)\right]\bar{d}_\pi\left(\frac{x}{y},Q_V^2\right)$$

$$\bar{u}^{NP}(x,Q_V^2) = \int_x^1 \frac{dy}{y}\frac{1}{2}P_{\pi\Delta}(y)\;\;\bar{u}_\pi\left(\frac{x}{y},Q_V^2\right),$$

data from the E866/NuSea Collaboration



receive contributions from the $|\pi^+ n\rangle$ and $|\pi^- \Delta^{++}\rangle$ Fock states and explains the origin of the Gottfried sum rule violation

(J.M, H.R.C; PRD61 (2000) 054006)

and, in general

$$
\begin{aligned}
q(x,Q_0^2) &= N_q \sum_q \int_x^1 \frac{dy}{y}\left[P_{M_q}(y) v_{qM}\left(\frac{x}{y}\right) + P_{B_q}(y) v_{qB}\left(\frac{x}{y}\right)\right] \\
g(x,Q_0^2) &= P_{\pi^0} = N_g \frac{(1-x)^{a_g}}{x}\int_0^x dy\; y\bar{q}(y)(y-x)v_q(y-x)
\end{aligned}
$$

The strange sea of the proton

- The strange sea of the proton at the Q_0^2 scale comes from the $|KH\rangle$ Fock state.
- The strange sea quark and anti-quark pdfs are

Hyperon probability density

$$s^{NP}(x) = \int_x^1 \frac{dy}{y} P_H(y)\; s_H(x/y)$$

s-quark inside the H

$$\bar{s}^{NP}(x) = \int_x^1 \frac{dy}{y} P_K(y)\; \bar{s}_K(x/y)$$

$\bar{s}$-quark inside the K

Kaon probability density

For the valence strange quarks inside the Kaon and the Hyperon we use the simple forms

$$\bar{s}_K(x) = \frac{1}{\beta(a_{\rm K}+1, b_{\rm K}+1)} x^{a_{\rm K}} (1-x)^{b_{\rm K}}$$

$$s_H(x) = \frac{1}{\beta(a_{\rm H}+1, b_{\rm H}+1)} x^{a_{\rm H}} (1-x)^{b_{\rm H}}$$

Normalization constants to ensure one strange valence quark inside the Kaon and the Hyperon

Fit to experimental data

Instead of using the full form of the recombination model, we parameterize the P_H and P_K inside-hadron probability densities as

$$P_{\mathrm{K}}(x) = \frac{1}{\beta(a_{\mathrm{KN}}+1, b_{\mathrm{KN}+1)}} x^{a_{\mathrm{KN}}}(1-x)^{b_{\mathrm{KN}}}$$

$$P_{\mathrm{H}}(x) = \frac{1}{\beta(a_{\mathrm{HN}}+1, b_{\mathrm{HN}}+1)} x^{a_{\mathrm{HN}}}(1-x)^{b_{\mathrm{HN}}}$$

Normalized to one Kaon and one Hyperon in the |KH⟩ Fock state

- The use of the above expressions for P_K and P_H does not imply to loose generality.
- Remembering that

$$P_M(x) = \int_0^x \frac{dy}{y} \int_0^{x-y} \frac{dz}{z} F(y,z) R(x,y,z)$$

with

$$F(y,z) = \beta y v(y) z \bar{q}(z) \rho(y,z)$$

is possible to choose $\rho(y,z)$ such that

$$P_M(x) = \mathbf{N}\, x^a (1-x)^b$$

The momentum sum rule

$$\int_0^1 dx \left[x P_B(x) + x P_M(x) \right] \; = \; 1$$

requires that

$$\frac{\Gamma(a_{\rm KN} + b_{\rm KN} + 2)\Gamma(a_{\rm KN} + 2)}{\Gamma(a_{\rm KN} + 1)\Gamma(a_{\rm KN} + b_{\rm KN} + 3)} + \frac{\Gamma(a_{\rm HN} + b_{\rm HN} + 2)\Gamma(a_{\rm HN} + 2)}{\Gamma(a_{\rm HN} + 1)\Gamma(a_{\rm HN} + b_{\rm HN} + 3)} = 1$$

fixing one parameter in the meson probability density

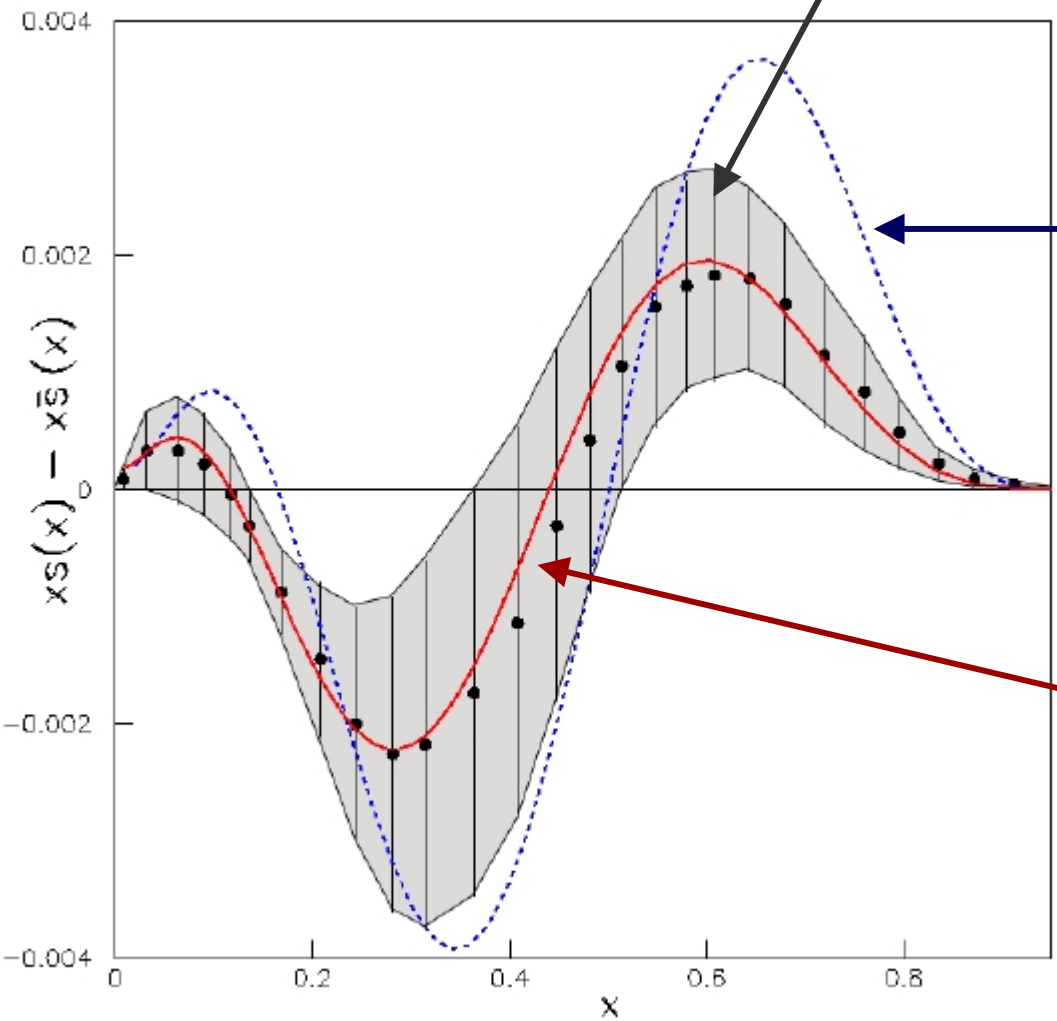
Then our fitting function

$$xs(x) - x\bar{s}(x) = N^2 \left[xs^{NP}(x) - x\bar{s}^{NP}(x) \right]$$

has 8 free parameters:

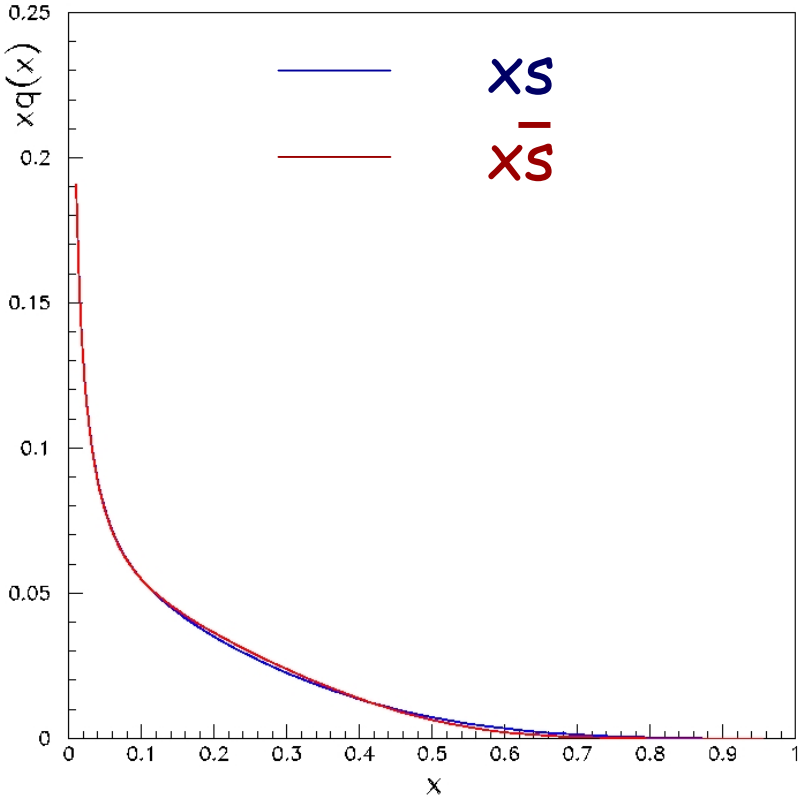
- global normalization N.
- 4 parameters from the $\bar{s}_K$ and s_H valence probability densities.
- 3 parameters from the Kaon and Hyperon probability densities.

Global fit to DIS data at $Q^2 = 20\ GeV^2$ (JHEP, 0601:006 (2006))



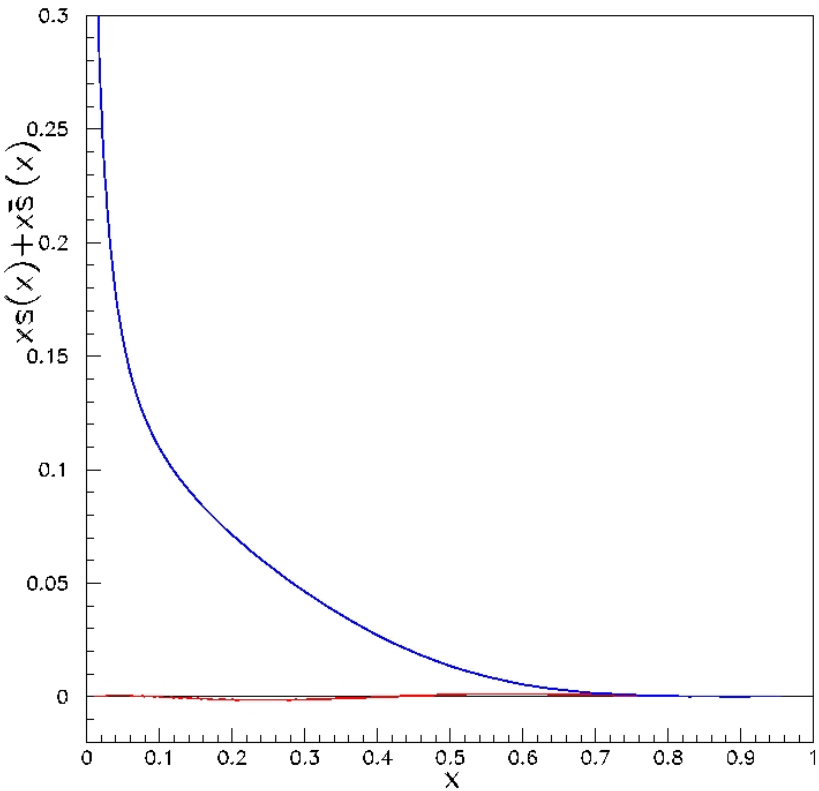
The model at $Q_0^2 = 0.49\ GeV^2$

Result of the fit at $Q^2 = 20\ GeV^2$



xs(x) and $x\bar{s}(x)$ parton distributions at $Q^2 = 20$ GeV^2

xs(x) + $x\bar{s}(x)$ and xs(x)-$x\bar{s}(x)$ at $Q^2 = 20$ GeV^2



Results of the fit

Parameter	Value
a_{KN}	$2.06 \pm 2.62 \times 10^{-7}$
b_{KN}	2.14 ± 0.11
a_K	5.14 ± 1.93
b_K	0.90 ± 0.34
a_{HN}	1.17 ± 0.35
a_H	9.47 ± 0.61
b_H	2.51 ± 0.61
N^2	0.04 ± 0.02

Probability of the $|KH\rangle$ Fock state

$b_{HN} = 1.1$ is fixed by the momentum sum rule

Conclusions

- The model provides a prescription for the sea quark and gluon pdfs at the scale Q_0^2, where pQCD evolution starts.

- The model describes quite well the results on the $xs - x\bar{s}$ asymmetry from the global fit to DIS data.

- The model describes quite well also the $\bar{e}$ experimental results about the Gottfried Sum Rule violation

- Need a global fit in order to fix all the parameters