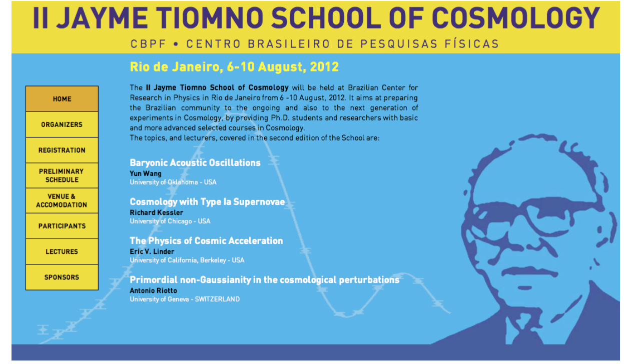

Lectures on Cosmology with Type Ia Supernovae:
Fitting the Hubble Diagram

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SN-only Hubble Fit Chi-Squared

Define reduced χ^2 to be

$$\chi_r^2 = \frac{1}{N_z}\sum_{i=1}^{N_z}\frac{\left[\mu(w,\Omega_{\rm M},z_i)-\mu(\bar{w},\overline{\Omega}_{\rm M},z_i)\right]^2}{\sigma_\mu^2}$$

- $\bar{w} = -1$ and $\overline{\Omega}_{\rm M} = 0.3$
- $\Omega_{\rm M} + \Omega_\Lambda = 1$ (flatness)
- $N_z = 10$ and $z_i = 0.05, 0.10, 0.15$ ... 0.95
- σ_μ is the uncertainty in each redshift bin.

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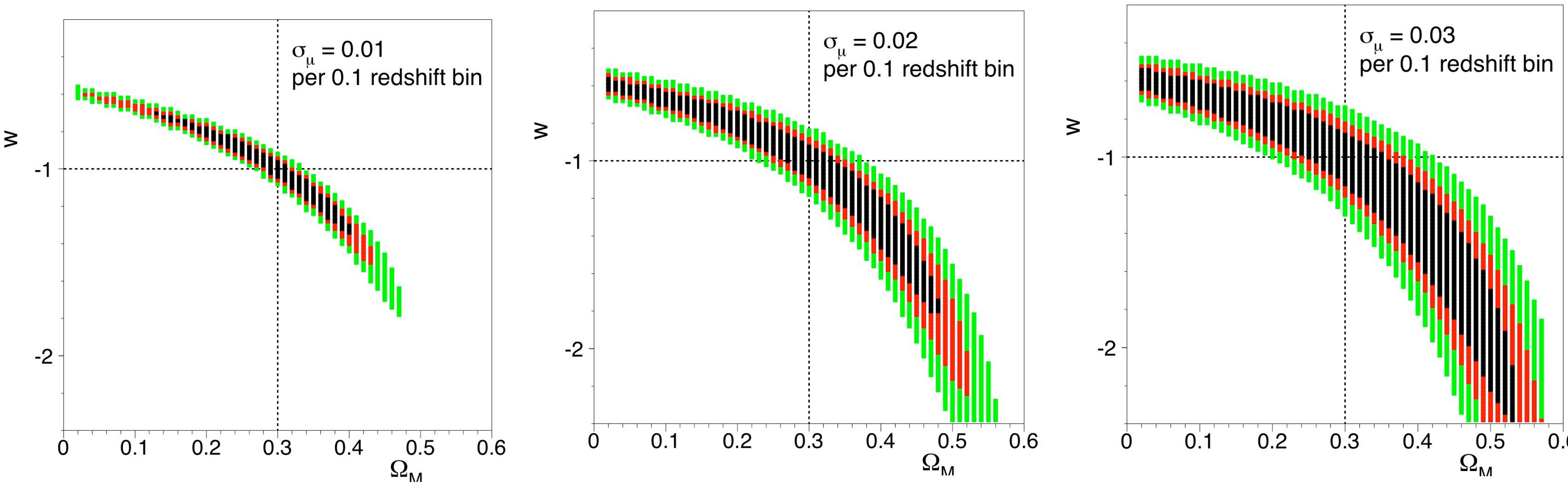
- $N_z = 10$ and $z_i = 0.05, 0.10, 0.15 \ldots 0.95$
- σ_μ is the uncertainty in each redshift bin.

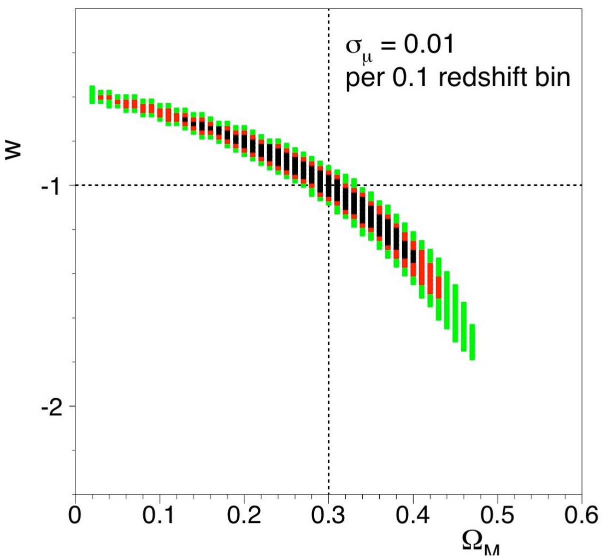
Exercise 

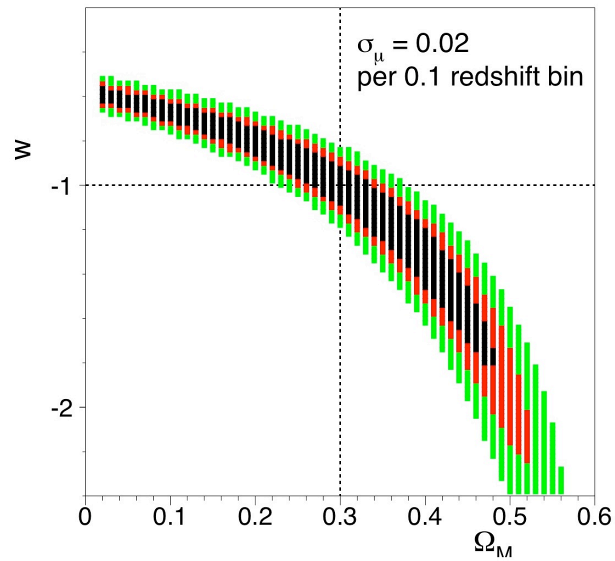
: for σ_μ = 0.01, 0.02 and 0.03, evaluate reduced χ^2 on a grid of w (-2.4 to -0.4) vs. Ω_M (0 to 0.6). map contours containing 67%, 90% and 99% of the probability.

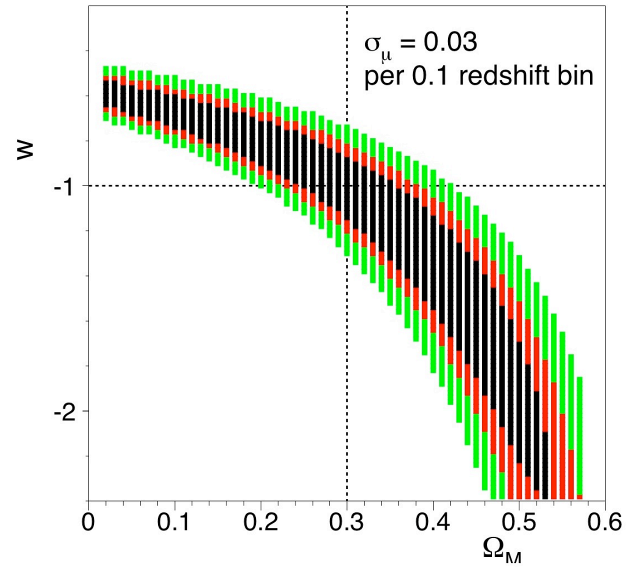
SN-only Contours

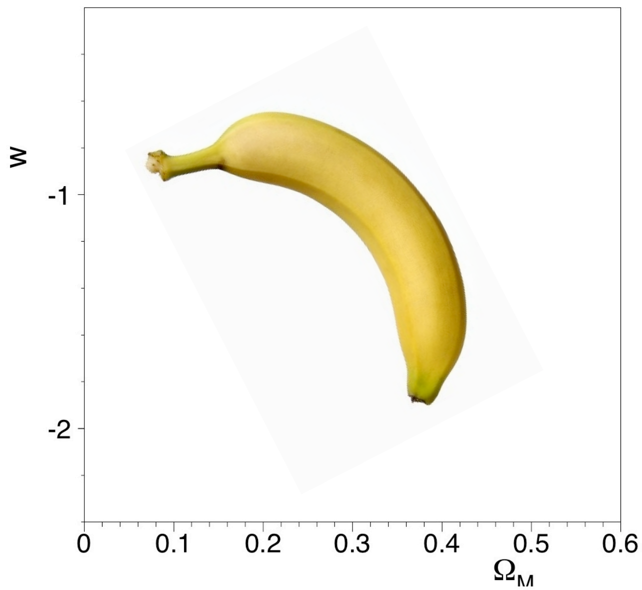


SN-only Contours









Affectionately referred to as "banana contours."

Full Hubble fit-χ^2 with Measured Distances

Measured distances

Model distances

$$\chi^2_\mu = \left\{ \sum_i \frac{[\mu_i - \mu(z_i\,;\;\Omega_{\rm M}, \Omega_{\rm DE}, w, H_0)]^2}{\sigma^2_\mu} \right\}$$

$$+\;\chi^2_{\rm BAO} + \chi^2_{\rm CMB} + \chi^2_{H_0}$$

$${\sigma_\mu}^2 = (\sigma_\mu^{\rm fit})^2 + (\sigma_\mu^{\rm int})^2 + (\sigma_\mu^z)^2,$$

Statistical error from LCFIT

Intrinsic scatter so that $\chi^2/N_{dof} = 1$

Contribution from redshift and peculiar-velocity uncertainties.

H_0 Marginalization

Exercise 16

Work through Appendix A.1 of
Goliath et al., A&A 380, 6 (2001)
and demonstrate the analytic marginalization over H_0 results in

$$\Delta\chi^2_{H0} = -\frac{B^2}{C} + \ln\left(\frac{C}{2\pi}\right)$$

$$B = \sum_{i=1}^{n} \frac{\Delta_i^2}{\sigma_i^2},$$

$$C = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}.$$

where $\Delta_i = \mu_i - \mu(z_i\,;\ \Omega_{\rm M}, \Omega_{\rm DE}, w, H_0)$

Priors from BAO & CMB

BAO (SDSS LRG, Eisenstein etal 05) :

$$A(z_1;w,\Omega_m,\Omega_{DE}) = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}}\left[\frac{1}{z_1\sqrt{|\Omega_k|}} S_k\left(|\Omega_k|^{1/2}\int_0^{z_1}\frac{dz}{E(z)}\right)\right]^{2/3}$$

with

$$\chi_{BAO}{}^2 = [(A(z_1;w,\Omega_m,\Omega_{DE}) - 0.469)/0.017]^2 \text{ for } z_1 = 0.35$$

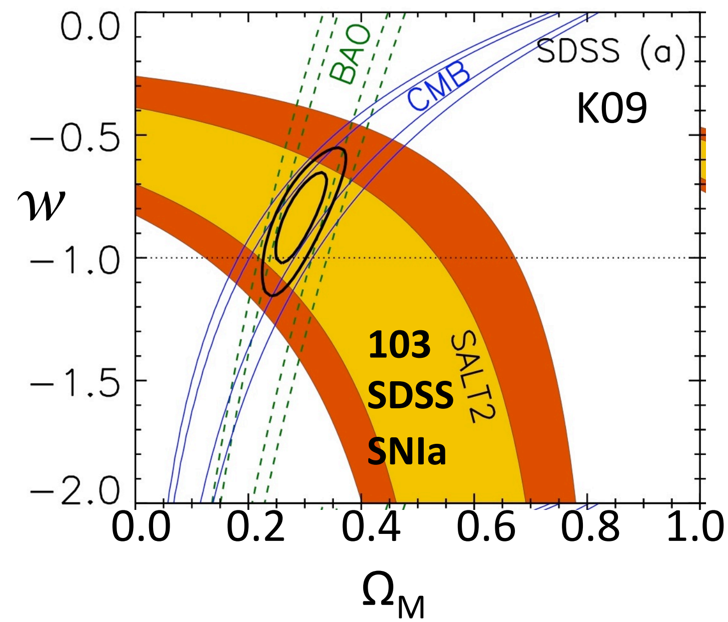
CMB (WMAP5, Komatsu etal 08) :

$$R(z_{CMB};w,\Omega_m,\Omega_{DE}) = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}}\left[S_k\left(|\Omega_k|^{1/2}\int_0^{z_{CMB}}\frac{dz}{E(z)}\right)\right]$$

with

$$\chi_{CMB}{}^2 = [(R(z_{CMB};w,\Omega_m,\Omega_{DE}) - 1.710)/0.019]^2 \text{ for } z_{CMB} = 1090$$

BAO+WMAP priors break degeneracy in w-Ω_M plane



Note: more recent priors from BAO-DR7 and WMAP7

Redshift Uncertainty

Exercise 

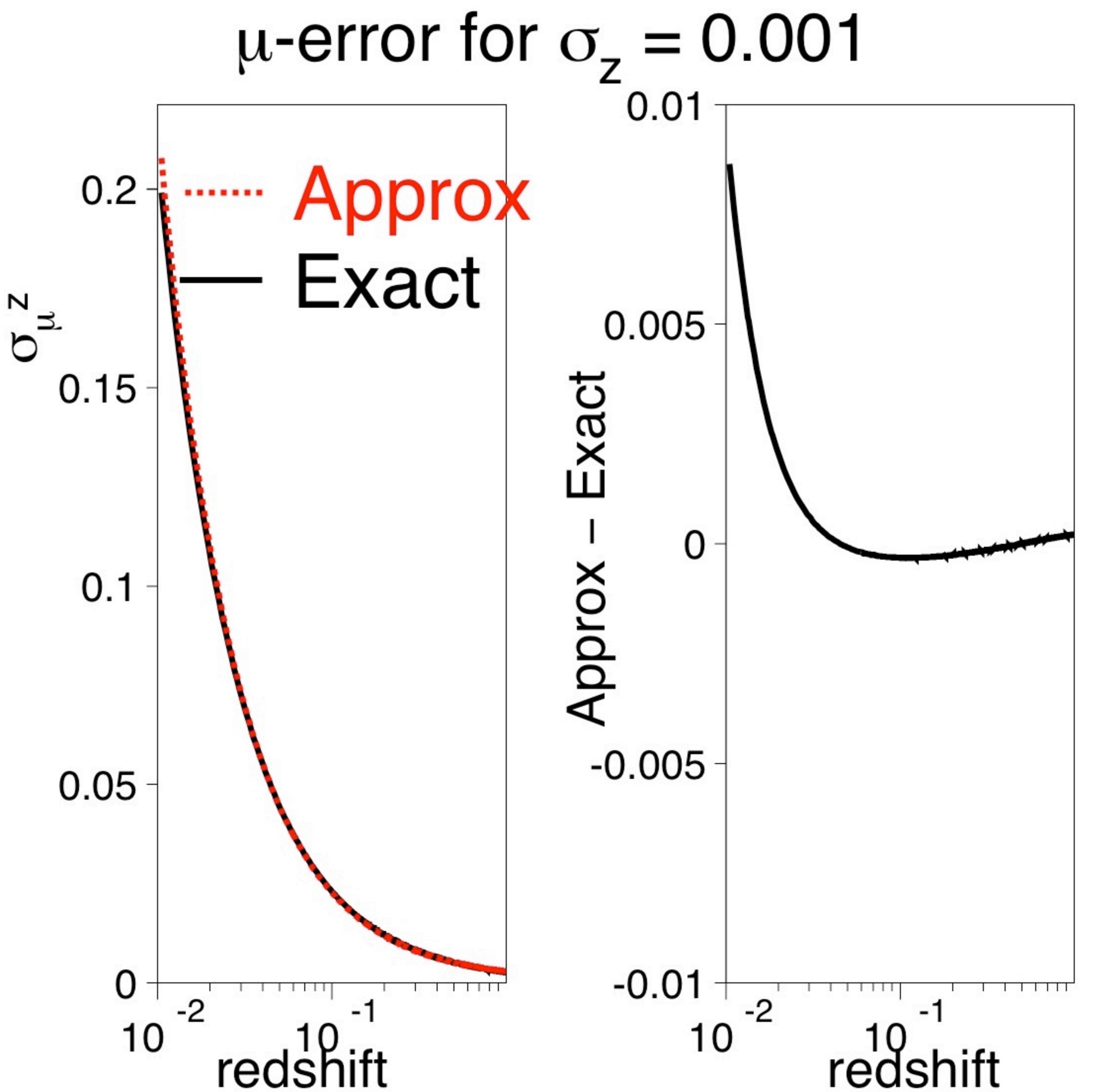
Compute $d\mu/d\sigma_z$ for an empty flat universe ($\Omega_M = \Omega_\Lambda = 0$, Ω_k=1) and show that

$$\sigma_\mu^z = \sigma_z \left(\frac{5}{\ln 10}\right) \left\{\frac{1}{1+z}\left[1 + \frac{1}{\ln(1+z)}\right]\right\}$$
$$\simeq \sigma_z \left(\frac{5}{\ln 10}\right) \left[\frac{1+z}{z(1+z/2)}\right]$$

Compare to exact calculation (Ω_M=0.3, Ω_Λ=0.7, w=−1) from z=0.01 to 1.

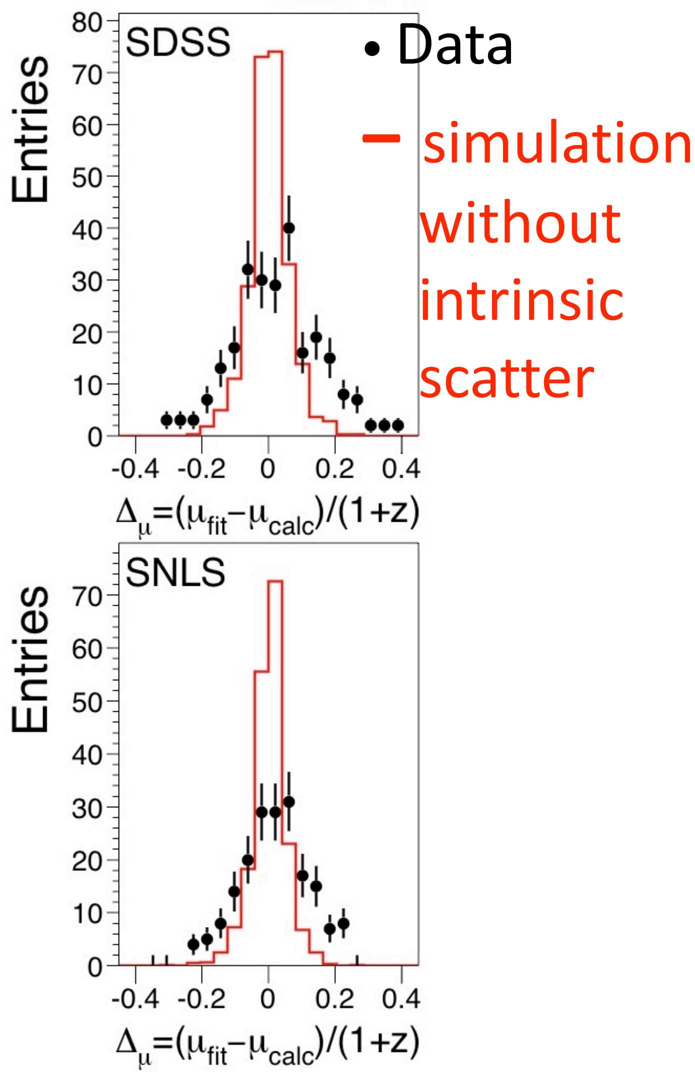
If we want σ_μ^z < the intrinsic dispersion (σ_{int}), show that we need a redshift cut $z > 2.2\, \sigma_z/\sigma_{int}$. Plug in typical numbers, noting that σ_z includes a peculiar velocity uncertainty.

Redshift Uncertainty



Intrinsic Dispersion

Hubble scatter residuals
(RK, paper in prep)



Observed Hubble scatter is much larger than expected from measured uncertainties.

Intrinsic Dispersion

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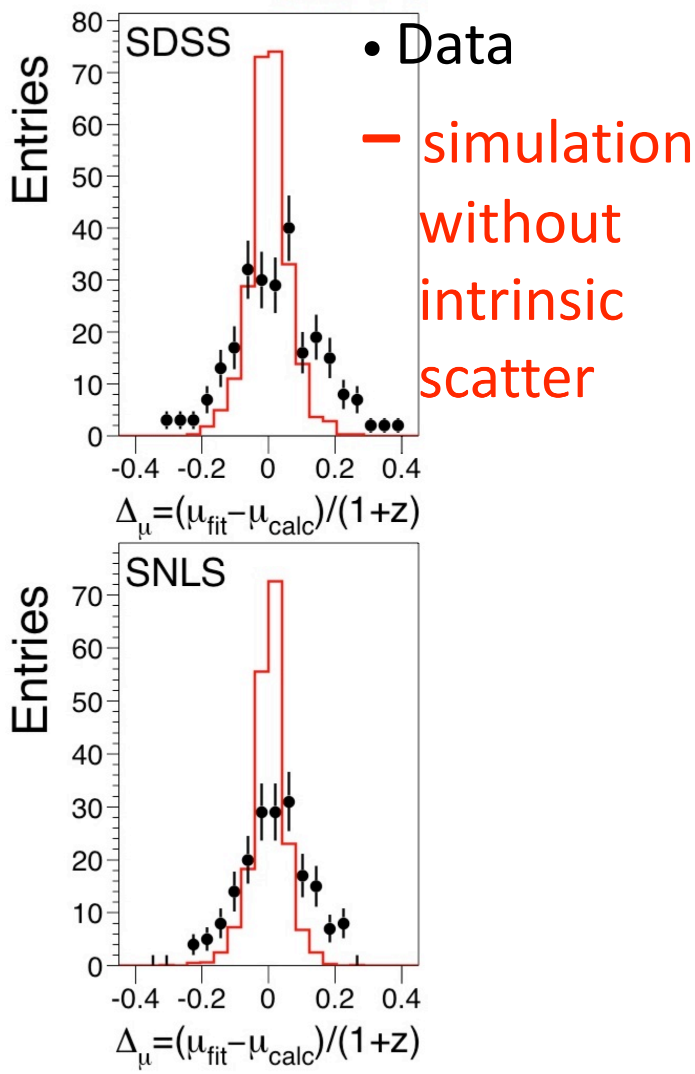


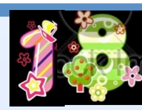
Observed Hubble scatter is much larger than expected from measured uncertainties.

Different cuts on color and incorrect error propagation lead to different estimates of σ_{int} : 0.07 to 0.18 !

σ_{int} often assumed to be constant, but it's not ! It depends on rest-frame wavelength (see error model in Guy 2010, and discussion in Marriner et al, 2011).

σ_{int} might also depend on epoch, stretch, color, redshift …

Incorrect σ_{int} model can bias Hubble fit.

Exercise 

Using the distance moduli from the HiZ team (Riess et al, 1998) listed in the next slide, carry out a χ^2 analysis to determine Ω_M and Ω_Λ.

Ignore BAO and CMB priors, and fix H_0.

Use peculiar velocity error of 200 km/s for nearby sample; high redshift σ_z = 0.005.

MLCS Fits: Riess et al., AJ 116, 1009 (1998)

EVIDENCE FOR AN ACCELERATING UNIVERSE

TABLE 10

NEARBY MLCS AND TEMPLATE-FITTING SN Ia PARAMETERS[a]

SN	log cz	MLCS Δ	MLCS A_B	MLCS μ_0 (σ)
1992bo	3.734	0.31	0.00	34.72(0.16)
1992bc	3.779	−0.50	0.00	34.87(0.11)
1992aq	4.481	0.05	0.00	38.41(0.15)
1992ae	4.350	−0.05	0.00	37.80(0.17)
1992P	3.896	−0.19	0.00	35.76(0.13)
1990af	4.178	0.09	0.18	36.53(0.15)
1994M	3.859	0.04	0.08	35.39(0.18)
1994S	3.685	−0.44	0.00	34.27(0.12)
1994T	4.030	0.11	0.22	36.19(0.21)
1995D	3.398	−0.42	0.00	33.01(0.13)
1995E	3.547	−0.61	2.67	33.60(0.17)
1995ac	4.166	−0.47	0.00	36.85(0.13)
1995ak	3.820	0.15	0.00	35.15(0.16)
1995bd	3.679	−0.29	2.52	34.15(0.19)
1996C	3.924	−0.07	0.24	35.98(0.20)
1996ab	4.572	−0.13	0.00	39.01(0.13)
1992ag	3.891	−0.50	0.77	35.37(0.23)
1992al	3.625	−0.35	0.00	33.92(0.11)
1992bg	4.024	−0.06	0.50	36.26(0.21)
1992bh	4.130	−0.16	0.28	36.91(0.17)
1992bl	4.111	−0.06	0.00	36.26(0.15)
1992bp	4.379	−0.26	0.04	37.65(0.13)
1992br	4.418	0.40	0.00	38.21(0.19)
1992bs	4.283	0.00	0.00	37.61(0.14)
1993H	3.871	0.16	0.67	35.20(0.26)
1993O	4.189	0.03	0.00	37.03(0.12)
1993ag	4.177	−0.19	0.64	36.80(0.17)

[a] Light curves restricted to B and V data within 40 days of B maximum.
[b] E = early-type host, L = late-type host.

TABLE 5

HIGH-z MLCS SN Ia LIGHT CURVE PARAMETERS

SN	z	m_B^{max}	m_V^{max}	Δ	A_B	μ_0 (σ_{μ_0})
1996E	0.43	22.81(0.21)	22.72(0.23)	−0.08(0.19)	0.31	41.74(0.28)
1996H	0.62	23.23(0.19)	23.56(0.18)	−0.42(0.16)	0.00	42.98(0.17)
1996I	0.57	23.35(0.28)	23.59(0.26)	−0.06(0.26)	0.00	42.76(0.19)
1996J	0.30	22.23(0.12)	22.21(0.11)	−0.22(0.10)	0.24	41.38(0.24)
1996K	0.38	22.64(0.12)	22.84(0.14)	0.29(0.06)	0.00	41.63(0.20)
1996U	0.43	22.78(0.22)	22.98(0.30)	−0.52(0.29)	0.00	42.55(0.25)
1997ce	0.44	22.85(0.09)	22.95(0.09)	0.07(0.08)	0.00	41.95(0.17)
1997cj	0.50	23.19(0.11)	23.29(0.12)	−0.04(0.11)	0.00	42.40(0.17)
1997ck	0.97	24.78(0.25)	...	−0.19(0.23)	...	44.39(0.30)
1995K	0.48	22.91(0.13)	23.08(0.20)	−0.33(0.26)	0.00	42.45(0.17)

NOTE.—Uncertainties in magnitudes are listed in parentheses.