



Centro Brasileiro de Pesquisas Físicas

Ministério da Ciência, Tecnologia e Inovação



UFRJ



Universidade Federal do Rio de Janeiro



**II JAYME TIOMNO SCHOOL OF COSMOLOGY**  
 CBPF • CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

**Rio de Janeiro, 6-10 August, 2012**

The II Jayme Tiomno School of Cosmology will be held at Brazilian Center for Research in Physics in Rio de Janeiro from 6 - 10 August, 2012. It aims at preparing the Brazilian community to the ongoing and also to the next generation of experiments in Cosmology, by providing Ph.D. students and researchers with basic and more advanced selected courses in Cosmology. The topics, and lecturers, covered in the second edition of the School are:


HOME
ORGANIZERS
REGISTRATION
PRELIMINARY SCHEDULE
VENUE & ACCOMMODATION
PARTICIPANTS
LECTURES
SPONSORS

**Baryonic Acoustic Oscillations**  
 Yun Wang  
 University of California - USA

**Cosmology with Type Ia Supernovae**  
 Richard Kessler  
 University of Chicago - USA

**The Physics of Cosmic Acceleration**  
 Eric V. Linder  
 University of California, Berkeley - USA

**Primordial non-Gaussianity in the cosmological perturbations**  
 Antonio Riotto  
 University of Geneva - SWITZERLAND



# Lectures on Cosmology with Type Ia Supernovae: Fitting the Hubble Diagram

R.Kessler (U.Chicago)

**II Jayme Tiomno School of Cosmology**  
**Rio de Janeiro, Brazil**  
**Aug 6-10, 2012**

# SN-only Hubble Fit Chi-Squared

Define reduced  $\chi^2$  to be

$$\chi_r^2 = \frac{1}{N_z} \sum_{i=1}^{N_z} \frac{[\mu(w, \Omega_M, z_i) - \mu(\bar{w}, \bar{\Omega}_M, z_i)]^2}{\sigma_\mu^2}$$

- $\bar{w} = -1$  and  $\bar{\Omega}_M = 0.3$
- $\Omega_M + \Omega_\Lambda = 1$  (flatness)
- $N_z = 10$  and  $z_i = 0.05, 0.10, 0.15 \dots 0.95$
- $\sigma_\mu$  is the uncertainty in each redshift bin.


# SN-only Hubble Fit Chi-Squared

Define reduced  $\chi^2$  to be

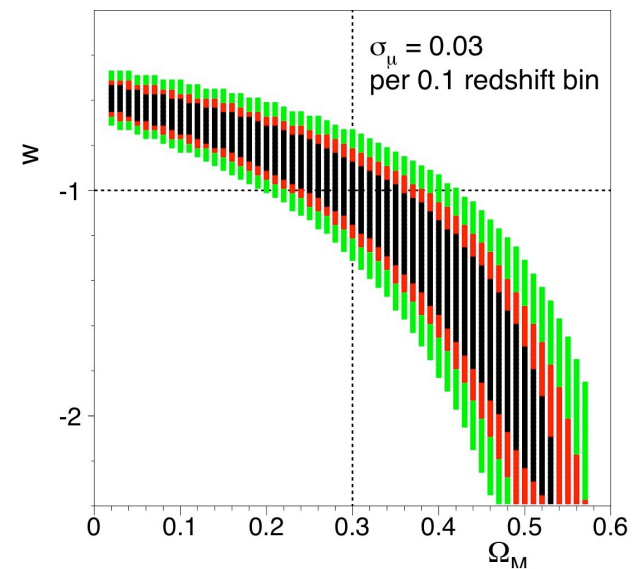
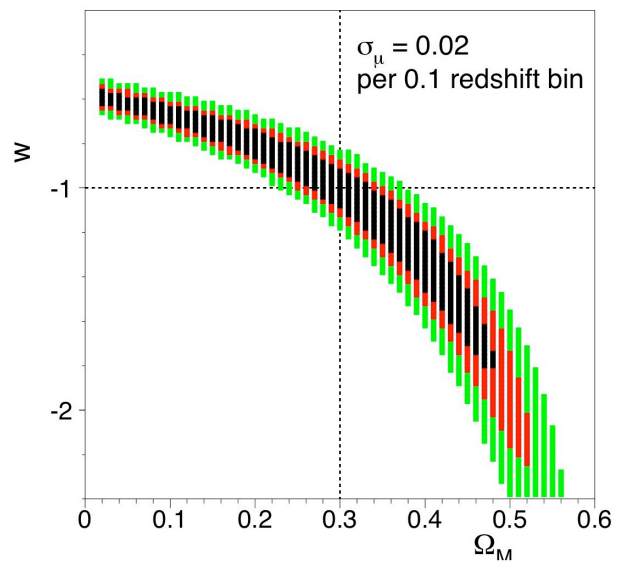
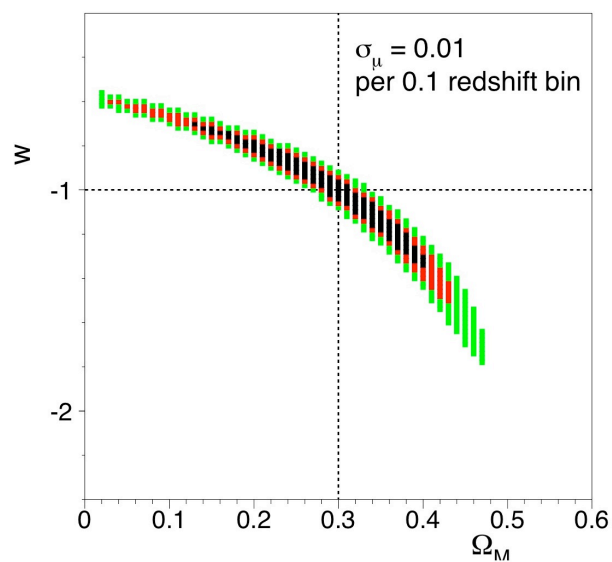
$$\chi_r^2 = \frac{1}{N_z} \sum_{i=1}^{N_z} \frac{[\mu(w, \Omega_M, z_i) - \mu(\bar{w}, \bar{\Omega}_M, z_i)]^2}{\sigma_\mu^2}$$

- $\bar{w} = -1$  and  $\bar{\Omega}_M = 0.3$
- $\Omega_M + \Omega_\Lambda = 1$  (flatness)
- $N_z = 10$  and  $z_i = 0.05, 0.10, 0.15 \dots 0.95$
- $\sigma_\mu$  is the uncertainty in each redshift bin.

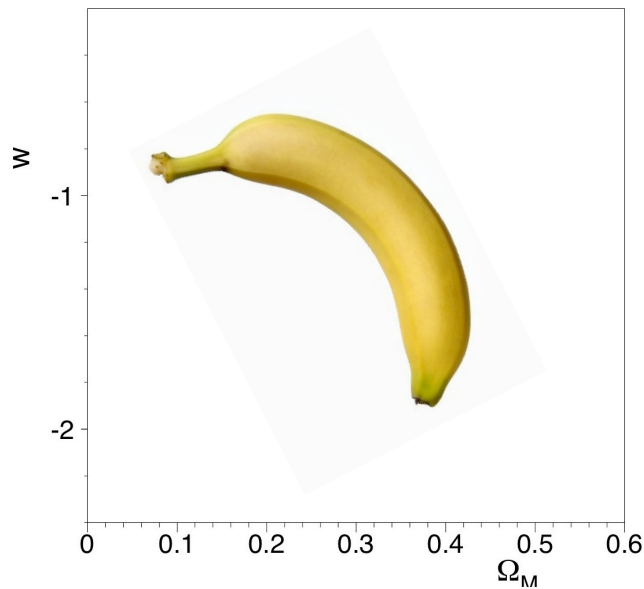
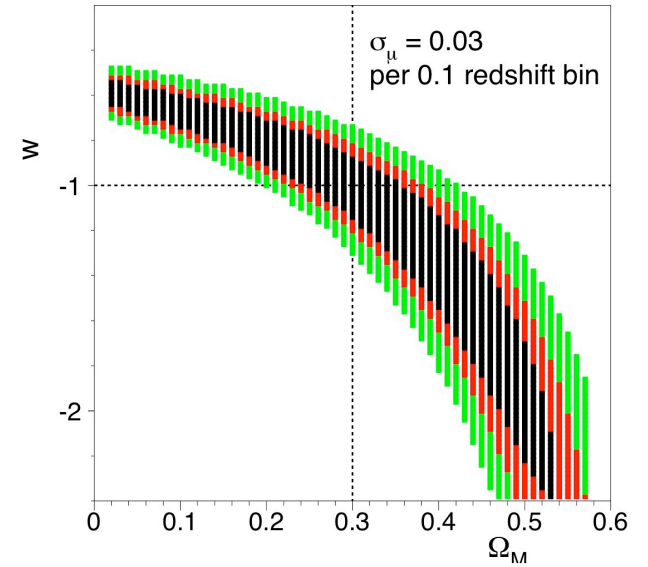
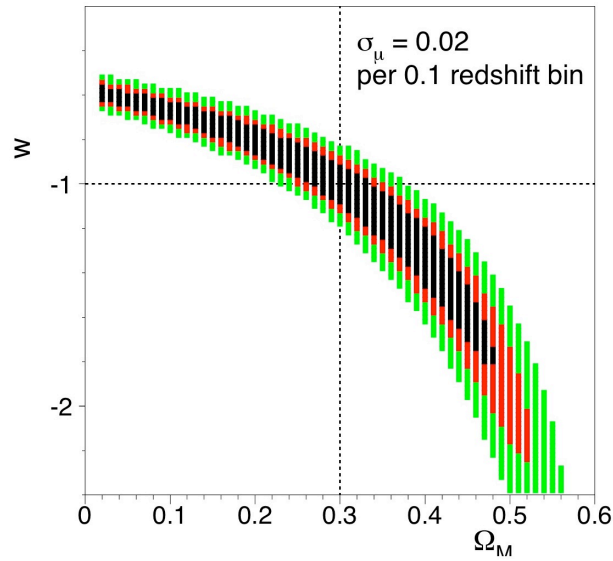
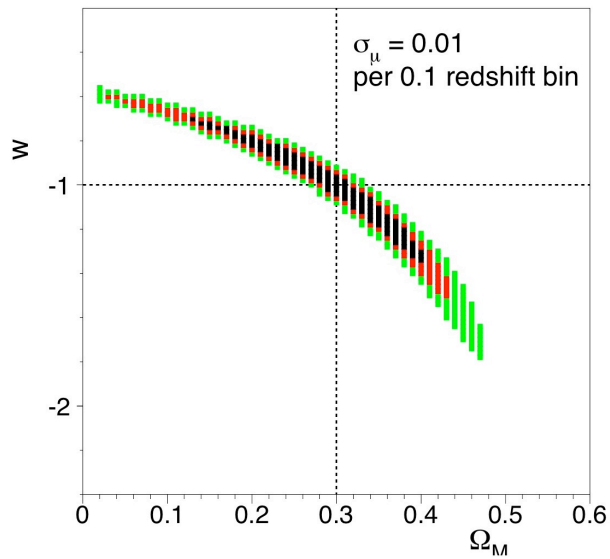


**Exercise**  : for  $\sigma_\mu = 0.01, 0.02$  and  $0.03$ , evaluate reduced  $\chi^2$  on a grid of  $w$  (-2.4 to -0.4) vs.  $\Omega_M$  (0 to 0.6). map contours containing 67%, 90% and 99% of the probability.

# SN-only Contours



# SN-only Contours



Affectionately  
referred to as  
“banana contours.”

# Full Hubble fit- $\chi^2$ with Measured Distances

Measured distances

Model distances

$$\chi_{\mu}^2 = \left\{ \sum_i \frac{[\mu_i - \mu(z_i; \Omega_M, \Omega_{DE}, w, H_0)]^2}{\sigma_{\mu}^2} \right\} + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2 + \chi_{H_0}^2$$

$$\sigma_{\mu}^2 = (\sigma_{\mu}^{\text{fit}})^2 + (\sigma_{\mu}^{\text{int}})^2 + (\sigma_{\mu}^z)^2,$$

Statistical error from LCFIT

Intrinsic scatter so that  $\chi^2/N_{\text{dof}}=1$

Contribution from redshift and peculiar-velocity uncertainties.

# $H_0$ Marginalization

## Exercise

Work through Appendix A.1 of Goliath et al., A&A 380, 6 (2001) and demonstrate the analytic marginalization over  $H_0$  results in

$$\Delta\chi^2_{H_0} = -\frac{B^2}{C} + \ln\left(\frac{C}{2\pi}\right)$$

$$B = \sum_{i=1}^n \frac{\Delta_i^2}{\sigma_i^2},$$

$$C = \sum_{i=1}^n \frac{1}{\sigma_i^2}.$$

where  $\Delta_i = \mu_i - \mu(z_i; \Omega_M, \Omega_{DE}, w, H_0)$

# Priors from BAO & CMB

BAO (SDSS LRG, Eisenstein et al 05):

$$A(z_1; w, \Omega_m, \Omega_{DE}) = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left[ \frac{1}{z_1 \sqrt{|\Omega_k|}} S_k \left( |\Omega_k|^{1/2} \int_0^{z_1} \frac{dz}{E(z)} \right) \right]^{2/3}$$

with

$$\chi_{BAO}^2 = [(A(z_1; w, \Omega_m, \Omega_{DE}) - 0.469)/0.017]^2 \text{ for } z_1 = 0.35$$

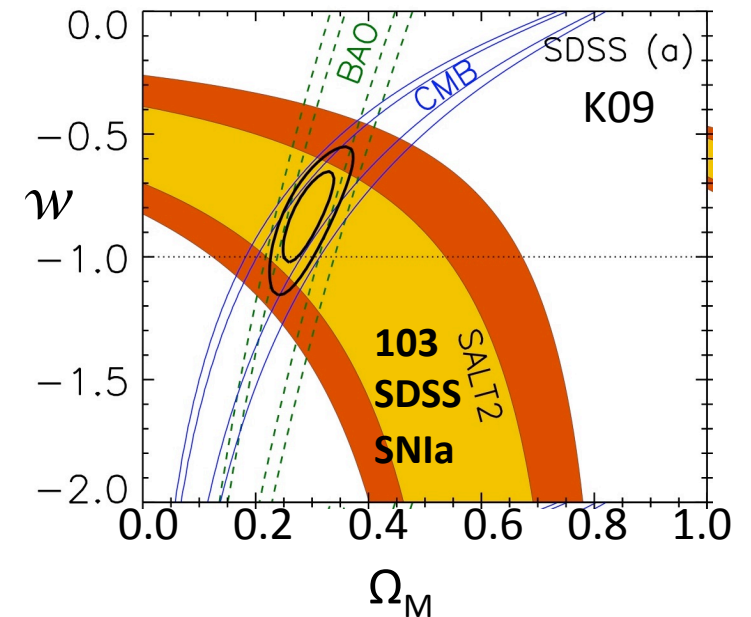
CMB (WMAP5, Komatsu et al 08):

$$R(z_{CMB}; w, \Omega_m, \Omega_{DE}) = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}} \left[ S_k \left( |\Omega_k|^{1/2} \int_0^{z_{CMB}} \frac{dz}{E(z)} \right) \right]$$

with

$$\chi_{CMB}^2 = [(R(z_{CMB}; w, \Omega_m, \Omega_{DE}) - 1.710)/0.019]^2 \text{ for } z_{CMB} = 1090$$

BAO+WMAP priors  
break degeneracy  
in  $w$ - $\Omega_M$  plane



Note: more recent priors from  
BAO-DR7 and WMAP7



# Redshift Uncertainty

## Exercise



Compute  $d\mu/d\sigma_z$  for an empty flat universe ( $\Omega_M = \Omega_\Lambda = 0$ ,  $\Omega_k=1$ ) and show that

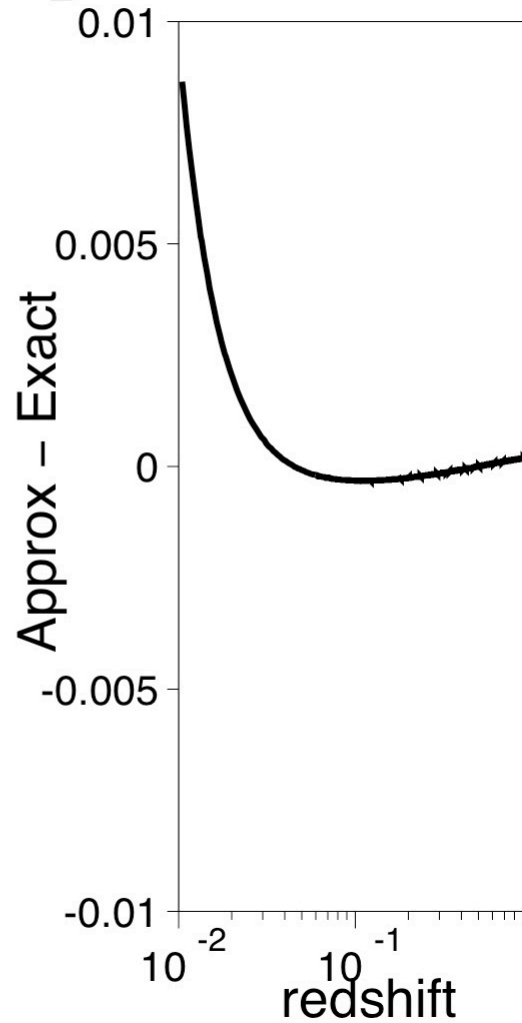
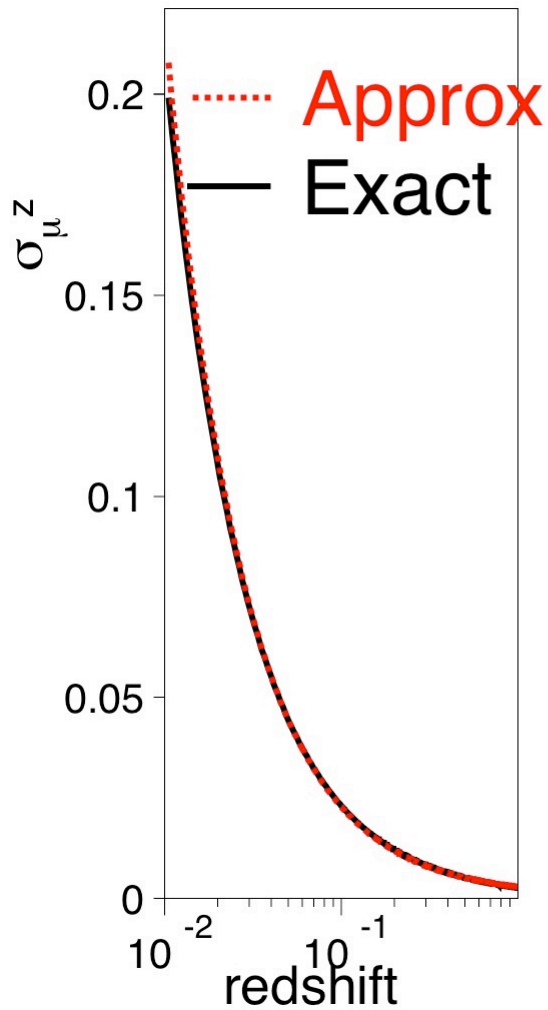
$$\begin{aligned}\sigma_\mu^z &= \sigma_z \left( \frac{5}{\ln 10} \right) \left\{ \frac{1}{1+z} \left[ 1 + \frac{1}{\ln(1+z)} \right] \right\} \\ &\simeq \sigma_z \left( \frac{5}{\ln 10} \right) \left[ \frac{1+z}{z(1+z/2)} \right]\end{aligned}$$

Compare to exact calculation ( $\Omega_M=0.3$ ,  $\Omega_\Lambda=0.7$ ,  $w=-1$ ) from  $z=0.01$  to 1.

If we want  $\sigma_\mu^z <$  the intrinsic dispersion ( $\sigma_{\text{int}}$ ), show that we need a redshift cut  $z > 2.2 \sigma_z/\sigma_{\text{int}}$ . Plug in typical numbers, noting that  $\sigma_z$  includes a peculiar velocity uncertainty.

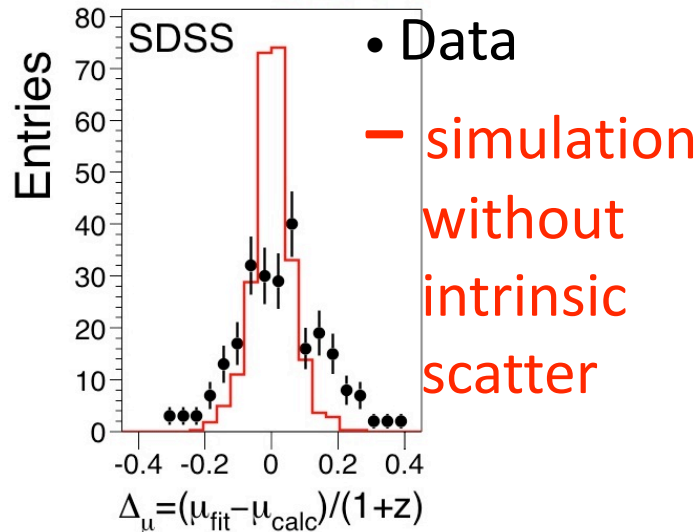
# Redshift Uncertainty

$\mu$ -error for  $\sigma_z = 0.001$

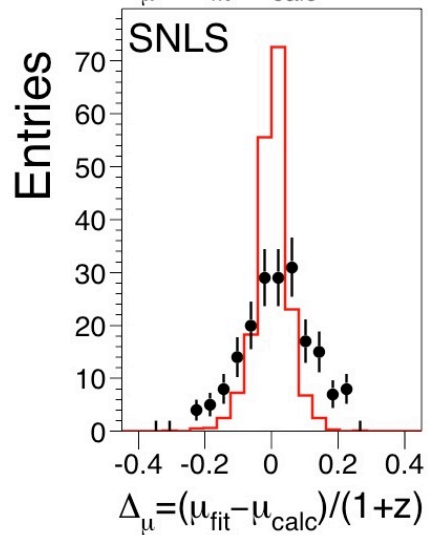


# Intrinsic Dispersion

Hubble scatter residuals  
(RK, paper in prep)

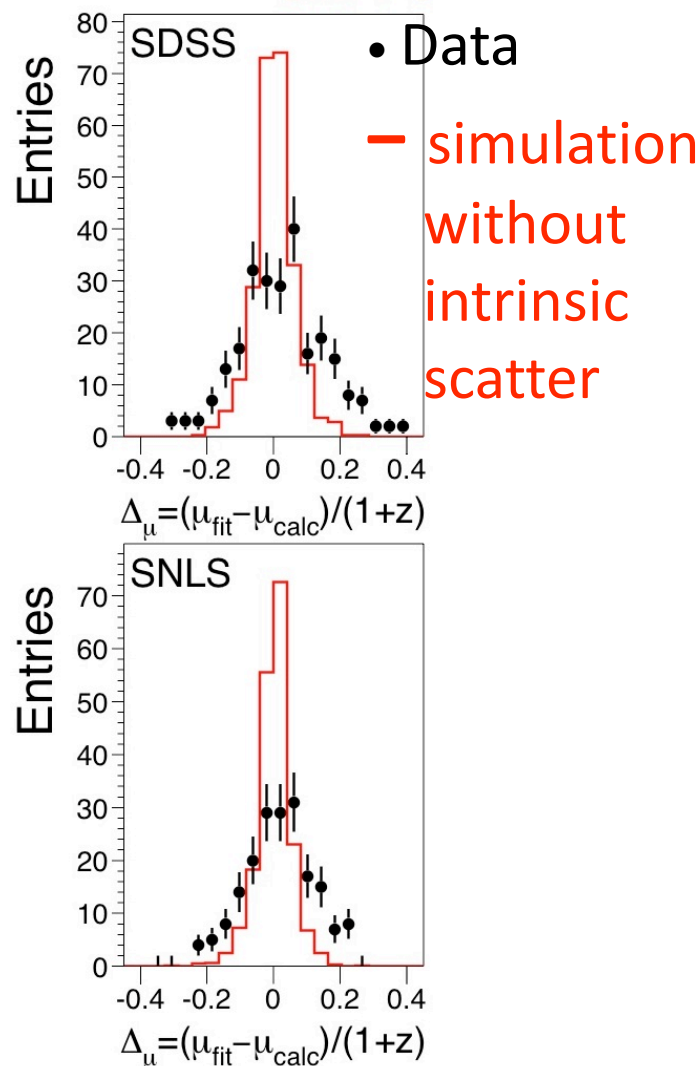


Observed Hubble scatter is much larger than expected from measured uncertainties.



# Intrinsic Dispersion

Hubble scatter residuals  
(RK, paper in prep)



Observed Hubble scatter is much larger than expected from measured uncertainties.

Different cuts on color and incorrect error propagation lead to different estimates of  $\sigma_{\text{int}}$  : 0.07 to 0.18 !

$\sigma_{\text{int}}$  often assumed to be constant, but it's not ! It depends on rest-frame wavelength (see error model in Guy 2010, and discussion in Marriner et al, 2011).

$\sigma_{\text{int}}$  might also depend on epoch, stretch, color, redshift ...

**Incorrect  $\sigma_{\text{int}}$  model can bias Hubble fit.**

## Exercise

Using the distance moduli from the HiZ team (Riess et al, 1998) listed in the next slide, carry out a  $\chi^2$  analysis to determine  $\Omega_M$  and  $\Omega_\Lambda$ .

Ignore BAO and CMB priors, and fix  $H_0$ .



Use peculiar velocity error of 200 km/s for nearby sample; high redshift  $\sigma_z = 0.005$ .

# MLCS Fits: Riess et al., AJ 116, 1009 (1998)

## EVIDENCE FOR AN ACCELERATING UNIVERSE

TABLE 10  
NEARBY MLCS AND TEMPLATE-FITTING SN Ia PARAMETERS<sup>a</sup>

SN	log cz	MLCS		
		$\Delta$	$A_B$	$\mu_0$ ( $\sigma$ )
1992bo .....	3.734	0.31	0.00	34.72(0.16)
1992bc .....	3.779	-0.50	0.00	34.87(0.11)
1992aq .....	4.481	0.05	0.00	38.41(0.15)
1992ae .....	4.350	-0.05	0.00	37.80(0.17)
1992P .....	3.896	-0.19	0.00	35.76(0.13)
1990af .....	4.178	0.09	0.18	36.53(0.15)
1994M .....	3.859	0.04	0.08	35.39(0.18)
1994S .....	3.685	-0.44	0.00	34.27(0.12)
1994T .....	4.030	0.11	0.22	36.19(0.21)
1995D .....	3.398	-0.42	0.00	33.01(0.13)
1995E .....	3.547	-0.61	2.67	33.60(0.17)
1995ac .....	4.166	-0.47	0.00	36.85(0.13)
1995ak .....	3.820	0.15	0.00	35.15(0.16)
1995bd .....	3.679	-0.29	2.52	34.15(0.19)
1996C .....	3.924	-0.07	0.24	35.98(0.20)
1996ab .....	4.572	-0.13	0.00	39.01(0.13)
1992ag .....	3.891	-0.50	0.77	35.37(0.23)
1992al .....	3.625	-0.35	0.00	33.92(0.11)
1992bg .....	4.024	-0.06	0.50	36.26(0.21)
1992bh .....	4.130	-0.16	0.28	36.91(0.17)
1992bl .....	4.111	-0.06	0.00	36.26(0.15)
1992bp .....	4.379	-0.26	0.04	37.65(0.13)
1992br .....	4.418	0.40	0.00	38.21(0.19)
1992bs .....	4.283	0.00	0.00	37.61(0.14)
1993H .....	3.871	0.16	0.67	35.20(0.26)
1993O .....	4.189	0.03	0.00	37.03(0.12)
1993ag .....	4.177	-0.19	0.64	36.80(0.17)

<sup>a</sup> Light curves restricted to  $B$  and  $V$  data within 40 days of  $B$  maximum.

<sup>b</sup> E = early-type host, L = late-type host.

TABLE 5  
HIGH- $z$  MLCS SN Ia LIGHT CURVE PARAMETERS

SN	$z$	$m_B^{\max}$	$m_V^{\max}$	$\Delta$	$A_B$	$\mu_0$ ( $\sigma_{\mu_0}$ )
1996E .....	0.43	22.81(0.21)	22.72(0.23)	-0.08(0.19)	0.31	41.74(0.28)
1996H .....	0.62	23.23(0.19)	23.56(0.18)	-0.42(0.16)	0.00	42.98(0.17)
1996I .....	0.57	23.35(0.28)	23.59(0.26)	-0.06(0.26)	0.00	42.76(0.19)
1996J .....	0.30	22.23(0.12)	22.21(0.11)	-0.22(0.10)	0.24	41.38(0.24)
1996K .....	0.38	22.64(0.12)	22.84(0.14)	0.29(0.06)	0.00	41.63(0.20)
1996U .....	0.43	22.78(0.22)	22.98(0.30)	-0.52(0.29)	0.00	42.55(0.25)
1997ce .....	0.44	22.85(0.09)	22.95(0.09)	0.07(0.08)	0.00	41.95(0.17)
1997cj .....	0.50	23.19(0.11)	23.29(0.12)	-0.04(0.11)	0.00	42.40(0.17)
1997ck .....	0.97	24.78(0.25)	...	-0.19(0.23)	...	44.39(0.30)
1995K .....	0.48	22.91(0.13)	23.08(0.20)	-0.33(0.26)	0.00	42.45(0.17)

NOTE.—Uncertainties in magnitudes are listed in parentheses.