



Centro Brasileiro de  
Pesquisas Físicas

Ministério da  
Ciência, Tecnologia  
e Inovação



UFRJ



Universidade Federal  
do Rio de Janeiro



**II JAYME TIOMNO SCHOOL OF COSMOLOGY**  
CBPF • CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

**Rio de Janeiro, 6-10 August, 2012**

The II Jayme Tiomno School of Cosmology will be held at Brazilian Center for Research in Physics in Rio de Janeiro from 6 - 10 August, 2012. It aims at preparing the Brazilian community to the ongoing and also to the next generation of experiments in Cosmology, by providing Ph.D. students and researchers with basic and more advanced selected courses in Cosmology. The topics, and lecturers, covered in the second edition of the School are:


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**Baryonic Acoustic Oscillations**  
Yun Wang  
University of Michigan - USA

**Cosmology with Type Ia Supernovae**  
Richard Kessler  
University of Chicago - USA

**The Physics of Cosmic Acceleration**  
Eric V. Linder  
University of California, Berkeley - USA

**Primordial non-Gaussianity in the cosmological perturbations**  
Antonio Riotto  
University of Geneva - SWITZERLAND



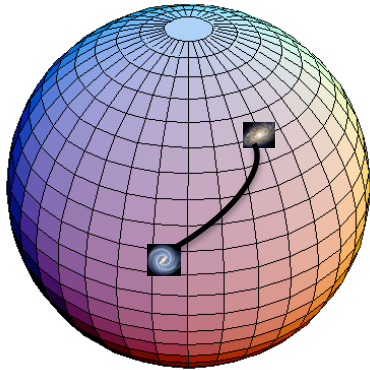
# Lectures on Cosmology with Type Ia Supernovae: Formalism

R.Kessler (U.Chicago)

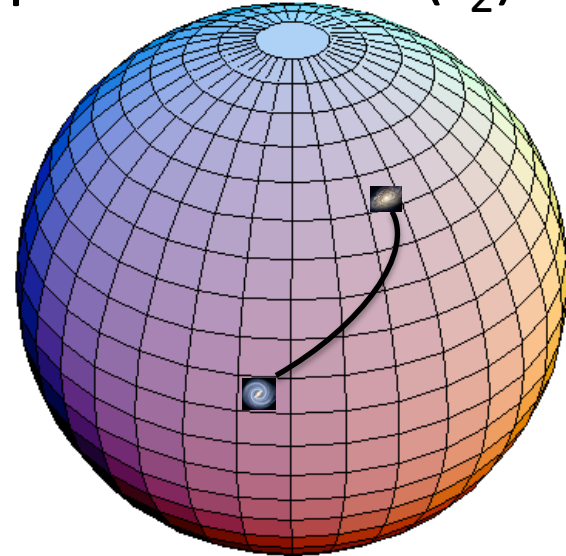
**II Jayme Tiomno School of Cosmology**  
**Rio de Janeiro, Brazil**  
**Aug 6-10, 2012**

# Expanding Universe

at time  $t_1$  galaxy  
separation is  $r a(t_1)$



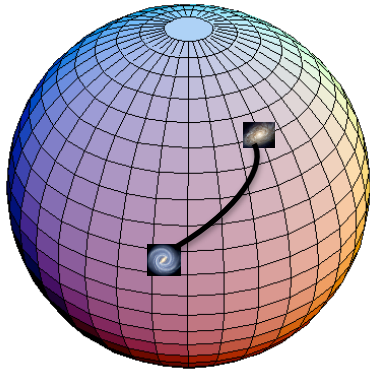
at time  $t_2$  galaxy  
separation is  $r a(t_2)$



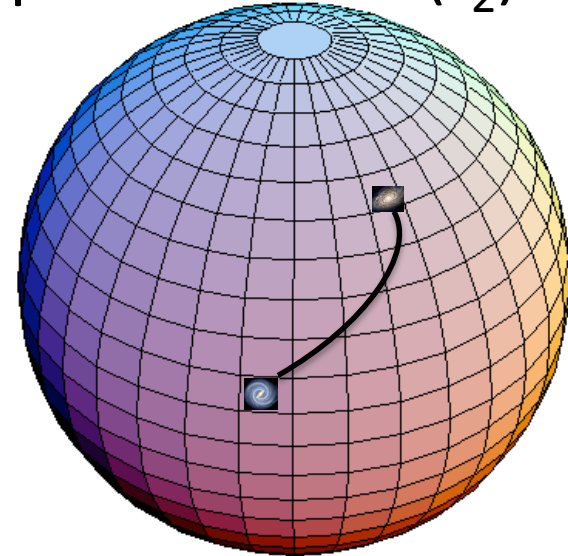
- $r$  = comoving separation
- $a(t)$  = scale factor
- Galaxy size determined by gravity (not affected by expansion)
- $a(t_2)/a(t_1) = \lambda_2/\lambda_1 = 1+z$ , where  $z$  = redshift

# Expanding Universe

at time  $t_1$  galaxy  
separation is  $r a(t_1)$



at time  $t_2$  galaxy  
separation is  $r a(t_2)$



Recesion velocity between galaxies,

$$\begin{aligned} v &= \frac{d(t_2) - d(t_1)}{t_2 - t_1} = r \frac{a(t_2) - a(t_1)}{t_2 - t_1} = \frac{d}{a} \cdot \frac{a(t_2) - a(t_1)}{t_2 - t_1} \\ &= (d/a) \dot{a} \equiv dH(t) \end{aligned}$$

$$H(t) \equiv \dot{a}/a$$

Today  $H(t_0) \approx 70 \text{ km/s/Mpc}$

Begin with Friedmann equation to describe evolution of cosmic scale factor  $a(t) = (1 + z)^{-1}$  where

$$a(t = t_0; \text{today}) = 1$$

$$a(t < t_0) < 1$$

$$[H(t)]^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

In terms of critical densities,

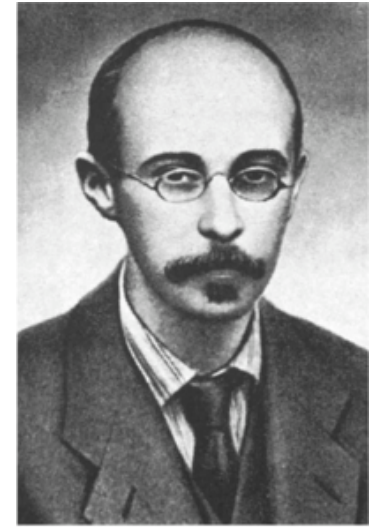
$$\Omega_M = \rho/\rho_c \quad , \quad \rho_c = \frac{3H_0}{8\pi G} \quad , \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_M + \Omega_\Lambda = 1 \quad (k = 0 \longrightarrow \text{flat})$$

Re-write Friedmann eq,

$$[H(t)]^2 = H_0^2 [\Omega_\Lambda(t) + \Omega_M(t) + \Omega_{\text{RAD}}(t)]$$

Today's  
Hubble  
parameter



*A. Friedmann*

k = curvature  
 $\rho$  = density  
 $\Lambda$  = cosmol. constant

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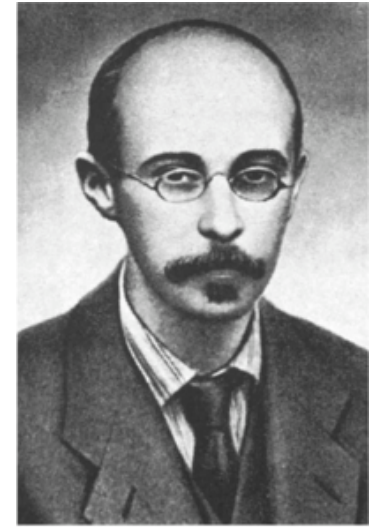
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Today's  
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parameter

$< 10^{-3}$  for  $z < 2$



*A. Friedmann*

k = curvature  
ρ = density  
Λ = cosmol. constant

matter :

$$\begin{aligned}\rho_M &\sim 1/a^3 \sim (1+z)^3 \\ \Omega_M(t) &= \Omega_M^0 (1+z)^3\end{aligned}$$

radiation :

$$\begin{aligned}\rho_{\text{RAD}} &\sim 1/a^4 \sim (1+z)^4 \\ \Omega_{\text{RAD}}(t) &= \Omega_{\text{RAD}}^0 (1+z)^4\end{aligned}$$

dark energy :  
( $w = p/\rho$ )

$$\begin{aligned}\rho_{\text{DE}} &\sim 1/a^{3(1+w)} \sim (1+z)^{3(1+w)} \\ \Omega_{\text{DE}}(t) &= \Omega_{\text{DE}}^0 (1+z)^{3(1+w)}\end{aligned}$$

For cosmological constant,  $w = -1$  and  
 $\Omega_\Lambda = \Omega_{\text{DE}}(t) = \text{constant}$

**HOT TOPIC:**

$w = -1$  (cosmological constant ?)

$w \neq -1$  (time varying DE ?)

$w = w(t)$  (time varying  $w$  ?)

# Expanding Universe Basics

$$H(z)^2 = H_0^2 \sum_i \Omega_i (1+z)^{3(1+w)}$$

Source of expansion	w	Evolution with z	$\Omega$ at z=0
Matter (dark, baryon, relic $\nu$ )	$v^2/c^2 \sim 0$	$\Omega_M(1+z)^3$	0.3
Radiation (CMB)	+1/3	$\Omega_\gamma(1+z)^4$	$\sim 10^{-5}$
Cosmological constant (?)	-1	$\Omega_\Lambda =$ constant	0.7
Curvature	-1/3	$\Omega_k(1+z)^2$	< few %

# Expanding Universe Basics

$$H(z)^2 = H_0^2 \sum_i \Omega_i (1+z)^{3(1+w)}$$

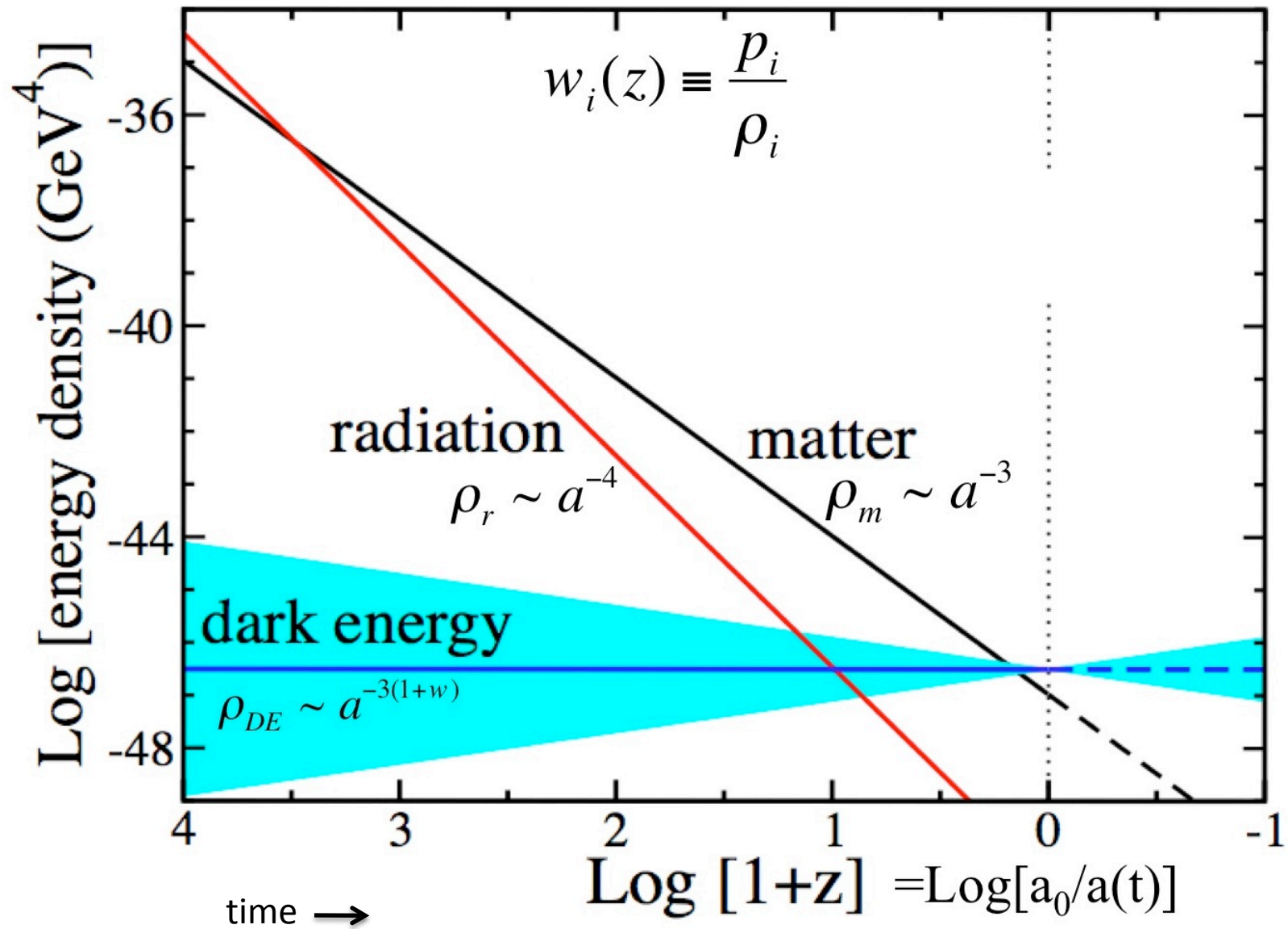
Note: 'w' refers to DE since other w are well known.

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$a = (1+z)^{-1}$  = cosmic scale factor

Equation of State parameter  $w$  determines Cosmic Evolution



**Exercise:**

Use Friedmann Eq. to solve for age of universe ( $t_0$ ) assuming  $\Omega_\Lambda$  is constant and  $\Omega_M + \Omega_\Lambda = 1$ ,

$$t_0 = \int_0^{t_0} dt = \frac{1}{H_0} \int da \text{ [bla bla]}$$

and show that

$$t_0 = \frac{2}{3H_0\sqrt{\Omega_\Lambda}} \ln \left[ \frac{1 + \sqrt{\Omega_\Lambda}}{\sqrt{1 - \Omega_\Lambda}} \right]$$

Plug in the numbers for  $\Omega_\Lambda = 0.0, 0.7, 1.0$

## Proper Motion Distance ( $D$ ).

How far does light travel from source to Earth in an expanding universe ? In terms of comoving coordinate  $r$ ,

$$a dr = c dt \quad (k = 0)$$

$$D = \int dr = c \int \frac{dt}{a} = \frac{c}{H_0} \int_{a_{\text{emit}}}^1 \left[ \frac{da'}{a'^2 \sqrt{\Omega_\Lambda + \Omega_M^0/a'^3}} \right]$$

Switch from  $a$  to observable  $1 + z = a^{-1}$ ,

$$D = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M^0(1 + z')^3}}$$

**Exercise:**



Use Taylor expansion to check nearby limit and show that

$$D(z \ll 1) \simeq \frac{v}{H_0} \left( 1 - \frac{v}{4c} \right)$$

where  $v = cz$ . At  $z = 0.1$  what is the correction to the linear Hubble law ?

## Key concept: Luminosity Distance “ $D_L$ ”

Absolute  
luminosity  
of source

$D_L$  is defined so that flux at Earth is

$$F_{\text{Earth}} = \mathcal{L} \cdot \frac{A_{\text{Earth}}}{4\pi D_L^2}$$

Proper  
motion  
distance

$D_L \neq D$  because

1. Flux = energy/second and  $\Delta T_{\text{Earth}} = (1 + z)\Delta T_{\text{source}}$
2.  $\lambda_{\text{Earth}} = (1 + z)\lambda_{\text{source}}$   
→ photons are less energetic at Earth.

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$$D_L^2 = D^2 \underbrace{(1 + z)}_{\text{time}} \underbrace{(1 + z)}_{\text{energy}}$$

$$\begin{aligned} D_L^{\text{energy}} &= D(1 + z) && \text{conventional definition} \\ D_L^{\text{count}} &= D\sqrt{1 + z} && \text{count – flux definition} \end{aligned}$$

## Definition of distance modulus “ $\mu$ ”

$$\mu \equiv 5 \log_{10} \left( \frac{D_L^{\text{energy}}}{10 \text{ pc}} \right)$$

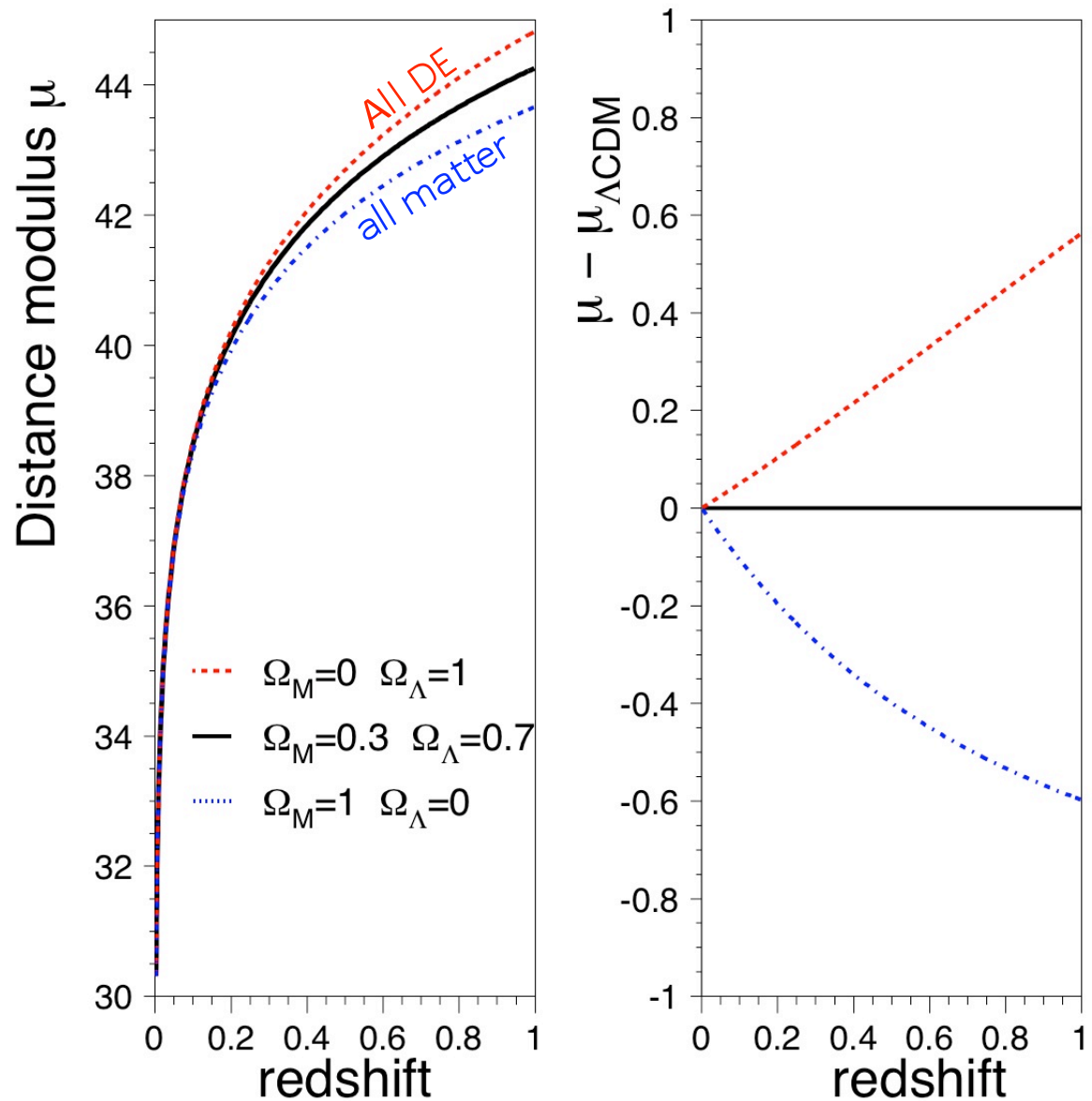
Examples:

$$D_L = 100 \text{ Mpc} \longrightarrow \mu = 35$$

$$D_L = 1 \text{ Gpc} \longrightarrow \mu = 40$$

Hubble diagram:  $\mu$  vs. redshift

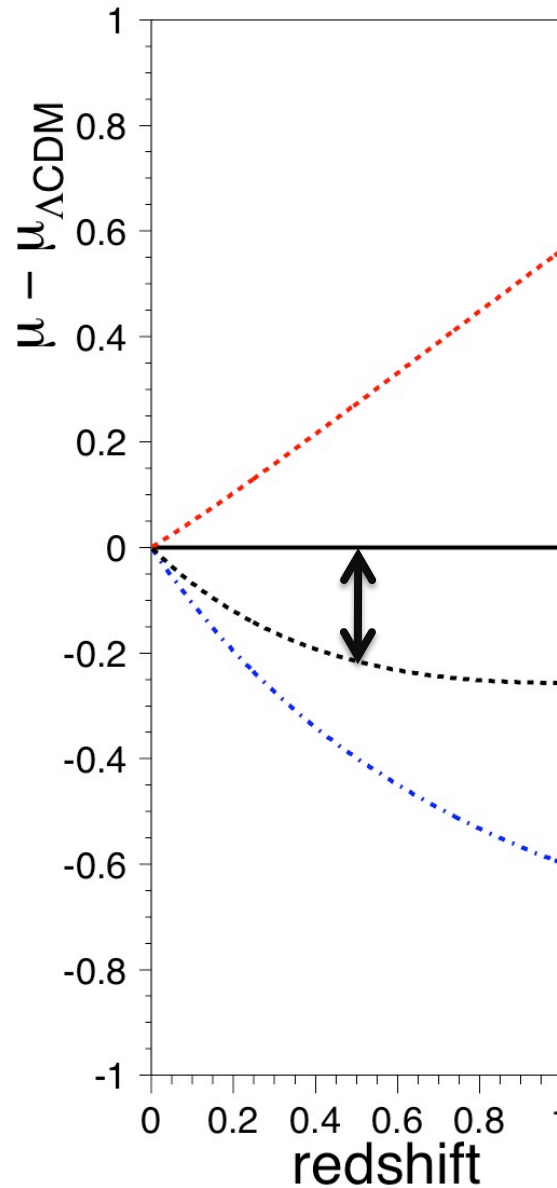
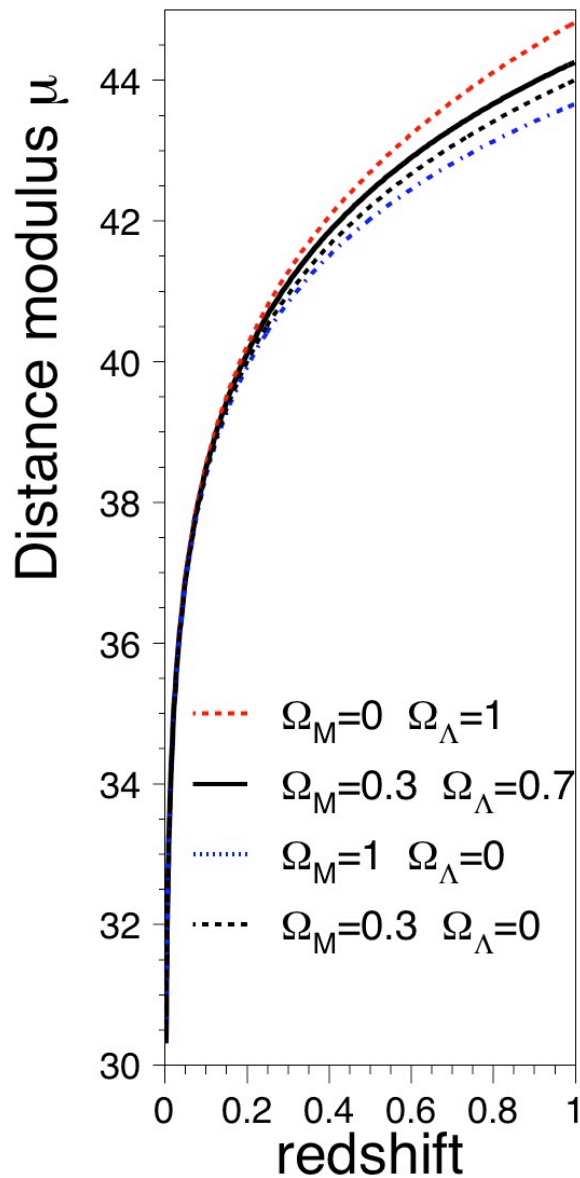
# Hubble Diagram Basics



Definition:

Distance modulus versus redshift

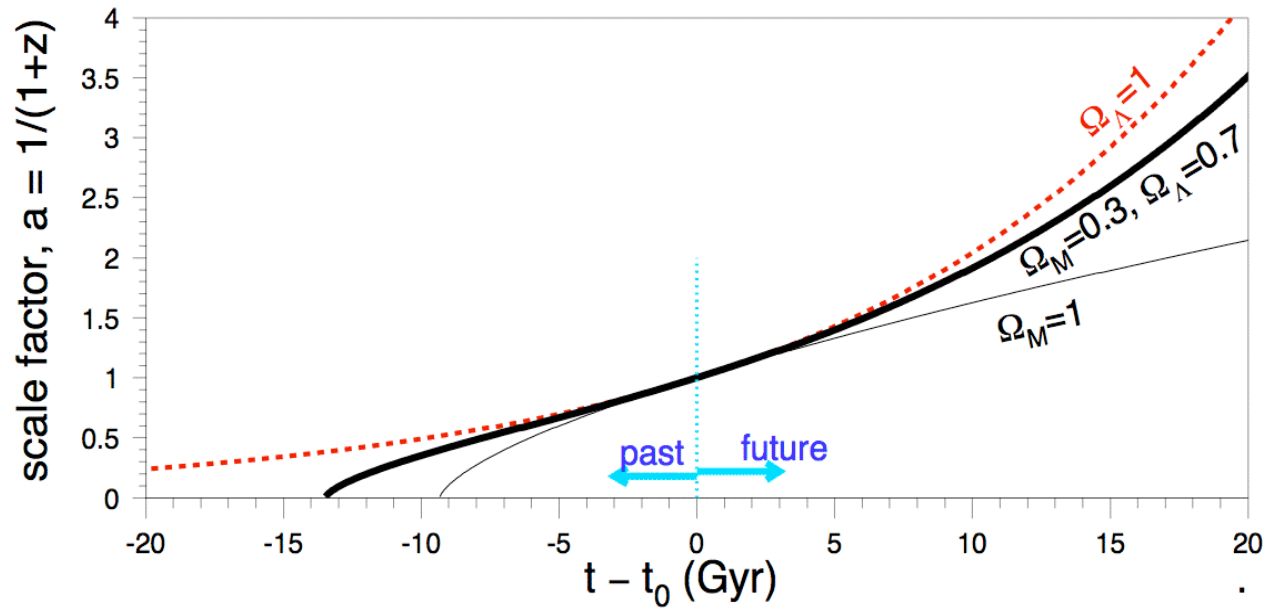
# Hubble Diagram Basics



Discovery of dark energy:  
distant SNe  
are 0.2 mag  
dimmer than  
expected  
( $\Omega_M=0.3 \quad \Omega_\Lambda=0$ )

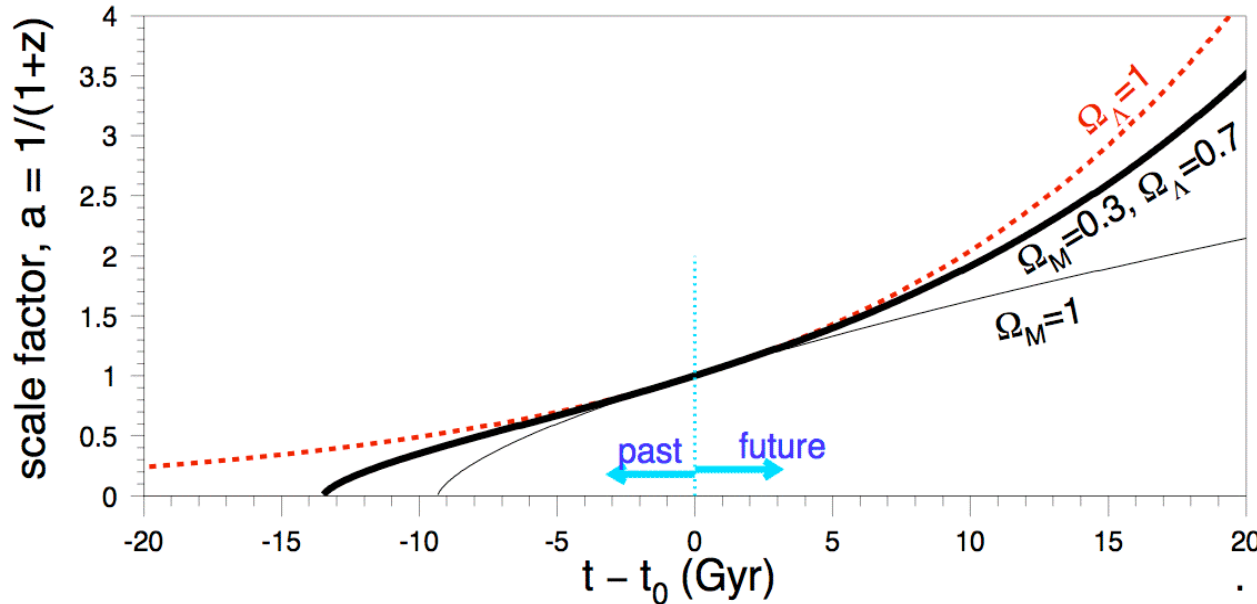


# Hubble Diagram Basics

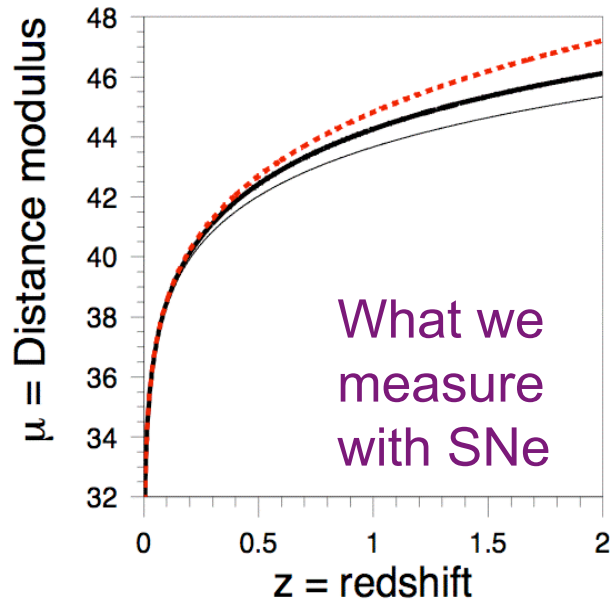


Expansion history depends on  $w$ ,  $\Omega_\Lambda$  and  $\Omega_M$

# Hubble Diagram Basics



Expansion history depends on  $w$ ,  $\Omega_\Lambda$  and  $\Omega_M$

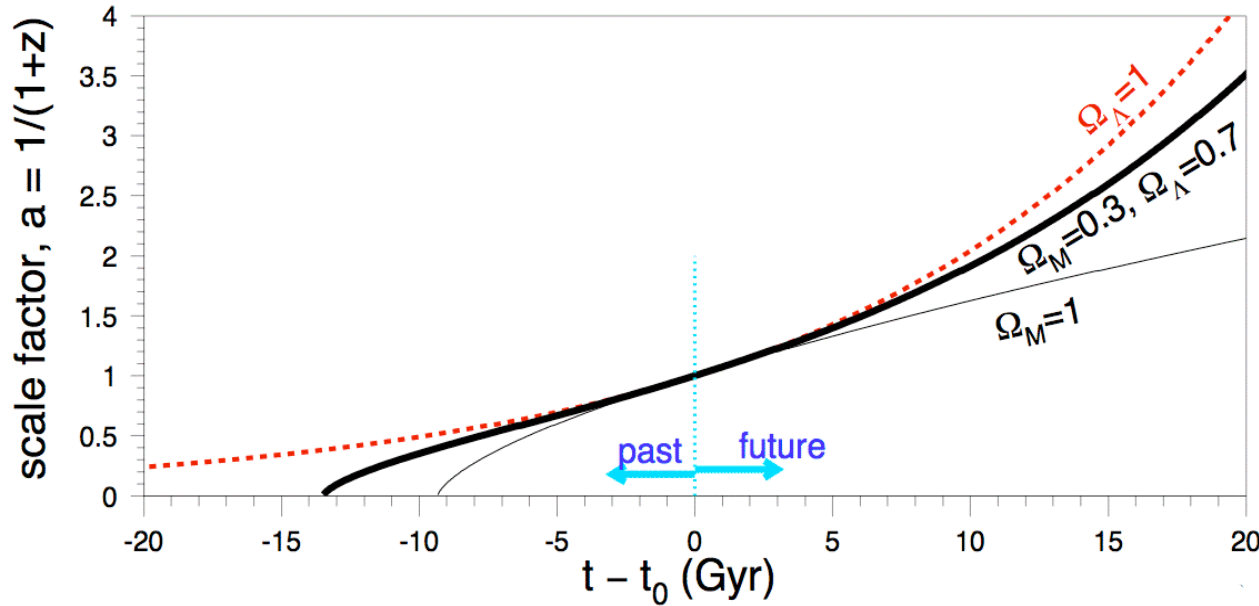


Flux  $\propto \mathcal{L}/4\pi D_L^2$ .

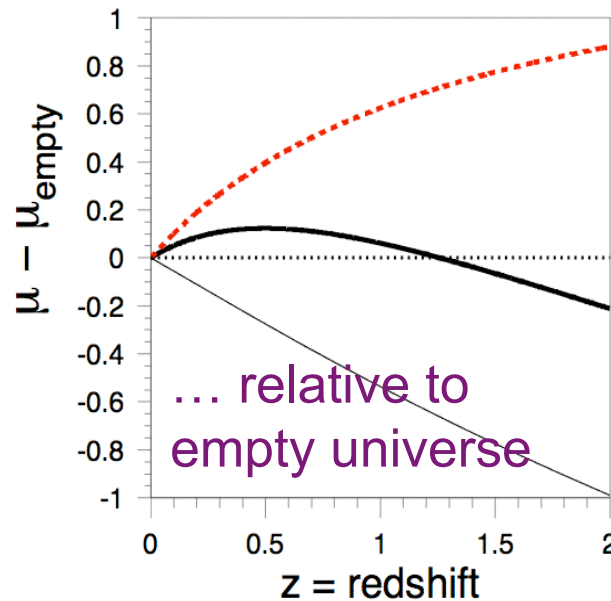
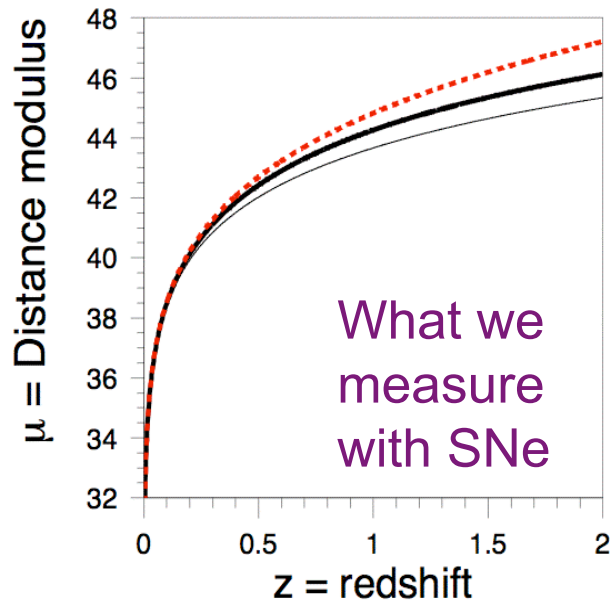
$D_L = (1+z) \int dz/H(z, \Omega_M, \Omega_\Lambda, w)$   
for flat universe.

Distance modulus:  $\mu = 5 \log(D_L/10\text{pc})$

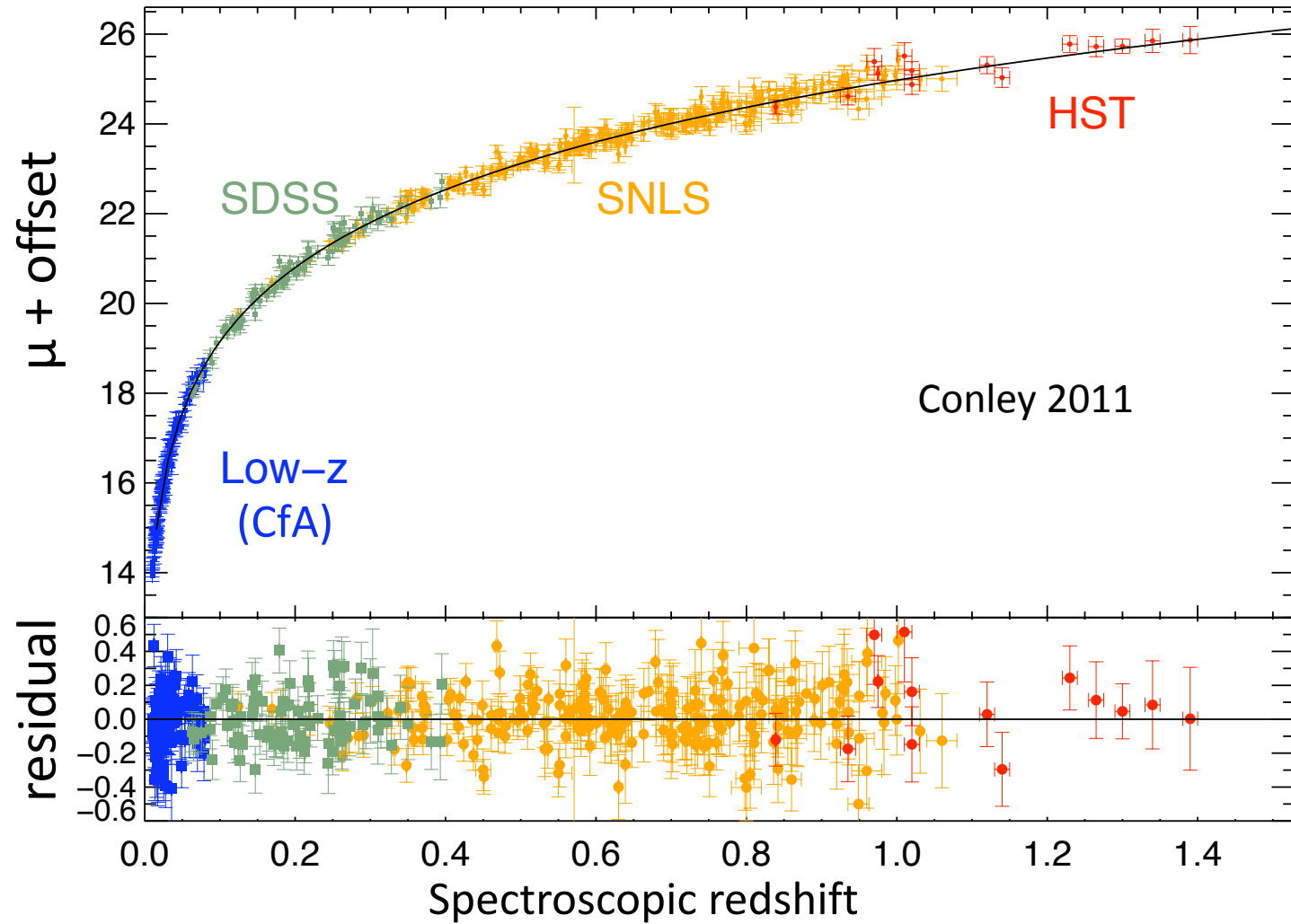
# Hubble Diagram Basics



Expansion history depends on  $w$ ,  $\Omega_\Lambda$  and  $\Omega_M$



# SN Ia Hubble Diagram



# Fun Facts About Dark Energy

- $\rho_{\Lambda} = 10^{-29} \text{ g/cm}^3$  everywhere.
- Earth volume contains 0.01g of dark energy.
- Assuming constant orbital velocity,  
dark energy decreases distance from sun by  
0.14 Å for Earth and 11 - 90 μm for Pluto
- Gravity and dark energy roughly cancel for Milky-Way and  
Andromeda galaxies (but galaxy-cluster gravity wins)
- $\Omega_{\Lambda} = 0.7$  today
- $\Omega_{\Lambda}/\Omega_M \sim 2.3$  today (compare  $\Omega_{\gamma}/\Omega_M < 10^{-4}$ ).
- $\Omega_{\Lambda} = \Omega_M$  at  $z=0.3$  (3-4 billion years ago, assumes  $w=-1$ ).
- Undetectable in terrestrial experiments (so far).
- Nobody knows what dark energy (or dark matter) is.

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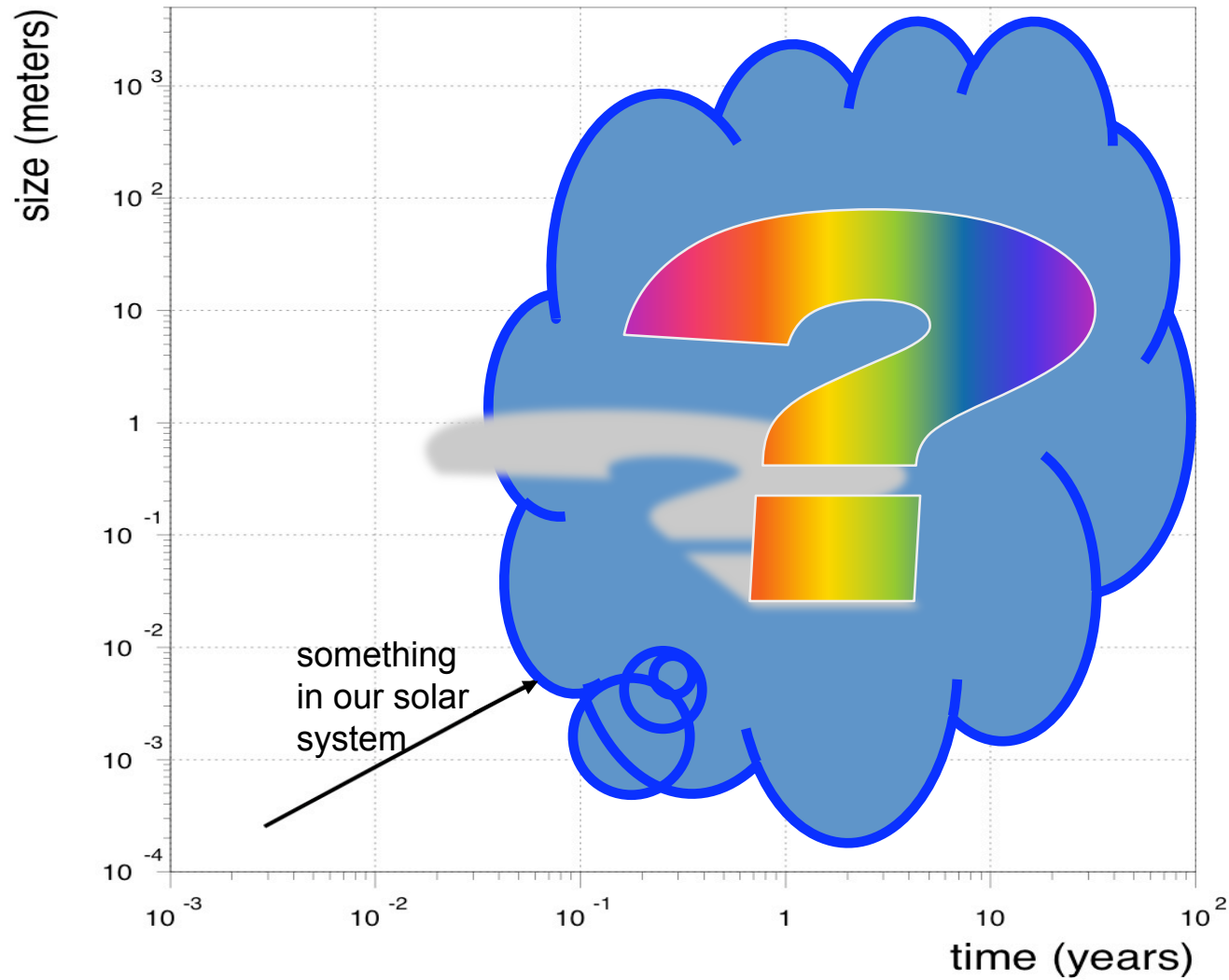
**Exercise:  
verify the  
numbers**

**Exercise clue:**  $\rho_{\text{eff}} = -\Lambda/(4\pi G)$

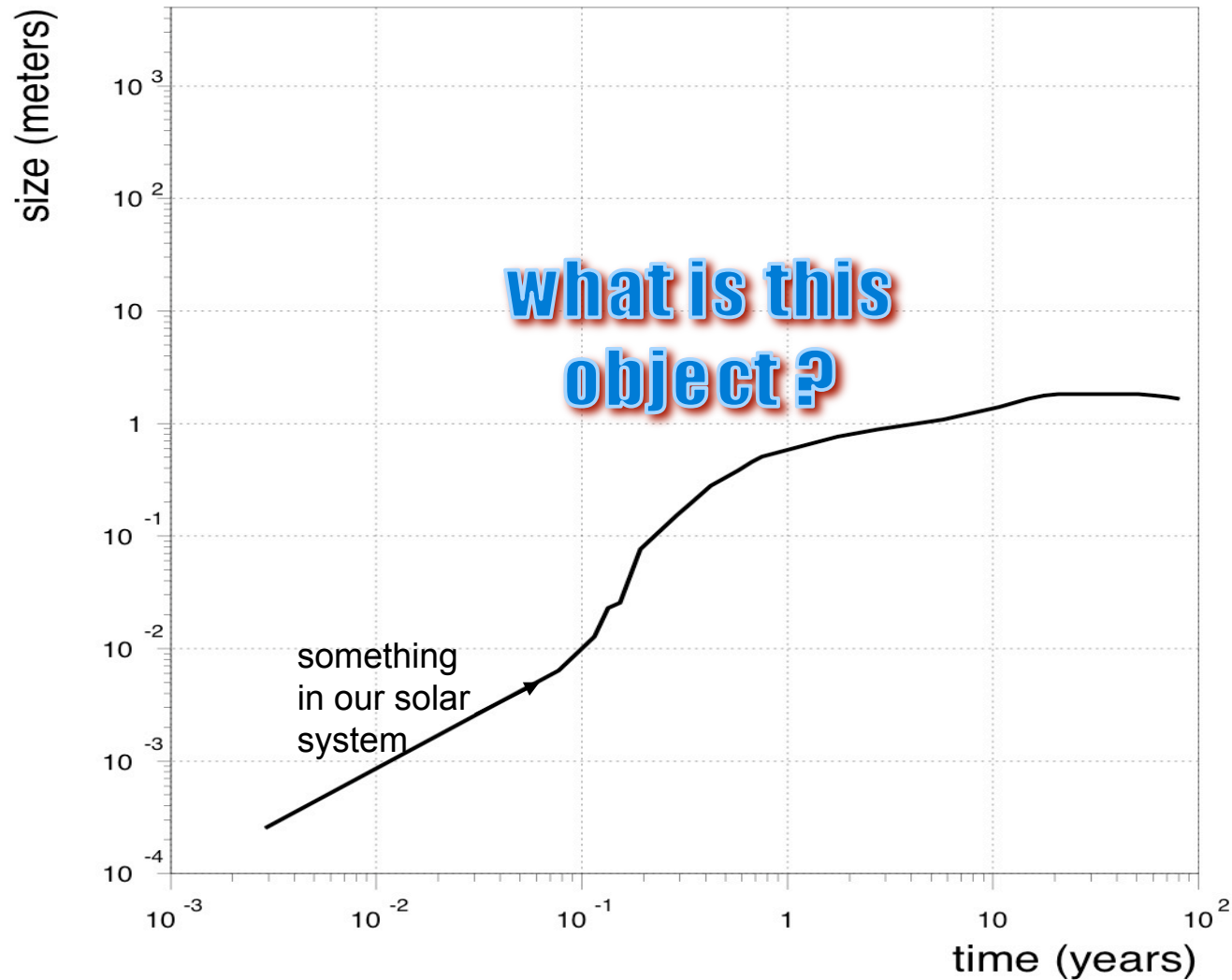
If we change the constant velocity assumption to constant orbital period, show that the change in radius is ×3 smaller.



# Understanding Expansion History is Tricky

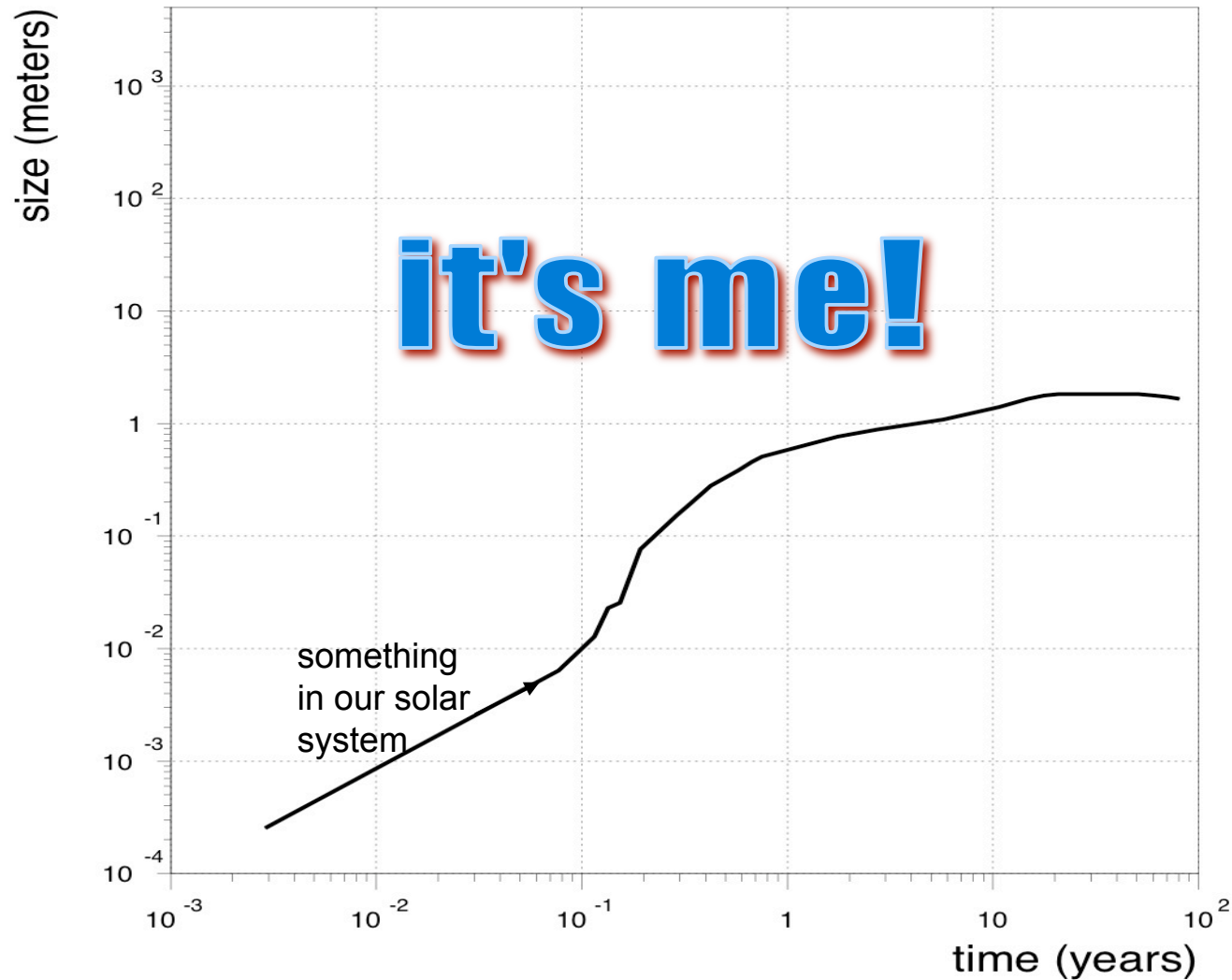


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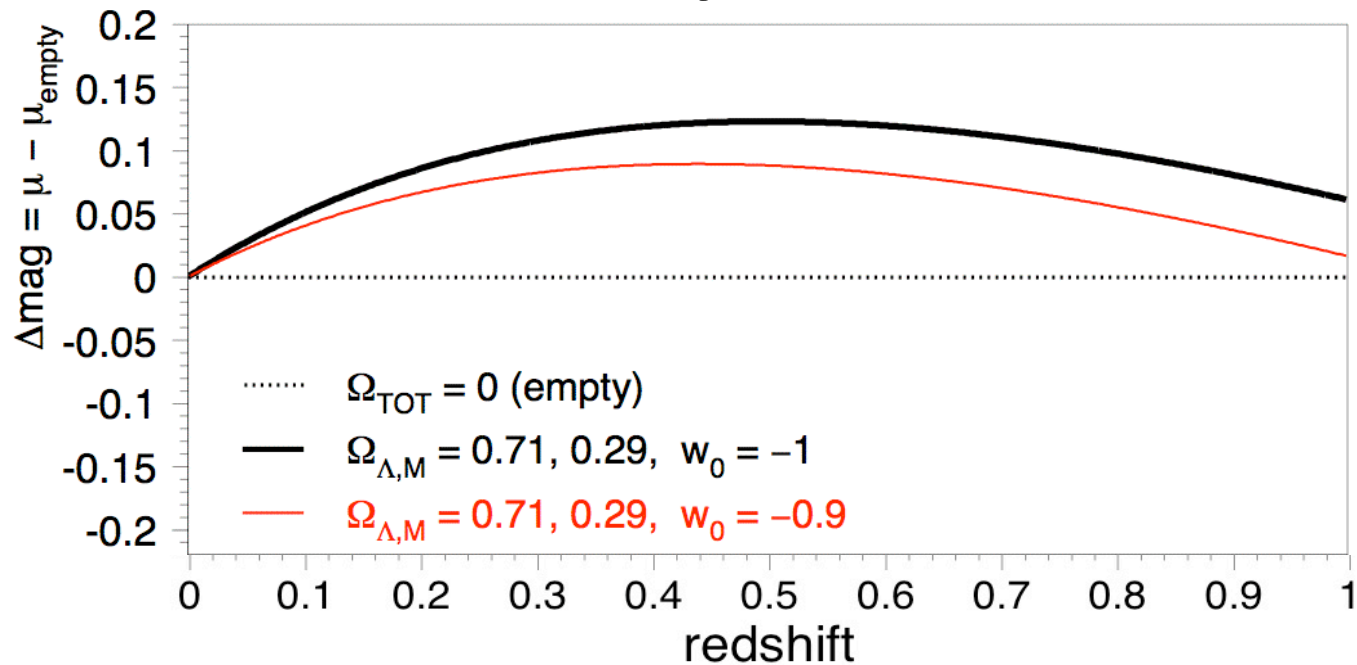




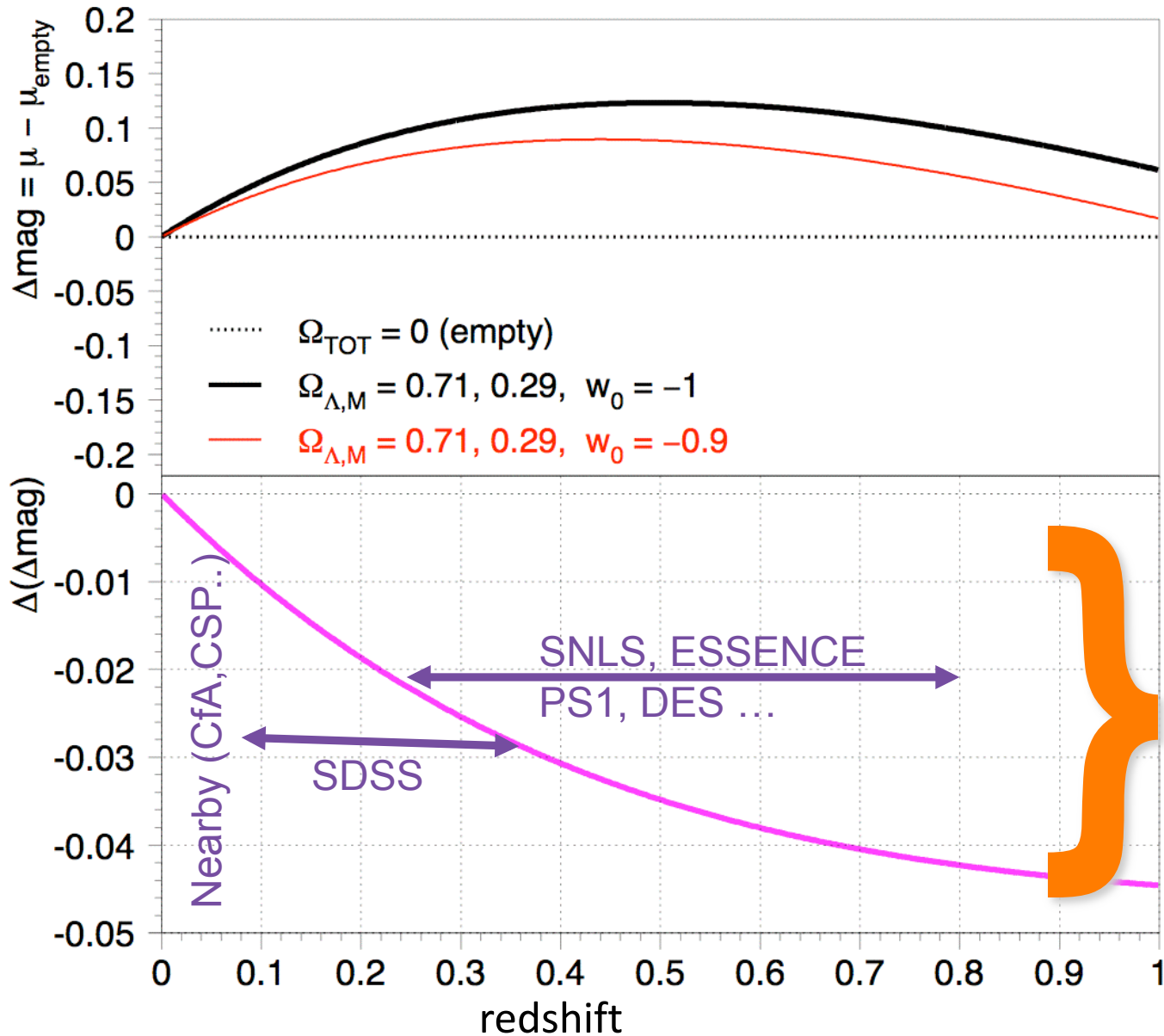
# Understanding Expansion History is Tricky



# w-sensitivity with SNIa distances



# w-sensitivity with SNIa distances



w = -0.9 gives  
4% variation  
from w = -1