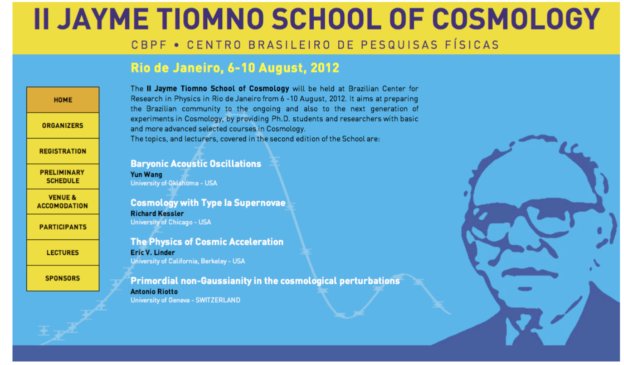


Lectures on Cosmology with Type Ia Supernovae: **Formalism**

R.Kessler (U.Chicago)

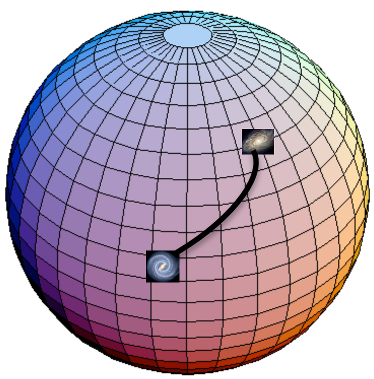
II Jayme Tiomno School of Cosmology

Rio de Janeiro, Brazil

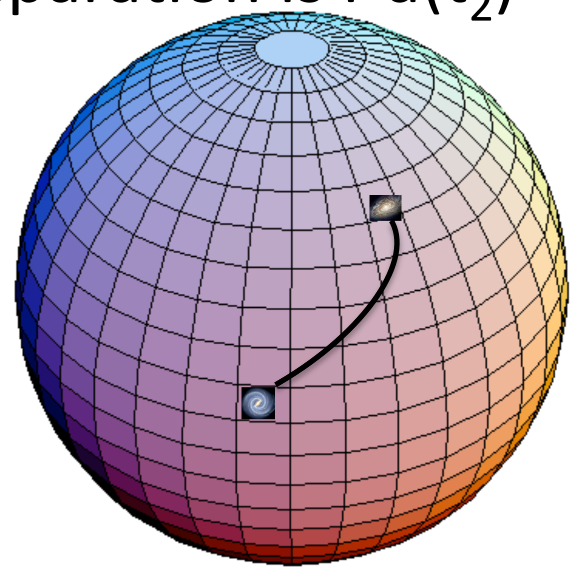
Aug 6-10, 2012

Expanding Universe

at time t_1 galaxy separation is r $a(t_1)$



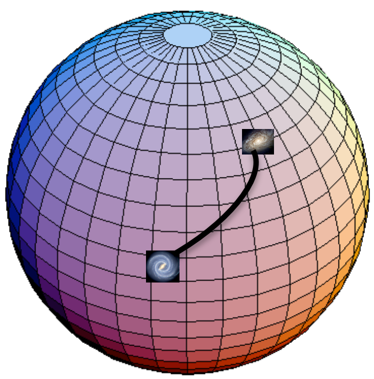
at time t_2 galaxy separation is r $a(t_2)$



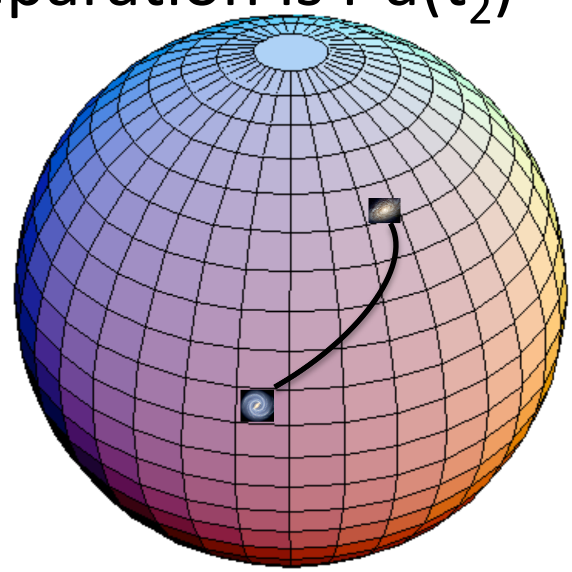
- r = comoving separation
- $a(t)$ = scale factor
- Galaxy size determined by gravity (not affected by expansion)
- $a(t_2)/a(t_1) = \lambda_2/\lambda_1 = 1+z$, where z = redshift

Expanding Universe

at time t_1 galaxy separation is r $a(t_1)$



at time t_2 galaxy separation is r $a(t_2)$



Recesion velocity between galaxies,

$$v = \frac{d(t_2) - d(t_1)}{t_2 - t_1} = r\frac{a(t_2) - a(t_1)}{t_2 - t_1} = \frac{d}{a} \cdot \frac{a(t_2) - a(t_1)}{t_2 - t_1}$$

$$= (d/a)\dot{a} \equiv dH(t)$$

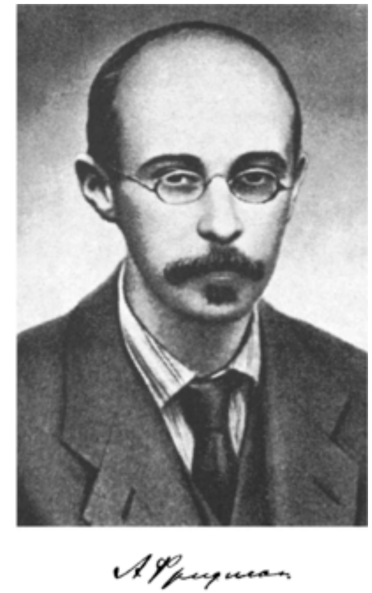
$$H(t) \equiv \dot{a}/a$$

Today $H(t_0) \approx 70$ km/s/Mpc

Begin with Friedmann equation to describe evolution of cosmic scale factor $a(t) = (1+z)^{-1}$ where

$$\begin{aligned} a(t = t_0; \text{today}) &= 1 \\ a(t < t_0) &< 1 \end{aligned}$$

$$[H(t)]^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$



In terms of critical densities,

$$\Omega_{\rm M} = \rho/\rho_c \quad , \quad \rho_c = \frac{3H_0}{8\pi G} \quad , \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_{\rm M} + \Omega_\Lambda = 1 \quad (k = 0 \longrightarrow \text{flat})$$

k = curvature
ρ = density
Λ = cosmol. constant

Re-write Friedmann eq,

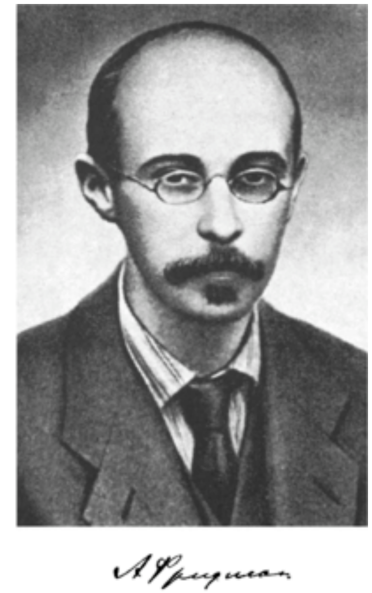
$$[H(t)]^2 = H_0^2 \left[\Omega_\Lambda(t) + \Omega_{\rm M}(t) + \Omega_{\rm RAD}(t)\right]$$

↑ Today's Hubble parameter

Begin with Friedmann equation to describe evolution of cosmic scale factor $a(t) = (1+z)^{-1}$ where

$$a(t = t_0; \text{today}) = 1$$
$$a(t < t_0) < 1$$

$$[H(t)]^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$



k = curvature

ρ = density

Λ = cosmol. constant

In terms of critical densities,

$$\Omega_{\mathrm{M}} = \rho/\rho_c \quad , \quad \rho_c = \frac{3H_0}{8\pi G} \quad , \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_{\mathrm{M}} + \Omega_\Lambda = 1 \quad (k = 0 \longrightarrow \text{flat})$$

Re-write Friedmann eq,

$$[H(t)]^2 = H_0^2 \left[\Omega_\Lambda(t) + \Omega_{\mathrm{M}}(t) + \Omega_{\mathrm{RAD}}(t)\right]$$

Today's Hubble parameter

$< 10^{-3}$ for $z < 2$

matter :

$$\rho_M \sim 1/a^3 \sim (1+z)^3$$
$$\Omega_{\mathrm{M}}(t) = \Omega^0_{\mathrm{M}}(1+z)^3$$

radiation :

$$\rho_{\mathrm{RAD}} \sim 1/a^4 \sim (1+z)^4$$
$$\Omega_{\mathrm{RAD}}(t) = \Omega^0_{\mathrm{RAD}}(1+z)^4$$

dark energy :
$(w = p/\rho)$

$$\rho_{\mathrm{DE}} \sim 1/a^{3(1+w)} \sim (1+z)^{3(1+w)}$$
$$\Omega_{\mathrm{DE}}(t) = \Omega^0_{\mathrm{DE}}(1+z)^{3(1+w)}$$

For cosmological constant, $w = -1$ and
$\Omega_\Lambda = \Omega_{\mathrm{DE}}(t) = \text{constant}$

HOT TOPIC:

$$w = -1 \quad \text{(cosmological constant ?)}$$
$$w \neq -1 \quad \text{(time varying DE ?)}$$
$$w = w(t) \quad \text{(time varying } w \text{ ?)}$$

Expanding Universe Basics

$$H(z)^2 = H_0^2 \Sigma_i \Omega_i (1+z)^{3(1+w)}$$

Source of expansion	**w**	**Evolution with z**	**Ω at z=0**
Matter (dark, baryon, relic ν)	$v^2/c^2 \sim 0$	$\Omega_M(1+z)^3$	0.3
Radiation (CMB)	+1/3	$\Omega_\gamma(1+z)^4$	$\sim 10^{-5}$
Cosmological constant (?)	−1	Ω_Λ = constant	0.7
Curvature	−1/3	$\Omega_k(1+z)^2$	< few %

Expanding Universe Basics

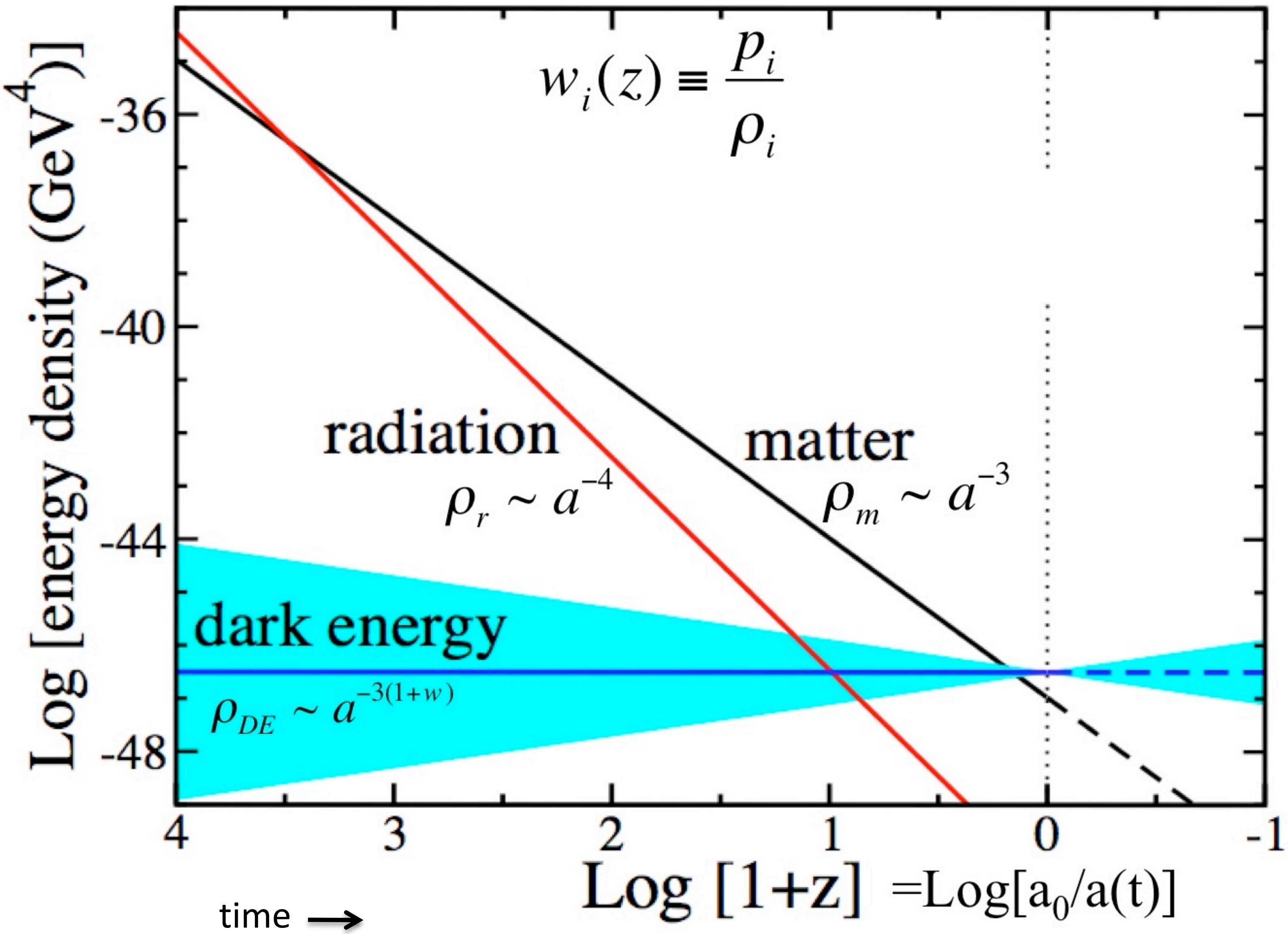
$H(z)^2 = H_0^2 \, \Sigma_i \, \Omega_i \, (1+z)^{3(1+w)}$

Note: 'w' refers to DE since other w are well known.

Source of expansion	w	Evolution with z	Ω at z=0
Matter (dark, baryon, relic ν)	$v^2/c^2 \sim 0$	$\Omega_M(1+z)^3$	0.3
Radiation (CMB)	+1/3	$\Omega_\gamma(1+z)^4$	$\sim 10^{-5}$
Cosmological constant (?)	−1	Ω_Λ = constant	0.7
Curvature	−1/3	$\Omega_k(1+z)^2$	< few %

$a = (1+z)^{-1}$ = cosmic scale factor

Equation of State parameter *w* determines Cosmic Evolution



Exercise: 

Use Friedmann Eq. to solve for age of universe (t_0) assuming Ω_Λ is constant and $\Omega_M + \Omega_\Lambda = 1$,

$$t_0 = \int_0^{t_0} dt = \frac{1}{H_0} \int da \, [\text{bla bla}]$$

and show that

$$t_0 = \frac{2}{3H_0\sqrt{\Omega_\Lambda}} \ln\left[\frac{1+\sqrt{\Omega_\Lambda}}{\sqrt{1-\Omega_\Lambda}}\right]$$

Plug in the numbers for $\Omega_\Lambda = 0.0, 0.7, 1.0$

Proper Motion Distance (D).

How far does light travel from source to Earth in an expanding universe ? In terms of comoving coordinate r,

$$adr = cdt \quad (k = 0)$$

$$D = \int dr = c \int \frac{dt}{a} = \frac{c}{H_0} \int_{a_{\rm emit}}^{1} \left[\frac{da'}{a'^2 \sqrt{\Omega_\Lambda + \Omega_{\rm M}^0 / a'^3}} \right]$$

Switch from a to observable $1 + z = a^{-1}$,

$$D = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_{\rm M}^0 (1 + z')^3}}$$

Exercise:

Use Taylor expansion to check nearby limit and show that

$$D(z << 1) \simeq \frac{v}{H_0} \left(1 - \frac{v}{4c} \right)$$

where $v = cz$. At $z = 0.1$ what is the correction to the linear Hubble law ?

Key concept: Luminosity Distance "D_L"

Absolute luminosity of source

D_L is defined so that flux at Earth is

$$F_{\text{Earth}} = \mathcal{L} \cdot \frac{A_{\text{Earth}}}{4\pi D_L^2}$$

Proper motion distance

$D_L \neq D$ because

1. Flux = energy/second and $\Delta T_{\text{Earth}} = (1+z)\Delta T_{\text{source}}$
2. $\lambda_{\text{Earth}} = (1+z)\lambda_{\text{source}}$
   $\longrightarrow$ photons are less energetic at Earth.

Absolute luminosity of source

Proper motion distance

Key concept: Luminosity Distance "D_L"

D_L is defined so that flux at Earth is

$$F_{\text{Earth}} = \mathcal{L} \cdot \frac{A_{\text{Earth}}}{4\pi D_L^2}$$

$D_L \neq D$ because

1. Flux = energy/second and $\Delta T_{\text{Earth}} = (1+z)\Delta T_{\text{source}}$
2. $\lambda_{\text{Earth}} = (1+z)\lambda_{\text{source}}$
   $\longrightarrow$ photons are less energetic at Earth.

$$D_L^2 = D^2 \underbrace{(1+z)}_{\text{time}} \underbrace{(1+z)}_{\text{energy}}$$

$$D_L^{\text{energy}} = D(1+z) \quad \text{conventional definition}$$

$$D_L^{\text{count}} = D\sqrt{1+z} \quad \text{count} - \text{flux definition}$$

Definition of distance modulus "μ"

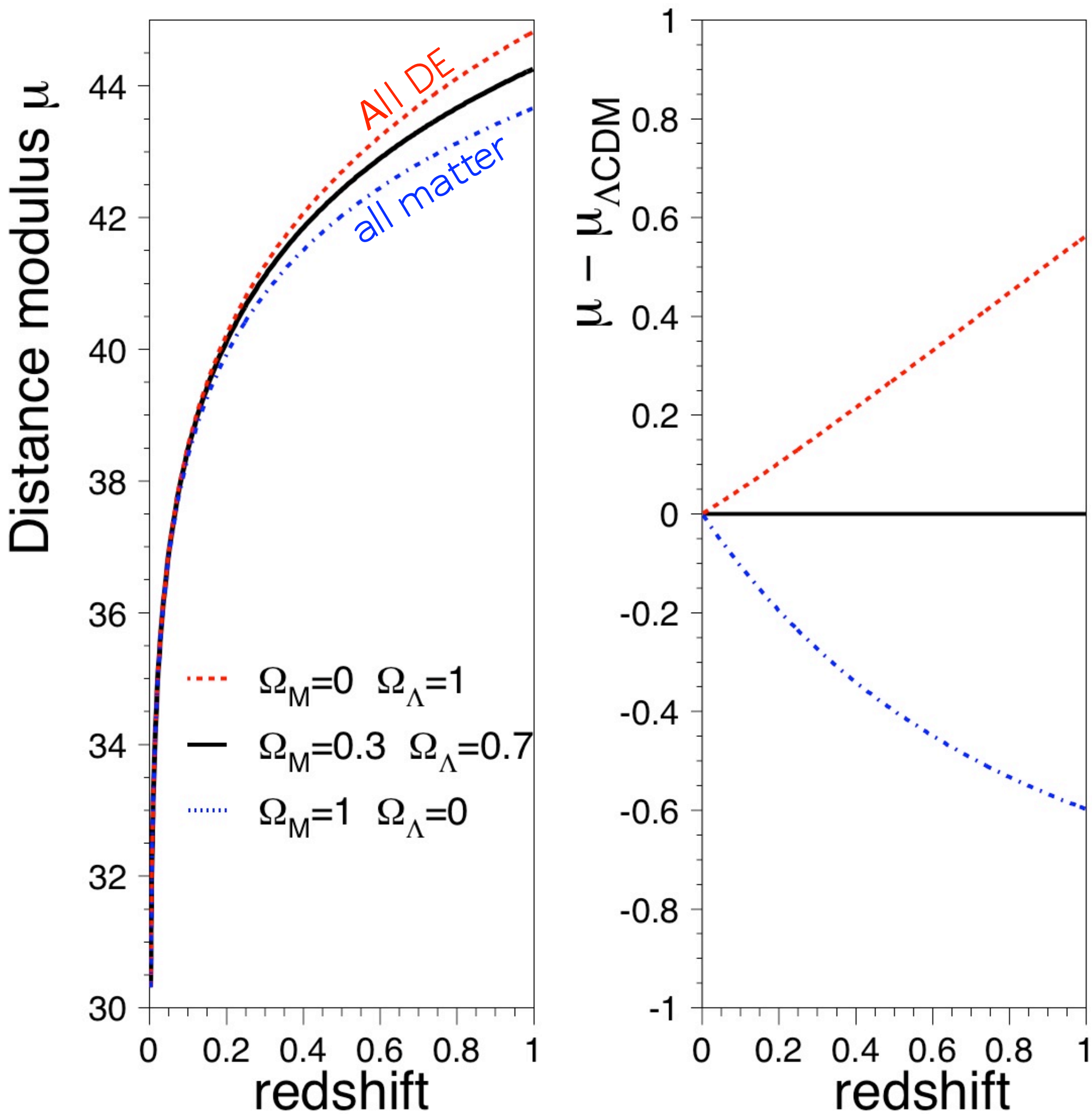
$$\mu \equiv 5\log_{10}\left(\frac{D_L^{\text{energy}}}{10\ \text{pc}}\right)$$

Examples:

$$D_L = 100\ \text{Mpc} \longrightarrow \mu = 35$$
$$D_L = 1\ \text{Gpc} \longrightarrow \mu = 40$$

Hubble diagram: μ vs. redshift

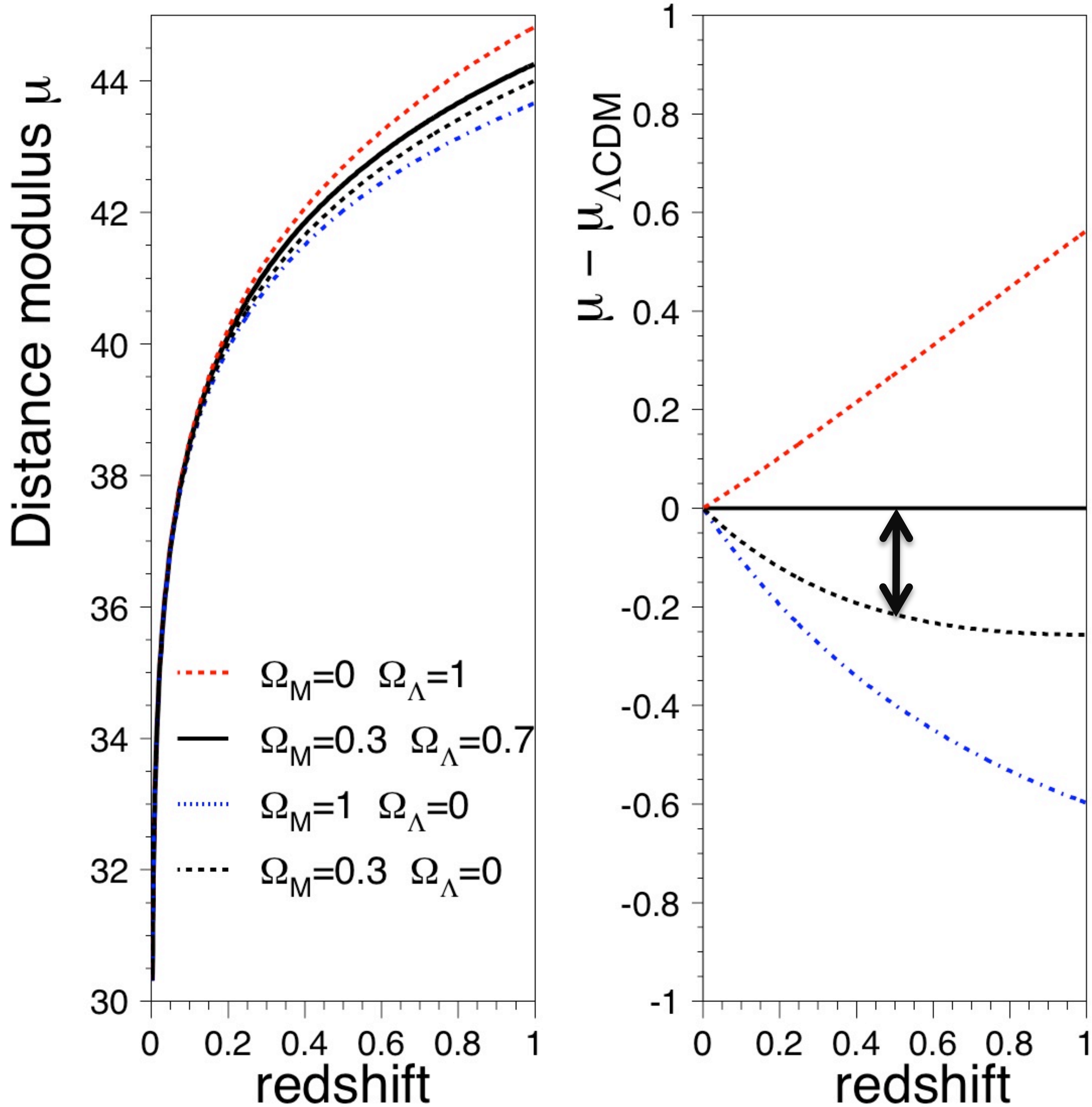
Hubble Diagram Basics



Definition:

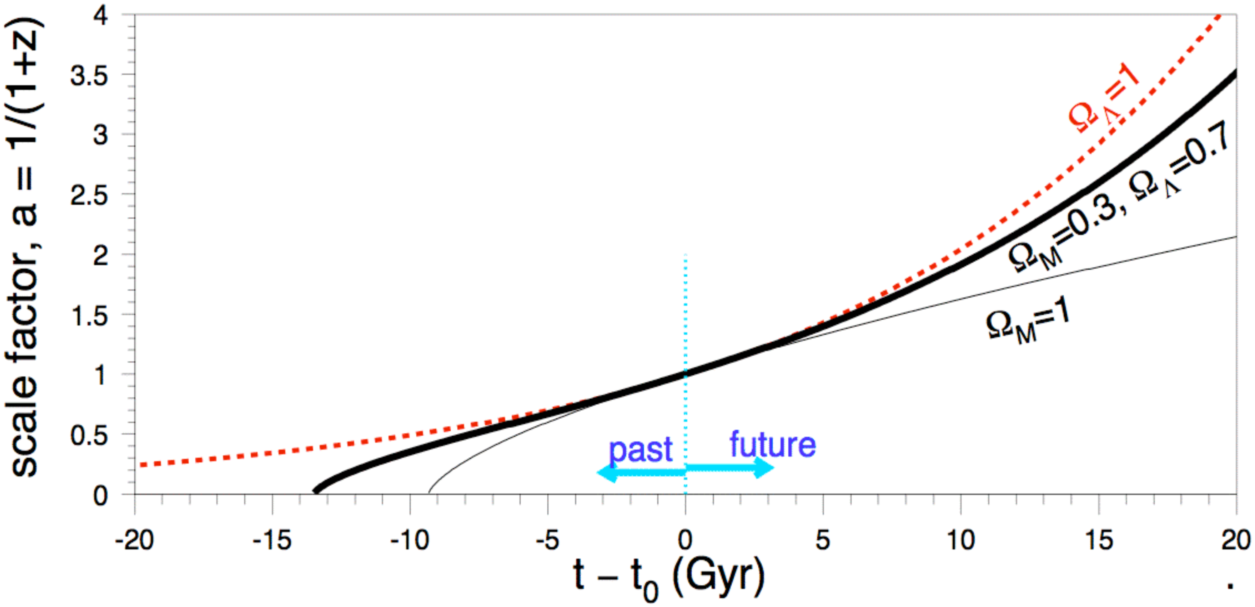
Distance modulus versus redshift

Hubble Diagram Basics



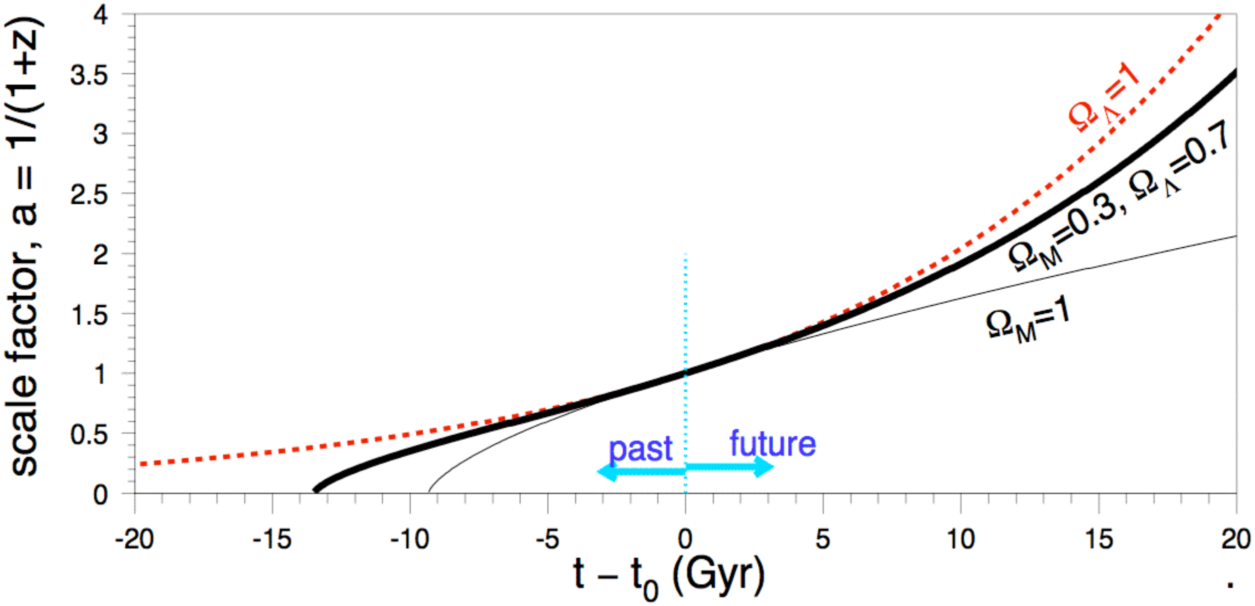
Discovery of dark energy: distant SNe are 0.2 mag dimmer than expected (Ω_M=0.3 Ω_Λ=0)

Hubble Diagram Basics

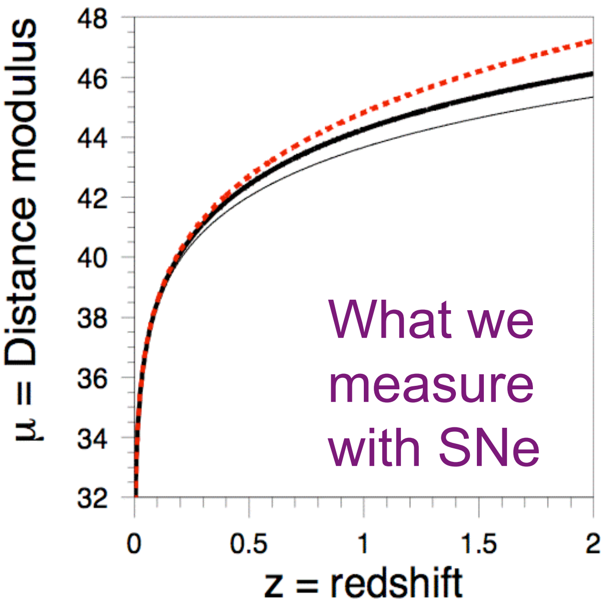


Expansion history depends on w, Ω_Λ and Ω_M

Hubble Diagram Basics



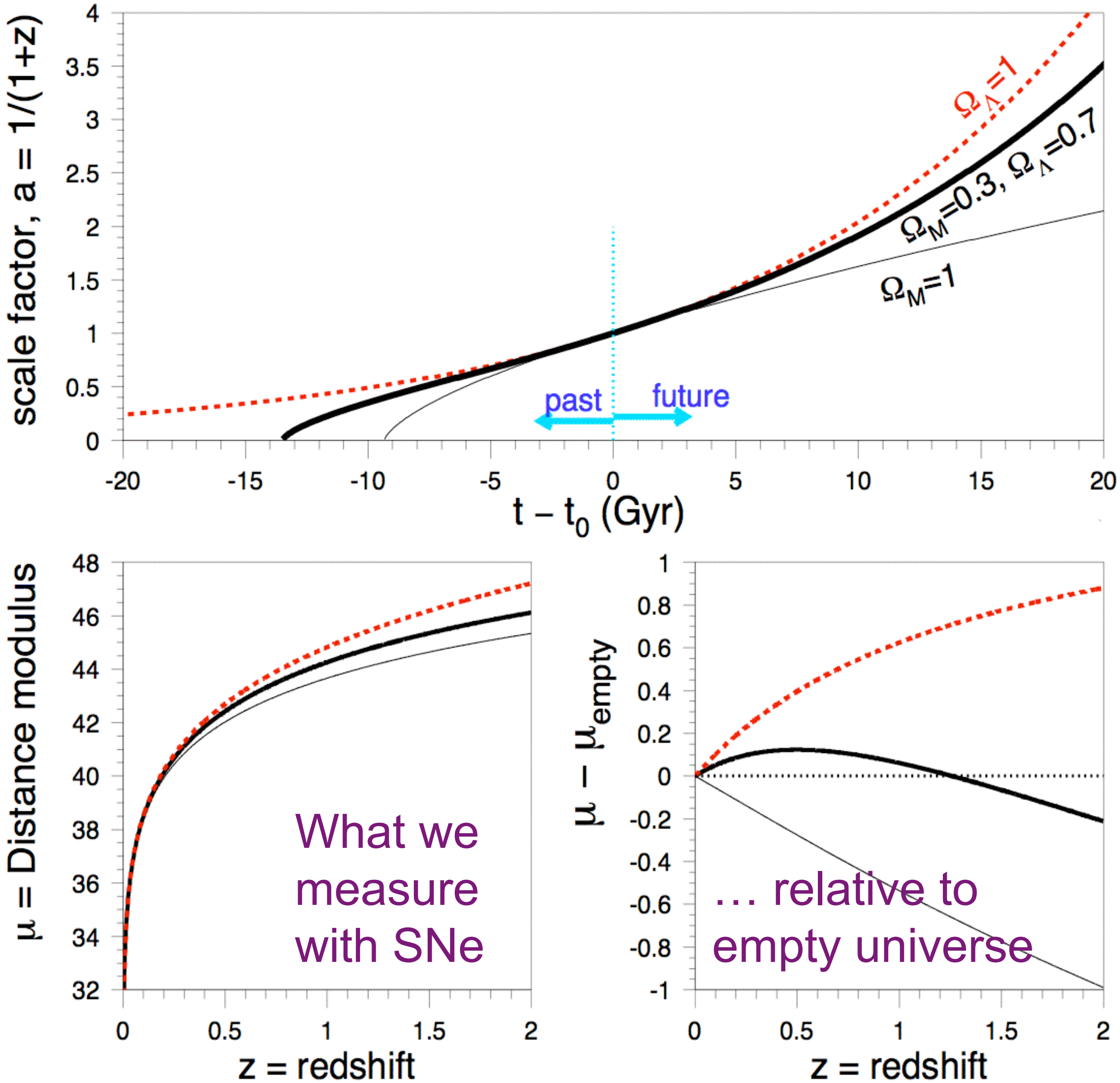
Expansion history depends on w, Ω_Λ and Ω_M



Flux $\propto \mathscr{L}/4\pi D_L^2$.

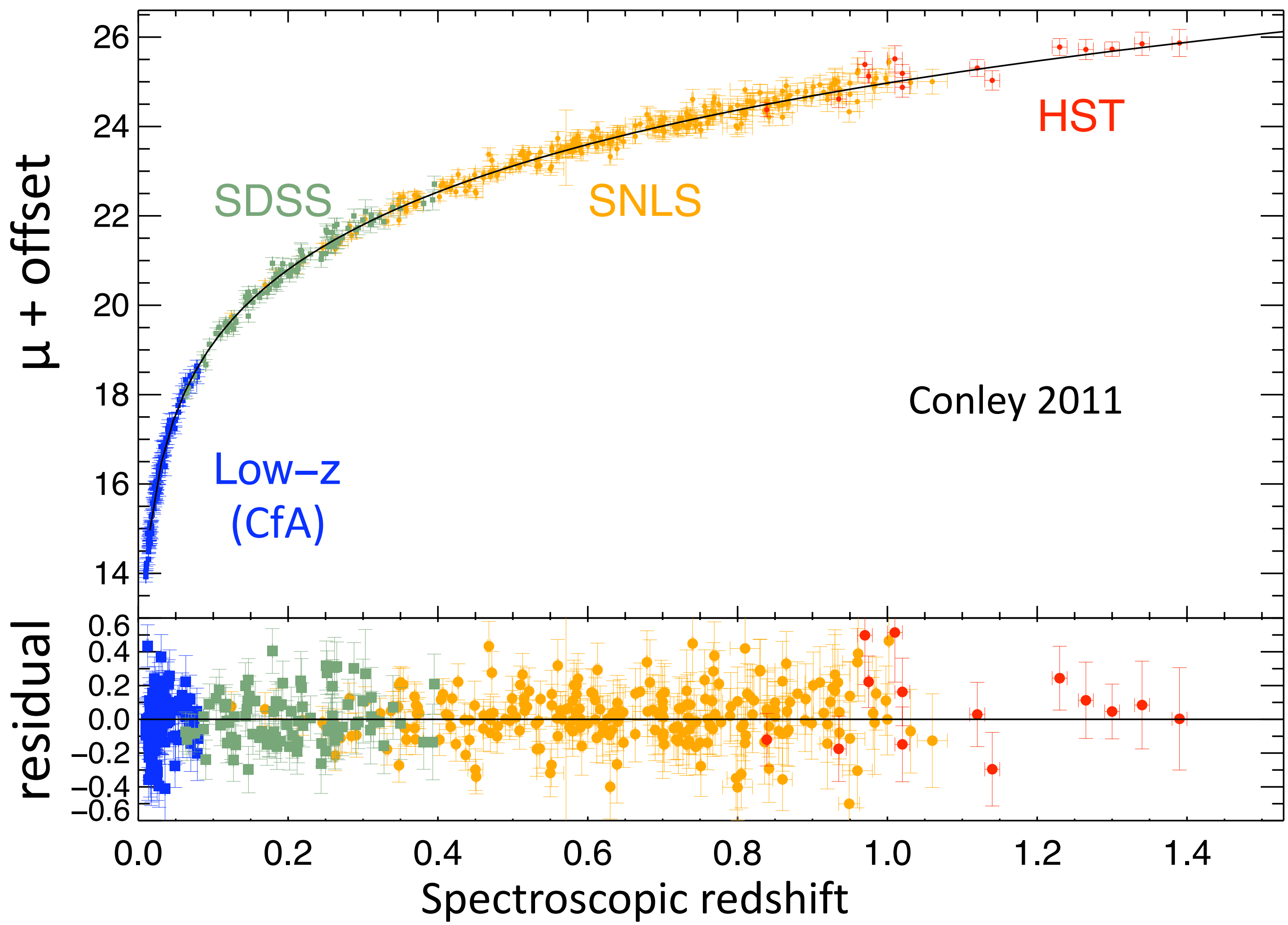
$D_L = (1+z)\int dz/H(z,\Omega_M,\Omega_\Lambda,w)$
for flat universe.
Distance modulus: $\mu=5\log(D_L/10pc)$

Hubble Diagram Basics



Expansion history depends on w, Ω_Λ and Ω_M

SN Ia Hubble Diagram



Fun Facts About Dark Energy

- $\rho_\Lambda = 10^{-29}$ g/cm^3 everywhere.
- Earth volume contains 0.01g of dark energy.
- Assuming constant orbital velocity,

  dark energy decreases distance from sun by

  0.14 Å for Earth and 11 - 90 μm for Pluto
- Gravity and dark energy roughly cancel for Milky-Way and Andromeda galaxies (but galaxy-cluster gravity wins)
- $\Omega_\Lambda = 0.7$ today
- $\Omega_\Lambda/\Omega_M \sim 2.3$ today (compare $\Omega_\gamma/\Omega_M < 10^{-4}$).
- $\Omega_\Lambda = \Omega_M$ at z=0.3 (3-4 billion years ago, assumes w=-1).
- Undetectable in terrestrial experiments (so far).
- Nobody knows what dark energy (or dark matter) is.

Fun Facts About Dark Energy

- $\rho_\Lambda = 10^{-29}$ g/cm^3 everywhere.
- Earth volume contains 0.01g of dark energy.
- Assuming constant orbital velocity,

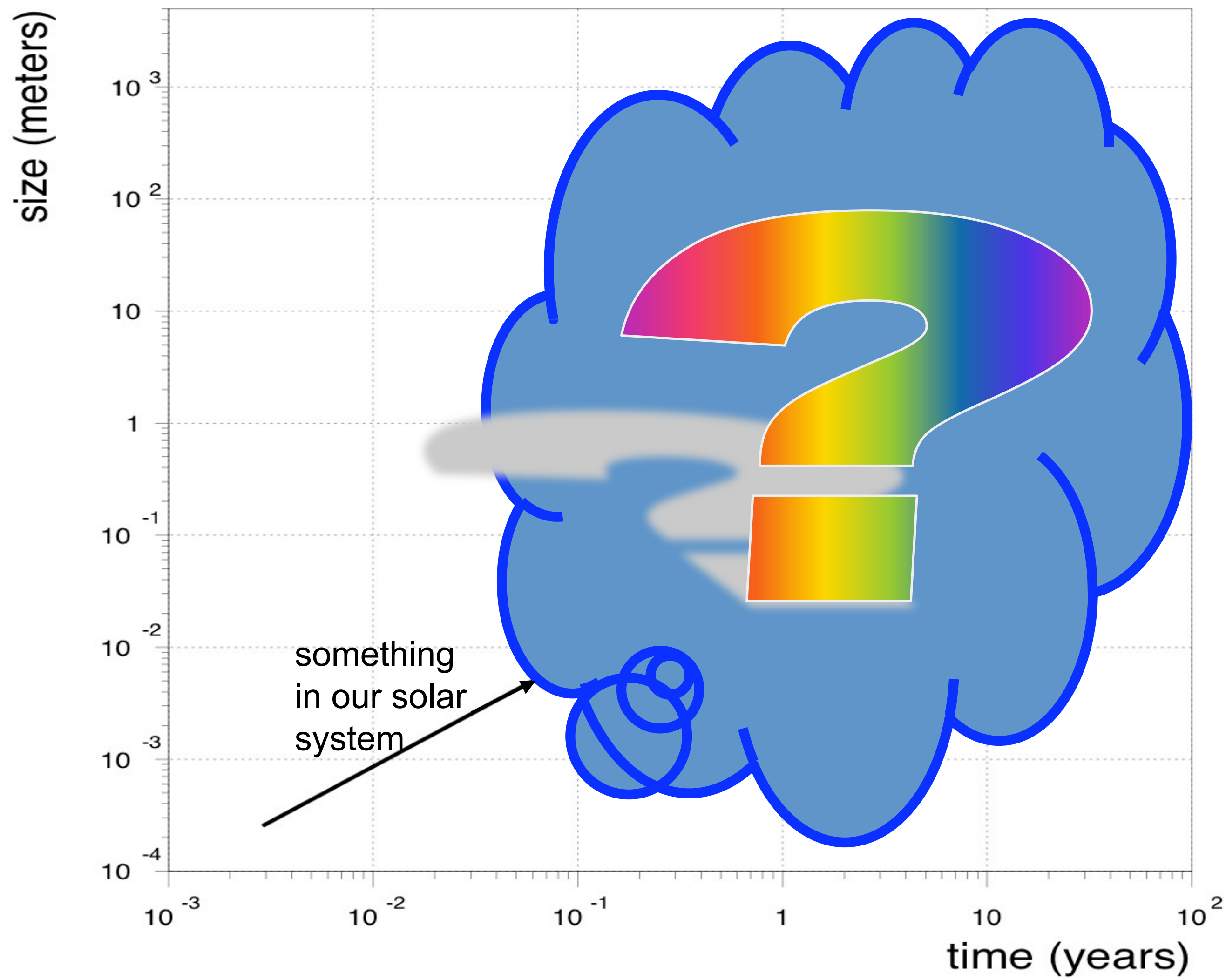
  dark energy decreases distance from sun by

  0.14 Å for Earth and 10 - 90 μm for Pluto
- Gravity and dark energy roughly cancel for Milky-Way and Andromeda galaxies (but galaxy-cluster gravity wins)

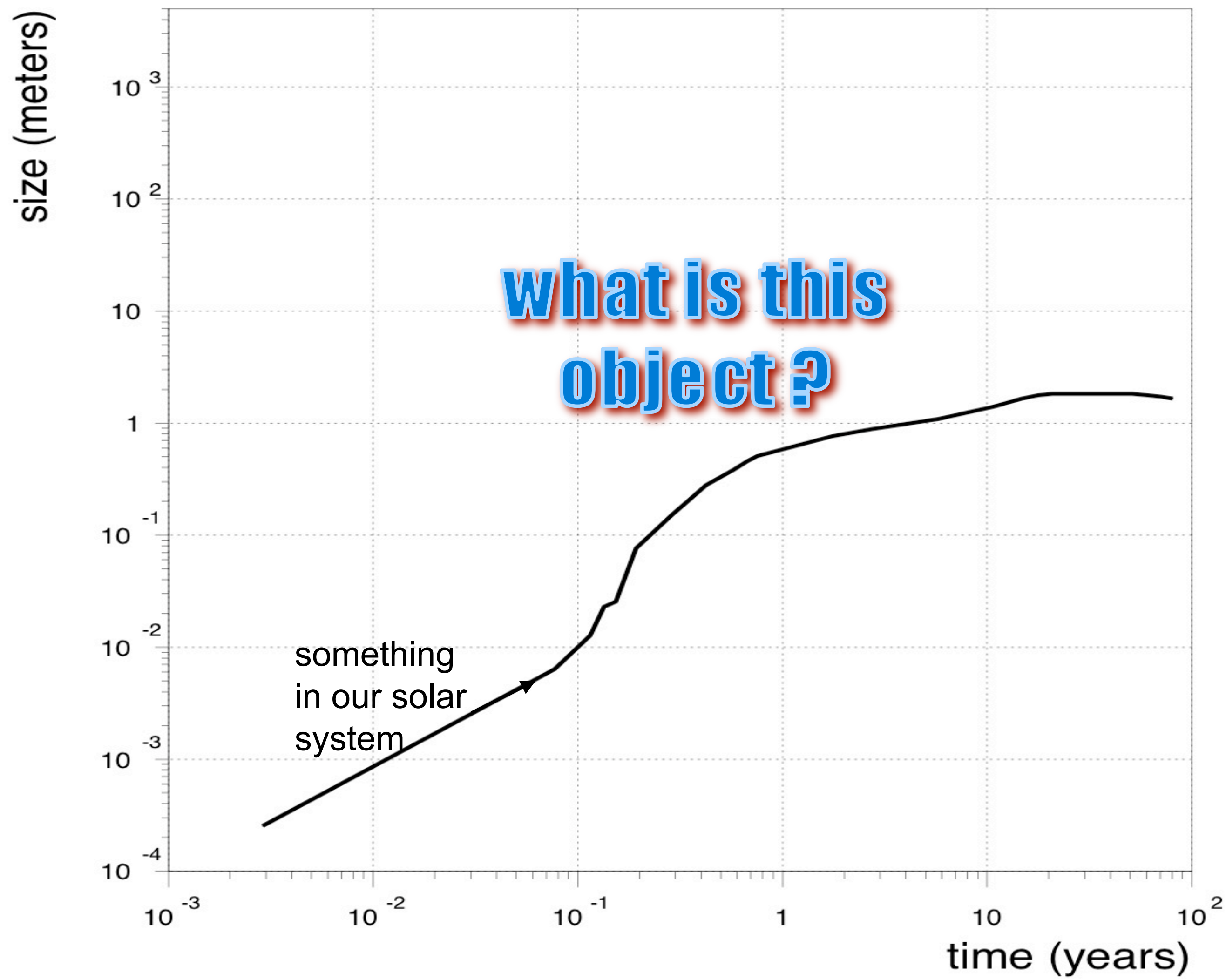
Exercise: verify the numbers

Exercise clue: $\rho_{eff} = -\Lambda/(4\pi G)$
If we change the constant velocity assumption to constant orbital period, show that the change in radius is ×3 smaller.

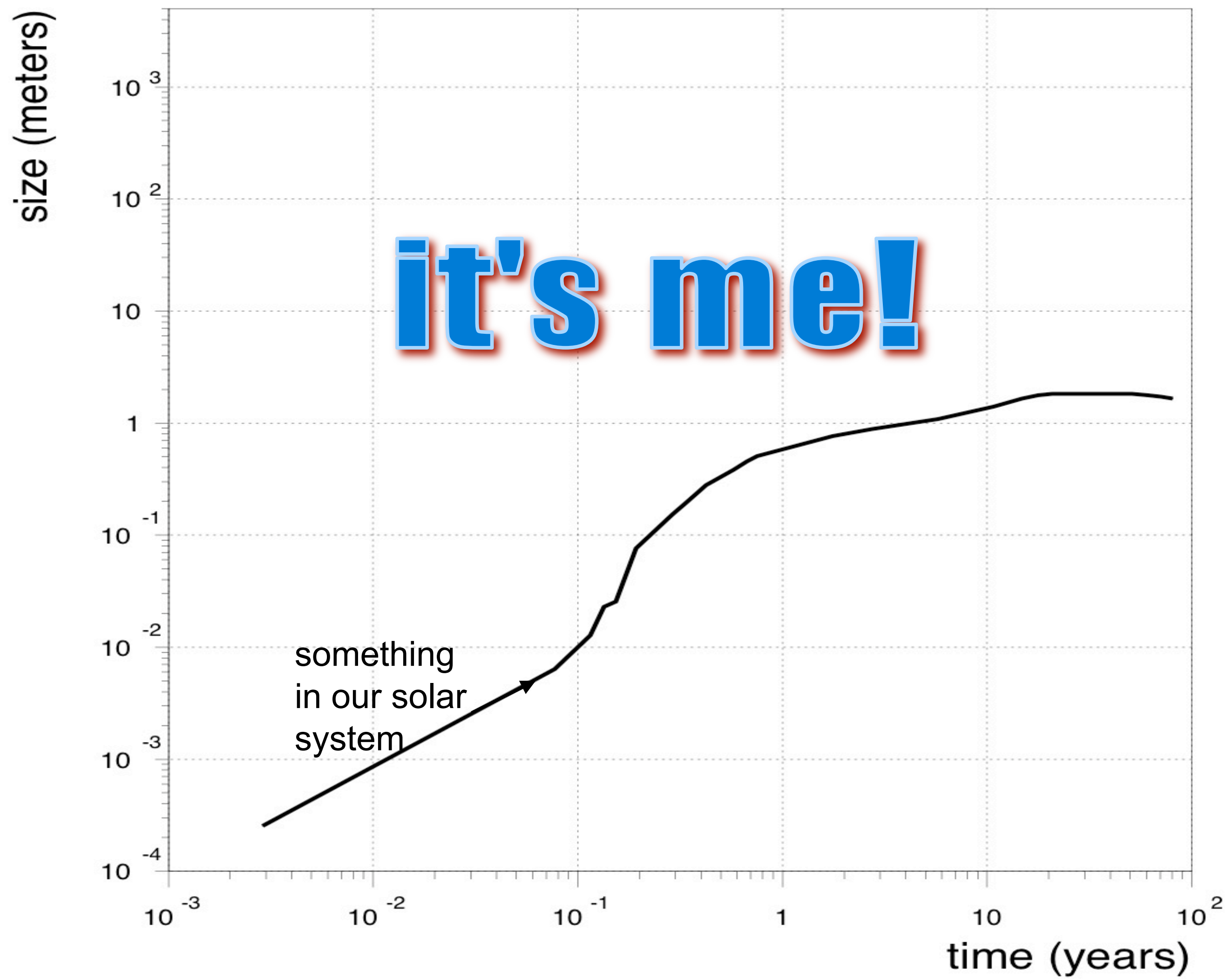
Understanding Expansion History is Tricky



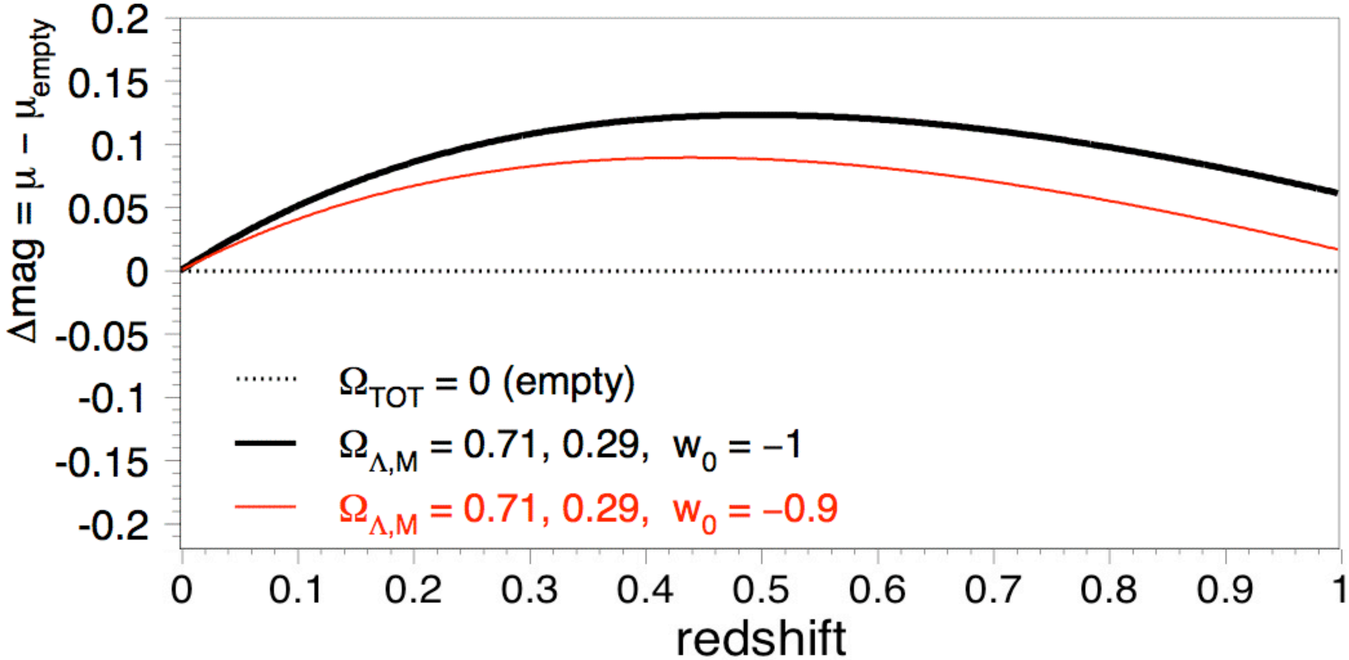
Understanding Expansion History is Tricky



Understanding Expansion History is Tricky



w-sensitivity with SNIa distances



w-sensitivity with SNIa distances