Physics of
Cosmic Acceleration

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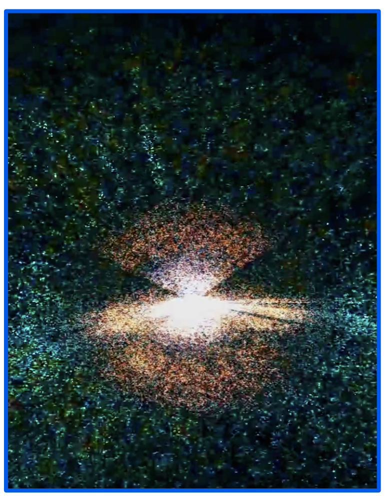
II Tiomno School (Rio 2012)



UC Berkeley & Berkeley Lab
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Exploring Cosmology

An experimenter and a theorist go on a hike...

These Lectures are not on Dark Energy

Rene Magritte

Plan of Lectures

1. Cosmic Expansion and Growth
2. Dark Energy as a Field
3. Dark Energy as Gravity
4. Chasing Down Cosmic Acceleration

The first 2/3 of each part will be lecture, the last 1/3 will be questions, discussion, and exercises.

Acceleration

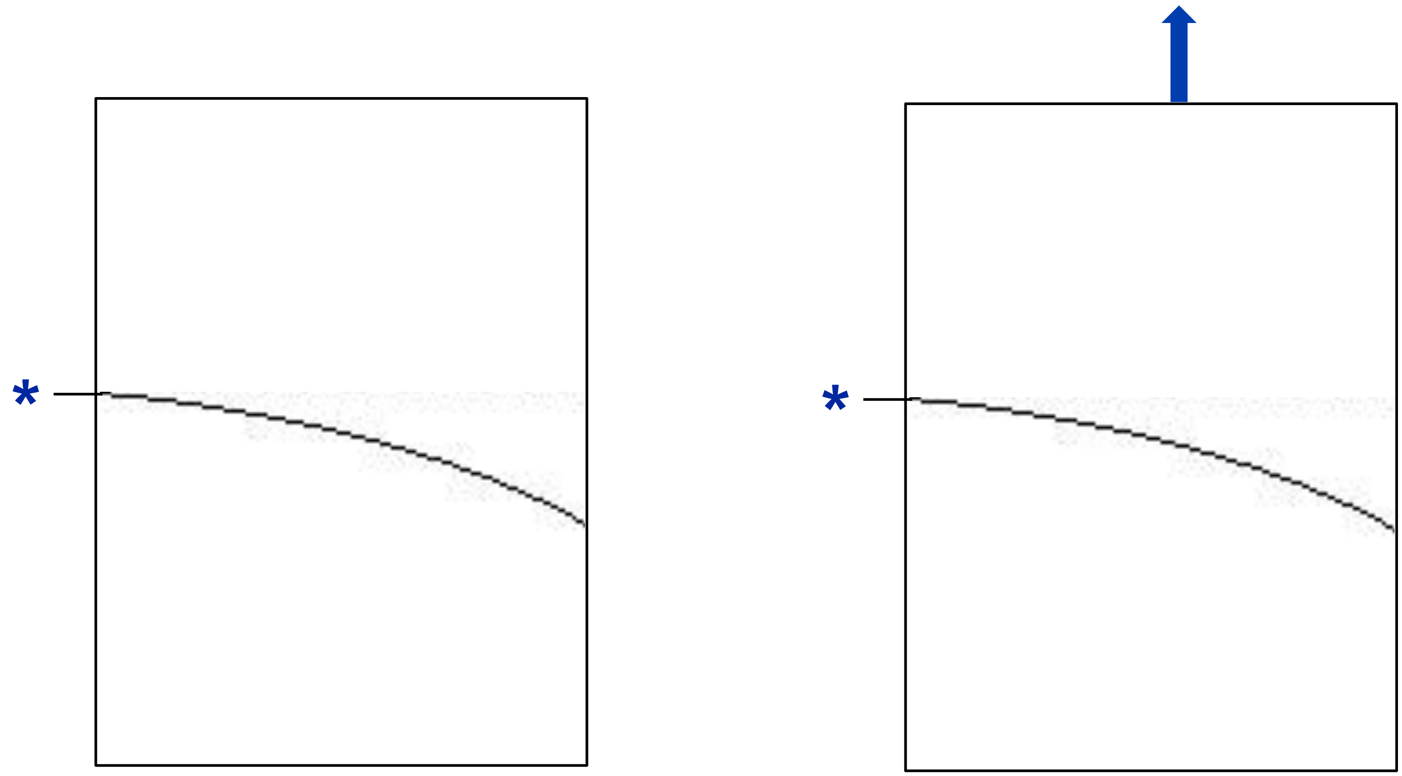
Acceleration is a key element of physics, central to Einstein's Equivalence Principle.

Gravity = Curvature = Acceleration

Gravity is equivalent to the curvature of spacetime geometry, and determines the motions of particles along geodesics.

Forces (acceleration) change the motions of particles can be viewed as affecting spacetime geometry. Locally, acceleration is equivalent to gravity.

Acceleration = Gravity

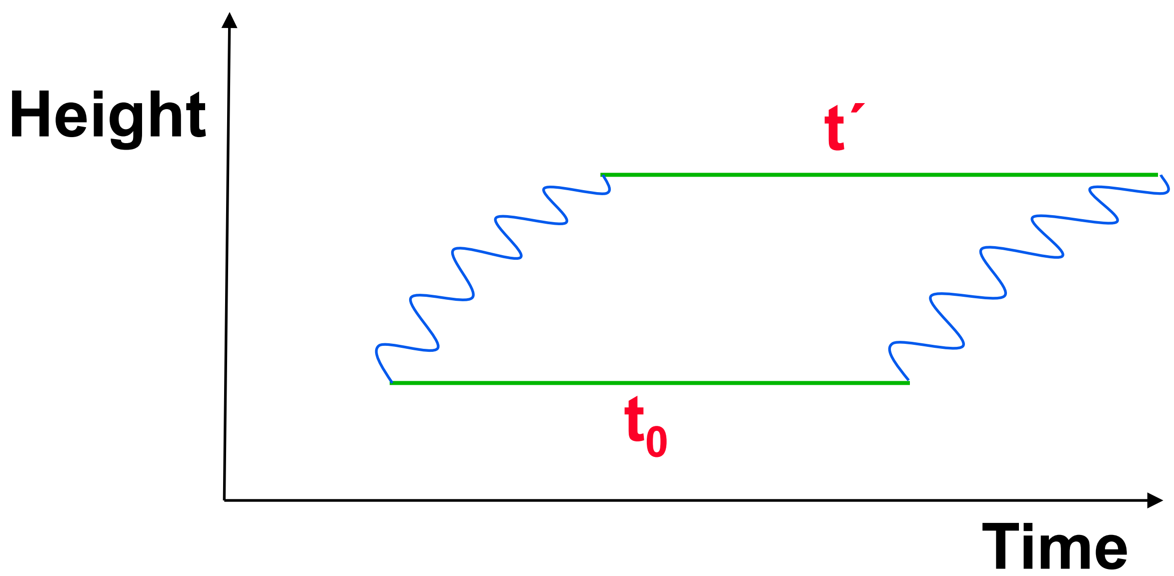


In the presence of gravity or of acceleration, light follows a curved path. Locally, they are equivalent.

Acceleration = Curvature

The Principle of Equivalence teaches that

Acceleration = Gravity = Curvature



Acceleration ⇒ over time will get $v=gh/c$,
so $z = v/c = gh/c^2$ (gravitational redshift).

But, $t' \neq t_0$ ⇒ parallel lines not parallel (curvature)!

Cosmic Acceleration

Acceleration has:

- Direct (kinematic) effect on spacetime through a(t)

- Dynamic effects on objects within spacetime, e.g. growth, ISW

What appears in the metric is the cosmic scale factor a(t).

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

The metric can be spatially flat (k=0) but the *spacetime* is curved if $\ddot{a} \neq 0$

This is exactly the Equivalence Principle:

Gravity = Curvature = Acceleration

Spacetime Geometry

Homogeneity and isotropy determine the spacetime to be maximally symmetric and the metric takes the Robertson-Walker form.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Spherical symmetry is obvious because the spatial sections involve two-spheres: for constant r the angular dependence is just $d\omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

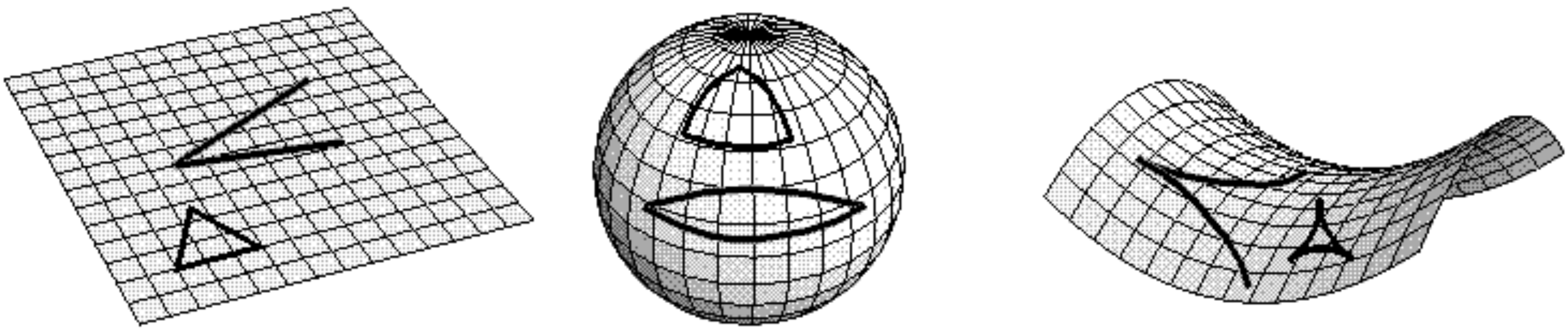
The key ingredients are

constant parameter k – spatial curvature,

function of time a(t) – scale (expansion) factor.

Spatial Curvature

k isinverse square radius of curvature, $k=1/R_c^2$.
If $k=0$ then $R_c=\infty$ and space is flat.
$k>0$ indicates positive curvature (like a sphere),
$k<0$ negative curvature (like a hyperboloid/saddle).



k=0 k>0 k<0

We can also choose to make r dimensionless (giving dimensions to a) and normalize k=0, +1, -1.

Cosmic Expansion

In front of the spatial part of the metric is the scale factor a(t), scaling all distances. If a increases with time, this indicates cosmic expansion.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

If r is dimensionful then a is dimensionless and we can normalize $a_{today}=a_0=1$. Cannot simultaneously normalize k and a!

2nd derivatives of the metric g_{ab} $\quad ds^2 = g_{ab}\, dx^a dx^b$
form the Ricci tensor, determining spacetime curvature. This is proportional to $\ddot{a}$

Cosmic Expansion

Space flatness: k=0

Spacetime flatness: $\ddot{a} = 0$

Exercise 1.1: Show that $\ddot{a} = 0$ is equivalent to a flat (Minkowski) spacetime.

All results coming directly from the metric (spacetime symmetries) are called kinematics.

We have not had to specify any laws of gravity!

Results that require force laws are called dynamics.

Light Propagation

Light signals travel on null geodesics (ds=0) and measure ∫dt/a = ∫da dt/da (a/a^2) = ∫da^{-1} dt/dlna = ∫dz/H. Distances are directly affected by acceleration.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

If source and observer are comoving, the distance r_c is constant. Thus ∫dt/a = const.

Imagine a source pulsing with frequency ν~1/dt. The emission at t_e+dt_e is observed at t_o+dt_o. But

$$\int_{t_o}^{t_e} \frac{dt}{a} = \int_{t_o+dt_o}^{t_e+dt_e} \frac{dt}{a} \qquad \Longrightarrow \frac{dt_e}{a(t_e)} - \frac{dt_o}{a(t_o)} = 0$$

Redshift

Redshift is given by

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{\lambda_o}{\lambda_e} = \frac{a_o}{a_e}$$

Note this is a purely kinematic effect.

General formula for redshift is

$$1 + z = \frac{(g_{ab} u^a k^b)_e}{(g_{ab} u^a k^b)_o}$$

where u^a is source 4-velocity, k^b is photon 4-momentum

Exercise 1.2: What else can affect redshift?

Acceleration in Redshift

Since acceleration is a property of a(t), can we detect acceleration directly in redshift?

Redshifts are changes in scale/position ("velocities"):

$$z=[a(t_0)-a(t_e)]/a(t_e) \rightarrow H_0\,(t_0-t_e)$$

Redshift shifts are changes in changes ("acceleration"):

$$dz/dt_0 = [\dot{a}_0-\dot{a}_e]/a_e = H_0(1+z)-H(z) \rightarrow \Delta z = -zq_0H_0\,\Delta t$$

Redshift drift (Sandage 1962; McVittie 1962; Linder 1991,1997)

$\Delta z=10^{-8}$ over 100 years

BAO for Acceleration

Acceleration can be seen directly through redshift drift.

$$\dot{z} = H_0\,(1+z) - H(z)$$

McVittie/Sandage 1962

Europe wants to build a 40m telescope to stare at quasars for 10 years and measure z to 10^{-10}.

Instead, use radial BAO of galaxies 10^{10} years apart.

Technique	Equation	Nuisance	Sign
z Drift	$\dot{z}_2 - \dot{z}_1 = H_0\,(z_2 - z_1) - (H_2 - H_1)$	H_0	w < -1/3
radial BAO	$\mathrm{rBAO}_2 - \mathrm{rBAO}_1 = s(H_2 - H_1)$	s	w < -1

Exercise 1.3: Show the sign of z drift gives the sign of acceleration; show the sign of rBAO gives the sign of 1+w.

Measuring Acceleration

Distances are directly affected by acceleration. They are the most practical kinematic way to measure cosmic acceleration.

If we introduce dynamics (forces, interactions) there are many other ways – but we also need to be sure we actually understand the forces, not just the spacetime symmetry.

Direct dynamical detection?

But... *Dark energy in solar system = 3 hours of sunlight*.

Co-dependence?

Variations of fundamental constants; lab/accelerator/universe (highly model dependent).

What is Dark Energy?

How many dark rectangles do you see?

Beyond Kinematics

Once you go beyond kinematics to dynamics, you have a lot of questions to answer!

What is its dynamics?

Does dark energy interact?

Does dark energy have internal degrees of freedom?

Can we split off matter and radiation?

In these lectures we will mostly assume that dark energy can be treated as a single, independent quantity (so we can talk about matter etc. separately).

Cosmological Framework

Equivalence Principle

→ Metric description of spacetime

Homogeneity and Isotropy

→ Metric is Robertson-Walker

→ Energy-momentum has perfect fluid form (ρ,p)

Gravitational Field Eqs (General Relativity) + Homogeneity and Isotropy

→ Friedmann equations for evolution of spacetime

Equations of State + Friedmann equations

→ Evolution of energy densities

Gravitating Energy

Einstein says gravitating mass depends on energy-momentum tensor:

both energy density ρ and pressure p, as ρ+3p

Negative pressure can give negative "mass"

Newton's 2nd law: Acceleration = Force / mass

$$\ddot{R} = -GM/R^2 = -(4\pi/3)G\,\rho\,R$$

Einstein/Friedmann equation:

$$\ddot{a} = -(4\pi/3)G\,(\rho+3p)\,a$$

Negative pressure can accelerate the expansion

Negative Pressure

Relation between ρ and p (*equation of state*) is crucial:

$$w = p / \rho$$

Acceleration possible for $p < -(1/3)\rho$ or $w < -1/3$

What does negative pressure mean?

Consider 1st law of thermodynamics:

$$dU = -p\ dV$$

But for a spring $dU = +k\ xdx$
or a rubber band $dU = +T\ dl$

Vacuum Energy

Quantum physics predicts that the very structure of the vacuum should act like springs.

Space has a "springiness", or tension, or vacuum energy with negative pressure.

Review --
Einstein: expansion acceleration depends on $\rho+3p$
Thermodynamics: pressure p can be negative
Quantum Physics: vacuum energy has negative p

Cosmological observations can map the expansion history, measure acceleration, detect vacuum energy.

Friedmann Equations

Equations of motion for the homogeneous background.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - ka^{-2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\dot{\rho}_i = -3\frac{\dot{a}}{a}(\rho_i + p_i)$$

Only two equations independent because Bianchi identity redundant.

Notation

$$H(a) = \frac{\dot{a}}{a} \qquad q(a) = -\frac{a\ddot{a}}{\dot{a}^2} \qquad \Omega_i(a) = \frac{8\pi G \rho_i(a)}{3H^2(a)}$$

$$\Omega_{\rm tot}(a) = \sum_i \Omega_i(a) = 1 - \Omega_k(a) = 1 + \frac{k}{a^2 H^2}$$

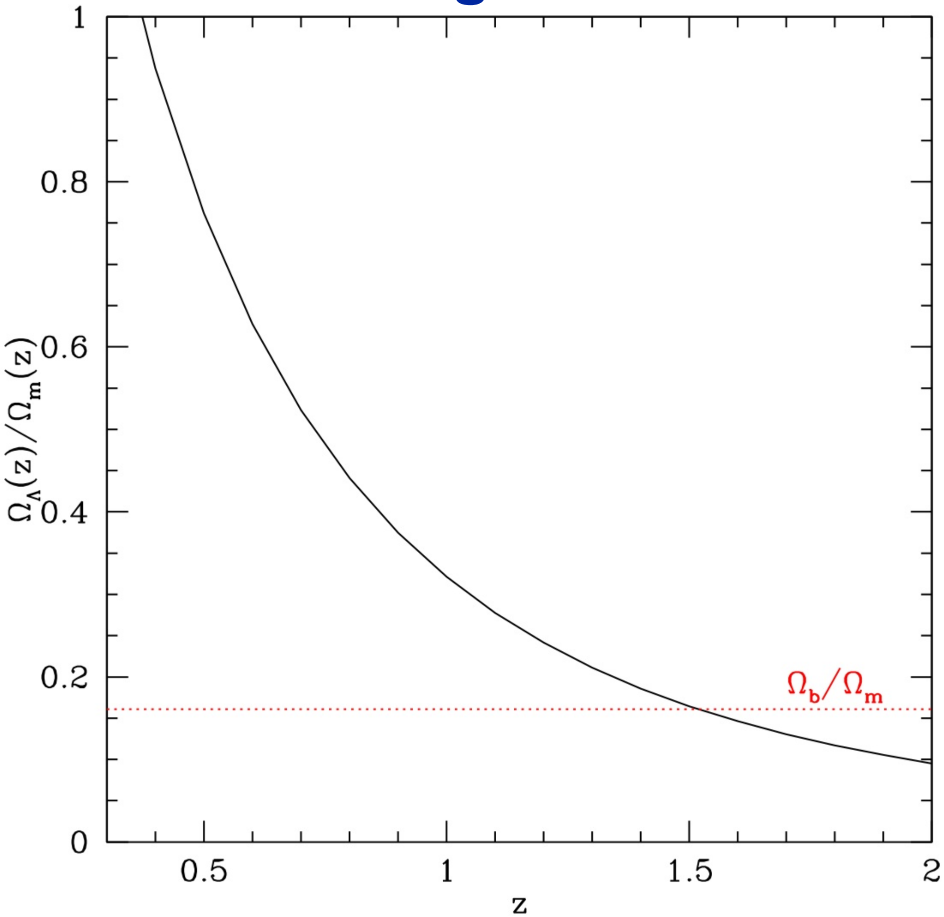
$$\rho_i(a) = \rho_i\, e^{-3\int_0^{\ln a} d\ln a'\,[1+w_i(a')]} \sim a^{-3(1+w_i)}$$

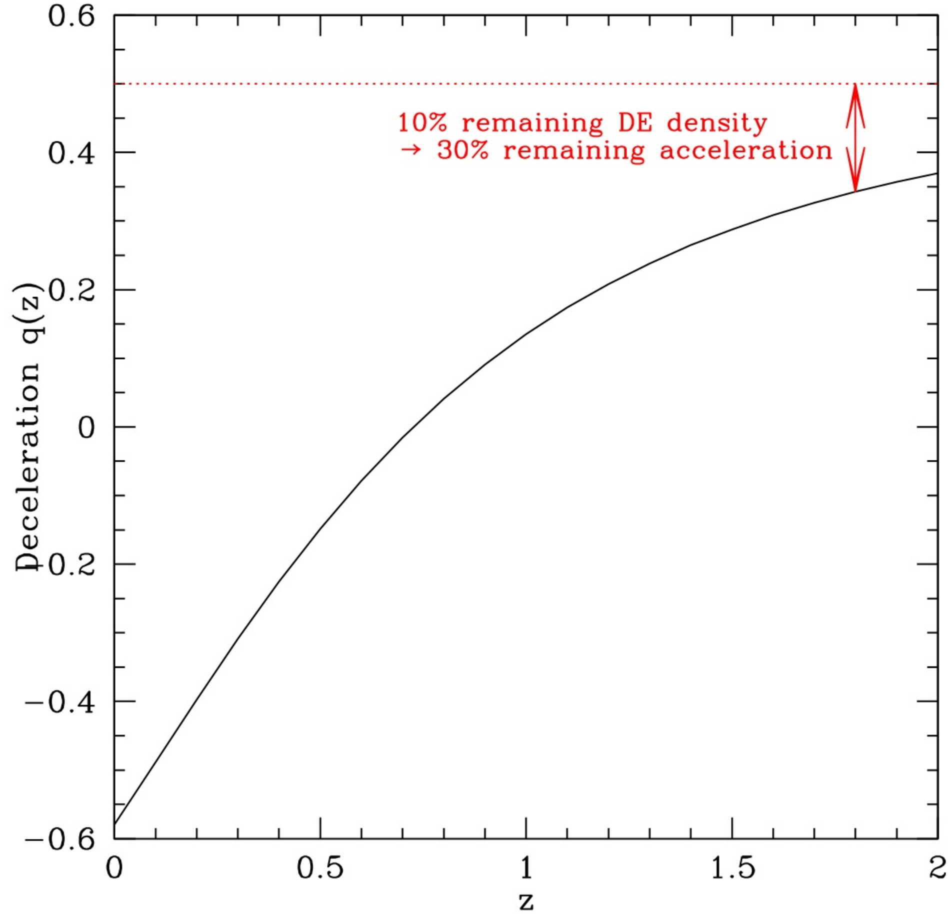
$$H(a)/H_0 = \left[\Sigma\, \Omega_i a^{-3(1+w_i)} + 1 - \Omega_{\rm tot}(a)\right]^{1/2}$$

$$q(a) = \frac{1}{2} \Sigma\, \Omega_i (1+3w_i) a^{-3(1+w_i)} / [H(a)/H_0]^2$$

Redshift Range for Acceleration

Acceleration is not just "recent universe", z<<1. Over what redshift range should we measure it?



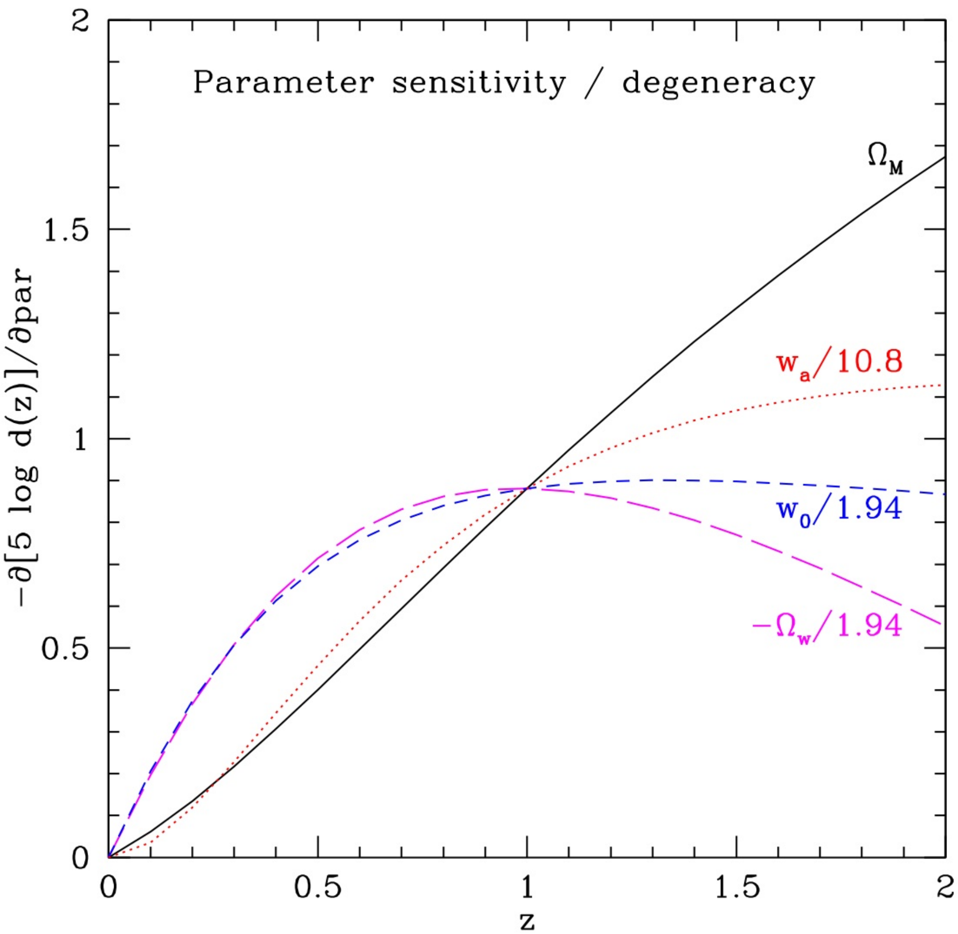


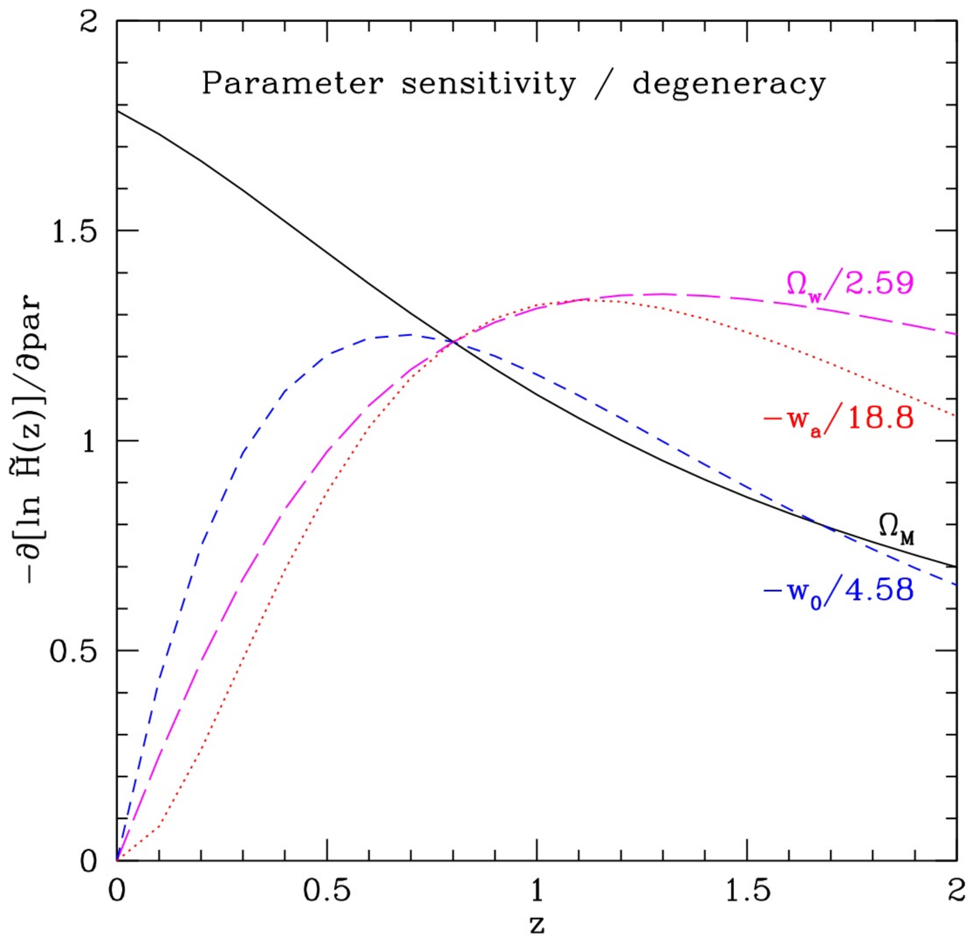
Deep enough that is less than 10% energy density? Not next-to-dominant?

Deep enough that have accounted for >2/3 of the acceleration?

Distance Complementarity

Distances relative to **low** and **high** redshift





have different degeneracies, hence complementarity

e.g. Supernovae (R. Kessler) and BAO (Y. Wang)

Growth of Structure

Equations of motion for linearly perturbed quantities gives growth of structure.

Newtonian approach (doesn't always work!): Perturb the acceleration equation by

$$R = R_0\, a(t)\, [1 - \delta(t)/3]$$

that conserves mass

$$(\rho + \delta\rho)\, R^3 = \rho\, R_0^3 a^3(t)$$

This determines growth of density inhomogeneities δ=δρ/ρ

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta = 0$$

Note expansion (H) slows exponential (Jeans instability) growth to power law in time (δ~a in matter domination).

Physics of Growth

Growth $g(a)=(\delta\rho/\rho)/a$ depends purely on the expansion history H(z) – *and* gravity theory.

$$g'' + [5 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a}]g'a^{-1} + [3 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a} - \tfrac{3}{2}G\,\Omega_m(a)]\,ga^{-2} = 0$$

Within general relativity ($G=G_N=1$),
expansion determines growth and vice versa.

Acceleration suppresses growth in two ways:

1) the friction term ~ (3-q) so q<0 slows growth,

2) the source term $\Omega_m(a)$ is diminished.

Observational Leverage

Exercise 1.1: Show that $\ddot{a}$ = 0 is equivalent to a flat (Minkowski) spacetime.

Exercise 1.2: What else can affect redshift?

Exercise 1.3: Show the sign of z drift gives the sign of acceleration; show the sign of rBAO gives the sign of 1+w.

For more dark energy resources, see

http://supernova.lbl.gov/~evlinder/scires.html

Resource Letter on Dark Energy **http://arxiv.org/abs/0705.4102**

Mapping the Cosmological Expansion **http://arxiv.org/abs/0801.2968**

and the references cited therein.