Part IV: Introduction to Gravitational Lensing



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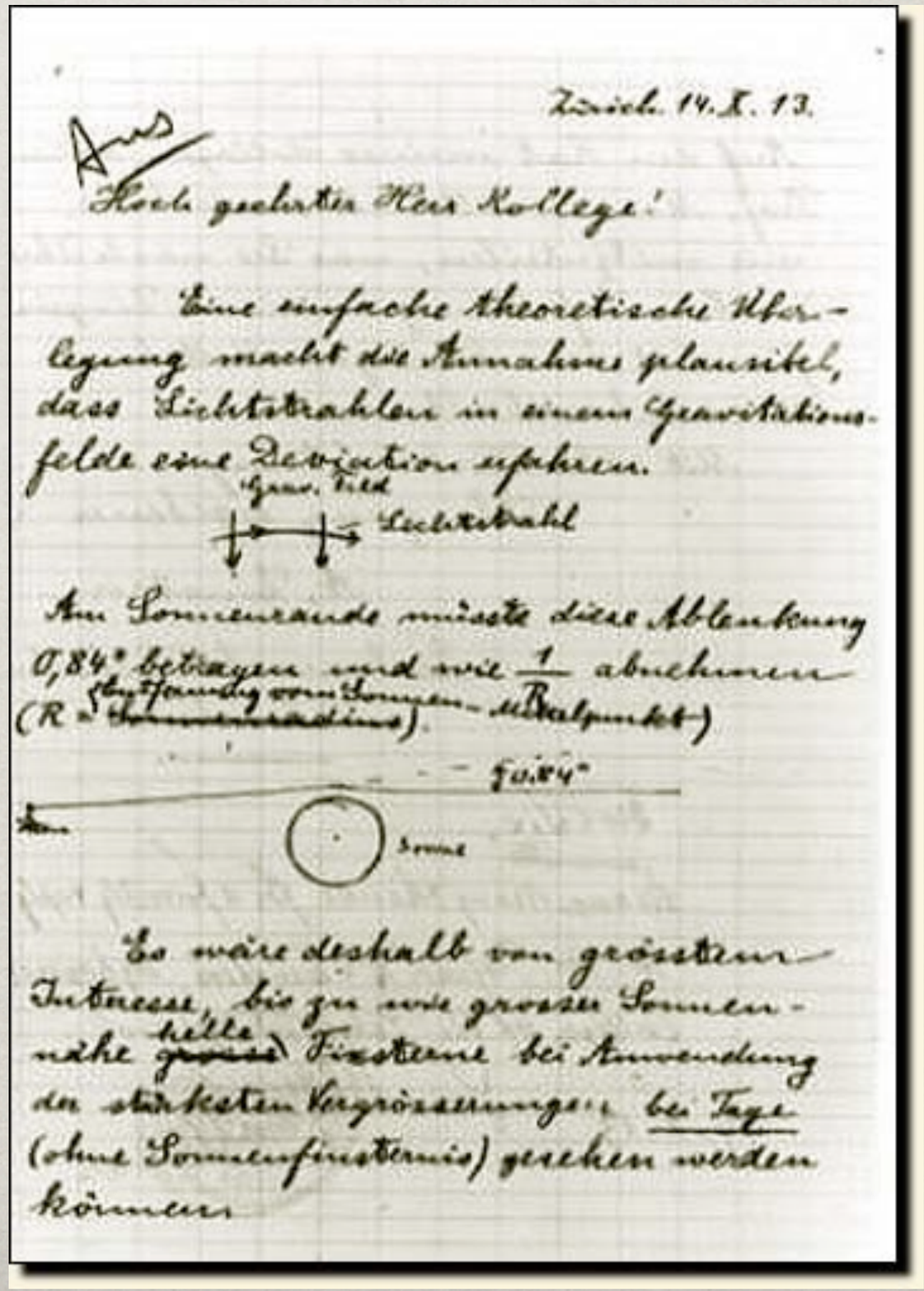
Detection of Light Deflection by Gravity

- 1911-1912: Einstein predicts light deflection and gravitational lensing effects

  Urges observational tests



Zürich. 14. X. 13.

Ans

Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Grav. Feld

Lichtstrahl

Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie $\frac{1}{R}$ abnehmen (R = ~~Sonnenradius~~ Entfernung vom Sonnen-Mittelpunkt).

0,84"

Sonne

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe ~~grosse~~ helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

Detection of Light Deflection by Gravity

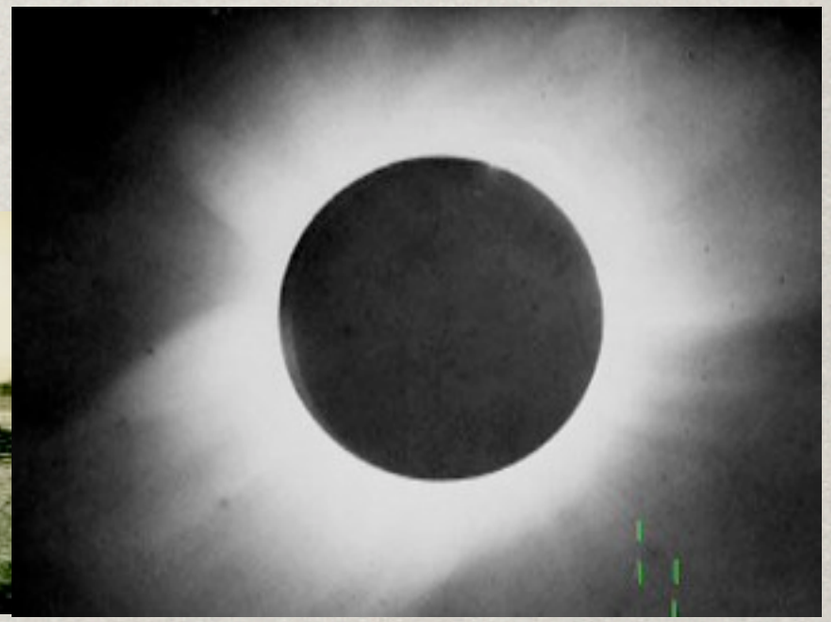
- 1912: Argentinean expedition to observe solar eclipse in Brazil
  - Got rained out



Detection of Light Deflection by Gravity

- 1912: Argentinean expedition to observe solar eclipse in Brazil
  - Got rained out
- 1914: Erwin Freundlich's expedition to observe an eclipse in Crimea (Russia)
  - Detained because WWI break up
- 1919: Sobral (Brazil)
  - Detection of the bending of light (better weather than Prince Island)





"The question that my mind formulated was answered by the sunny sky of Brazil"

A. Einstein

Breve Histórico

- Einstein 1911-12: Einstein's predictions and the first intent of observation
- Sobral 1919: light deflection by gravity
- Chwolson 1924: first paper on gravitational lensing
- Einstein 1936: lensing by stars

"Of course, there is not much hope of observing this phenomenon directly"

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,
PRINCETON, N. J.

Brief History

- Einstein 1911-12: Einstein's predictions and the first intent of observation
- Sobral 1919: light deflection by gravity
- Chwolson 1924: first paper on gravitational lensing
- Einstein 1936: lensing by stars

"Of course, there is not much hope of observing this phenomenon directly"



- Zwicky 1937: optimistic

"The probability that nebulae which act as gravitational lenses will be found becomes practically a certainty"





- Walsh, Carswell, Weymann 1979:
  discovery of the first lensed quasar
  (double image of QSO 0957+561)

Discoveries

- Sobral 1919: light deflection
- Walsh, Carswell, Weymann 1979: first lensed quasar (double image of QSO 0957+561)
- Roger Lynds e Vahe Petrosian 1986, Soucail, Fort, & Picat 1987: discovery of gravitational arcs

arcs in A370, A2218, CL2244-02,
not clearly predicted...

- Hewitt, et al. 1988: Einstein ring (radio)
- Irwin et al. 1989: Quasar microlensing
- EROS & MACHO collaborations, 1993:

  first stellar microlensing

- Bond et al., 2003: first planet detection
  - ~2 Jupiter mass planets @ ~3 AU

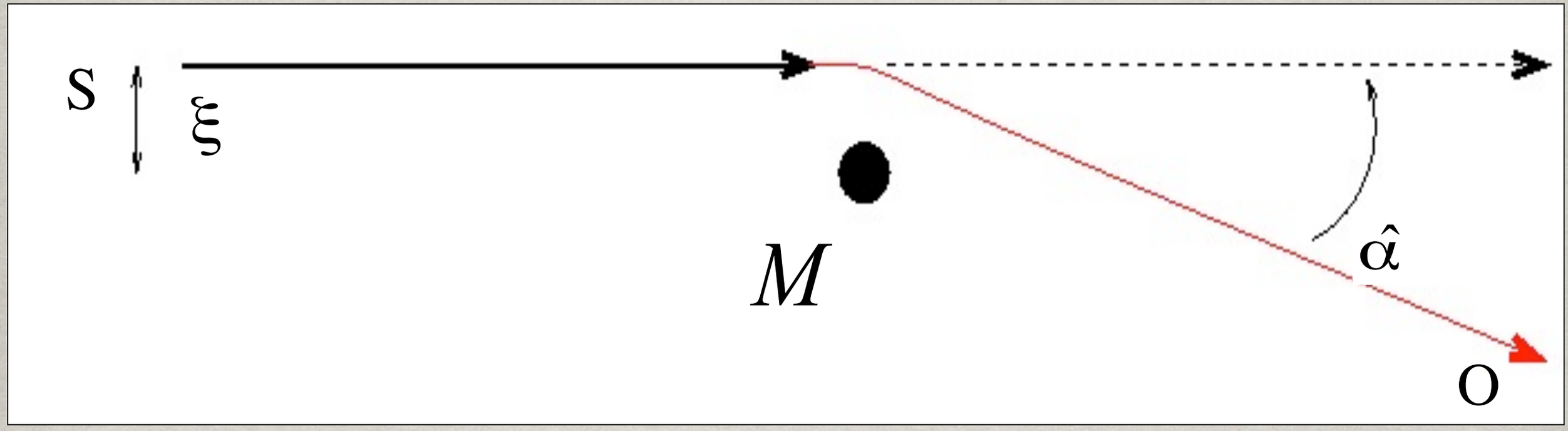


Bending of light by gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

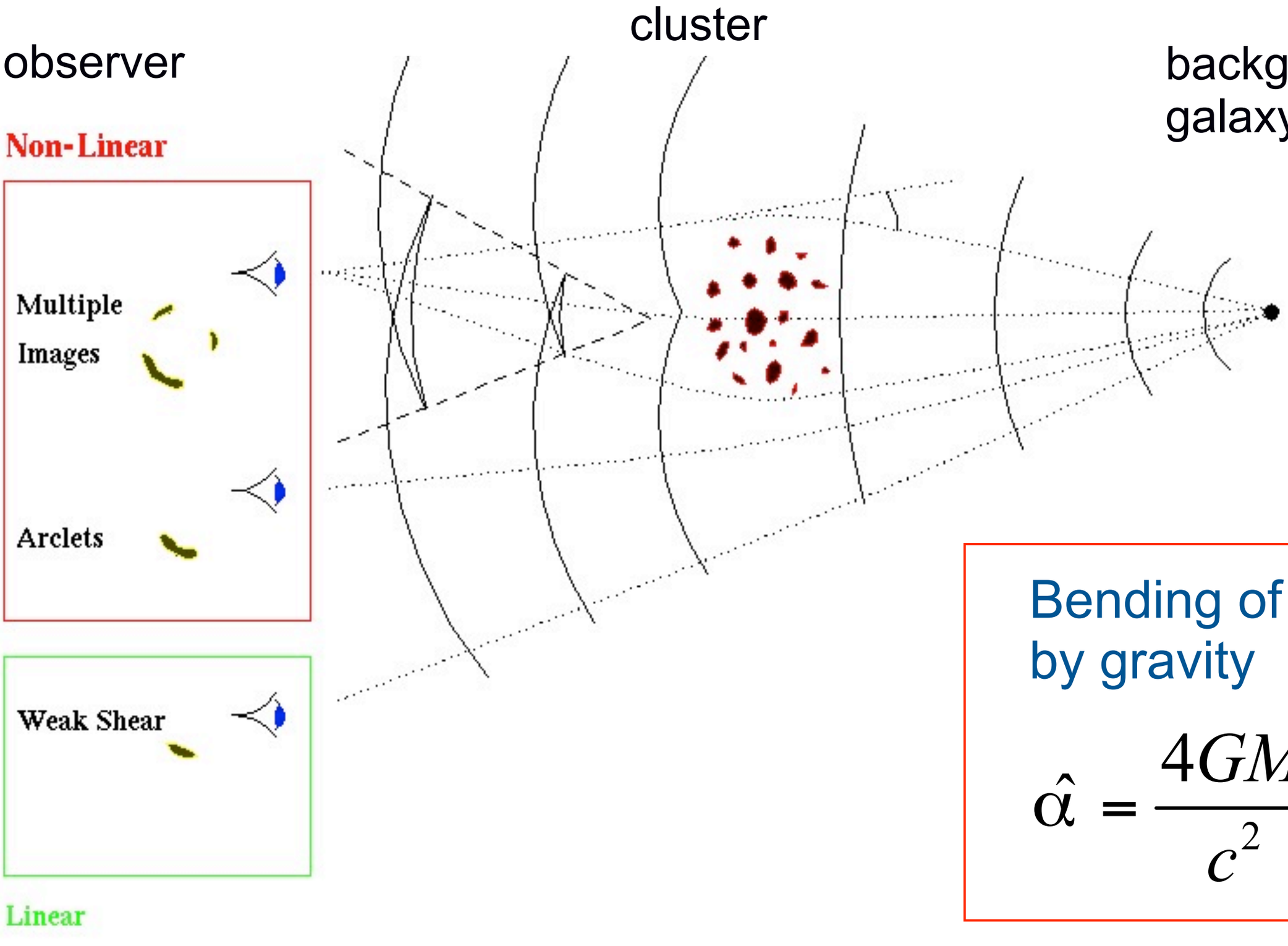
$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c(1 + 2\phi/c^2)$$



Deflection angle (point source) $\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{\xi}$

Achromatic

Weak and Strong Lensing Effects



Bending of light by gravity

$$\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{\xi}$$

A plethora of phenomena

Strength

- Strong lensing
  - Strong magnifications
  - Multiple images
  - Distortions
    - Rings
    - Arcs
- Weak Lensing
  - Small twist
  - Small magnification
  - Detected statistically

Angular scale

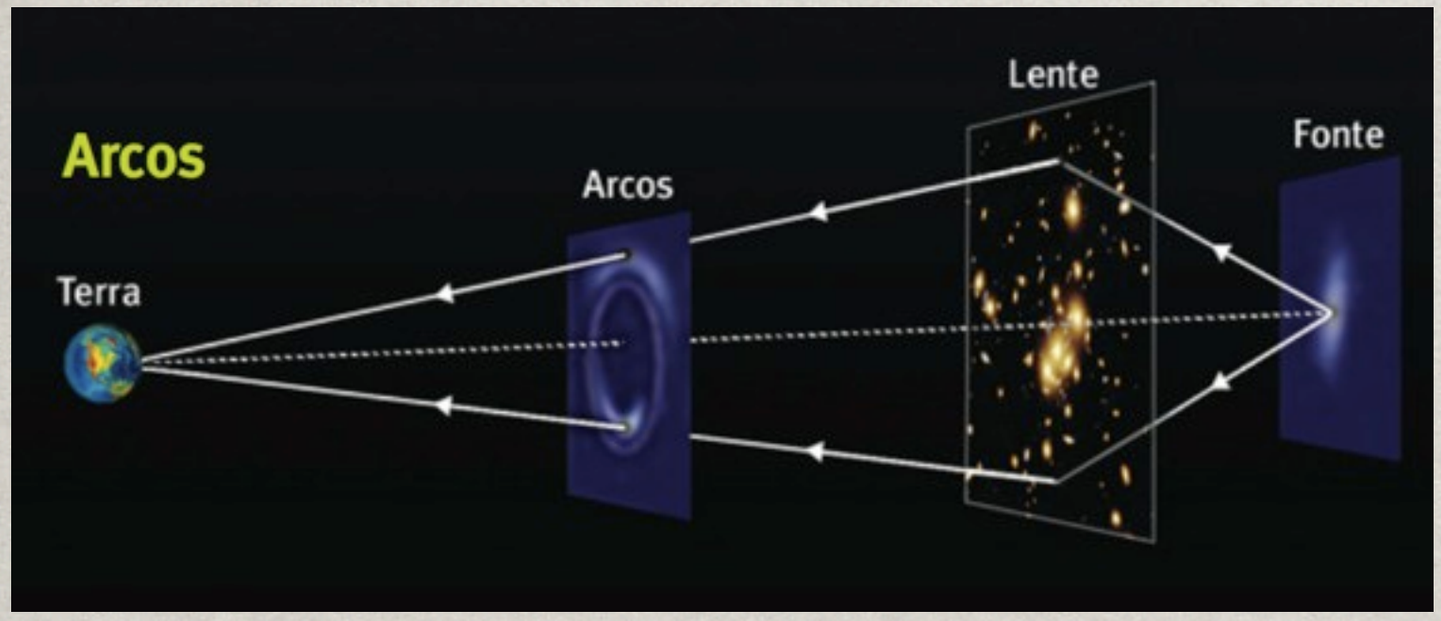
- Micro-lensing
  - MACHOS
  - Planetary search
- Micro and mili-lensing
  - Quasars
- “Macro-lensing”
  - Galaxies
  - Clusters
  - Large-scale structure

Strong Lensing

- Multiple images, strong distortions, large magnifications, time delays
- Null geodesics
  - surface brightness conservation
  - achromatic

} → Gravitational telescopes

  - Unique probe of inner structure of galaxy clusters ➔ DM, *b*
  - Provide complementary cosmological probes and tests of gravity



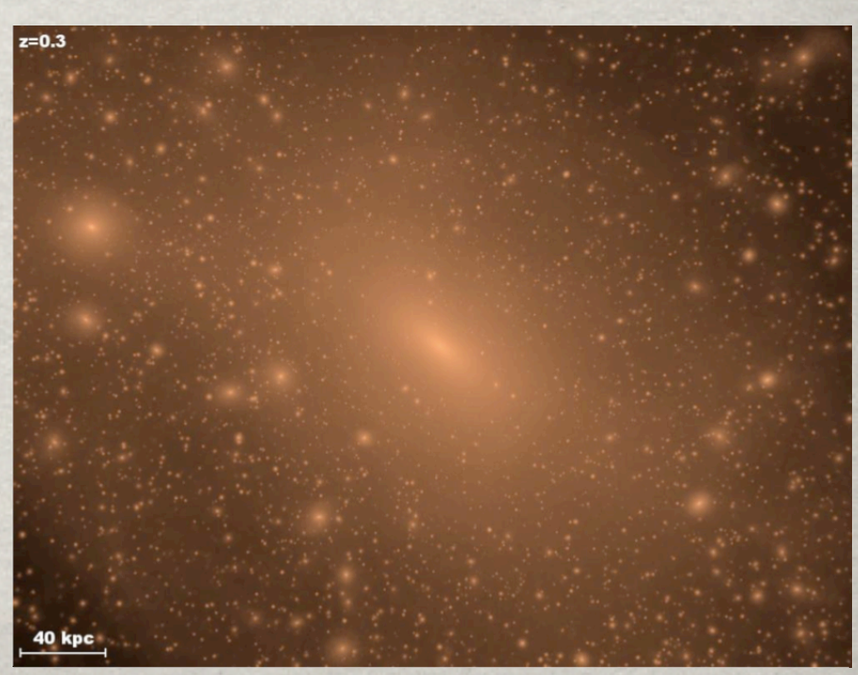
strong lensing, **weak gravity**



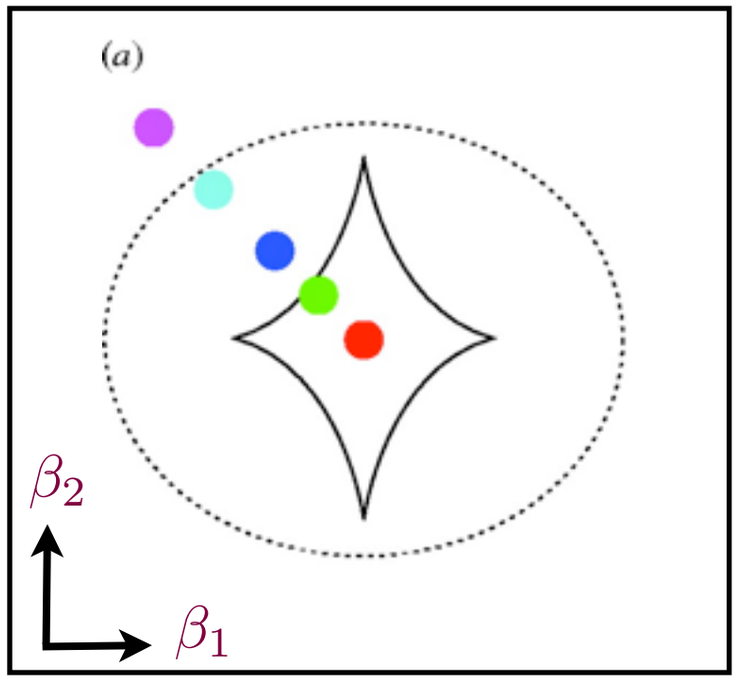
Gravitational arcs

Mass Distribution from Strong Lensing

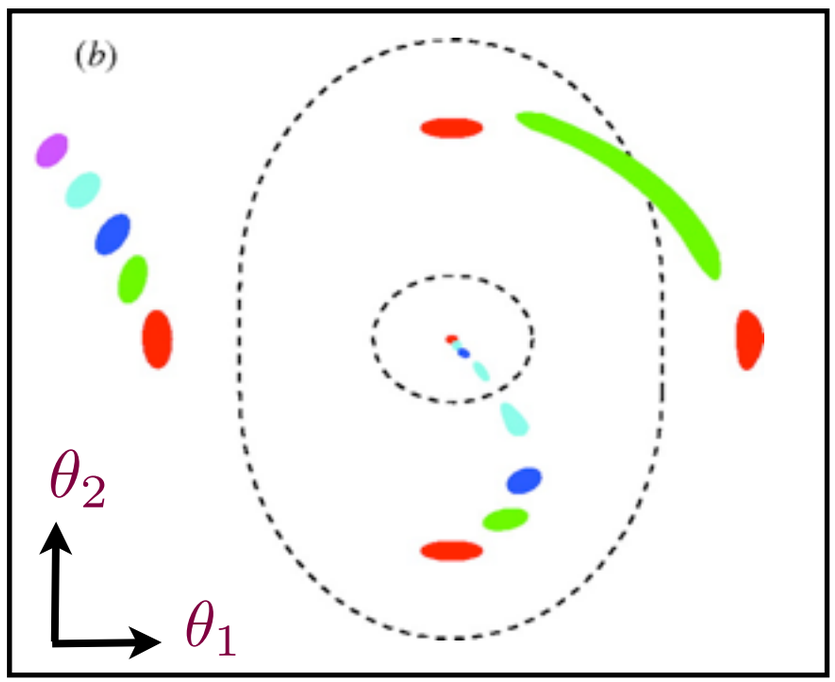
- Constraints on the particle properties of Dark Matter
- Density profiles of galaxies and inner cluster regions:
  - cuspy halo profiles? $\rho(r) \propto r^{-\gamma} \quad \gamma \sim 1$ Bartelmann, astro-ph/0207032
  - baryonic processes: cooling, AGN feedback,… Mead et al. 1001.2281
  - dark matter interactions
- Substructure Moustakas et al. 0902.3219, Koopmans et al. 0902.3186
  - overabundance of satellites
  - primordial spectrum
- Luminous x Matter
  - e.g. DM cross section Williams and Saha, 1102.3943



Geometry of lensing by a single plane

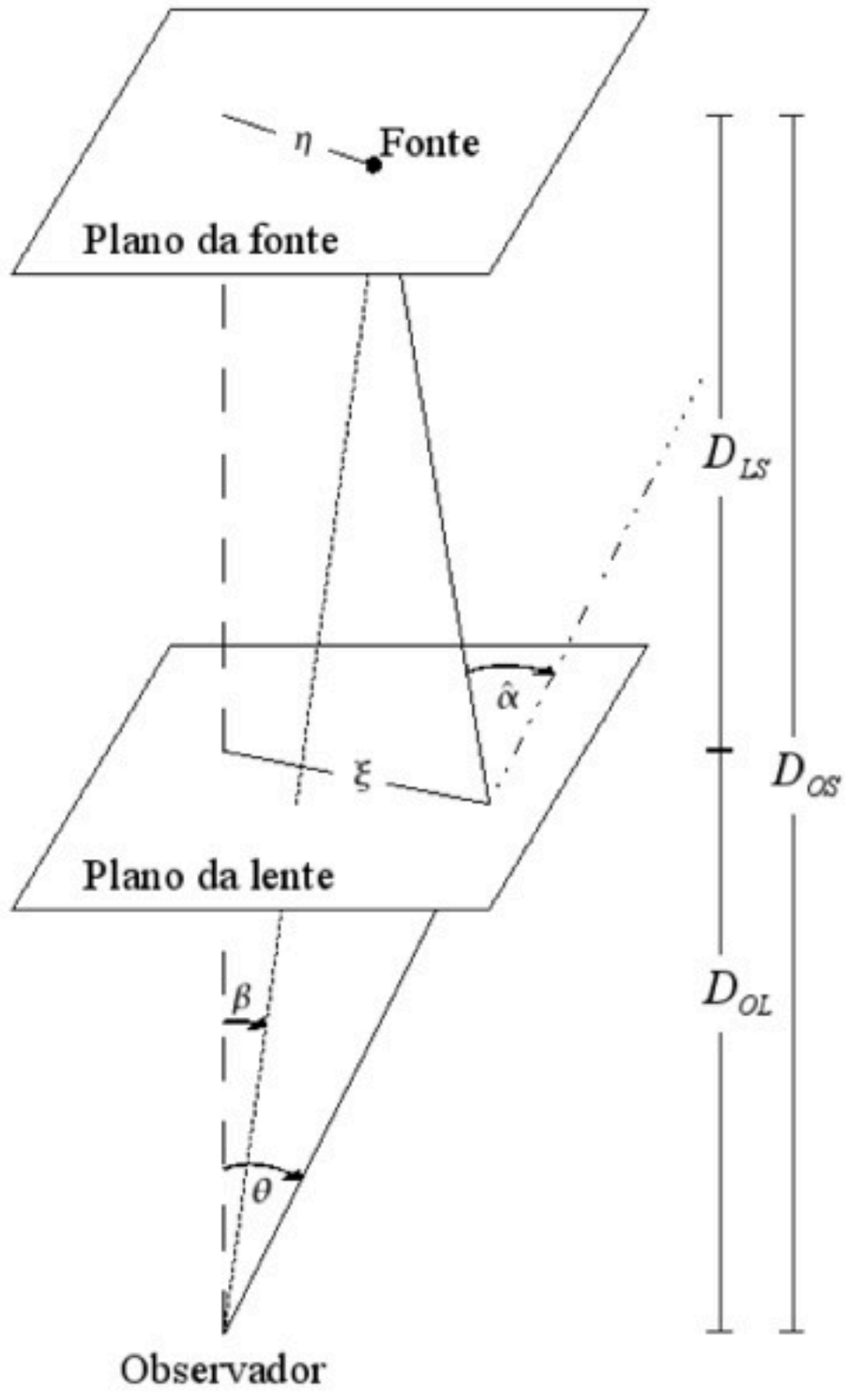


Source plane

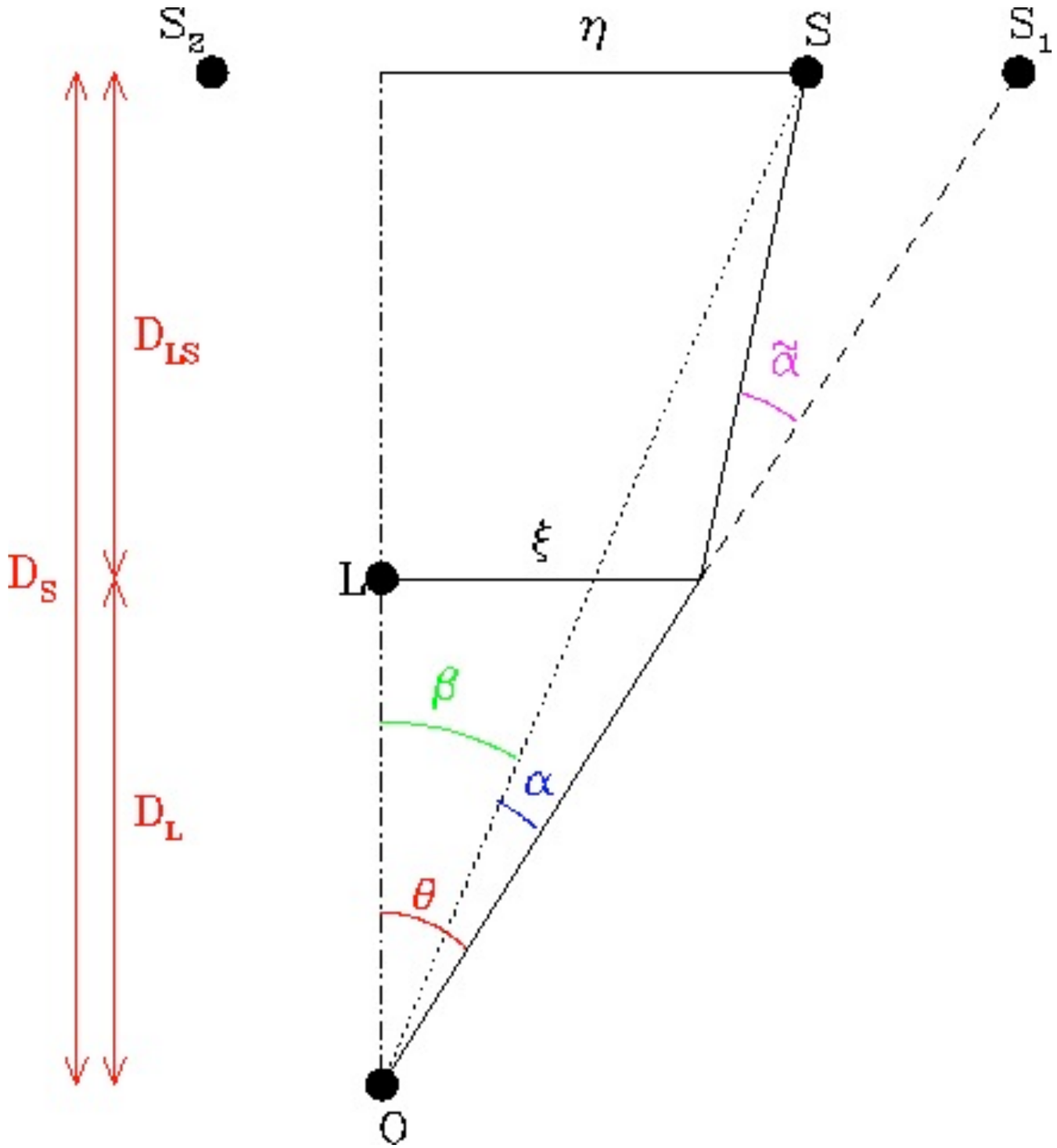


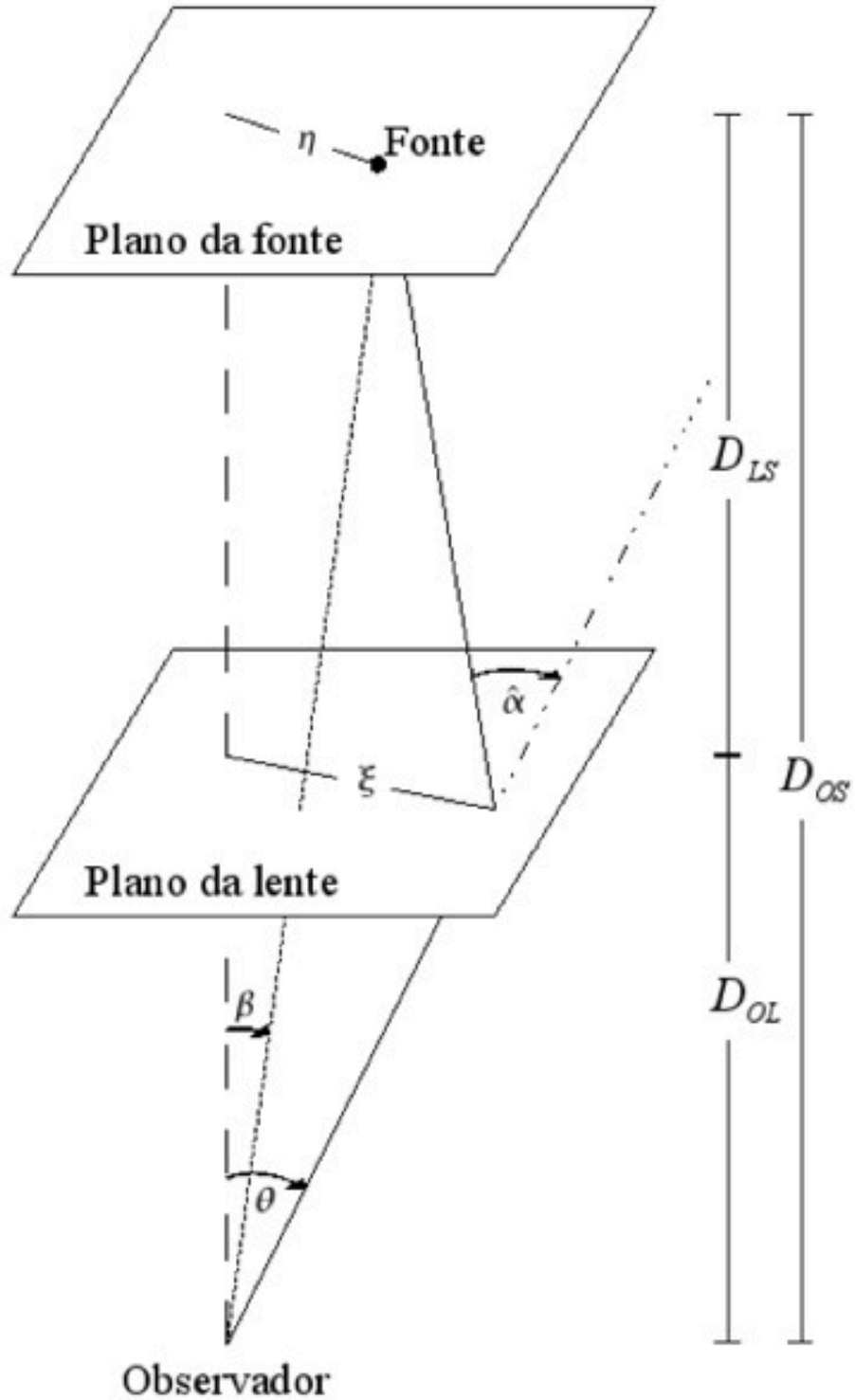
Lens plane

Physical coordinates (distance)	η	ξ
Angular coordinates	β	θ
dimensionless coordinates	y	x

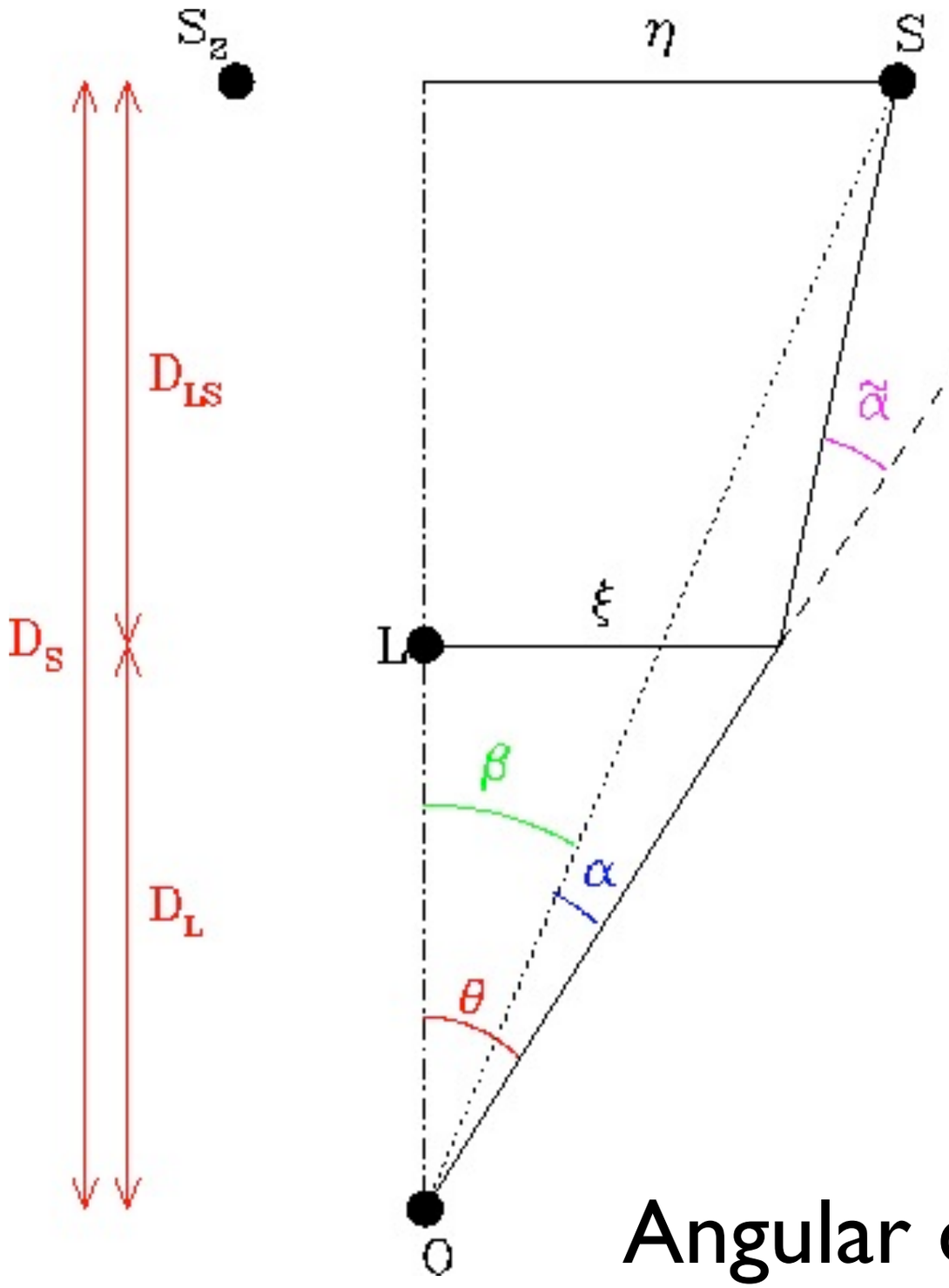


Geometry of lensing by a single plane





The Lens Equation



$$\vec{\beta} D_{OS} = \vec{\theta} D_{OS} - \hat{\vec{\alpha}} D_{LS}\left(\vec{\theta}\right)$$

Reduced deflexion angle

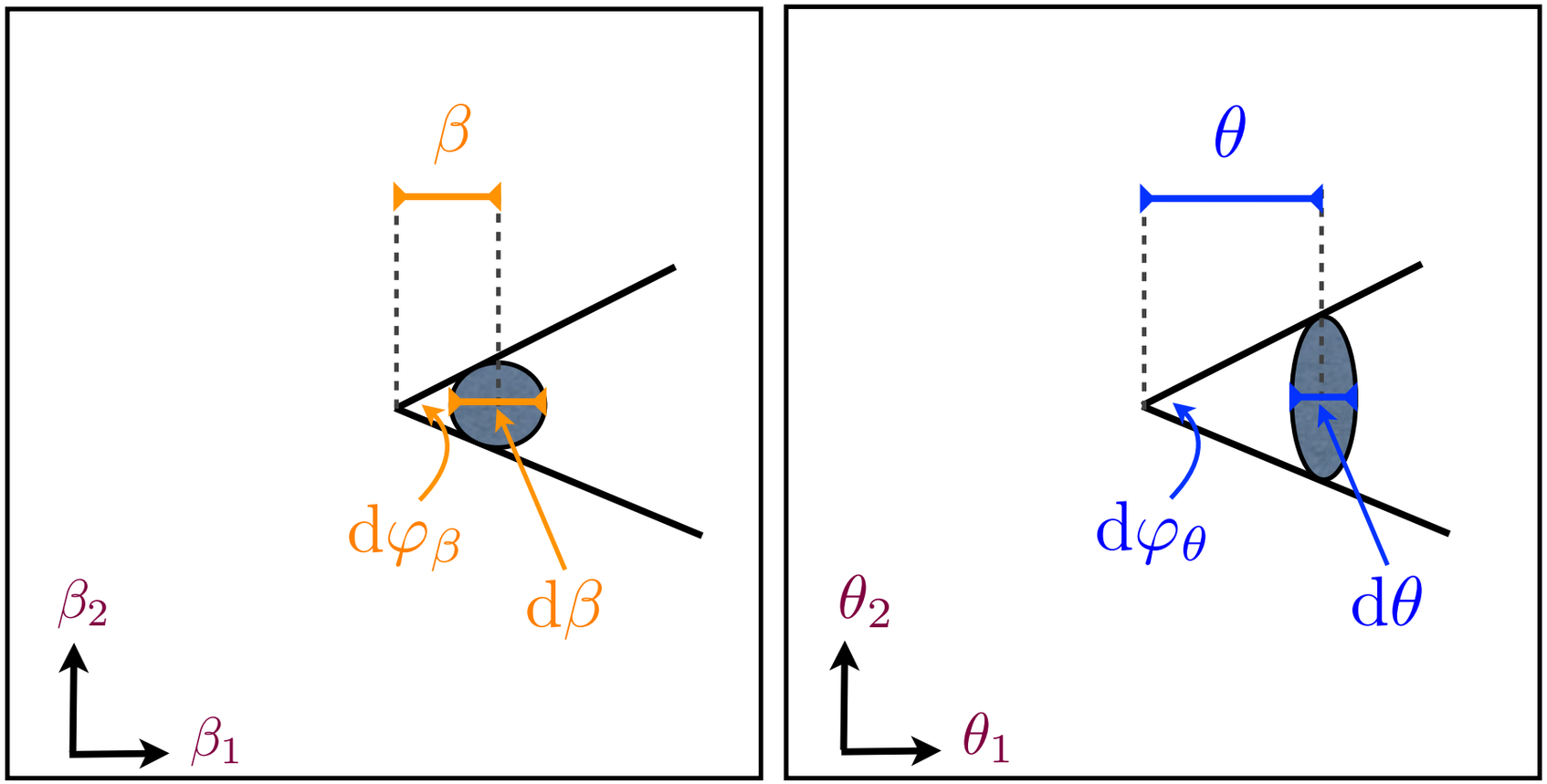
$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

Lens equation

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right)$$

Angular diameter distances

Magnification in the axially symmetric case



Source plane

Lens/images

Radial length	$\mathrm{d}\beta$	$\mathrm{d}\theta$	Radial magnification	$\dfrac{\mathrm{d}\theta}{\mathrm{d}\beta}$
Tangential length	$\beta\,\mathrm{d}\varphi_\beta$	$\theta\,\mathrm{d}\varphi_\theta$	Tangential magnification	$\dfrac{\mathrm{d}\varphi_\theta}{\mathrm{d}\varphi_\beta}\dfrac{\theta}{\beta}$

$= 1$

Lens Equation and Point Mass

Point Mass Lens

Point mass $$\hat{\vec{\alpha}} = \frac{4GM}{c^2\xi}$$

Reduced deflexion angle $$\vec{\alpha} = \frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2\theta}$$

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Einstein angle $$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM}{c^2}}$$

Images and magnification

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Solutions $$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

Magnification $$\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta}$$

Images and magnification

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Solutions $$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

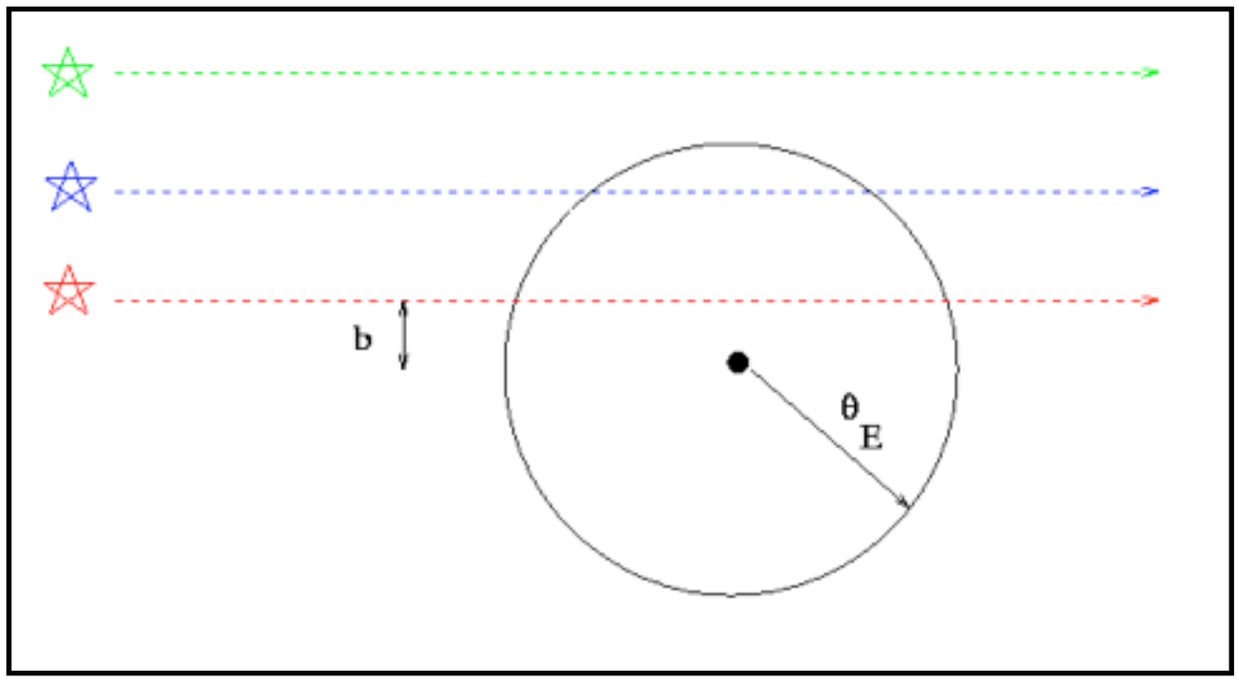
Magnification $$\mu_{1,2} = \left(1 - \left[\frac{\theta_E}{\theta_{1,2}}\right]^4\right)^{-1} = \frac{1}{2} \pm \frac{u^2+2}{2u\sqrt{u^2+4}}$$

Distance in units of the Einstein angle $$u = \beta/\theta_E$$

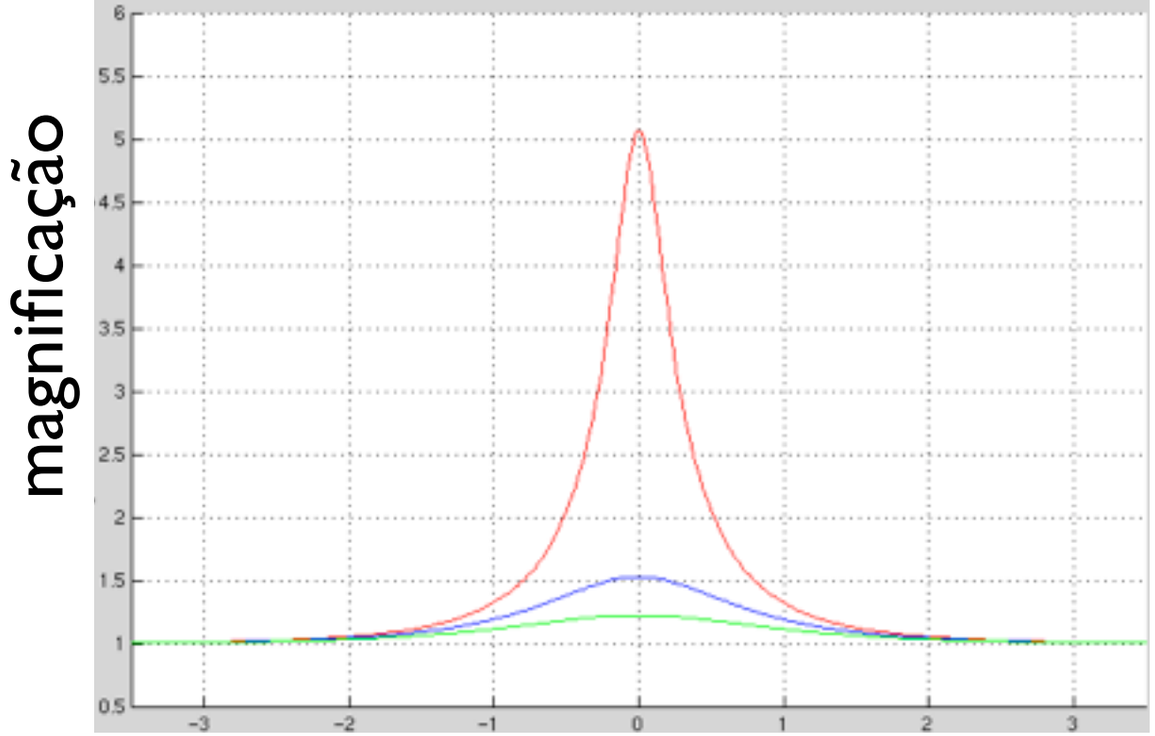
Total magnification $$\mu = |\mu_1| + |\mu_2| = \frac{u^2+2}{u\sqrt{u^2+4}}$$

Microlensing Point Mass

$$\mu = \frac{u^2+2}{u\sqrt{u^2+4}} \qquad u = \beta/\theta_E$$



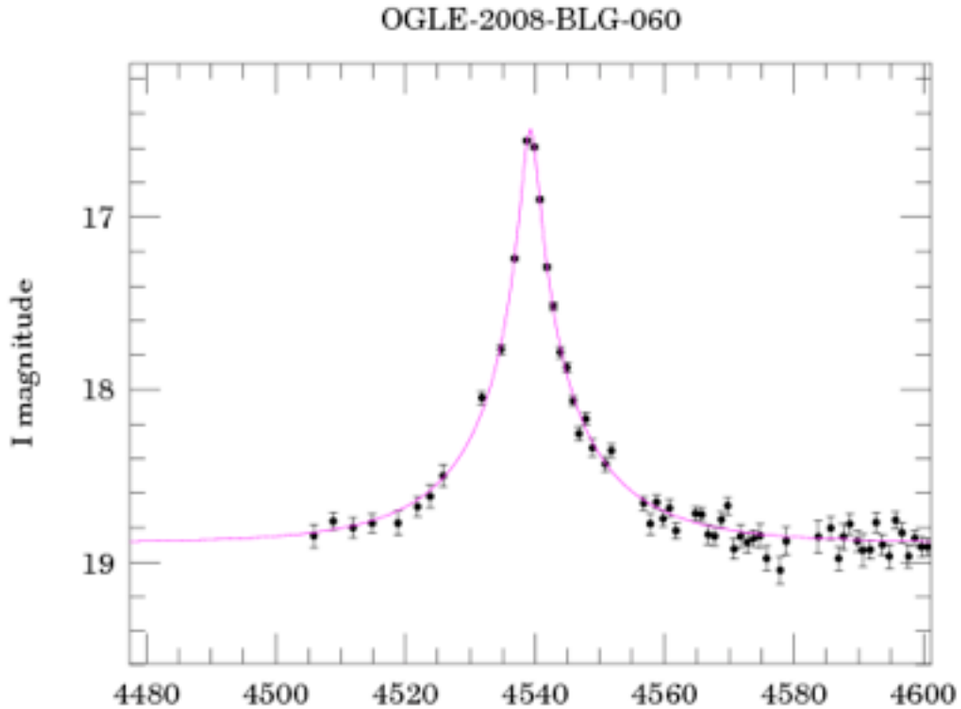
Source plane

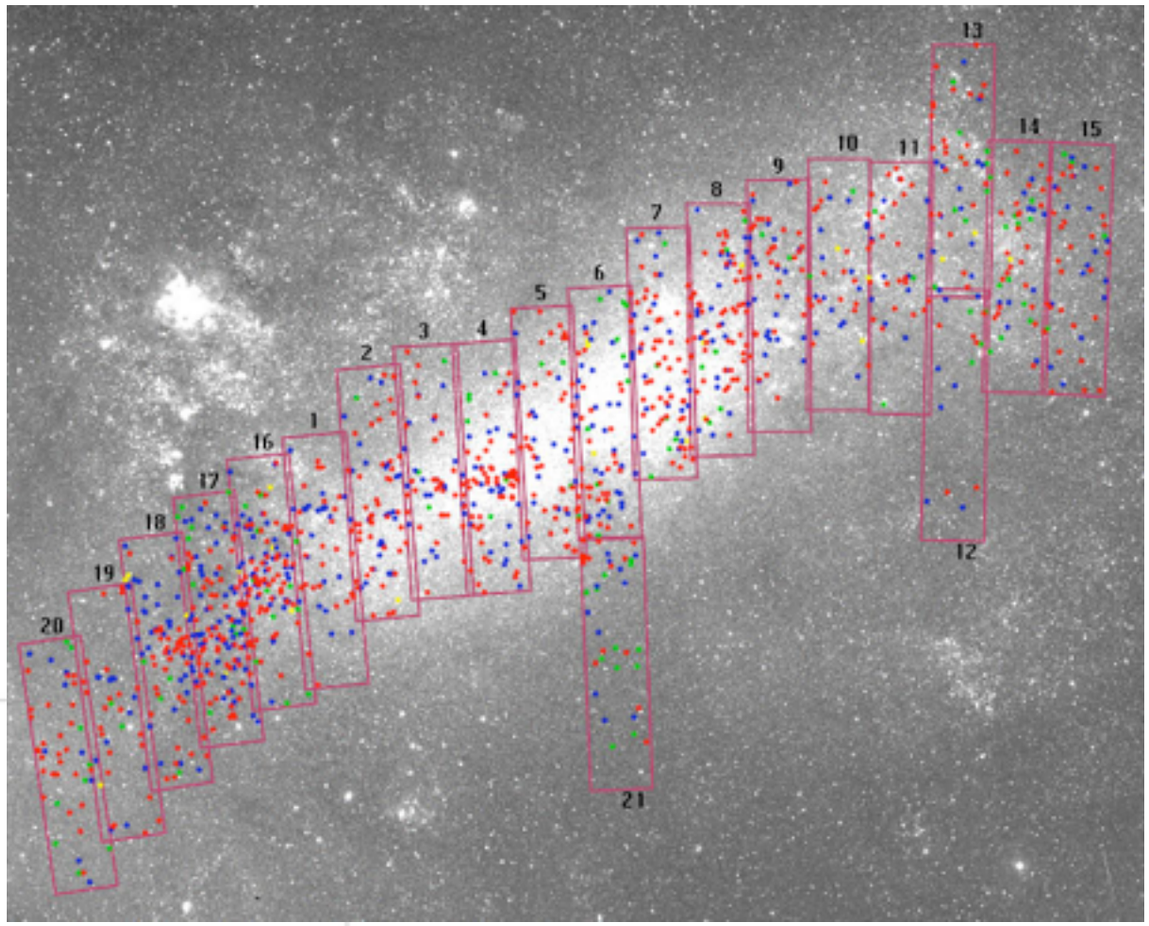


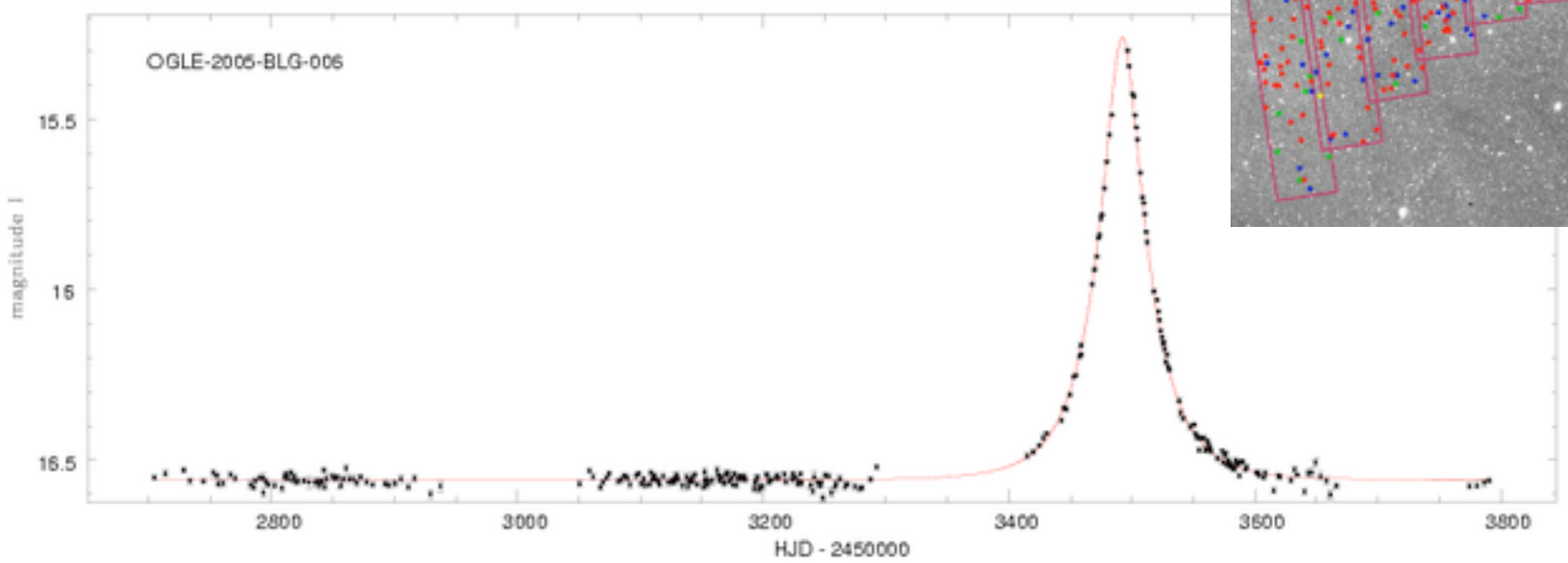
Distance to the lens in the x_1 direction in units of the Einstein angle

Microlensing observations

Projects MACHO, OGLE, EROS...

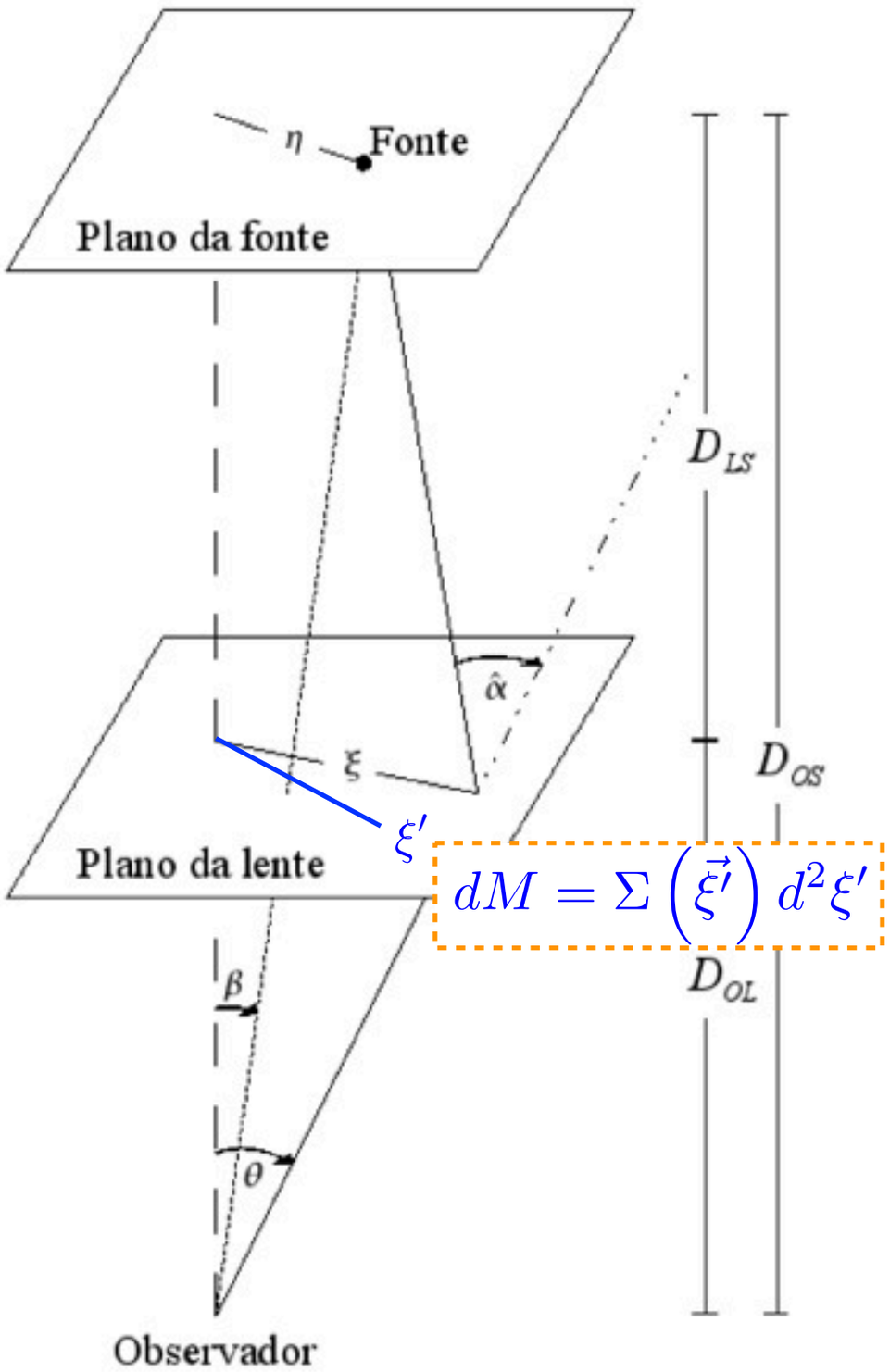






Extended Lenses

Extended Lenses



Point mass $$\hat{\vec{\alpha}} = \frac{4GM}{c^2\xi}$$

Surface mass density

$$\Sigma\left(\vec{\xi}\right) = \int_0^\infty dz\rho\left(\vec{\xi}, z\right)$$

Contribution of the area element

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2}\Sigma\left(\vec{\xi}'\right) d^2\xi' \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Deflexion angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2}\int d^2\xi'\Sigma\left(\vec{\xi}'\right) \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Lentes Extensas

Potencial projetado $\psi\left(\vec{\xi}\right) = \int dz \varphi(\vec{\xi}, z)$

Equação de Poisson $\nabla^2_\xi \psi\left(\vec{\xi}\right) = 4\pi G \Sigma\left(\vec{\xi}\right)$

Usando a função de Green 2D

$$\psi\left(\vec{\xi}\right) = 2G \int d^2\xi' \Sigma\left(\vec{\xi'}\right) \ln|\xi - \xi'|$$

Comparando com o ângulo de deflexão

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma\left(\vec{\xi'}\right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Obtemos $\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_\xi \psi\left(\vec{\xi}\right)$

Lentes Extensas

Ângulo de deflexão $\hat{\vec{\alpha}} = \frac{2}{c^2}\vec{\nabla}_\xi \psi\left(\vec{\xi}\right)$

convertendo variáveis $d\xi_i = D_{OL} d\theta_i$

Ângulo de deflexão reduzido

$$\vec{\alpha} = \hat{\vec{\alpha}}\frac{D_{LS}}{D_{OS}} = \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\vec{\nabla}_\theta \psi\left(\vec{\theta}\right)$$

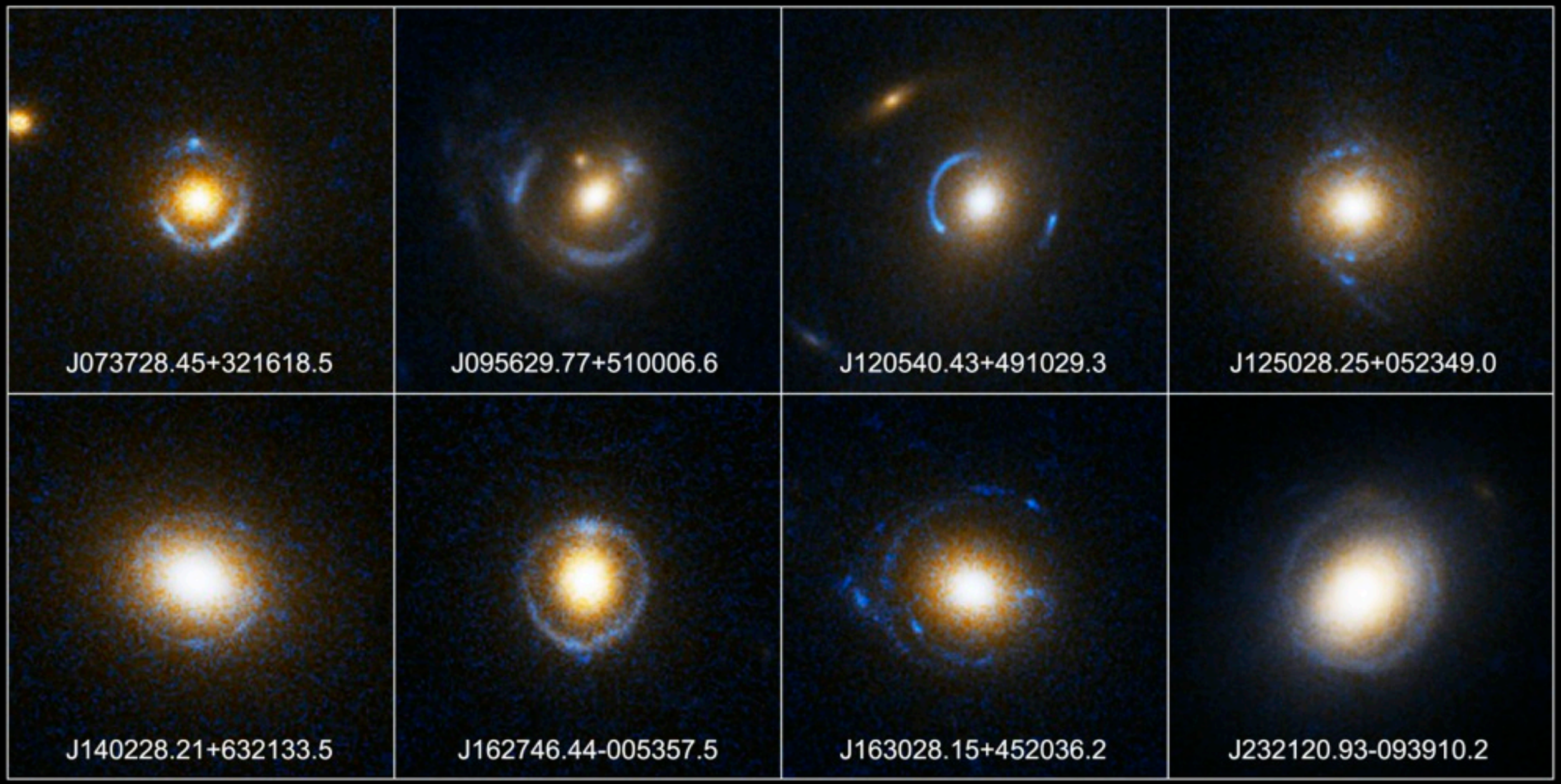
Potencial de Lente(amento) $\Psi(\vec{x}) \equiv \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\psi$

Equação da lente $\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right) = \vec{\theta} - \vec{\nabla}_\theta \Psi\left(\vec{\theta}\right)$

Gravitational Arcs

Strong Lensing
Macrolensing

Chwolson-Einstein rings



Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

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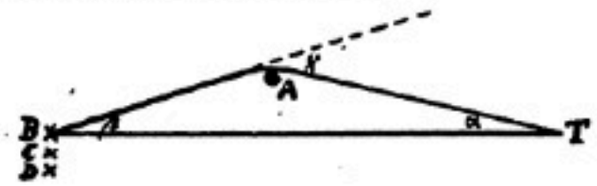
Chwolson-Einstein rings

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geqq \gamma \geqq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''.7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

Petrograd, 1924 Jan. 28.

Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

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A Relativistic Eclipse

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

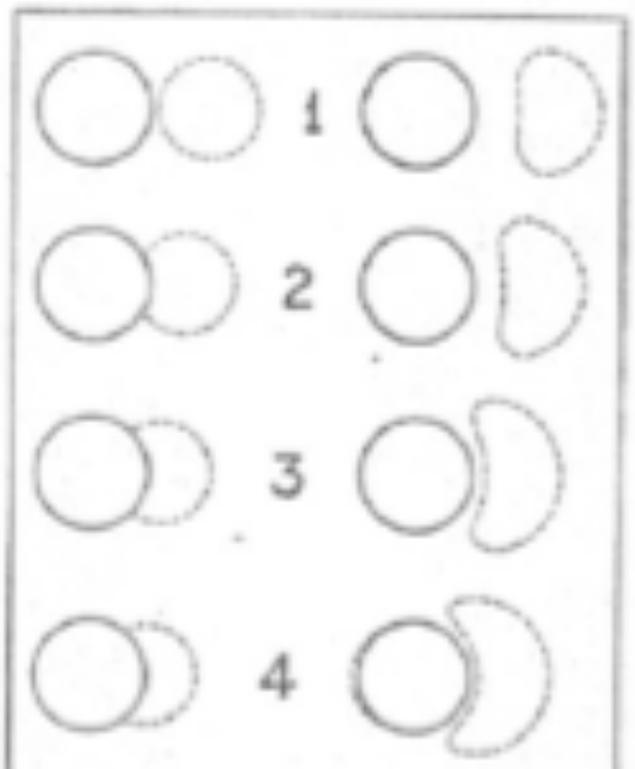
By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires long series of the

No one predicted gravitational arcs?





My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*

Singular Isothermal Sphere

Esfera Isotérmica Singular

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Densidade projetada (superficial)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Massa contida em raio ξ

$$M\left(\xi\right) = \int_0^\xi \Sigma\left(\xi'\right) 2\pi\xi' \, \mathrm{d}\xi' = \sigma_v^2 \frac{\pi}{G} \xi$$

Ângulo de deflexão

$$\Rightarrow \quad \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi}\hat{\xi} = 4\pi\left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

constante!

Equação da Lente

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \underbrace{4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{LS}}{D_{OS}}}_{\theta_E} \hat{\xi}$$

Equação da Lente

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad\qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{LS}}{D_{OS}} \hat{\xi} = \theta_E \, \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \, \hat{\theta} = \left(1 - \frac{\theta_E}{\theta}\right) \vec{\theta}$$

$$\Rightarrow \quad \beta = \left|1 - \frac{\theta_E}{\theta}\right| \theta$$

Soluções: 1) se $1 - \frac{\theta_E}{\theta} > 0$

então $\beta = \left(1 - \frac{\theta_E}{\theta}\right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

Equação da Lente

$$\beta = \left|1 - \frac{\theta_E}{\theta}\right| \theta$$

Soluções: I) se $1 - \frac{\theta_E}{\theta} > 0$

então $\beta = \left(1 - \frac{\theta_E}{\theta}\right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

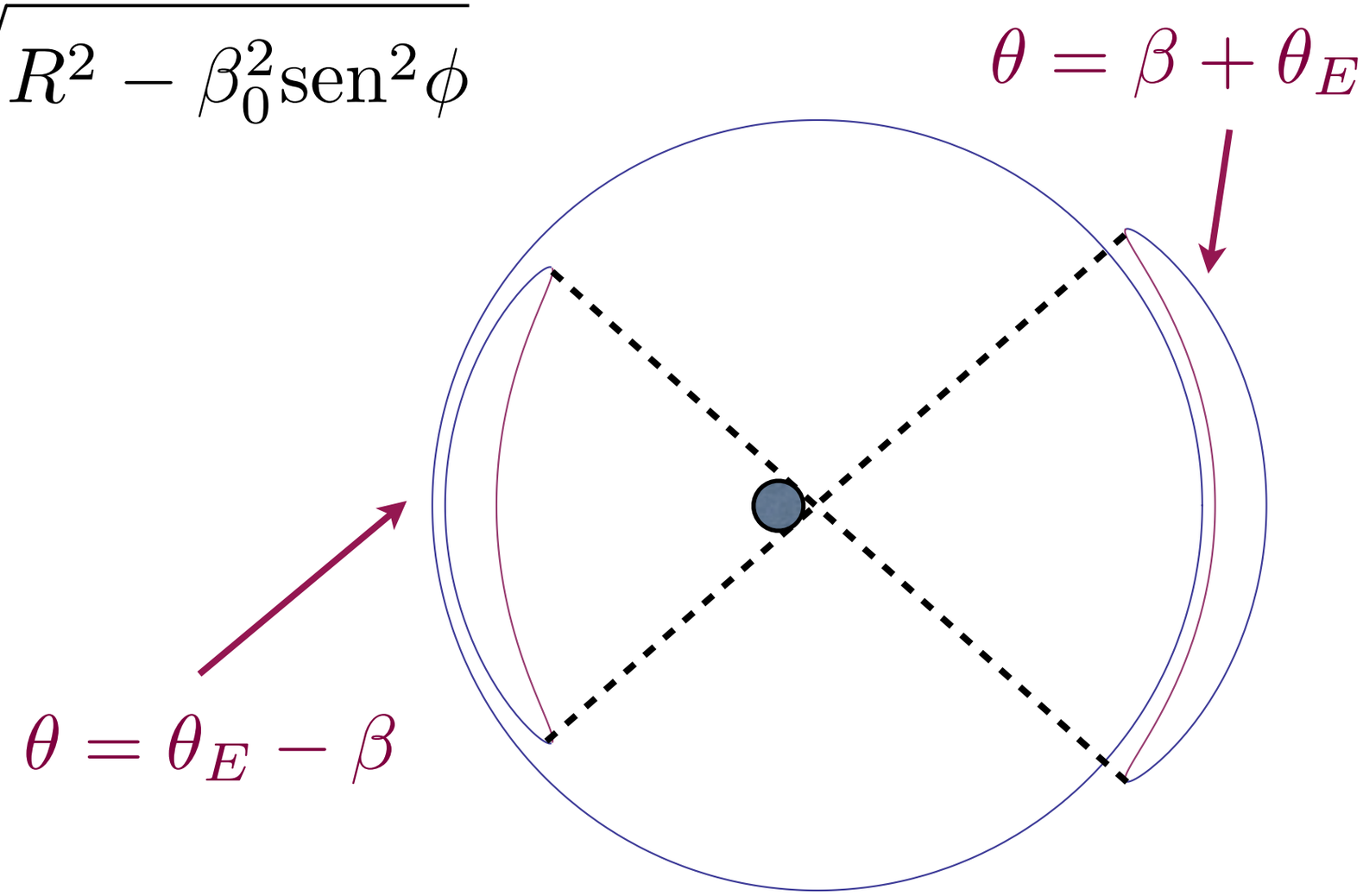
II) se $1 - \frac{\theta_E}{\theta} < 0$

então $\beta = -\left(1 - \frac{\theta_E}{\theta}\right) \theta = \theta_E - \theta \Rightarrow \boxed{\theta = \theta_E - \beta}$

Fonte circular

Círculo no plano das fontes: $\left|\vec{\beta} - \vec{\beta}_0\right| = R$

$$\beta = \beta_0 \cos\phi \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2\phi}$$



Caso geral $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2(\phi - \phi_0)}$

Modelos de Lentes

Modelos de potencial

Esfera isotérmica singular $\Psi = \theta_E \left|\vec{\theta}\right|$

Esfera isotérmica com "caroço" $\Psi = \theta_0 \sqrt{\theta + \theta_c^2}$

Convergência $\kappa = \theta_0 \dfrac{\theta^2 + 2\theta_c^2}{2\left(\theta^2 + \theta_c^2\right)^{3/2}}$

Perfil de densidade

Esfera isotérmica suavizada $$\rho(r) = \frac{\sigma^2}{2\pi G(r^2 + r_c^2)}$$

Densidade projetada $$\Sigma(\theta) = \frac{\sigma^2}{2GD_{OL}} \frac{1}{\sqrt{\theta^2 + \theta_c^2}}$$

Potencial de lente

$$\Psi(\vec{\theta}) = \theta_0 \left[\sqrt{\theta^2 + \theta_c^2} - \theta_c \ln \left(\sqrt{\theta^2 + \theta_c^2} + \theta_c \right) \right]$$

Outros modelos

Alguns modelos implementados no *gravlens* (+ Sérsic, Einasto, etc.)

Model	N_r	Density $\rho(r)$	Surface Density $\kappa(r)$
Point mass	0	$\delta(\mathbf{x})$	$\delta(\mathbf{x})$
Power law or α-models	2	$\left(s^2+r^2\right)^{(\alpha-3)/2}$	$\left(s^2+r^2\right)^{(\alpha-2)/2}$
Isothermal ($\alpha=1$)	1	$\left(s^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}$
$\alpha=-1$	1	$\left(s^2+r^2\right)^{-2}$	$\left(s^2+r^2\right)^{-3/2}$
Pseudo-Jaffe	2	$\left(s^2+r^2\right)^{-1}\left(a^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}-\left(a^2+r^2\right)^{-1/2}$
King (approximate)	1	$\cdots$	$2.12\left(0.75r_s^2+r^2\right)^{-1/2}$ $-1.75\left(2.99r_s^2+r^2\right)^{-1/2}$
de Vaucouleurs	1	$\cdots$	$\exp\left[-7.67(r/R_e)^{1/4}\right]$
Hernquist	1	$r^{-1}\left(r_s+r\right)^{-3}$	see eq. (47)
NFW	1	$r^{-1}\left(r_s+r\right)^{-2}$	see eq. (53)
Cuspy NFW	2	$r^{-\gamma}\left(r_s+r\right)^{\gamma-3}$	see eq. (57)
Cusp	3	$r^{-\gamma}\left(r_s^2+r^2\right)^{(\gamma-n)/2}$	see eq. (64)
Nuker	4	$\cdots$	see eq. (71)
Exponential disk	1	$\cdots$	$\exp[-r/R_d]$
Kuzmin disk	1	$\cdots$	$\left(r_s^2+r^2\right)^{-3/2}$

Elliptical models

Subsituir $\theta \rightarrow \sqrt{q_1\theta_1^2 + q_2\theta_2^2}$

- no potencial (modelos pseudo-elípticos)
- na densidade projetada (modelos elípticos)

Exemplo: Esfera isotérmica com "caroço" pseudo-elíptica

$$\Psi(\theta_1, \theta_2) = \frac{\Psi_0}{\theta_c}\sqrt{(1-\epsilon)\theta_1^2 + (1+\epsilon)\theta_2^2 + \theta_c^2}$$