Summary of part II: the homogeneous Universe

Two paths: Newtonian | General Relativity

Energy conservation (of test particle) in spherical gravitational potential

First law of thermodynamics

(+ special relativity)

Friedmann's Equation

$$\dot{a}^2 = \frac{8\pi G}{3} a^2 \rho - K$$

Energy Conservation

$$\frac{d\rho}{da} + \frac{3}{a}(\rho + p) = 0$$

$$H^2(a) = H_0^{\,2}\left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

©Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

Energy conservation (of test particle) in spherical gravitational potential

First law of thermodynamics

(+ special relativity)

General Relativity

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

\+ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ Einstein's equation

Friedmann's Equation

$$\dot{a}^2 = \frac{8\pi G}{3}a^2\rho - K$$

Energy Conservation

$$\frac{d\rho}{da} + \frac{3}{a}(\rho + p) = 0$$

$$H^2(a) = H_0^{\,2}\left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

© Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

General Relativity

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

© Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

Light propagation in co-moving coordinates ($c = 1$)

$$a(t)dr = dt$$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

General Relativity

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

© Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

General Relativity

Light propagation in co-moving coordinates ($c = 1$)

$$a(t)dr = dt$$

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

Flux: $$F = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

$$d_L = a(t_0) r_1 (1+z)$$

© Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

Light propagation in co-moving coordinates ($c = 1$)

$$a(t)dr = dt$$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

Flux: $$F = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

$$d_L = a(t_0) r_1 (1+z)$$

For $K = 0$

$$d_L = (1+z)\,\mathrm{sen}_K\left(H_0 \sqrt{|1-\Omega_0|} \int_0^z \frac{dz'}{H(z')} \right) \Big/ H_0 \sqrt{|1-\Omega_0|}$$

General Relativity

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2} dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2 \right)$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2} dr^2 \right)$

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-Kr^2}} \equiv f(r_1)$$

© Copyleft Martín Makler

Summary of part II: the homogeneous Universe

Two paths: Newtonian

Light propagation in co-moving coordinates ($c = 1$)

$$a(t)\,dr = dt$$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

Flux: $$F = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

For $K = 0$

$$d_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_r (1+z')^4 + \Omega_M (1+z')^3 + \Omega_\Lambda}}$$

General Relativity

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1$$

Flux: $$F = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

$$d_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_r (1+z')^4 + \Omega_M (1+z')^3 + \Omega_\Lambda}}$$

©Copyleft Martín Makler

The Dark Side of the Universe

Episode II

The Accelerating Universe



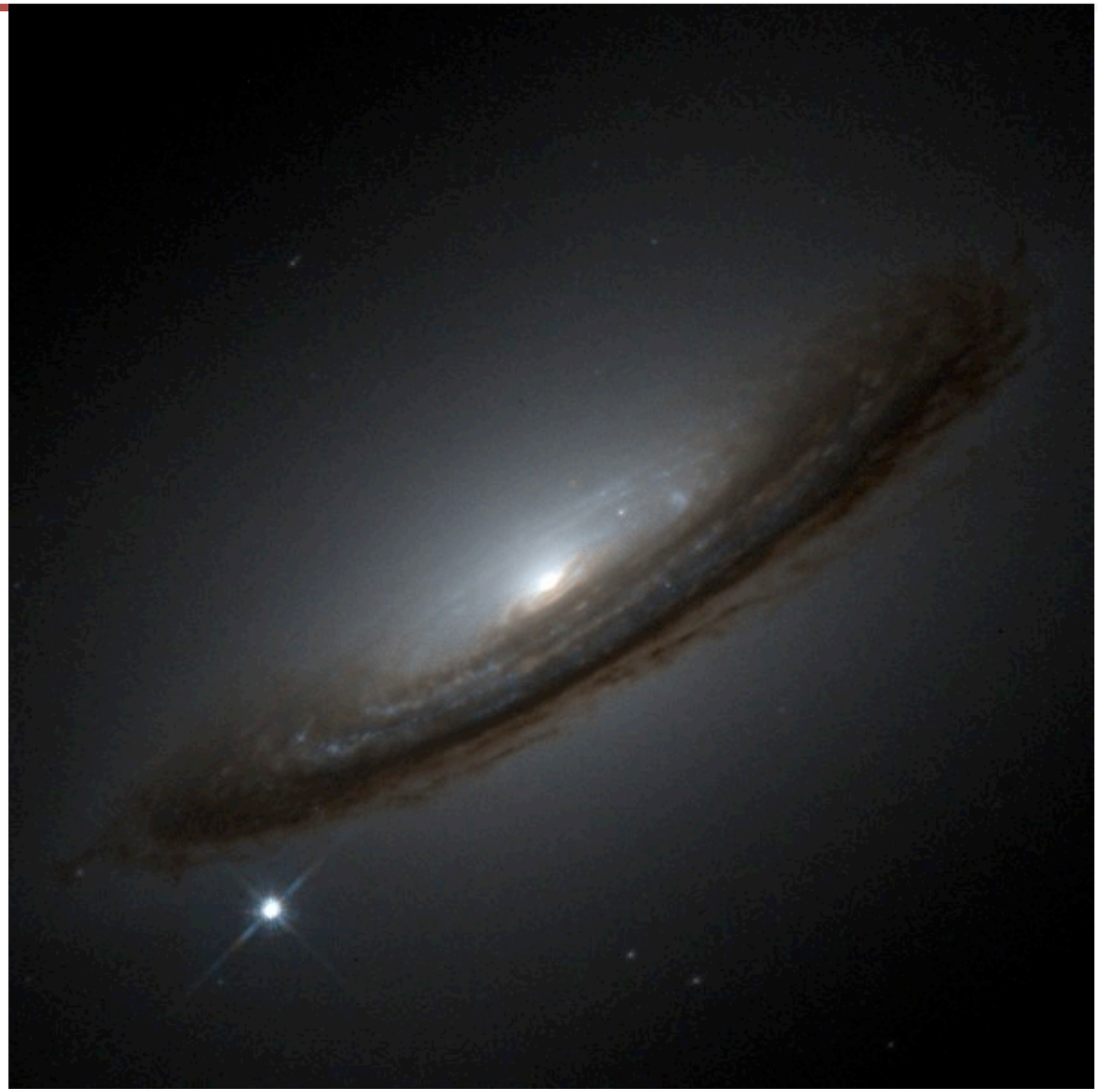
© Copyleft Martín Makler

Type Ia Supernovae and Cosmology

Advantages:

- Extreme Luminosities (10^9 - 10^{10} $L_\odot$)

➔ May be detected at large distances

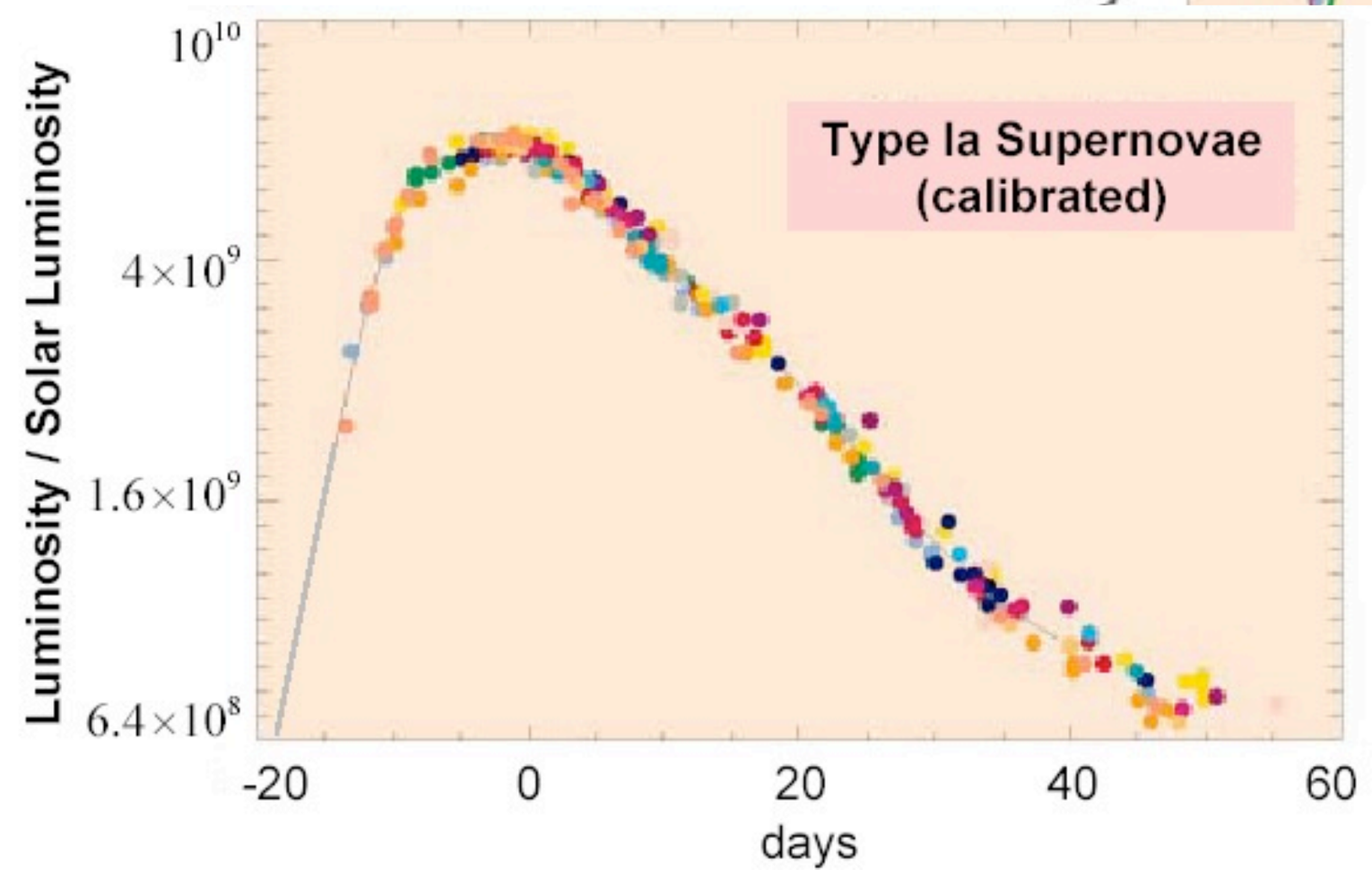


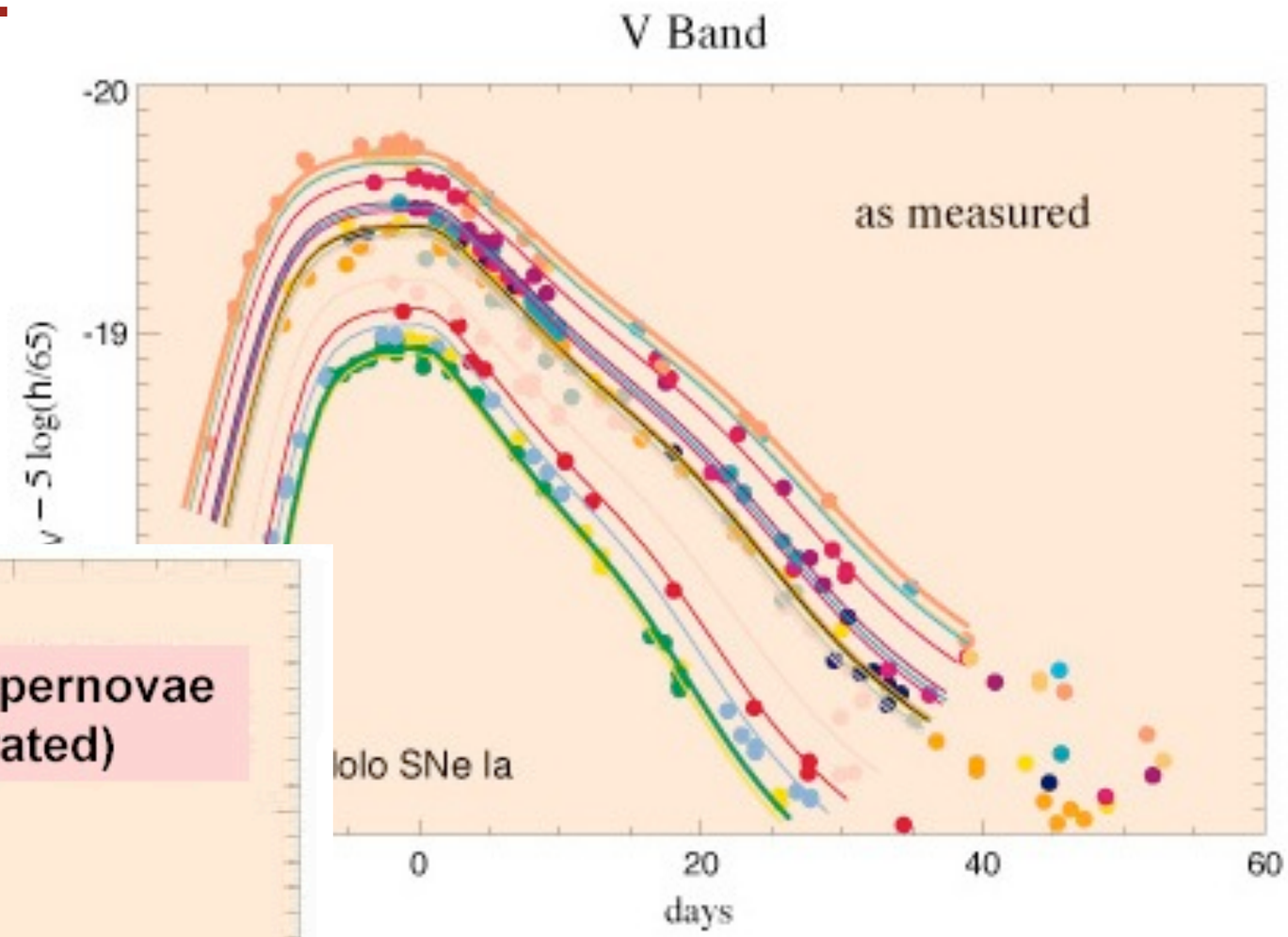
©Copyleft Martín Makler

Type Ia Supernovae Light Curves

Very homogeneous

➔ Standardizable candles





Type Ia Supernovae and Cosmology

Advantages:

- Extreme Luminosities (10^9 - 10^{10} $L_\odot$)
- Very homogeneous

➔ Standardizable candles

Disadvantages:

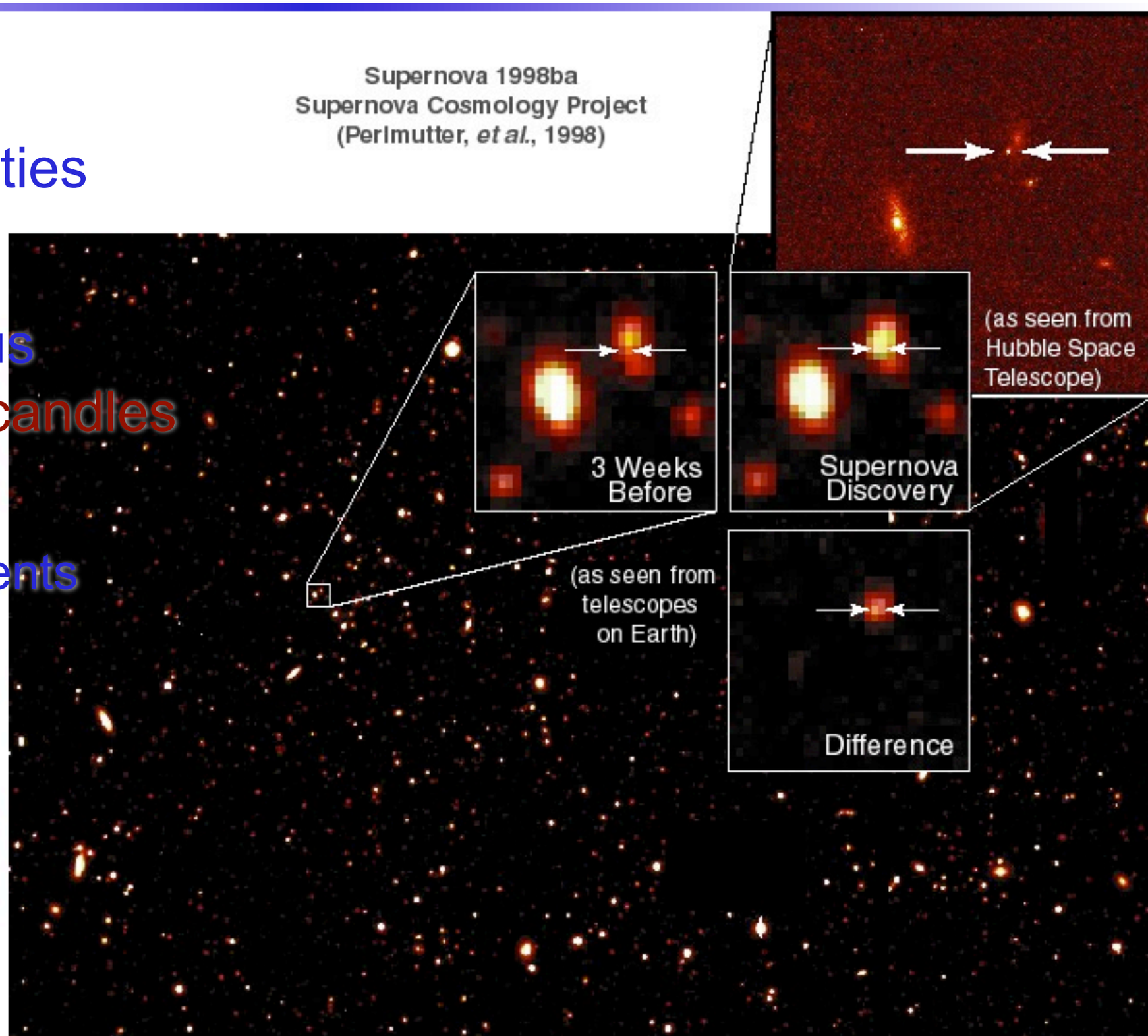
- Rare and random events

~ 1/500 yr/galaxy

- Short duration

Solution:

- Automated search
- SCP, High-z team



© Copyleft Martín Makler

The Accelerating Universe

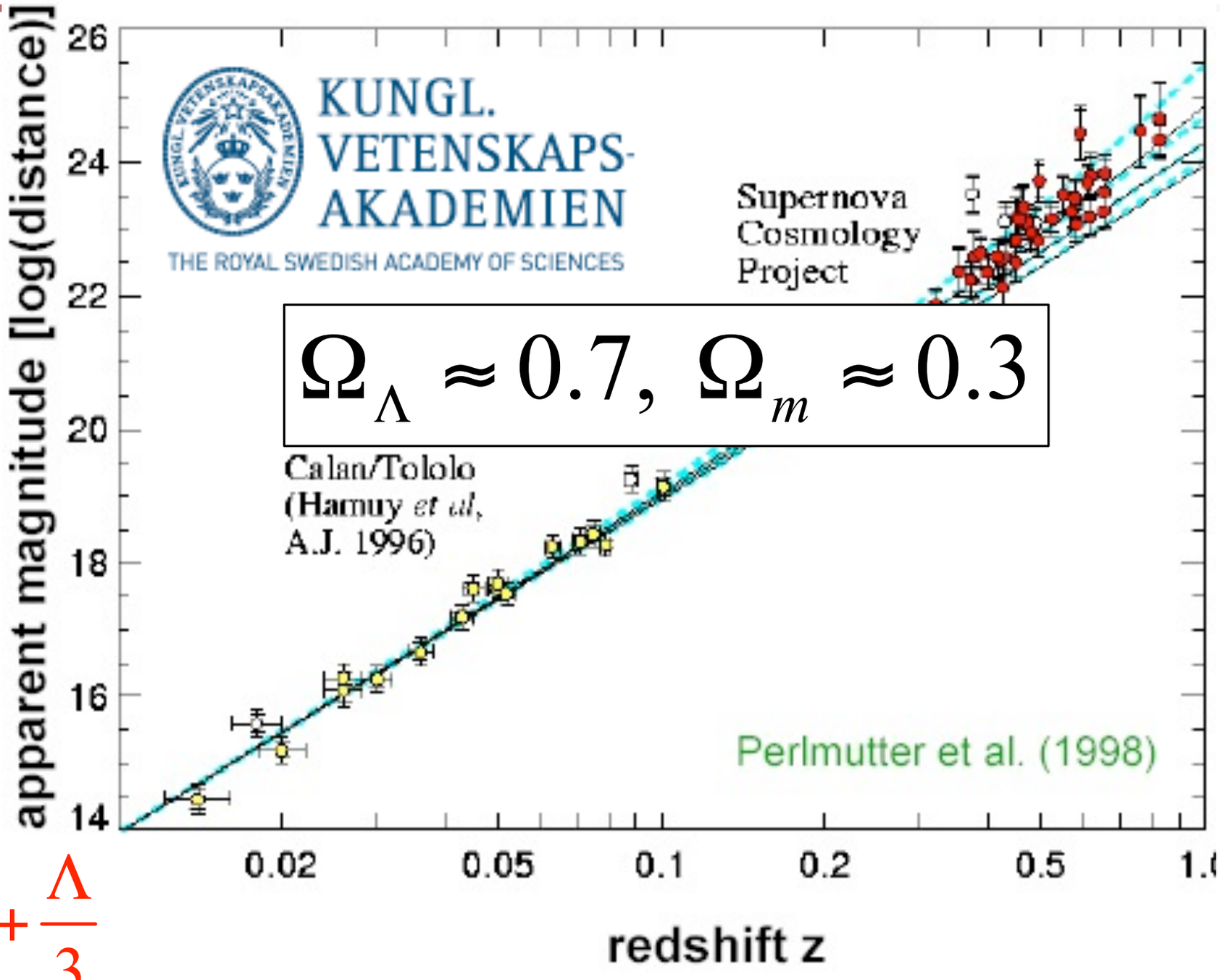
- Hubble diagram for large distances

➔ **The expansion is accelerating**

$$q_0 = -\frac{\ddot{a}_0}{a_0 H_0^2}$$

But

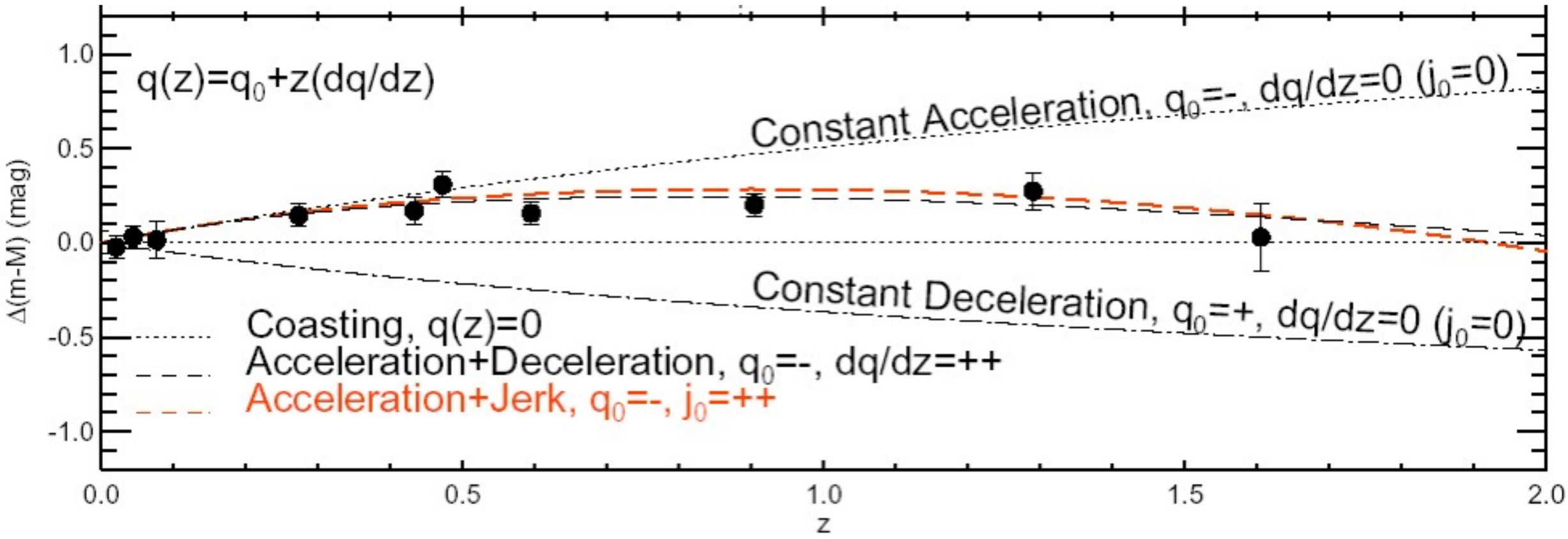
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$



⟹ **Dark Energy or Cosmological Constant**

© Copyleft Martín Makler

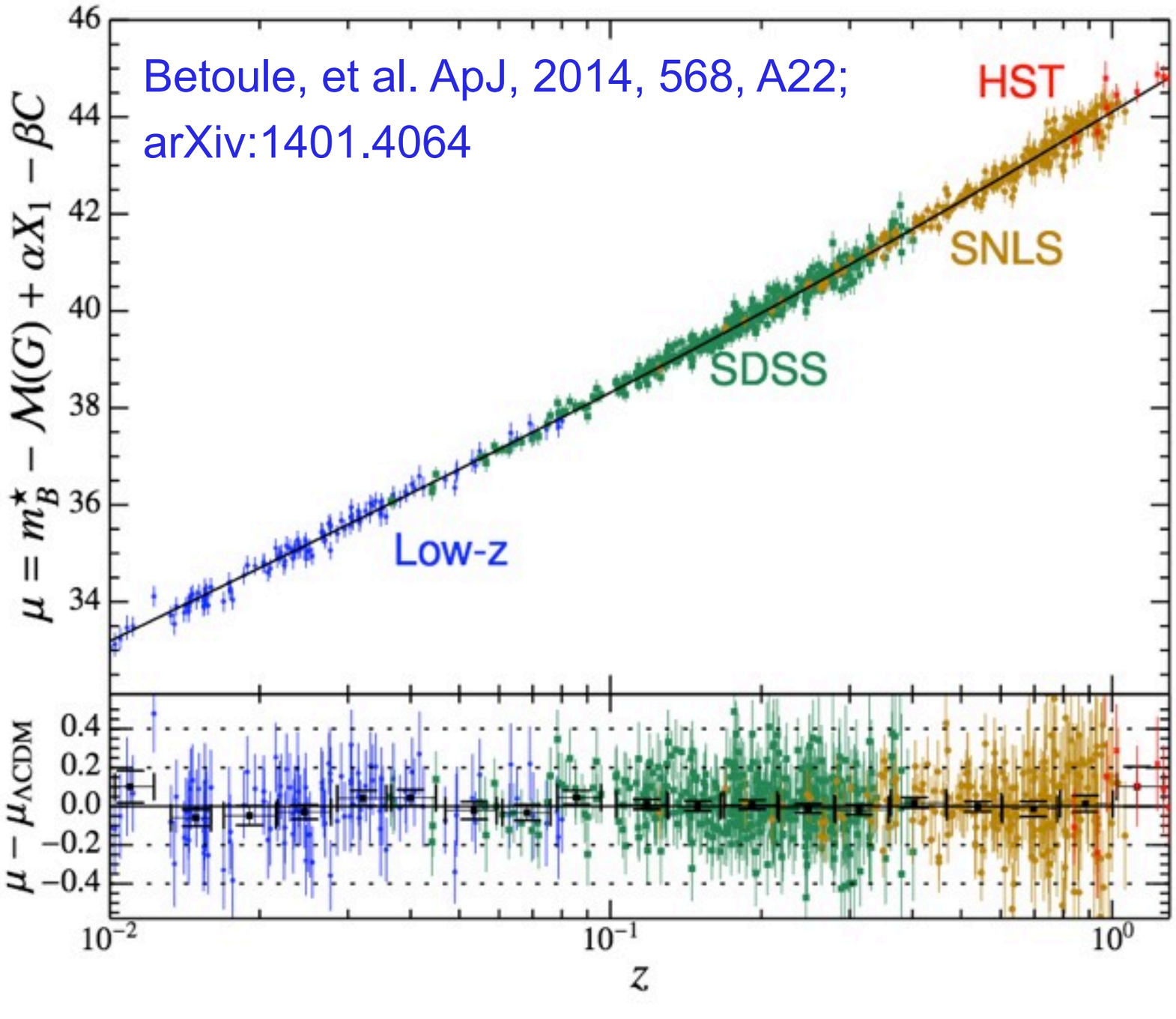
The Decelerated Universe!



© Copyleft Martín M

Current Results

Joint light-curve analysis (JLA)



$$\Omega_m = 0.295 \pm 0.034$$

$$p = w\rho$$

Combined with CMB (+flat):

$$w = -1.027 \pm 0.055$$

Combined with BAO (+flat):

$$w = -1.018 \pm 0.057$$

Age of the Universe

Age x H_0^{-1}

$$\dot{a}_0 = a_0 / t_{0\text{est}}$$

Evolution of the Universe ($\Lambda = 0$)

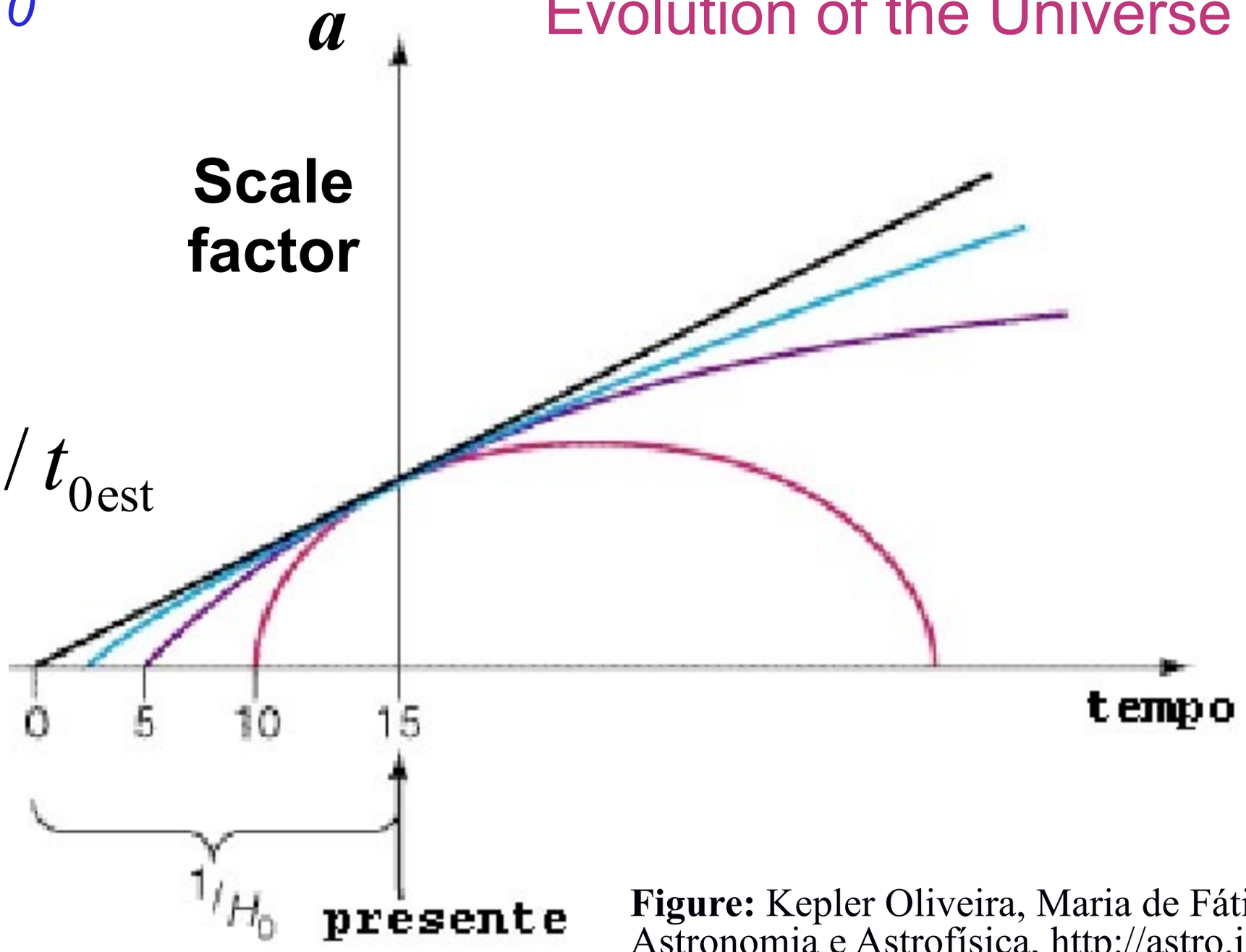


Figure: Kepler Oliveira, Maria de Fátima Saraiva Astronomia e Astrofísica, http://astro.if.ufrgs.br/

© Copyleft Martín Makler

Age of the Universe

- Acceleration solves the age problem

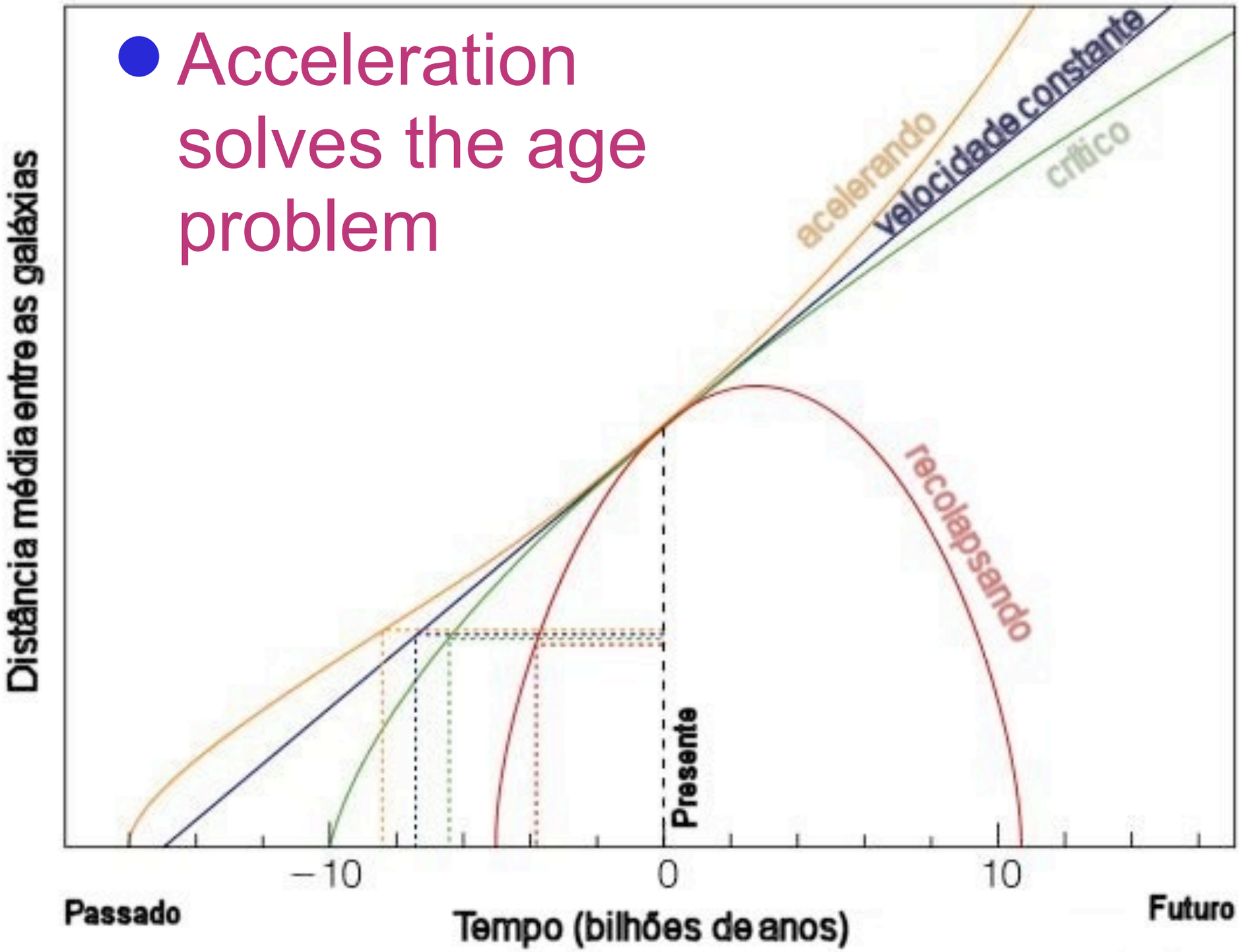


Figure: Kepler Oliveira, Maria de Fátima Saraiva, Astronomia e Astrofísica, http://astro.if.ufrgs.br/

© Copyleft Martín Makler

Dark Energy

- 2/3 of the energy density of the Universe are in the form of Dark Energy! (or $\Lambda \neq 0$)

Evidences:

- Accelerated expansion of distant galaxies
- Age of the Universe
- Small curvature
- Integrated Sachs-Wolfe effect
- Combined analyses of cosmological observables (cosmic concordance)

Candidates (Taxonomy of Dark Energy):

- Cosmological constant
- Scalar field:
  - Quintessence
  - Quartessence, k-essence, spintessencia, snot...

Modified gravity

© Copyleft Martín Makler

Part III

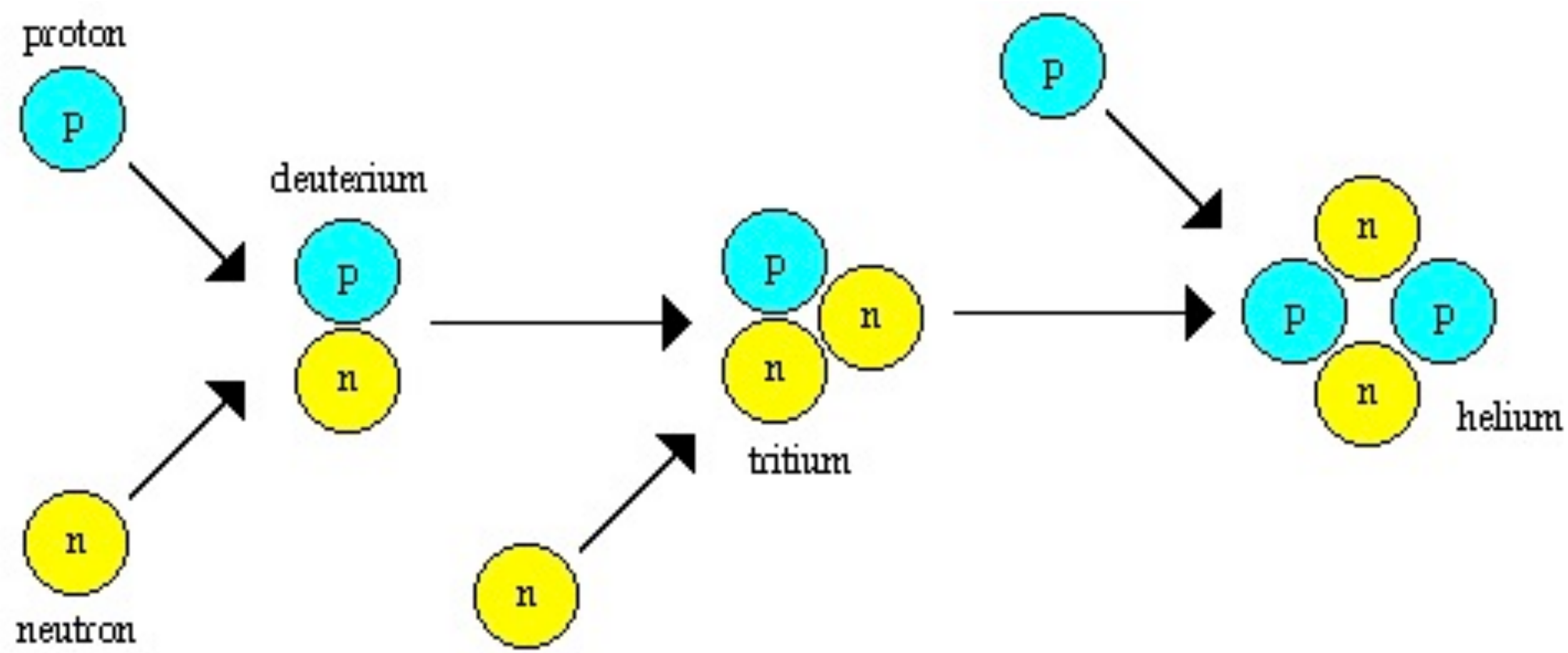
The Homogeneous Universe II:

A Brief Thermal History of the Universe

© Copyleft Martín Makler

Nucleosynthesis: Alchemy in the primordial Universe

Production of ^{7}Li, ^{3}He, D, ^{4}He



$z \sim 10^4$, 3MeV, $T \sim 10^{10}$ K - 10^9 K, *1*s a *3*min "after BB"

© Copyleft Martín Makler

Cartoon from http://blueox.uoregon.edu/~karen/astro123/lectures/lec20.html

Light Element Production

Radiation dominated Universe:

- *H = H(T)*
- interaction rate *f(T)* *depends only on* $\eta = n_b/n_\gamma$

(Radiation dominated Universe: $a \propto t^{1/2}$

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}$$)

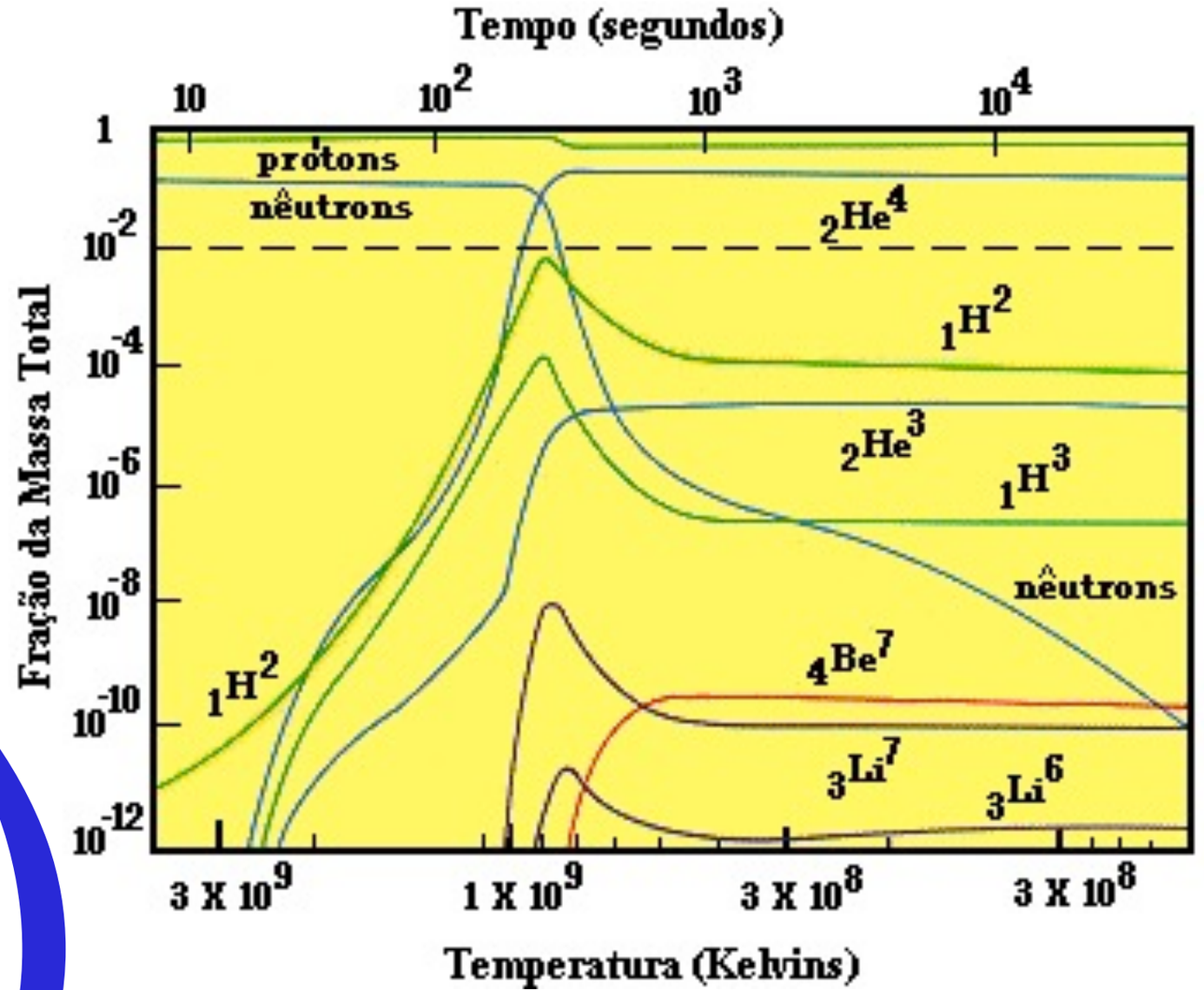
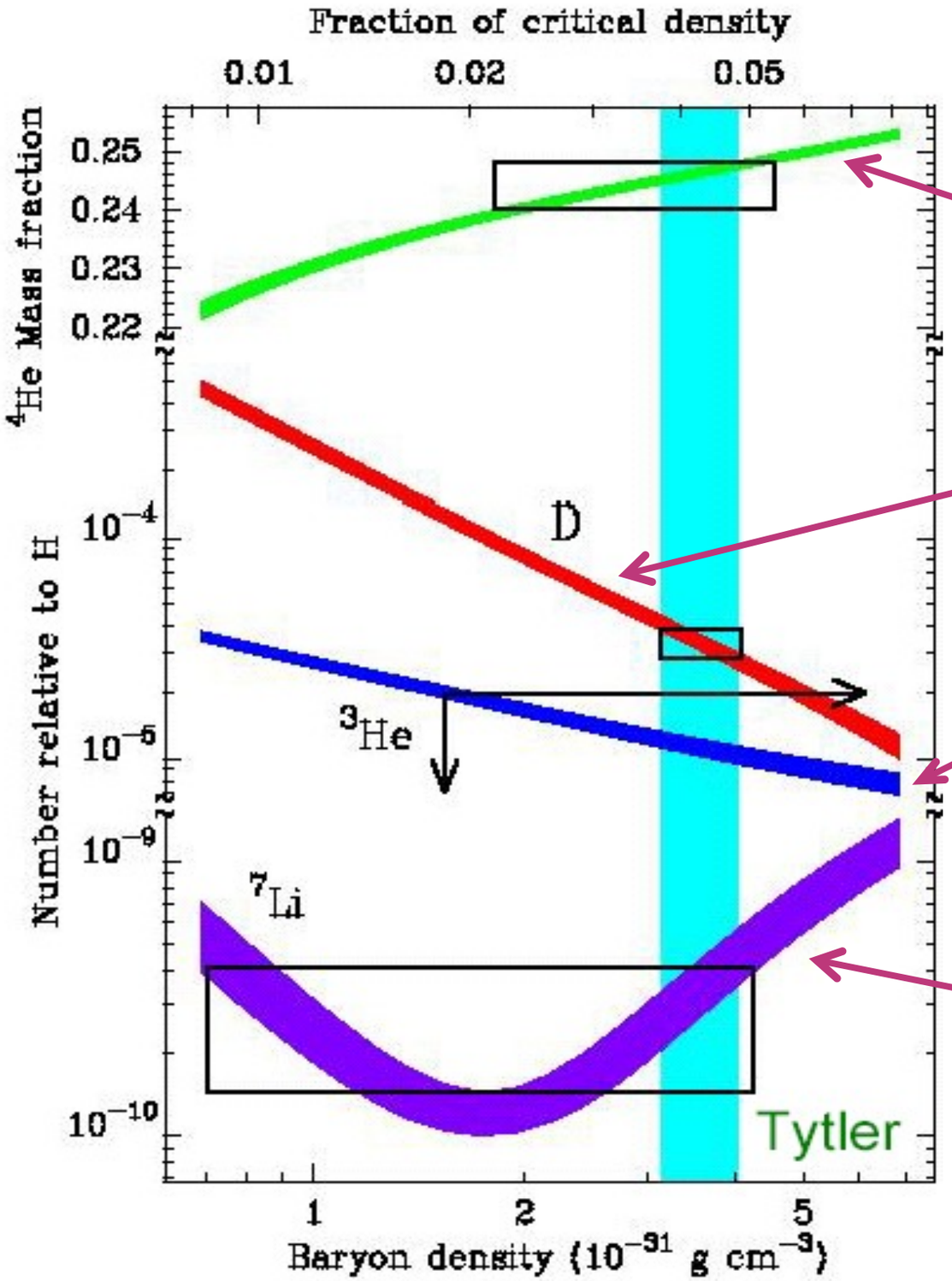


Figure: Kepler Oliveira, Maria de Fátima Saraiva Astronomia e Astrofísica, http://astro.if.ufrgs.br/

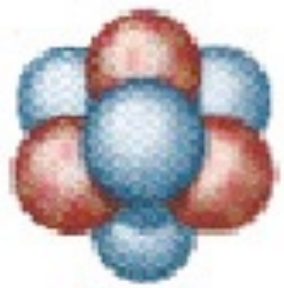
Light Element Abundance











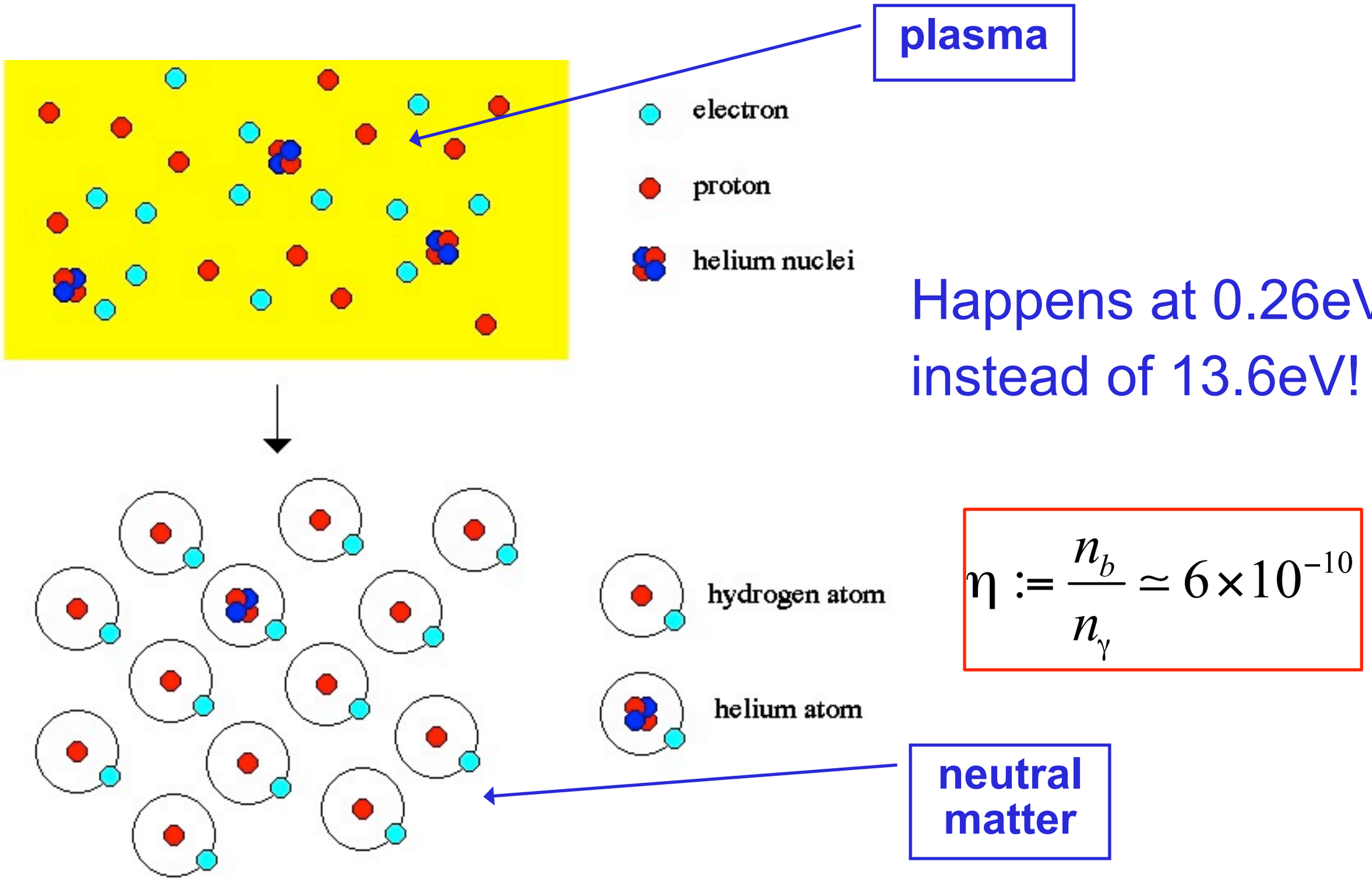
Production of ^{7}Li, ^{3}He, D, ^{4}He with the same η!

D is the best "baryometer"

$$\Omega_b \simeq 0,04$$

Independent of Dark Matter

Recombination



Happens at 0.26eV instead of 13.6eV!

$$\eta := \frac{n_b}{n_\gamma} \simeq 6 \times 10^{-10}$$

© Copyleft Martín Makler

Figure from http://blueox.uoregon.edu/~karen/astro123/lectures/lec20.html

Recombination

When the temperature drops below 3.000K the electrons get bounded to the nuclei

- The Universe becomes transparent
- Light propagates freely

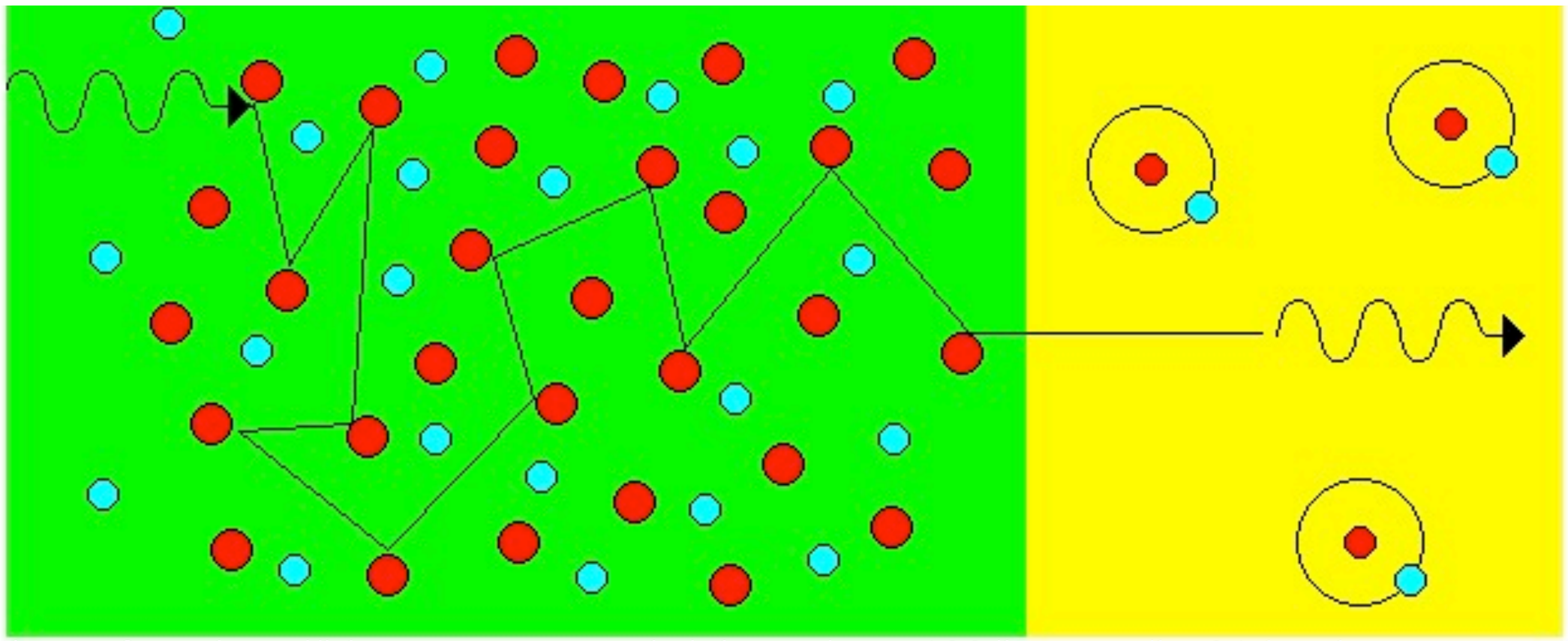


Figure from http://blueox.uoregon.edu/~karen/astro123/lectures/lec21.html

© Copyleft Martín Makler

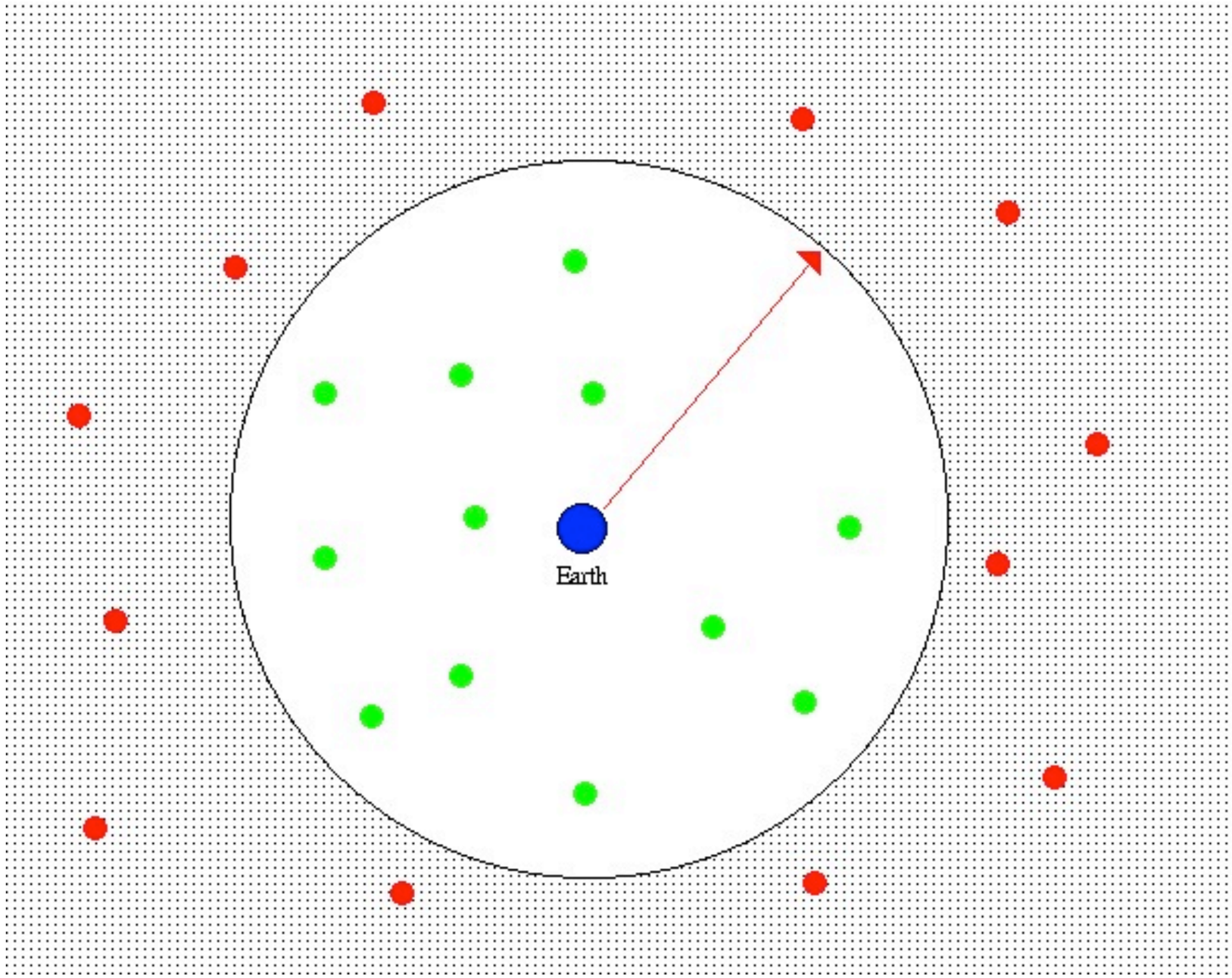
The Cosmic Microwave Background

"Photosphere" seen from our galaxy

Expect to see:

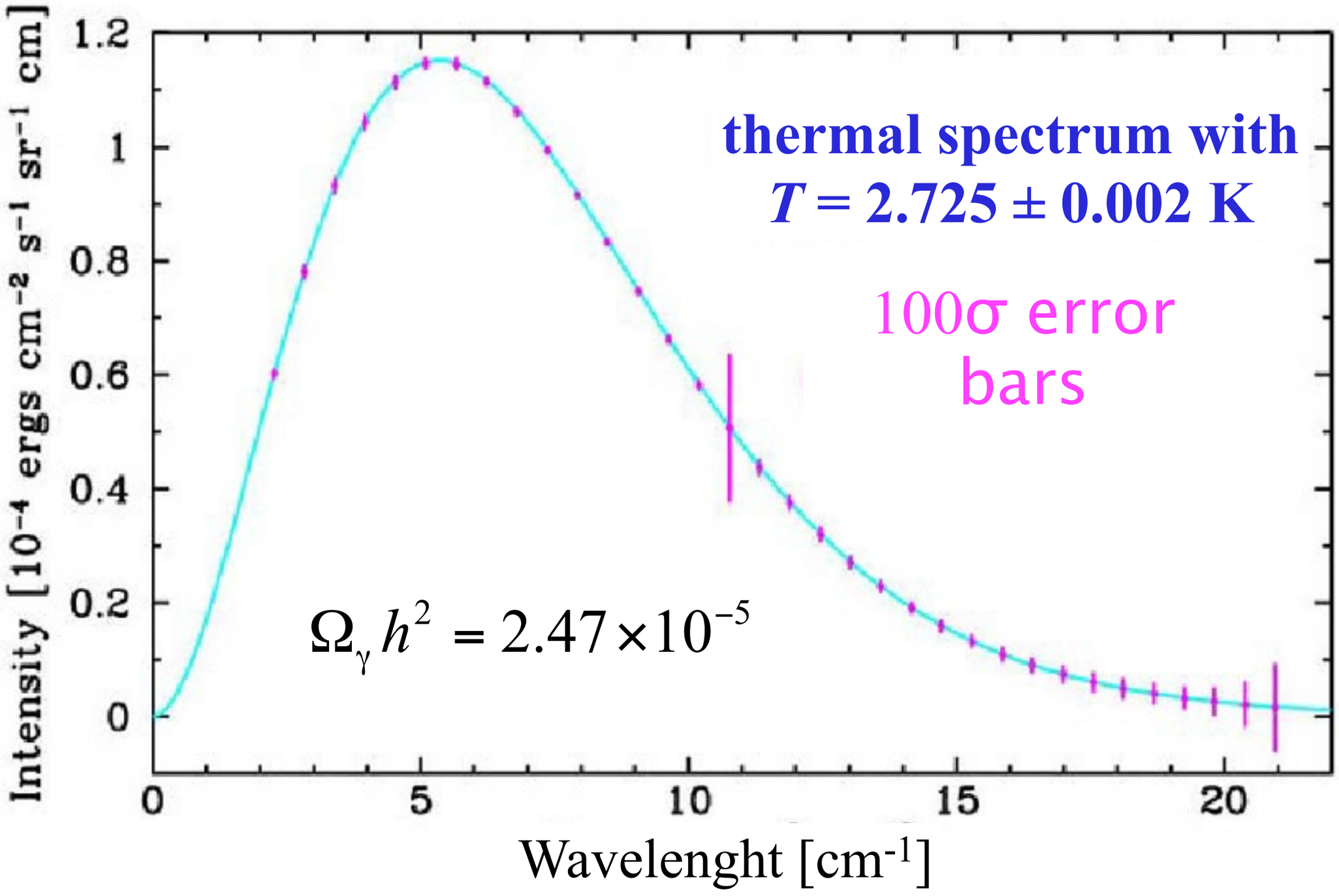
Black body with redshift

$z \sim 1000$



© Copyleft Martín Makler

Spectrum of the CMB



© Copyleft Martín Makler

Cosmic Microwave Background I

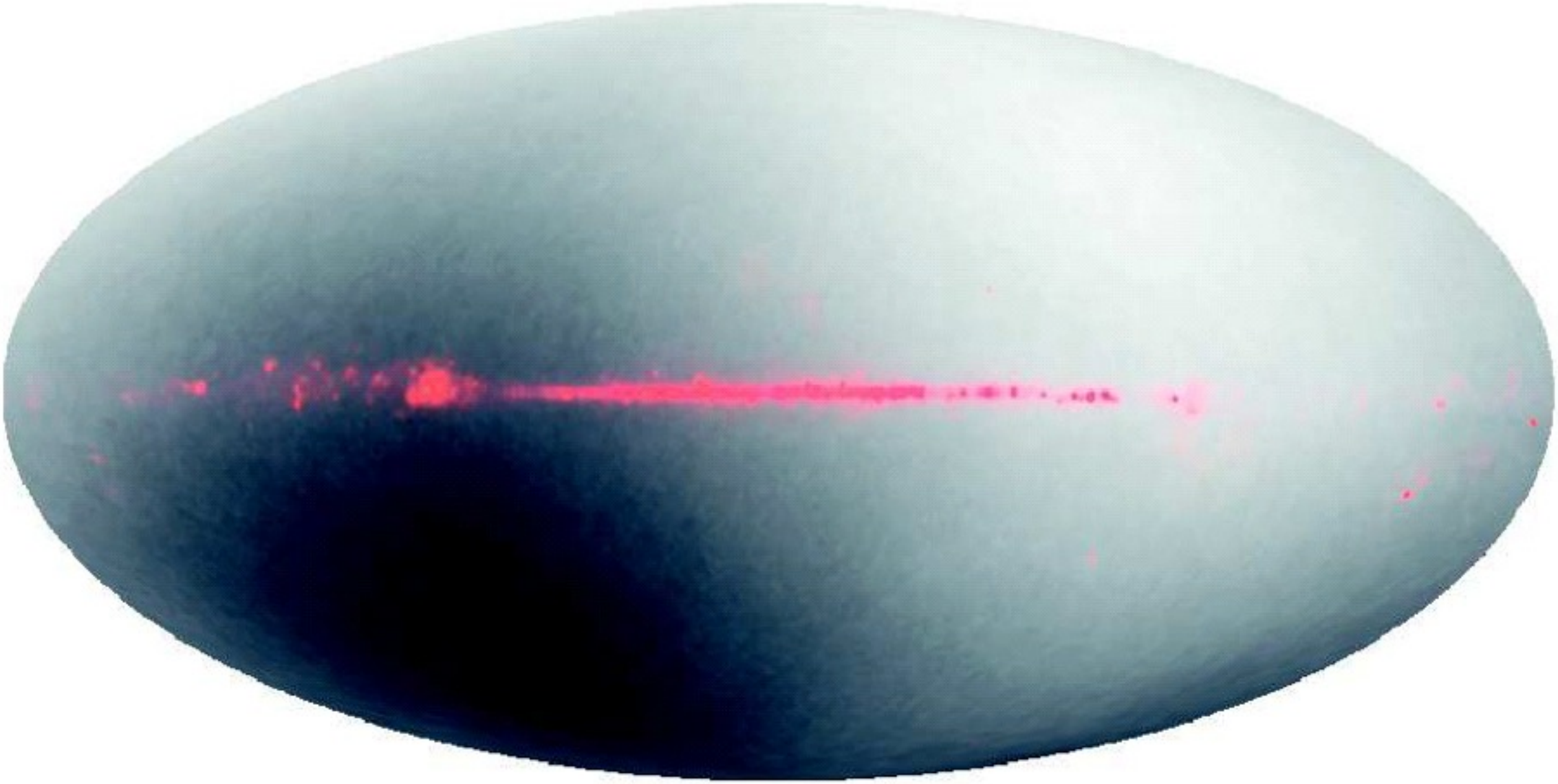
Cosmic Microwave Background Radiation Map



Contrast: 1x 400x

- Epoch of matter and radiation decoupling (about 380.000 years after the "*Big-Bang*").
- T_0 = 2.725 ± 0.002. Redshift, $z = 1089$.
- Highly homogeneous primordial Unverse
- Dipole: ΔT = 3.346 ± 0.017 mK ➔ v_{gal} = 360 Km/s

© Copyleft Martín Makler



- Epoch of matter and radiation decoupling (about 380.000 years after the "*Big-Bang*").
- T_0 = 2.725 ± 0.002. Redshift, $z = 1089$.
- Highly homogeneous primordial Unverse
- Dipole: ΔT = 3.346 ± 0.017 mK ➔ v_{gal} = 360 Km/s

© Copyleft Martín Makler

Some Milestones in the History of the Universe

kT (radiation)	Event
2 x 10^{-4} eV	Today
10^{-3} eV	Galaxy formation
0.26 eV	H recombination (matter-radiation decoupling)
10 eV	Matter domination
300 keV	Formation of light elements (He^4, He^3, D e Li) (primordial nucleosynthesis)
0.5 MeV	End of leptonic era (e^+ e^- annihilation)
100 MeV	End of hadronic era and beginning of leptonic era (hadronization, annihilation hadron anti-hadron)
1000 GeV	Electroweak phase transition
10^{15} GeV	Bariosynthesis? Great Unification?
10^{19} GeV	End of quantum era? Inflation?

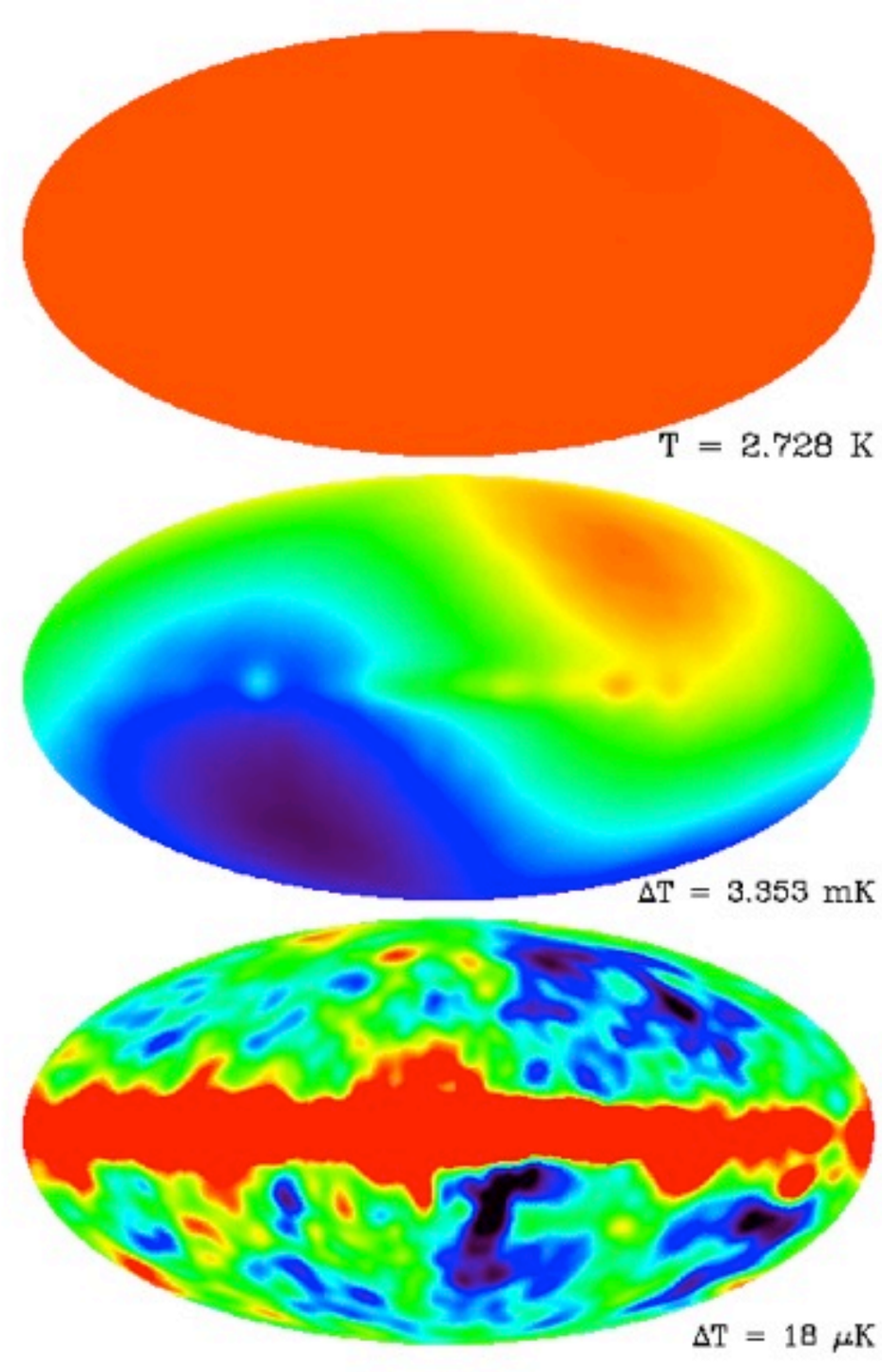
© Copyleft Martín Makler

The Perturbed Universe

CMB anisotropies and Large-scale Structure

© Copyleft Martín Makler

Anisotropies in the Cosmic Background

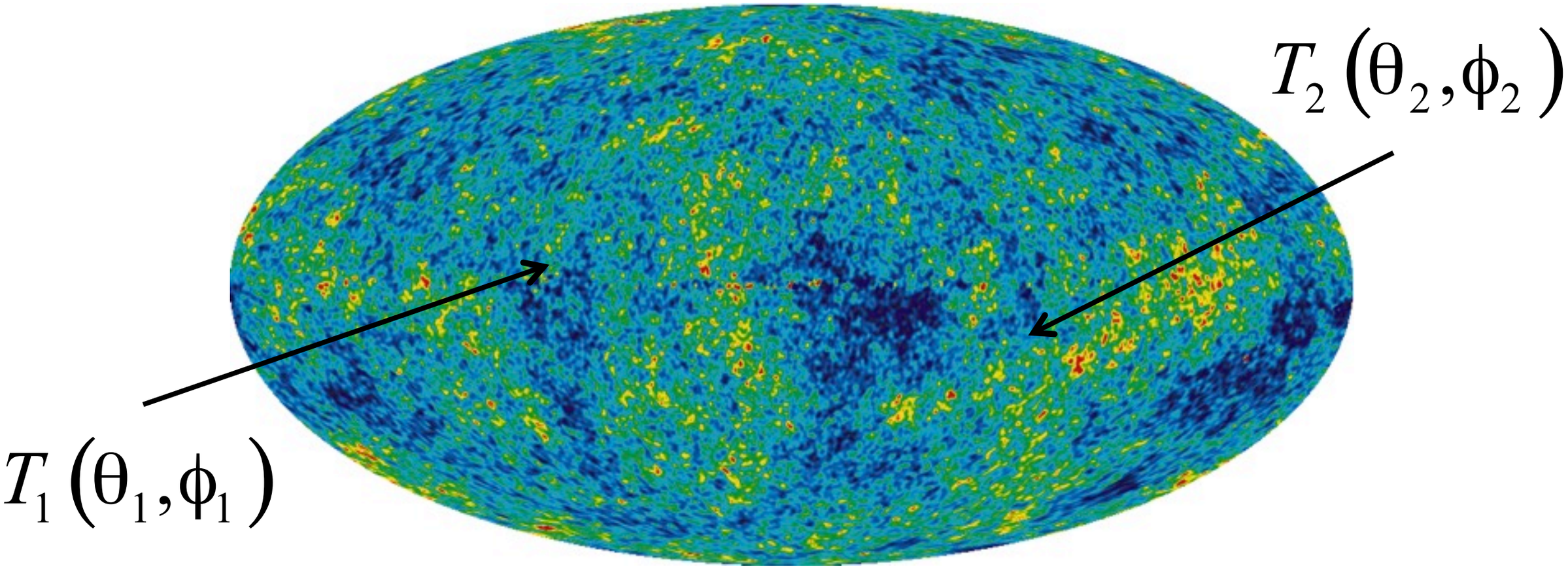


- $T_0 = 2.725 \pm 0.002$.
  redshift, $z = 1089$
- The primordial Universe was highly homogeneous
- Dipole:
  $\Delta T = 3.346 \pm 0.017$ mK
  $\Rightarrow v_{gal} = 360$ Km/s
- Temperature fluctuations:

$$\frac{\Delta T}{T} \approx 10^{-5}$$

Nobel prizes: 1978, 2006

Cosmic Microwave Background Power Spectrum



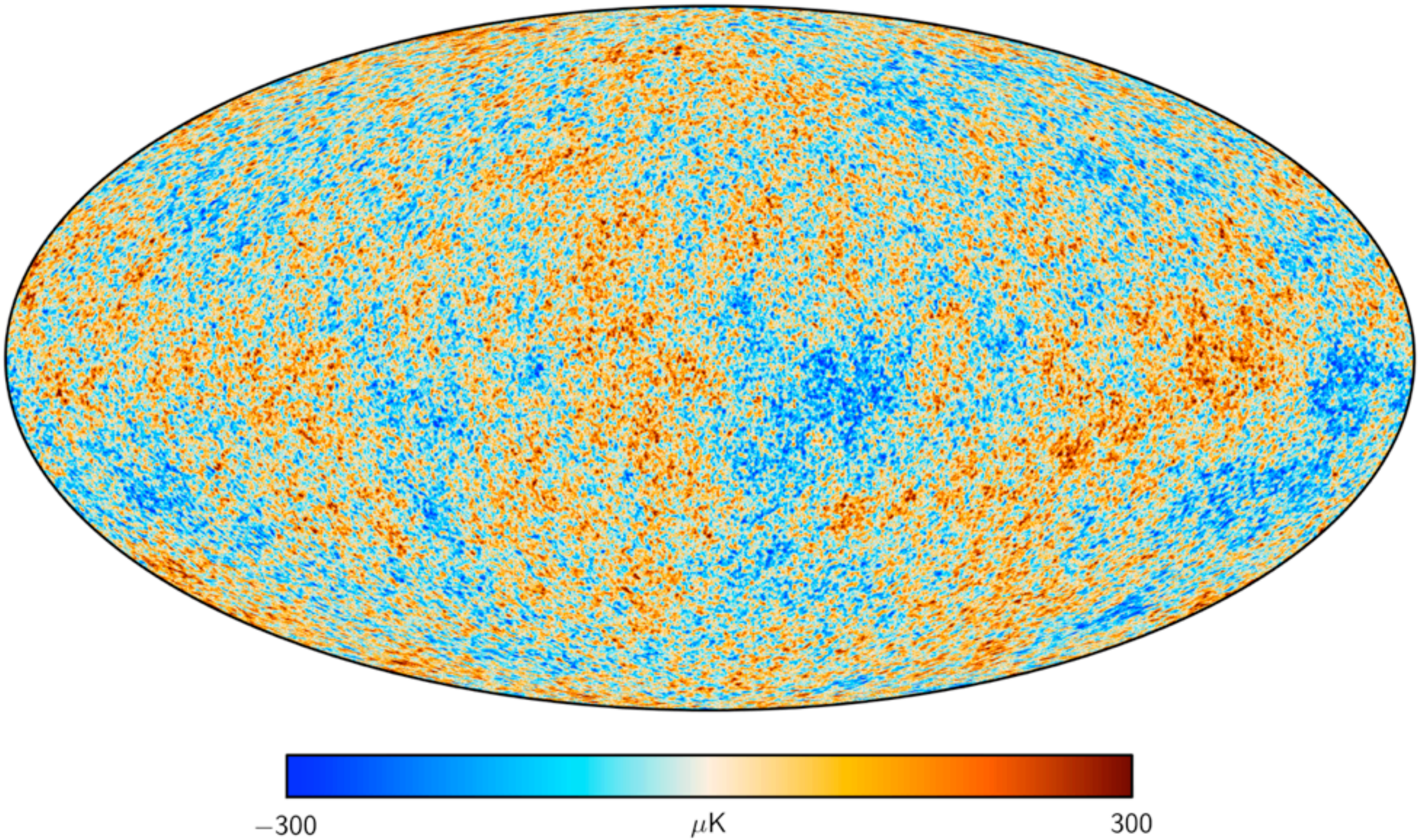
$$\langle T_1 T_2 \rangle = \sum a_{lm} Y_{lm}(\theta,\phi)$$

$$\langle |a_{lm}|^2 \rangle^{1/2} \equiv C_l$$

WMAP2008

© Copyleft Martín Makler

Planck 2015 Anisotropy Map



© Copyleft Martín Makler

Relativistic perturbation theory

- The starting point is the perturbed Robertson-Walker metric

$$\begin{aligned} ds^2 &= \left[g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}\right] dx^\mu dx^\nu \\ &= a^2(\tau)\left[-d\tau^2 + \gamma_{ij}(\vec{x}\,)dx^i dx^j + h_{\mu\nu}(\vec{x},\tau)dx^\mu dx^\nu\right] \ , \end{aligned}$$

- At the linear level modes decouple
- Scalar perturbations:

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\phi)\gamma_{ij}\,dx^i dx^j\right]$$

(for a perfect fluid $\Phi = \Psi$)

© Copyleft Martín Makler

Results from the linear analysis

Need for Dark Matter

- Baryonic matter: can only cluster after decoupling
  t_{dec} ~ 380.000 years
  and for $r > \lambda_J$ (Jeans length)
- Cold Dark Matter starts clustering at t_{eq} ~ 56.000 years (matter-radiation equality)
- Baryons follow the Dark Matter potential wells
- Silk damping decreases the amplitude of baryon perturbations

© Copyleft Martín Makler

CMB Anisotropies

- Primary Anisotropies

$$\frac{\Delta T}{T} = \left[\Psi - \hat{r}\cdot\vec{v} + \frac{1}{3}\delta_m\right]_{\eta_{ls}}$$

Sachs-Wolfe effect

Doppler effect

Intrinsic temperature fluctuation

- On large scales

$$\delta_m \simeq 2\Psi$$

➔ Sachs-Wolfe plateau

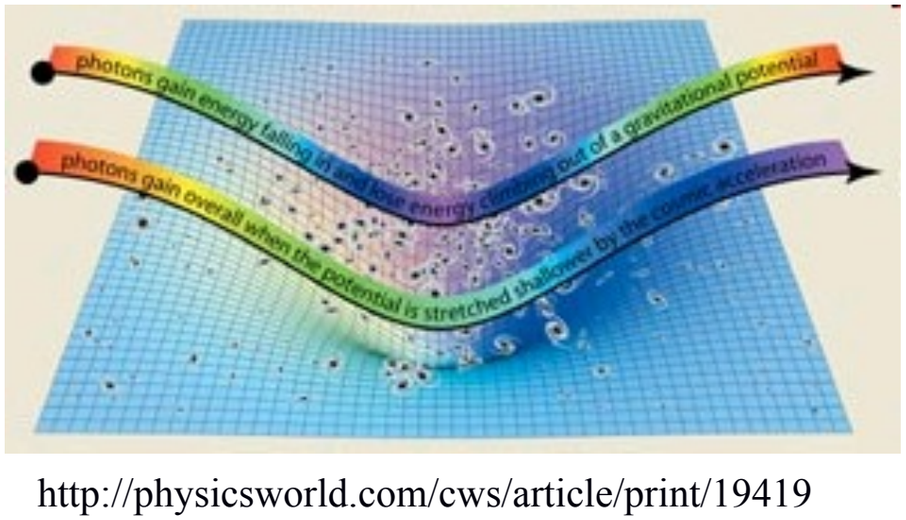
- On small scales:

baryon acoustic oscillations

© Copyleft Martin Makler

Integrated Sachs-Wolfe Effect

- Cumulative effect of gravitational shifts



http://physicsworld.com/cws/article/print/19419

$$\frac{\delta T_{ISW}}{T} = 2\int_{\tau_{dec}}^{\tau_0} \frac{\partial \phi}{\partial \tau} d\tau$$

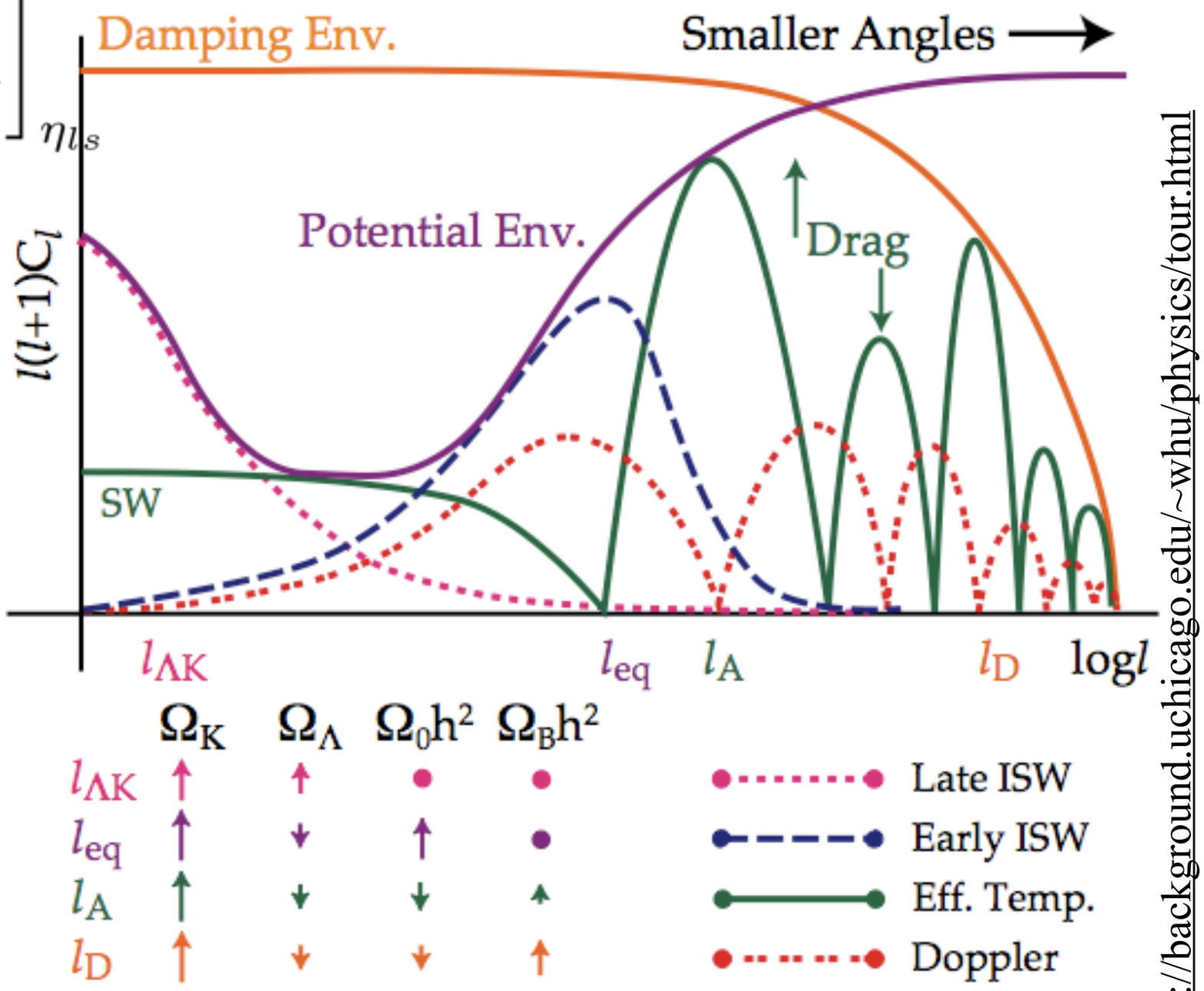
- Linear evolution, $\Omega_M = 1$ ➔ $\phi = const.$
- Late effect:
  - Correlation between the cosmic microwave background and large-scale structure!

⇨ **Dark Energy**

© Copyleft Martín Makler

Combining the Contributions

$$\frac{\Delta T}{T} = \left[\Psi - \hat{r}\cdot\vec{v} + \frac{1}{3}\delta_m\right]_{\eta_{ls}} + 2\int_{\tau_{dec}}^{\tau_0} \frac{\partial\phi}{\partial\tau} d\tau$$

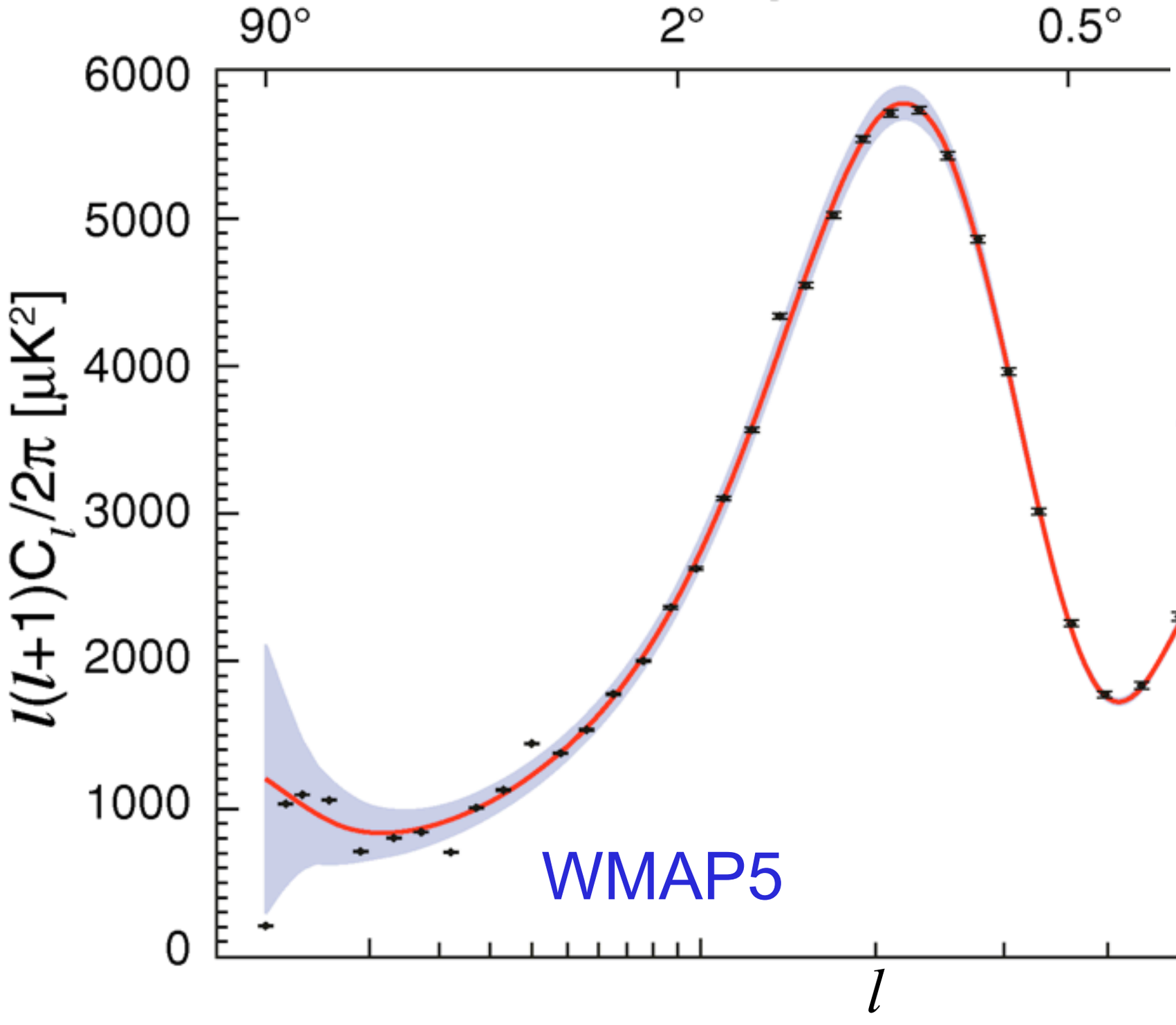


- Damping:
  - Photon diffusion in hot regions (Silk damping)
  - Finite thickness of last scattering surface

© Copyleft Martín Makler

http://background.uchicago.edu/~whu/physics/tour.html

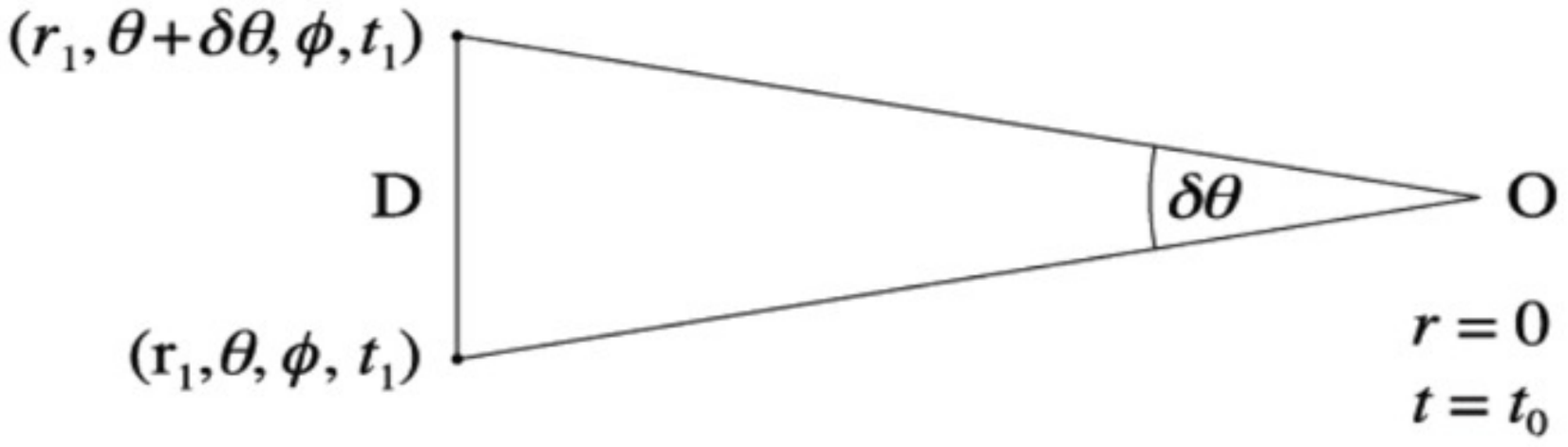
Angular Power Spectrum



- **As predicted**
- **Concordance model**
- **Only 6 parameters fit the data!**

© Copyleft Martín Makler

Angular Diameter Distance



$$d_A \equiv \frac{D}{\delta\theta}$$

$$(\text{for } \delta\theta \ll 1)$$

Standard ruler

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

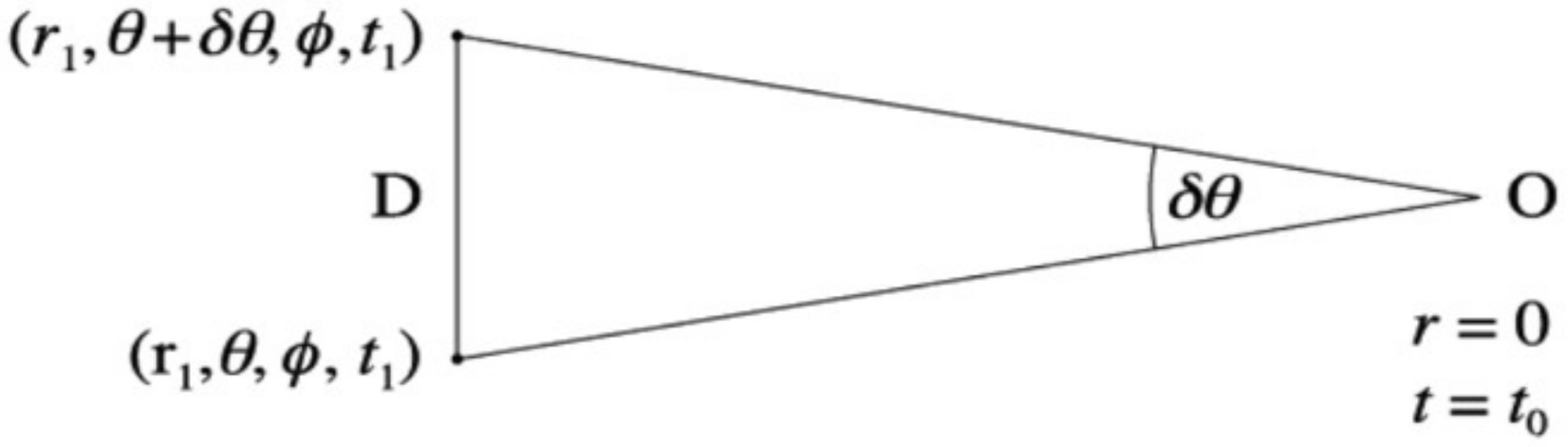
thus $D = a(t_1)\, r_1 \delta\theta$ such that $d_A = a(t_1)\, r_1$

$$d_A = c(1+z)^{-1}\frac{1}{\sqrt{|1-\Omega_0|}}S\left(\sqrt{|1-\Omega_0|}\int_0^z \frac{dz'}{H(z')}\right)$$

d_A may decrease with z!

© Copyleft Martín Makler

Angular Diameter Distance



$$d_A \equiv \frac{D}{\delta\theta}$$

$$(\text{for } \delta\theta \ll 1)$$

Standard ruler

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

thus $D = a(t_1)\, r_1 \delta\theta$ such that $d_A = a(t_1)\, r_1$

$$d_A = (1+z)^{-2} d_L$$ Valid for any space-time!

© Copyleft Martín Makler

Peaks in the Cosmic Microwave Background

Angular scale of acoustic horizon at decoupling:

$$\theta_A = \frac{r_s(z_{dec})}{d_A(z_{dec})}$$

Acoustic horizon

$$r_s = \int_0^{\tau_{dec}} c_s d\tau$$

where

Standard ruler

$$c_s = \frac{1}{\sqrt{3\left(1+\frac{3}{4}\frac{\rho_b}{\rho_\gamma}\right)}}$$

Angular diameter distance

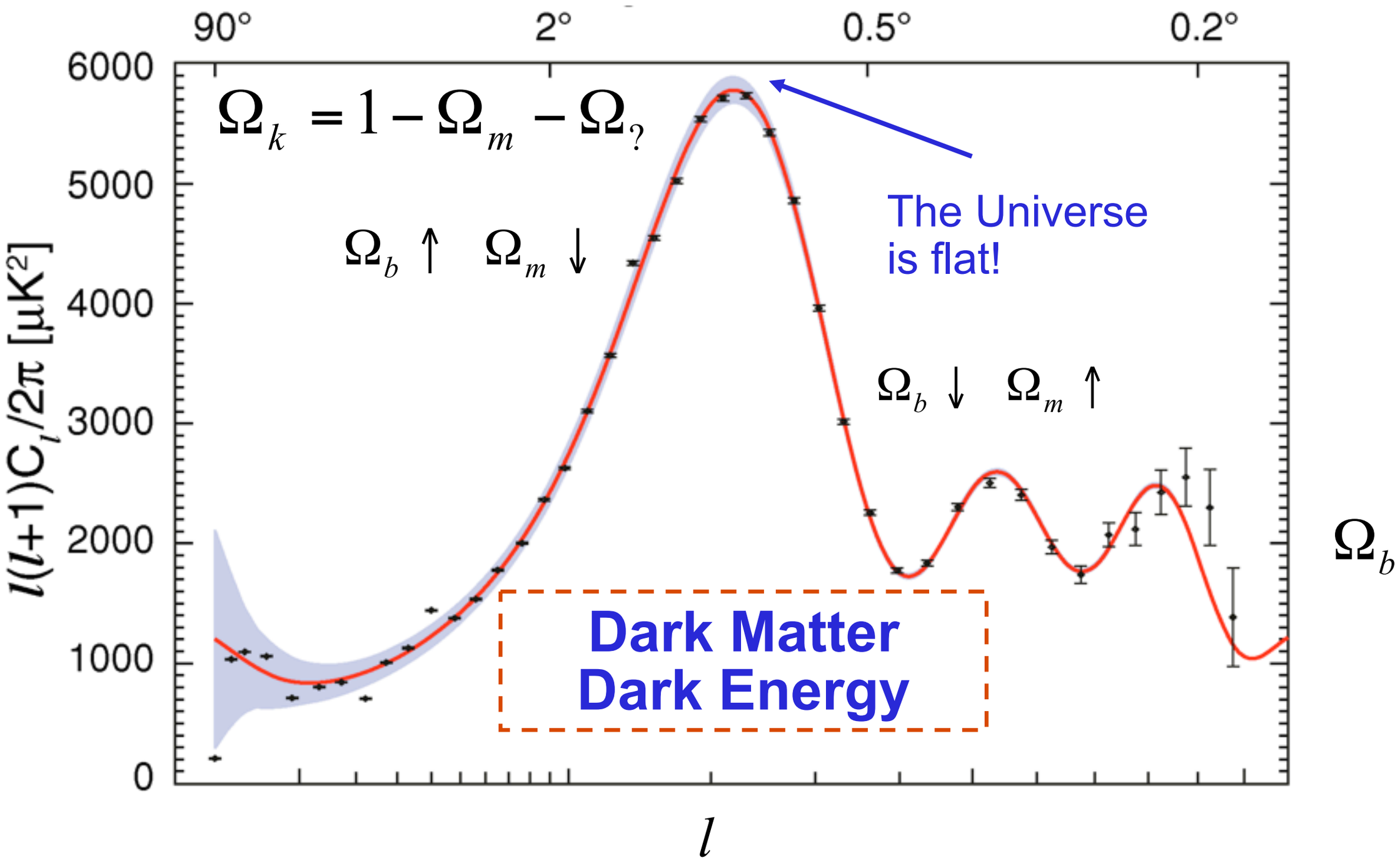
$$d_A = c(1+z)^{-1}\frac{1}{\sqrt{|1-\Omega_0|}}S\left(\sqrt{|1-\Omega_0|}\int_0^z \frac{dz'}{H(z')}\right)$$

At z ~ 1000 θ_A is mostly sensitive on $\Omega_K = 1 - \Omega_0$

For a flat Universe, $\theta_A \simeq 1^\circ$

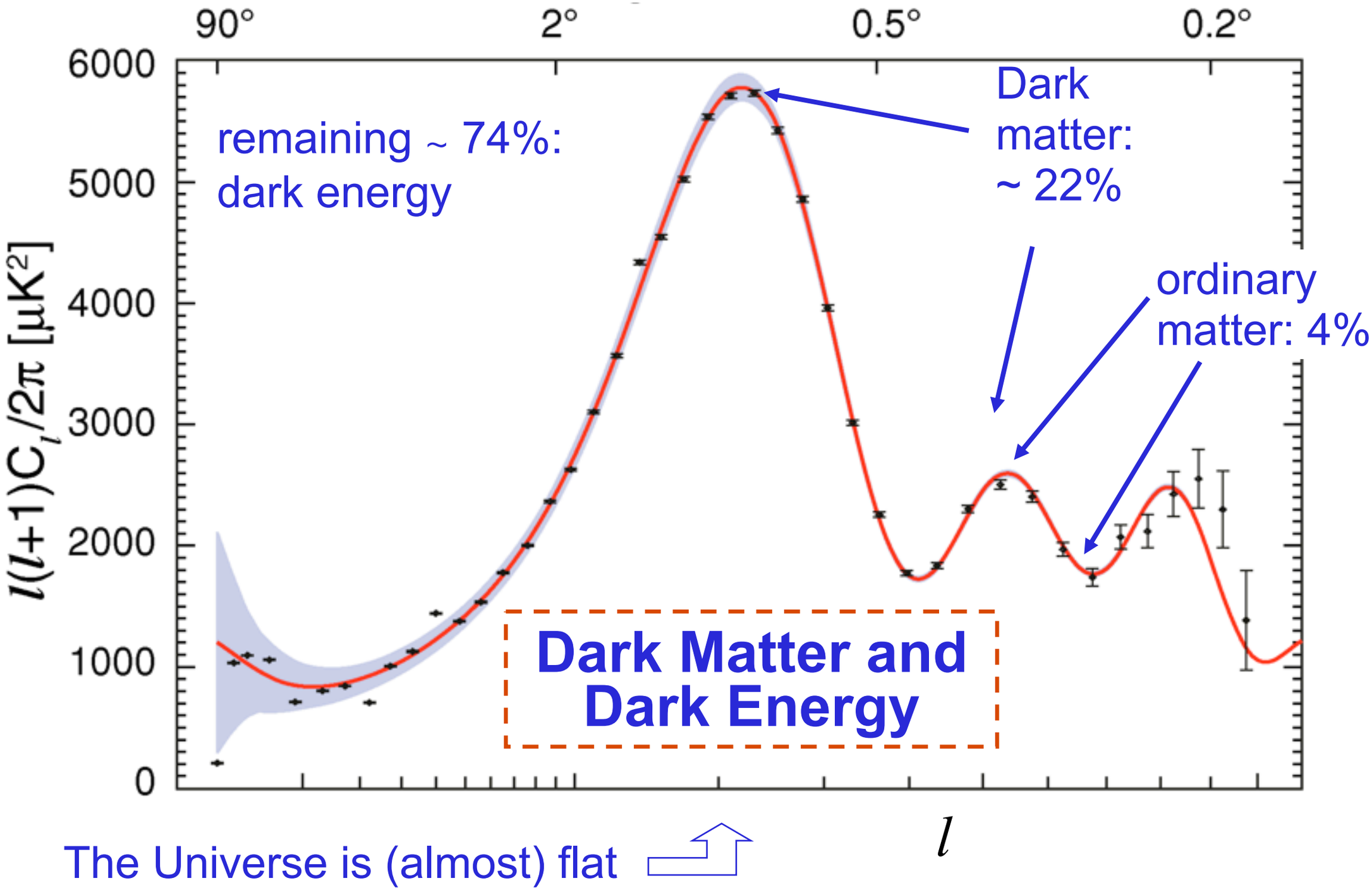
© Copyleft Martín Makler

Power spectrum according to WMAP5



© Copyleft Martín Makler

The wholly grail of Cosmology

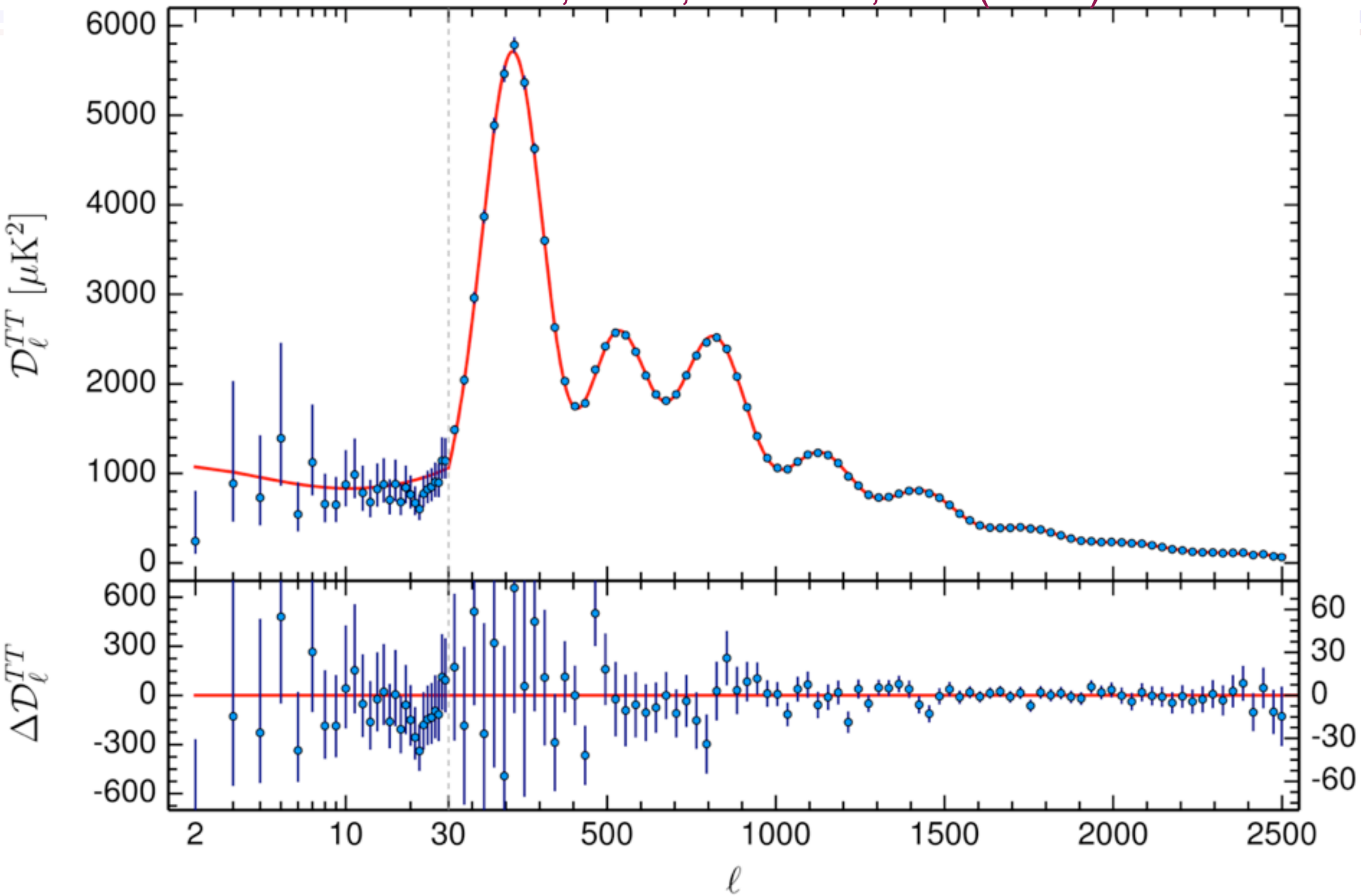


The Universe is (almost) flat

© Copyleft Martín Makler

Planck 2015 power spectrum

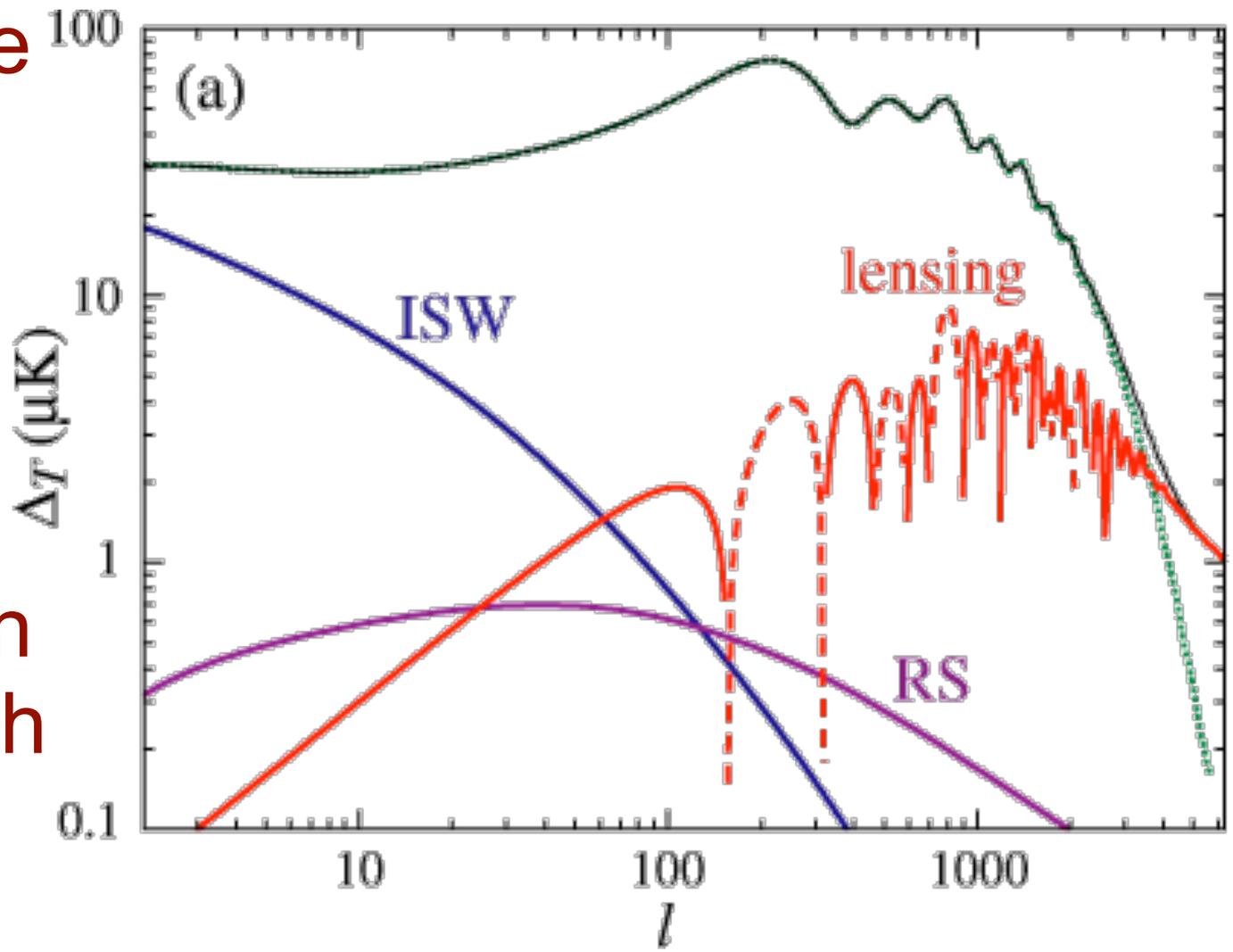
Planck collaboration, 2016, A&A 594, A11 (2016)



© Copyleft Martín Makler

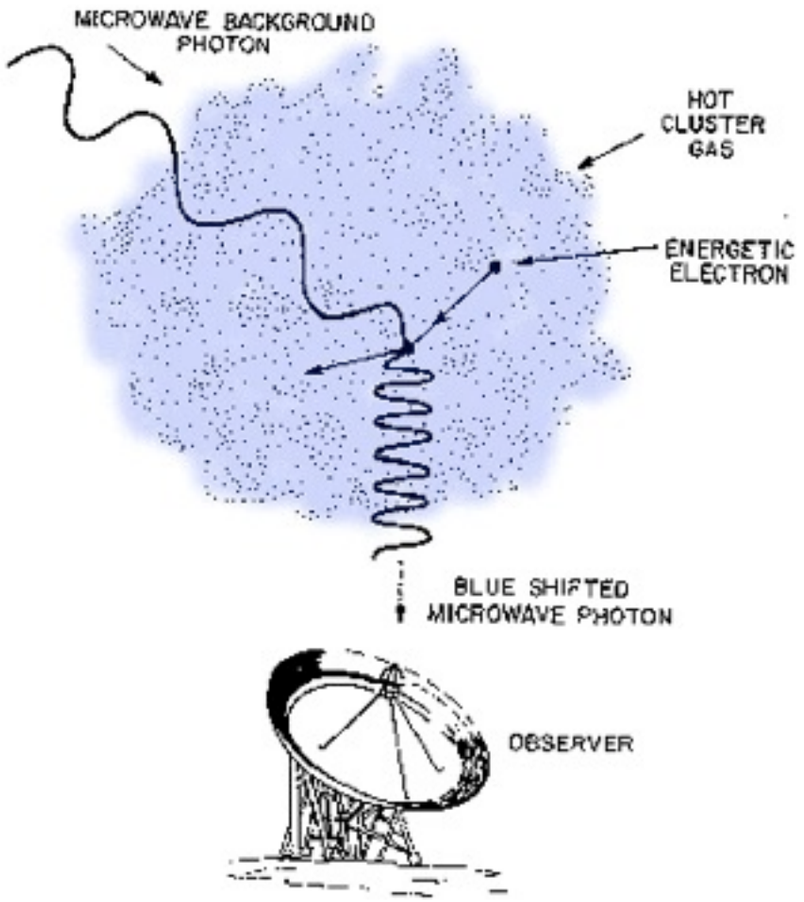
Secondary Anisotropies

- Integrated Sachs-Wolfe effect
- Gravitational Lensing
- Rees-Sciama effect
- Gravitational waves
- Scattering: reionization and Sunyaev Zel'dovich

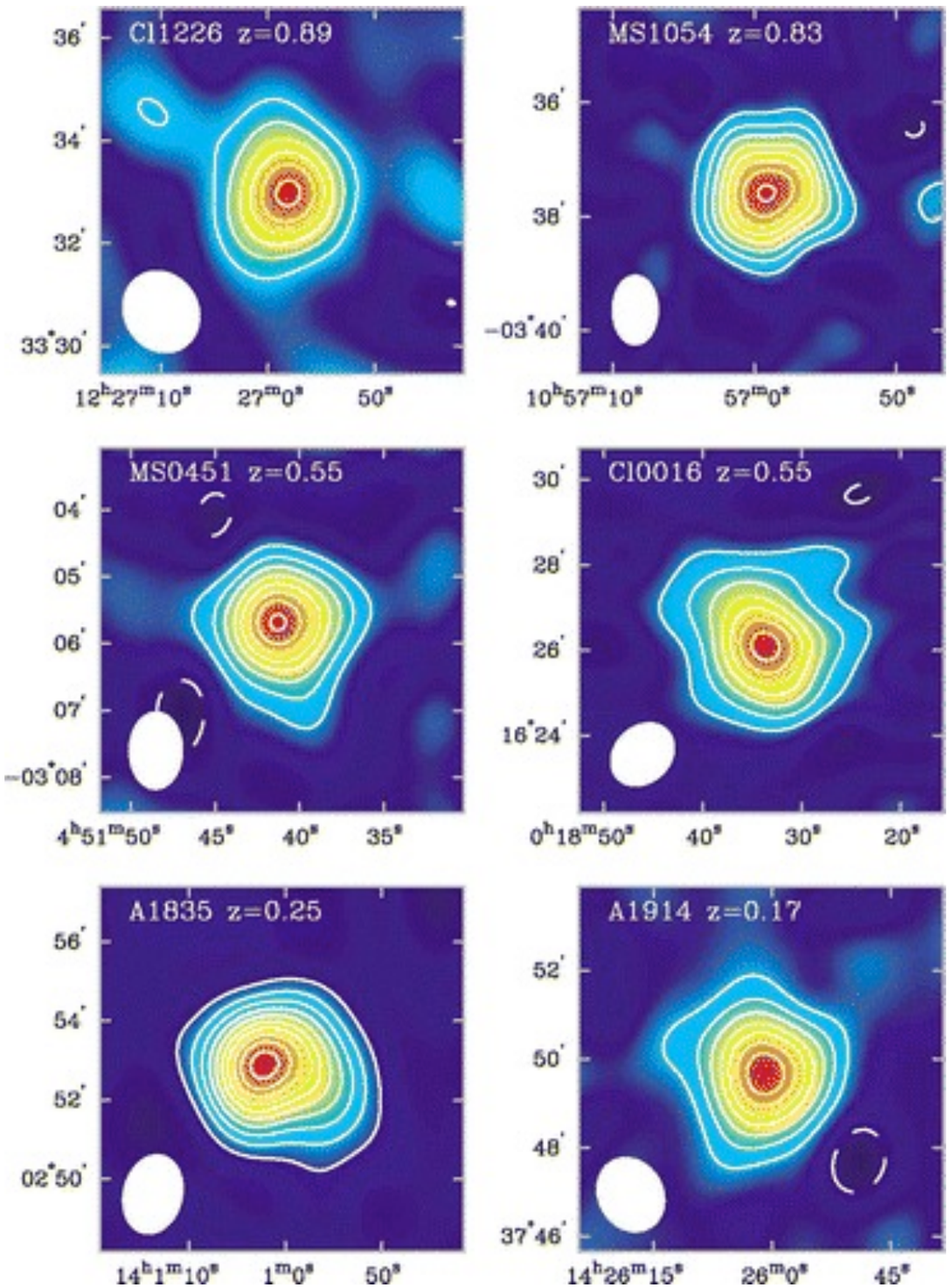


© Copyleft Martín Makler

Sunyaev Zel'dovich Effect



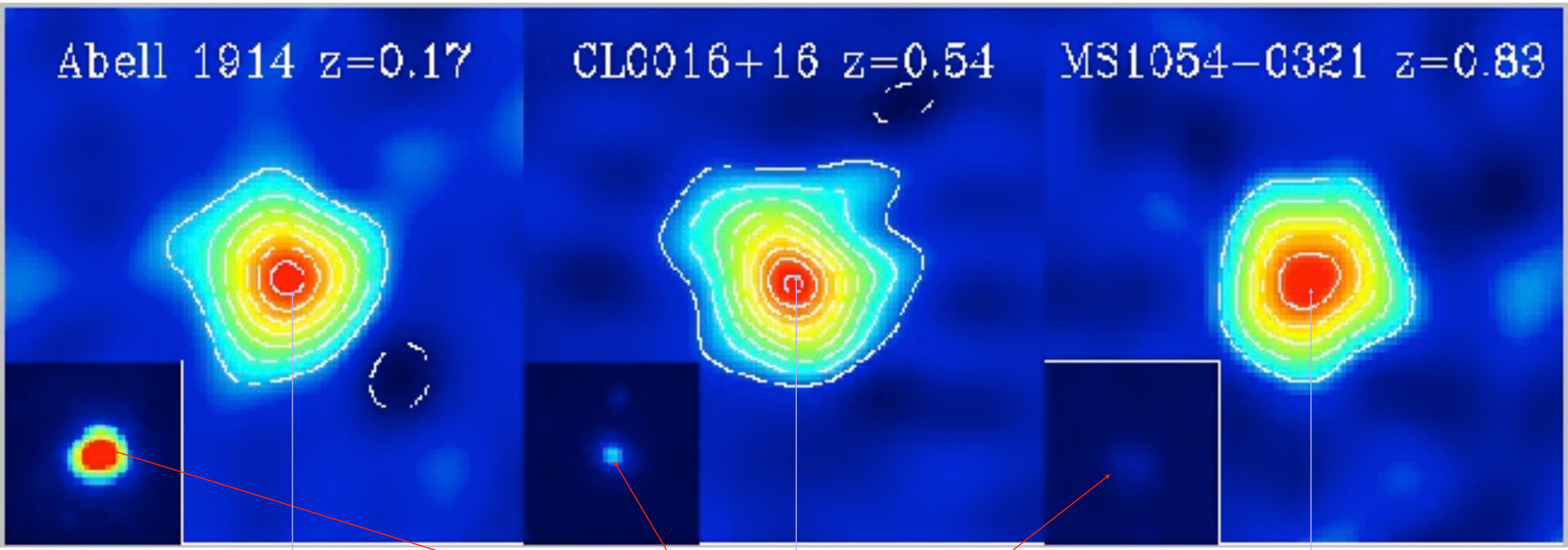




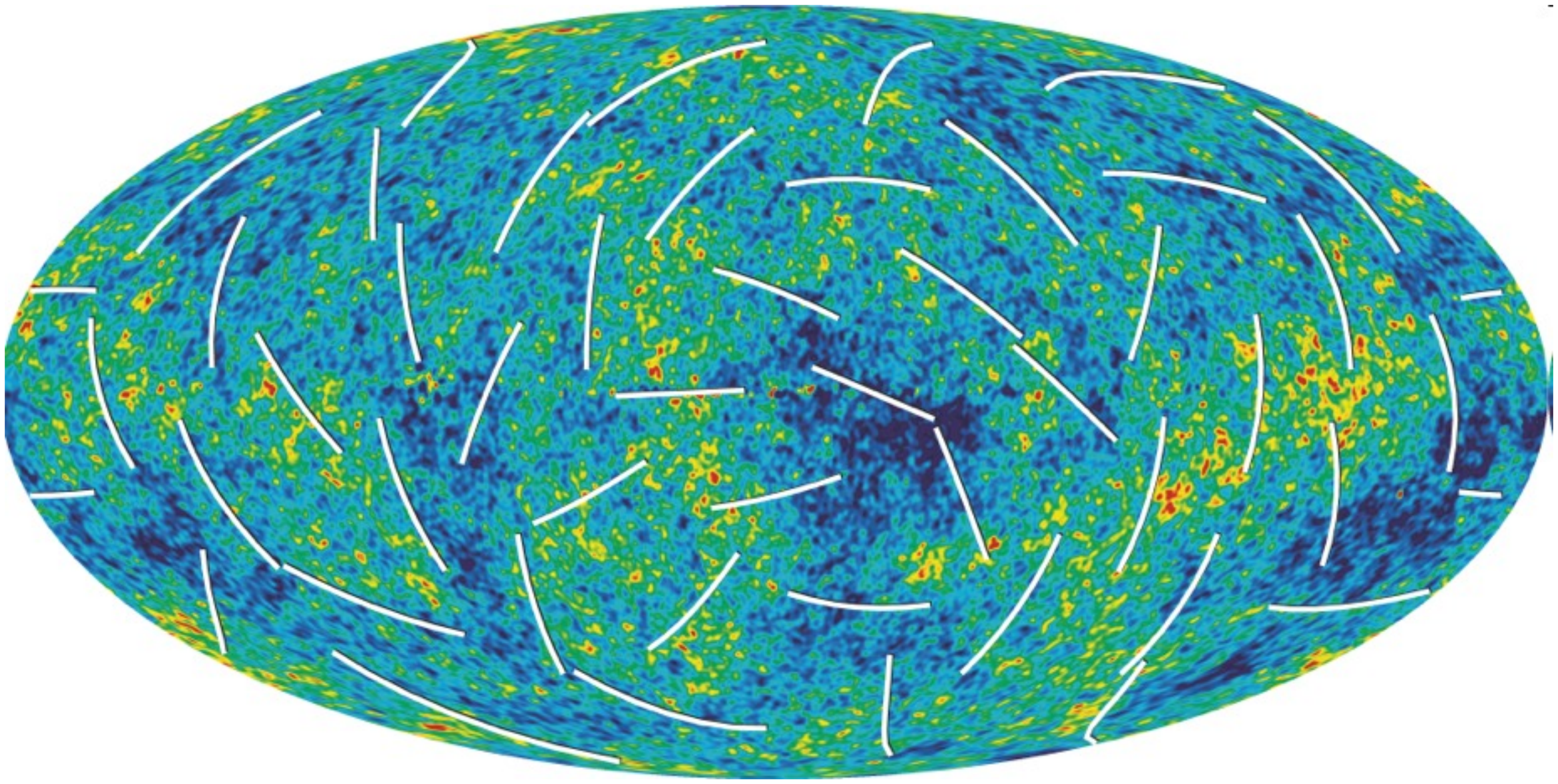
© Copyleft Martín Makler

Advantage of Sunyaev Zel'dovich Effect

- X-ray flux drops as $(1 + z)^{-4}$
- Decrement is independent from z

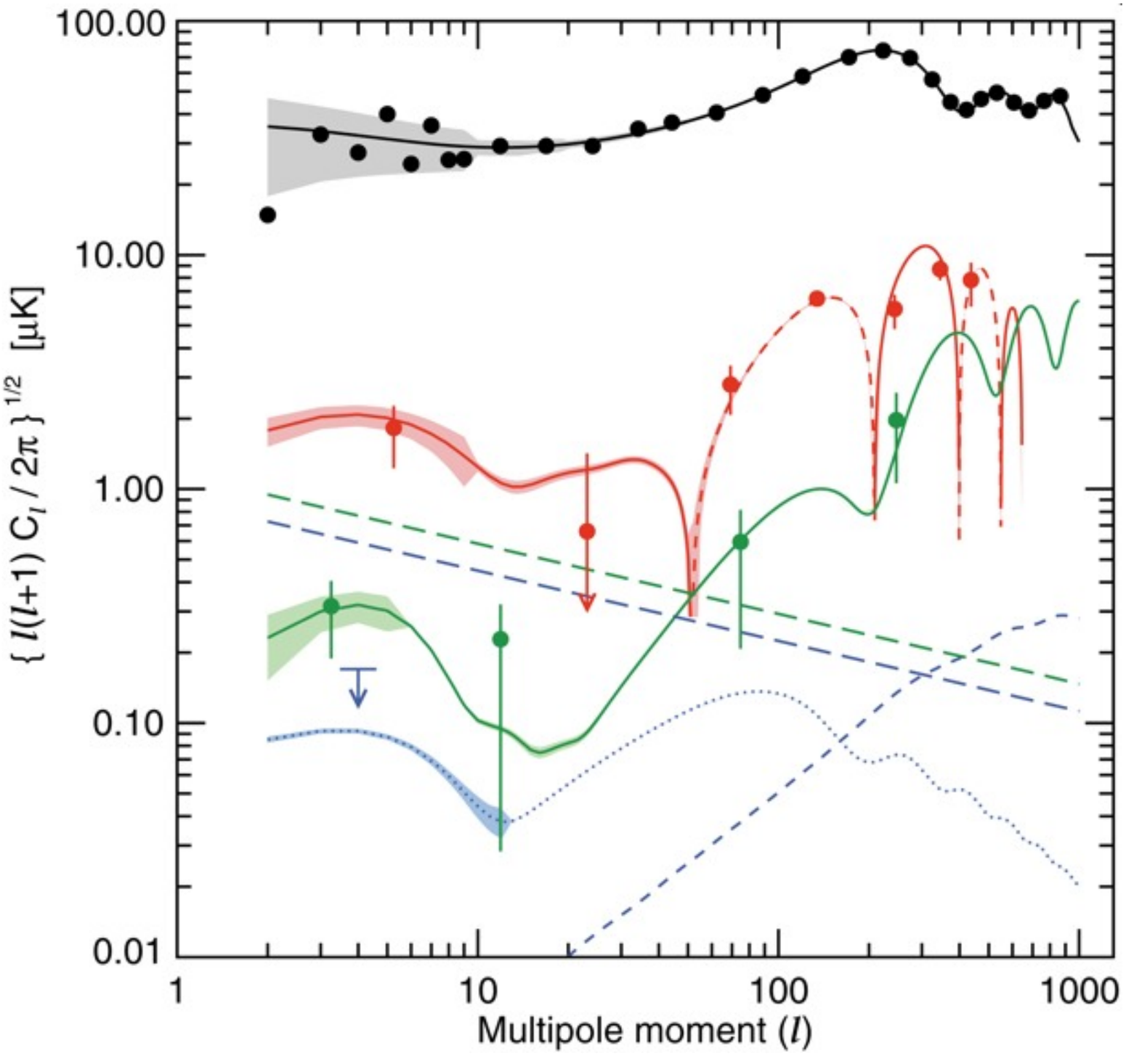


CMB Polarization



© Copyleft Martín Makler

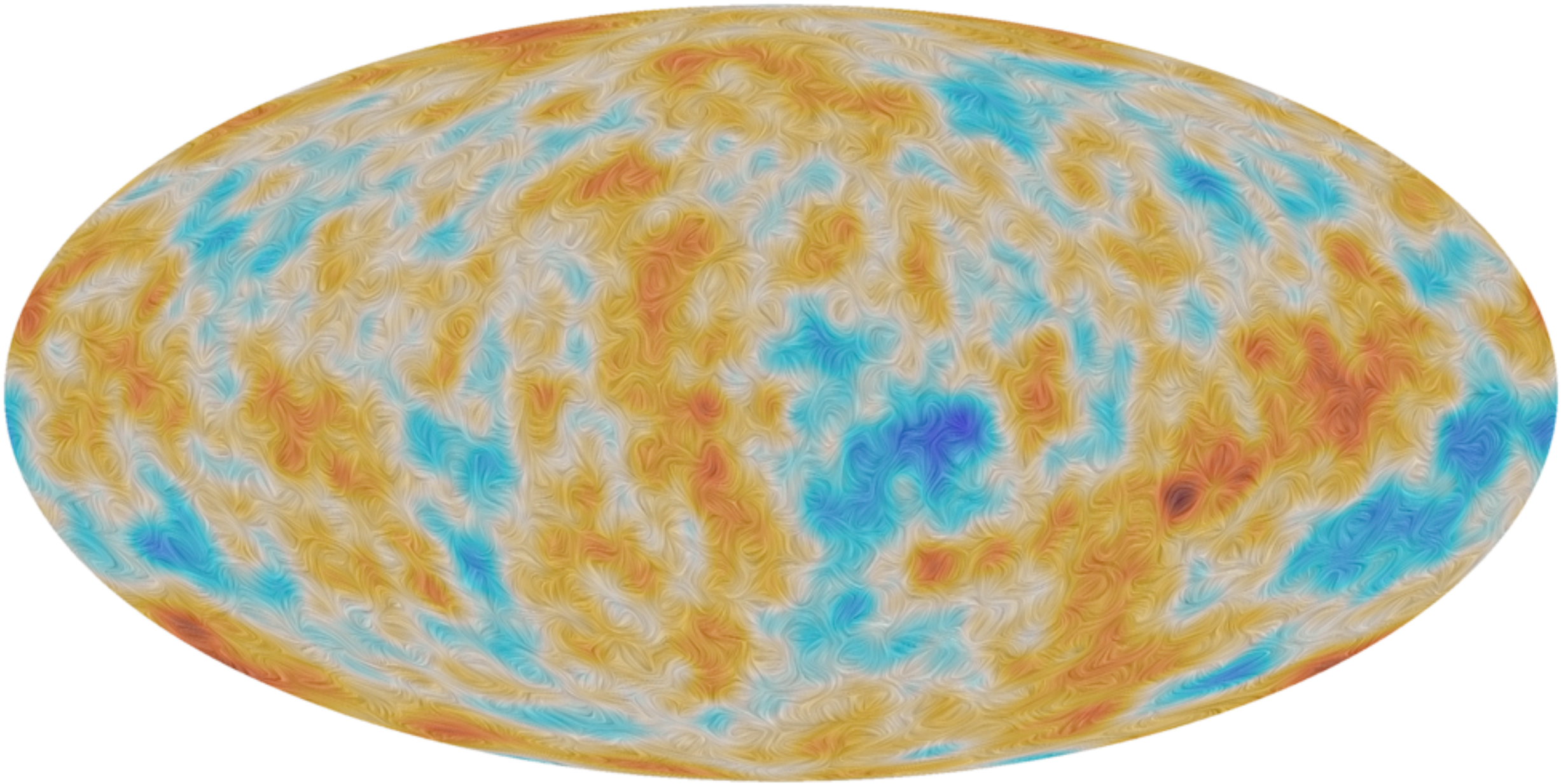
Polarization Spectrum



- TE and EE components
- BB component
- Gravitational waves

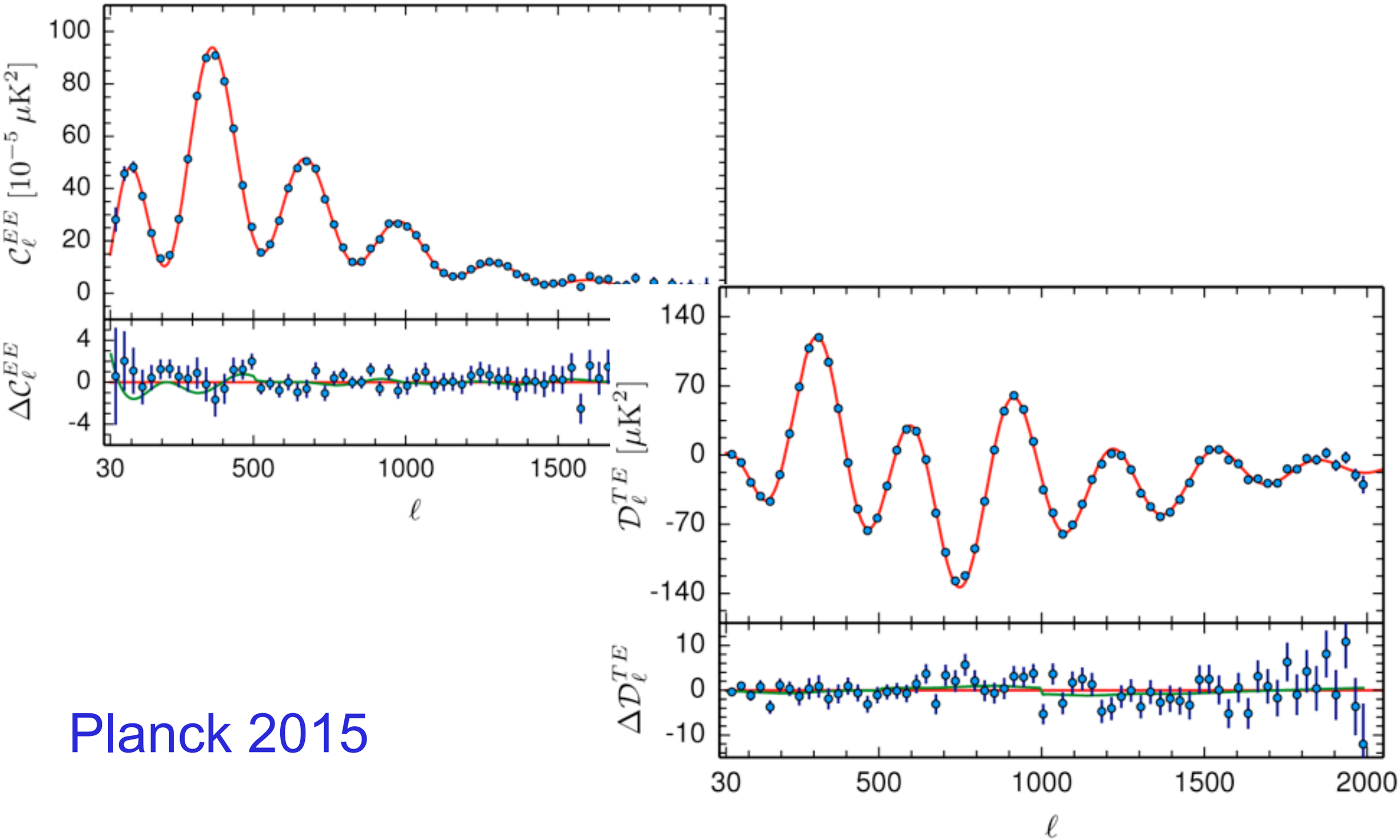
) Copyleft Martín Makler

Planck Polarization Map



© Copyleft Martín Makler

Polarization Power Spectra



Planck 2015

© Copyleft Martín Makler