Interações Fortes em Condições Extremas

(fifth lecture)

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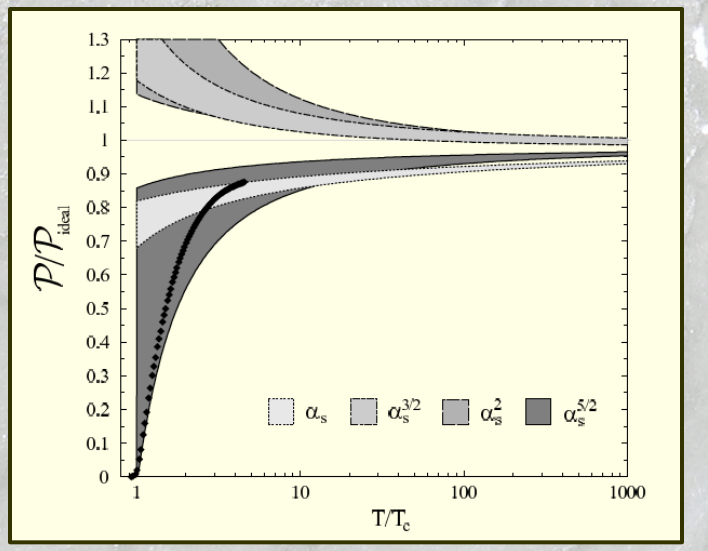
Fifth Lecture

- ★ Finite T x finite μ: pQCD, lattice, sign problem, etc
- ★ Nuclear EoS: relativistic and non-relativistic (brief)
- ★ pQCD at nonzero T and μ (brief)
- ★ Cold pQCD at high density for massless quarks
- ★ Nonzero mass effects
- ★ Compact stars and QCD at high density
- ★ Summary

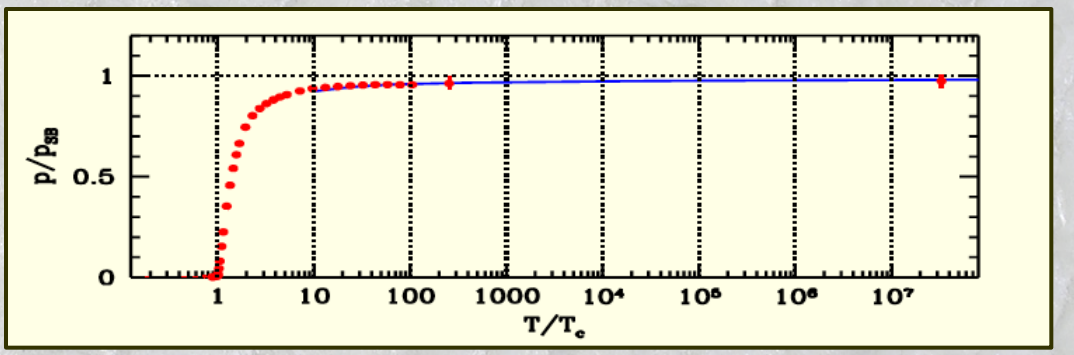
Finite T x finite μ: pQCD, lattice, sign problem, etc

- So far we have focused on the finite-temperature zero-density sector of the phase diagram for QCD. How about finite-density?

- We know that naïve perturbative finite-temperature QFT is plagued by infrared divergences, so that one has to improve the series by some sort of resummation.



[Andersen, Braaten & Strickland, 2000]



[From Fodor, Lattice 2007]

Link between lattice QCD and pQCD at finite T !!

• This is due to sum-integrals that yield a series involving $\alpha_s^{1/2}$ instead of α_s (+ logs). At the end, it generates huge oscillations in sign for the pressure as we include (large) corrections, the radius of convergence being essentially zero [Braaten, 2000]

• For T = 0 and $\mu > 0$, though, the perturbative series seems to be much more well-behaved, without oscillations and with decreasing corrections [there is still a bit of wishful thinking here, and the computation of α_s^3 corrections would help to corroborate it or not]

• Anyway, there seems to be room for sensible pQCD calculations for T = 0 and μ large (and not ridiculously large!)

• So, this is good news. Bad news is: it is really tough to compute QCD at finite μ on the lattice due to the Sign Problem:

- On the lattice, one does Monte Carlo calculations that rely on S_{eucl} being positive-definite to sample "relevant" contributions weighted by $\exp(-S_{eucl})$:

$$Z = \int [DA_\mu][D\bar{\psi}][D\psi] e^{-S_{eucl}}$$

- This method ("importance sampling") reduces dramatically the number of field configurations one has to consider [the total would go as exp (#V)]

- For QCD with $\mu \neq 0$, the effective S_{eucl} is complex, so that one does not have a well-defined weight factor any more...

The Sign Problem comes about in the following way:

Including fermions at finite μ, one has the action

$$S_{eucl} = S_{YM} + \int d^4x \; \bar{\psi}_f \hat{D} \psi_f$$

Integrating over the fermions, one obtains

$$e^{S^{eff}_{eucl}[A_\mu]} = e^{S_{YM}} \; \det(\hat{D})$$

$$\hat{D} = \begin{pmatrix} \sigma \cdot D & m_f \\ m_f & \sigma^\dagger \cdot D \end{pmatrix} - \mu_f$$

$$\sigma_\mu = (1, i\vec{\sigma}) \; ; \; D_\mu = \partial_\mu - iA_\mu$$

- Using $D^\dagger = -D$, one can write:

$$\det(\hat{D}) = \det \begin{pmatrix} \sigma \cdot D - \mu_f & m_f \\ m_f & -(\sigma \cdot D)^\dagger - \mu_f \end{pmatrix}$$

so that positivity and reality are lost, unless one assumes μ_f to be imaginary (analytic continuation) or that quarks are such that $m_u = m_d$ & $\mu_u = -\mu_d$ (isospin symmetry)



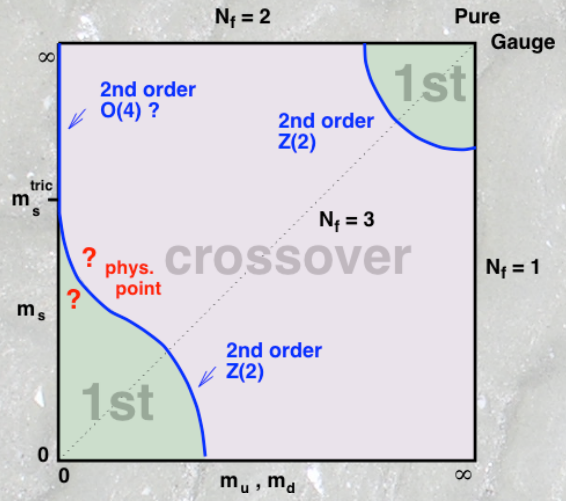
No well-defined criteria to choose configurations!

Exercise: fill in the gaps, i.e., do the calculations in detail.

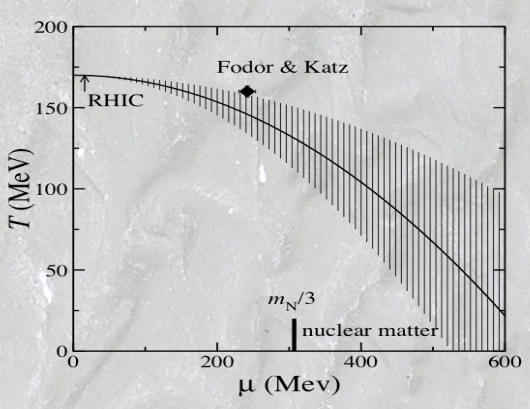
- There are many techniques, developed over the last few years, to try to bypass the Sign Problem. But there is no final established method for the general case yet, and the road is long and winding...

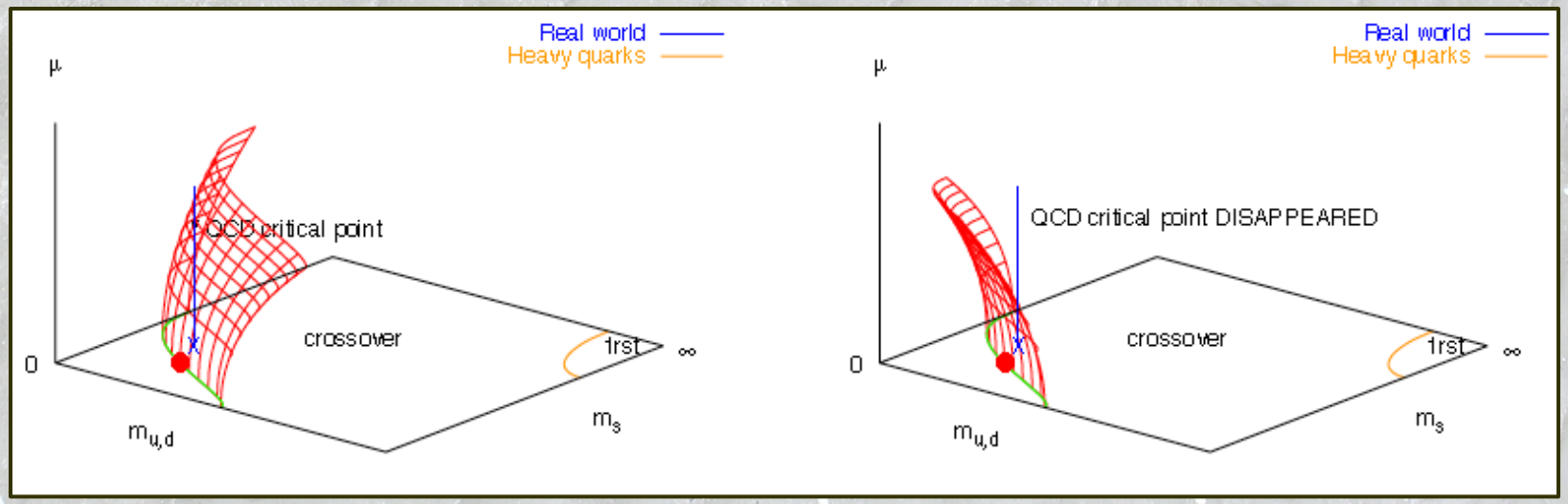
- Some techniques at finite μ:
  - Fodor et al.:
    Multi-parameter reweighting
  - Bielefeld-Swansea:
    Taylor exp. around μ=0
  - de Forcrand, Philipsen; D'Elia, Lombardo; ...
    Analytic continuation from imaginary μ

- Some results and possibilities:



[Allton et al., 2002]





[From: de Forcrand, Lattice 2007]

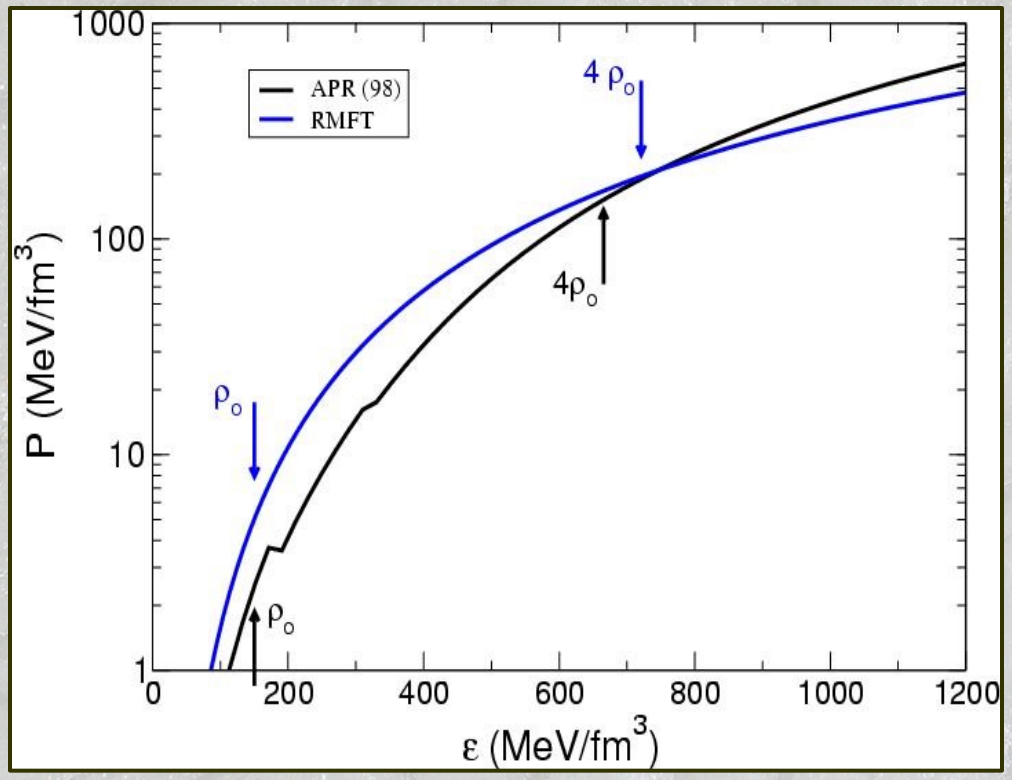
• The basic message is: at finite μ, we do not have yet the kind of guidance from the lattice as we have at finite T (μ=0). And most likely it will take a while...

• So, one has to compare different schemes of computation of thermodynamic quantities: pQCD, resummed pQCD (different possibilities), quasiparticle models, low-energy effective models, quantum hadrodynamics (QHD), non-relativistic many-body nuclear theory, etc.

• In general, one has to study the limits of low and high densities separately, matching results in the critical region where the phase transition happens.

• Observables have to be searched in astrophysics, more precisely in compact stars, where densities can be high enough to allow for a deconfined phase.

Nuclear EoS: relativistic and non-relativistic



[From Reddy, SEWM 2005]

- Very different from the bag model EoS
- Hard to trust for $\rho > 2\rho_0$
- Has to be matched onto high density EoS

pQCD at nonzero T and μ (in-medium QCD)

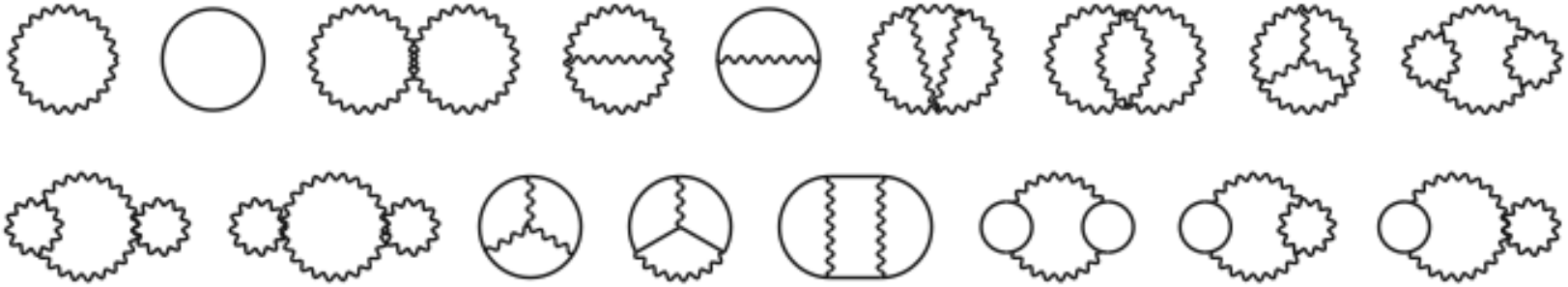
Freedman & McLerran, 1977-1978
Baluni, 1978 ; Toimela, 1980's
Braaten & Pisarski, Frenkel & Taylor, 1990-1992
Arnold & Zhai, 1994-1995
Andersen, Braaten & Strickland, 1999-2001
Kajantie et al, 2001 ; Peshier et al, 1999-2003
Blaizot, Iancu & Rebhan, 1999-2003
ESF, Pisarski & Schaffner-Bielich, 2001
Rebhan & Romatschke, 2003
Vuorinen, 2004-2007
... (many omissions)

Problems from theory side: really tough calculations, resummation methods, valid for high energy scales, misses vacuum, pushing perturbative methods to the limit,...

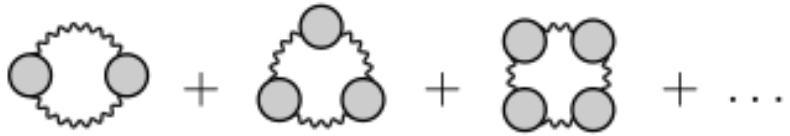
Pressure (diagrammatically speaking...)

(Vuorinen, 2004)

Up to three loops (g^4) the diagrams to be evaluated are (without ghosts)



Also at low T the first IR problems (for the pressure) appear at 3-loop order: on top of 2PI graphs must resum





+ HDL resummation & mass effects (really tough!)

Cold pQCD at high density for massless quarks

- Gas of massless u, d, s quarks
  [charge and β equilibrium achieved (no need for electrons)]
- T_{star} << typical μ in the core region -> T ~ 0
- m_s ~ 100 MeV << typical μ -> massless quarks (well,...)
- Interaction taken into account perturbatively up to α_s^2
- α_s^2 runs according to the renormalization group eqn.
- No bag constant
- Charge neutrality and β-equilibrium: $\mu_s = \mu_d = \mu_u$

Thermodynamic potential in MSbar scheme for N_f flavors and N_c=3

$$\Omega(\mu) = -\frac{N_f \mu^4}{4\pi^2}\left\{1 - 2\left(\frac{\alpha_s}{\pi}\right) - \left[G + N_f \ln\frac{\alpha_s}{\pi} + \left(11 - \frac{2}{3}N_f\right)\ln\frac{\Lambda}{\mu}\right]\left(\frac{\alpha_s}{\pi}\right)^2\right\}$$

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 u}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln(u)}{u} + \frac{4\beta_1^2}{\beta_0^4 u^2}\left(\left(\ln(u) - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]$$

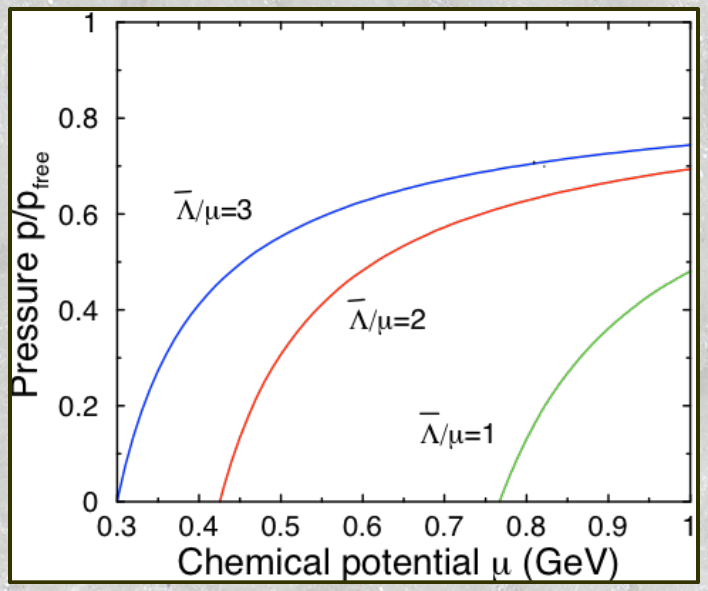
$$u = \ln(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})$$

- Λ: renormalization scale
- From PDG: α_s(2 GeV)= 0.3089 -> $\Lambda_{\rm MS}$ = 365 MeV for N_f = 3

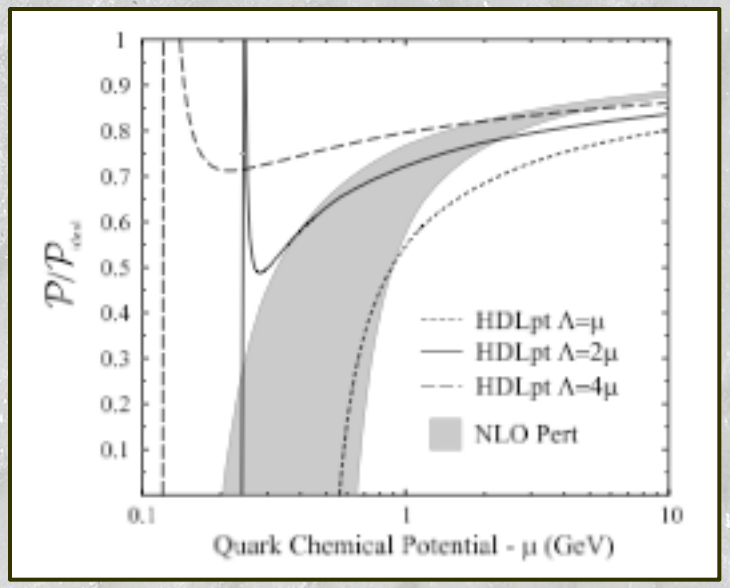
$$G = G_0 - 0.536 N_f + N_f \ln N_f$$

- G_0 = 10.374 ± 0.13
- In principle Λ/μ can be freely chosen (only freedom left). However, the choice is tightly constrained by physics to $2 \le \Lambda/\mu \le 3$
- Results are sensitive to the choice

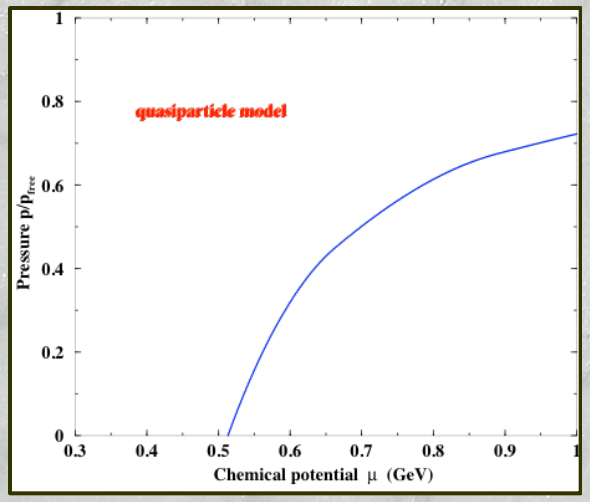
Results for the pressure:



(finite-density pQCD)



(Andersen & Strickland, 2002)



(Peshier, Kämpfer & Soff, 2002)

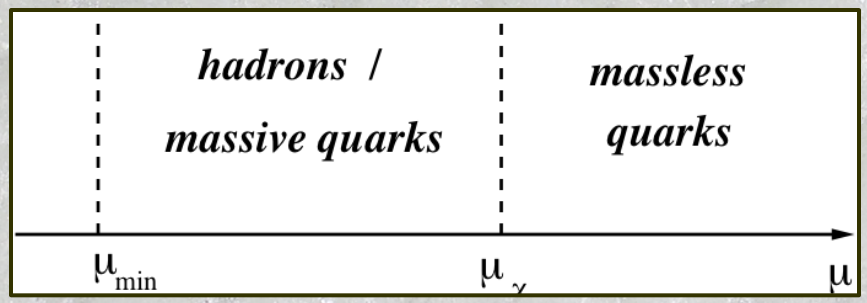
+ similar results from

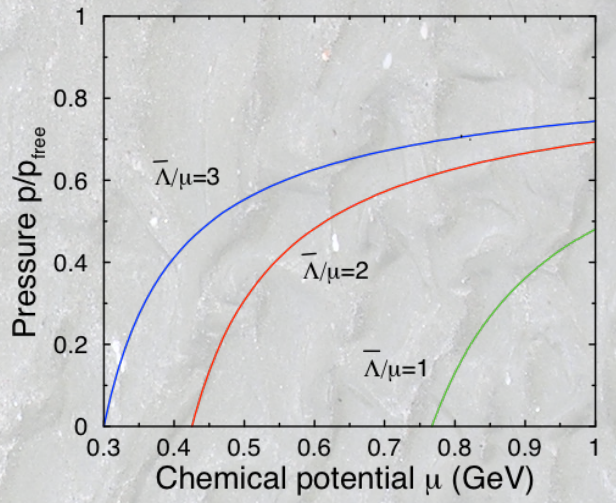
Blaizot, Iancu, Rebhan (2001)
Rebhan, Romatschke (2003)
...

pQCD, HDL, quasiparticle models, ...

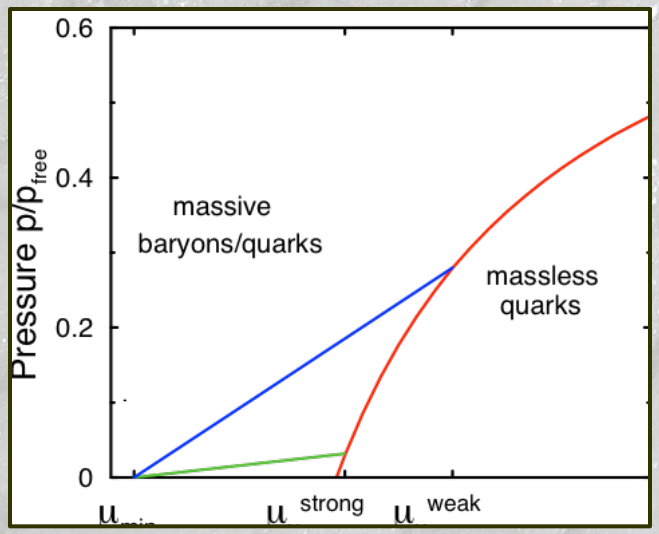
Matching to "low" density - Two scenarios:

Physical picture





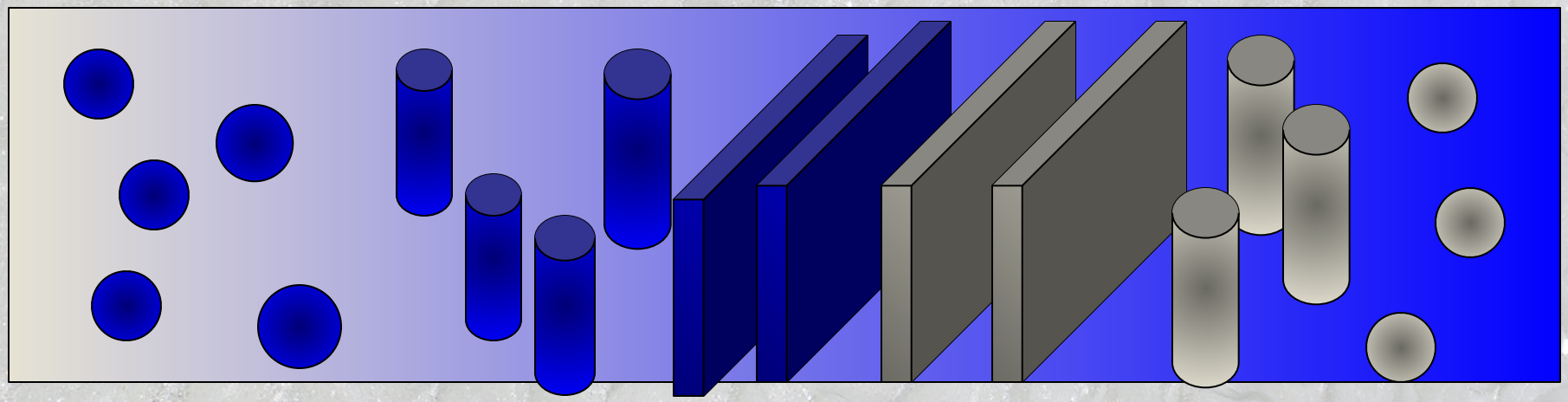
Two possibilities



- Possible scenarios for the intensity of the chiral transition can be related to the choice of Λ
- Moreover:
  - Pure neutron matter up to $\sim 2n_0$:
    [Akmal, Pandharipande, Ravelhall, 1998]
    $p/p_{free} \approx 0.04\ (n/n_0)^2$
  - Dilute nuclear matter in χPT at $\mu>0$

So: hadronic phase with small pressure viable!

Mixed phase structures ("pasta"):



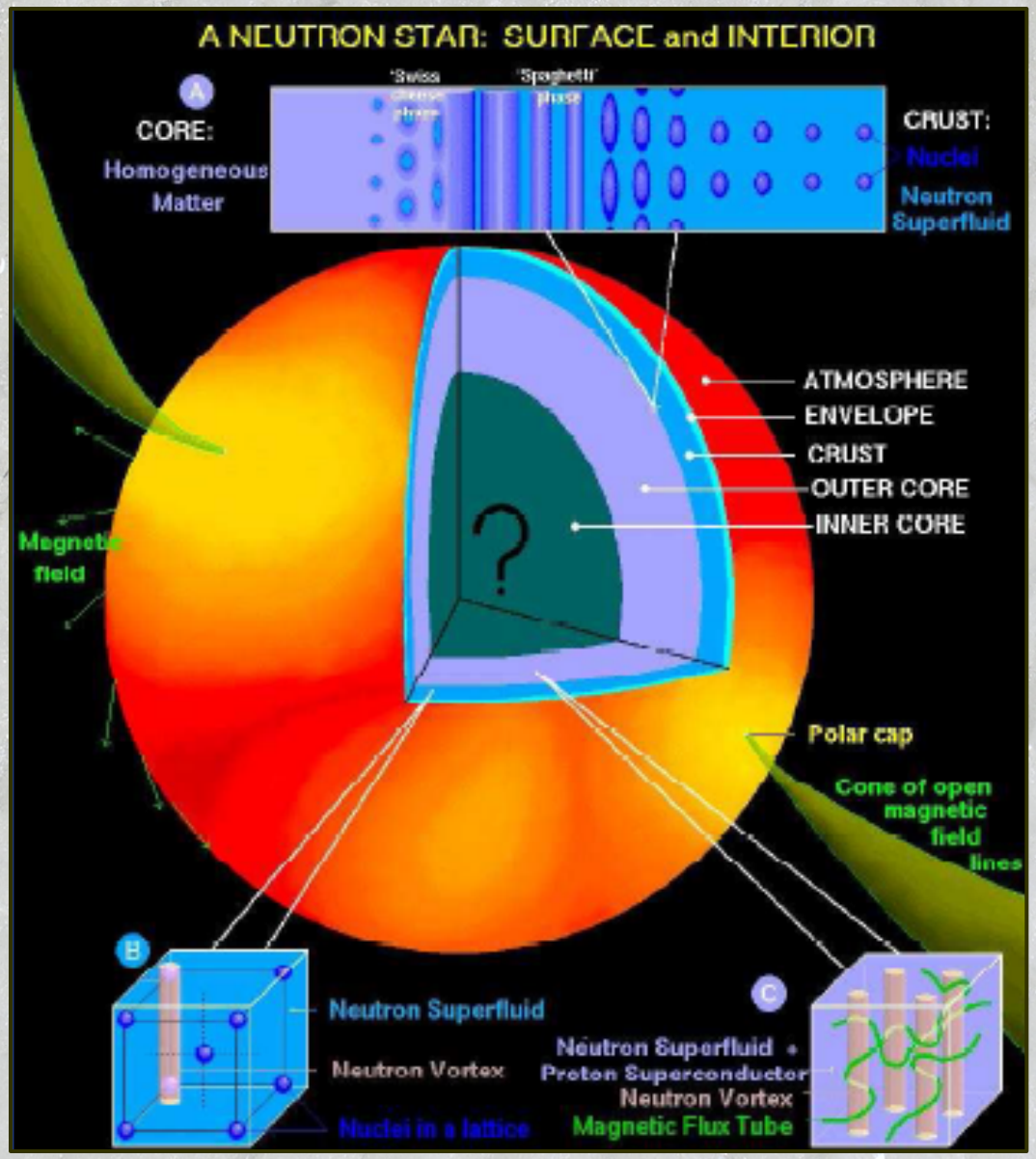
[From Reddy, SEWM 2005]

Nuclear

Exotic

- Typical size of defects ~ 5-10 fm

Crust structure:

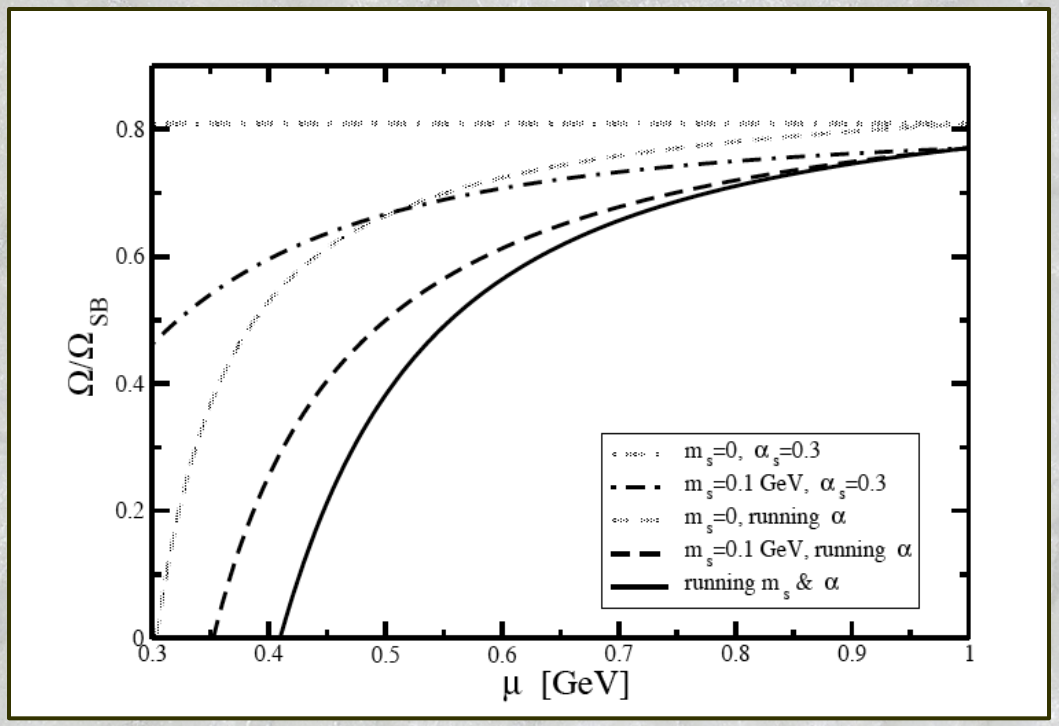


Nonzero quark mass effects

- Original approach: bag model + corrections ~ α_s from pQCD. In this case, corrections cancel out in the EoS for massles quarks.
- Quark mass effects were then estimated to modify the EoS by less than 5% and were essentially ignored for ~20 years.
- Quark masses & color SUC gaps neglected compared to typical values of μ in compact stars, ~400 MeV and higher.
- However, it was recently argued that both effects should matter in the lower-density sector of the EoS [Alford et al. (2004)].
- Although quarks are essentially massless in the core of quark stars, m_s runs up as one approaches the surface of the star !

This suggests the analysis of finite mass effects on the EoS for pQCD at high density including RG running of α_s and m_s !

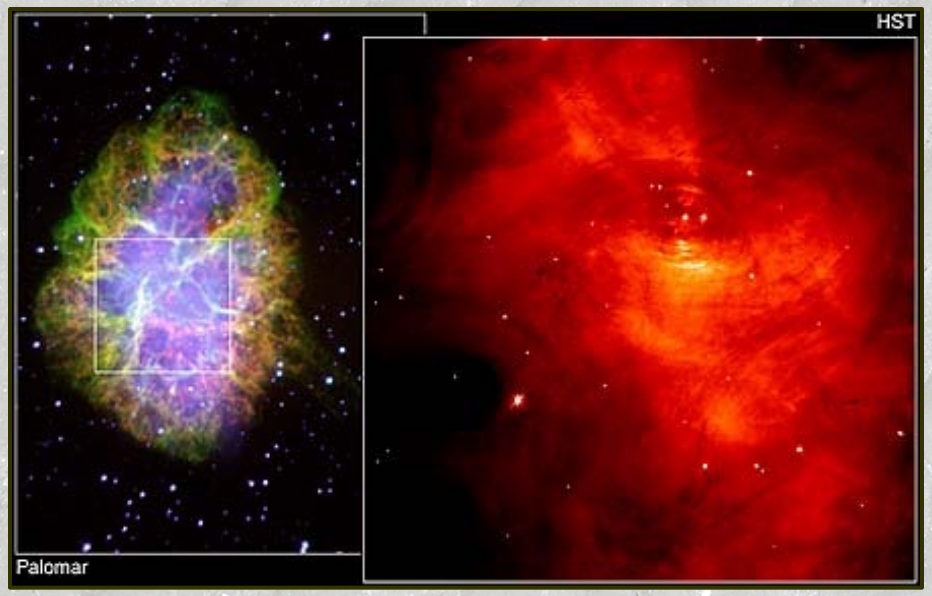
Thermodynamic potential for one massive flavor



(massive finite-density pQCD)

- Finite quark mass effects can dramatically modify the EoS for cold and dense QCD -> astrophysical effects!
- Numbers here are just illustrative of the strength of the effect. $O(\alpha_s^2)$ corrections will modify them significantly

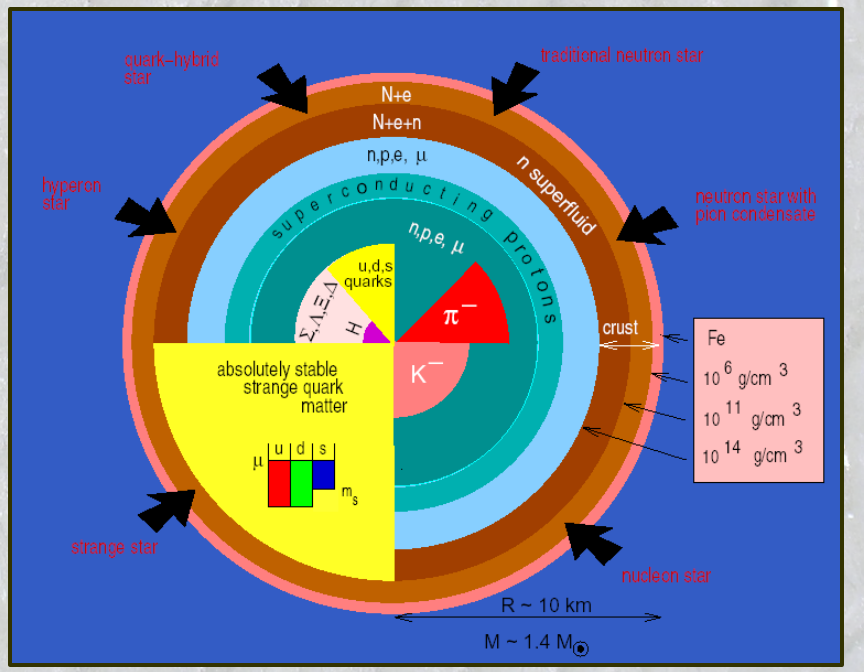
Compact stars and QCD at high density



(NASA)

- More than a thousand pulsars known today
- Several different scenarios for the core, including stable strange quark matter (Bodmer-Witten)

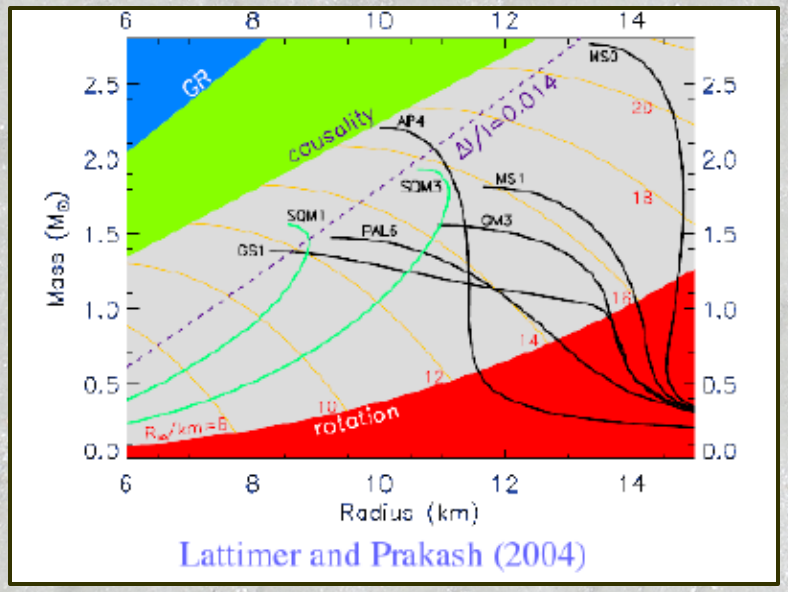
- Created by supernova explosion
- One possible final stage of evolution of massive stars
- Extreme central densities



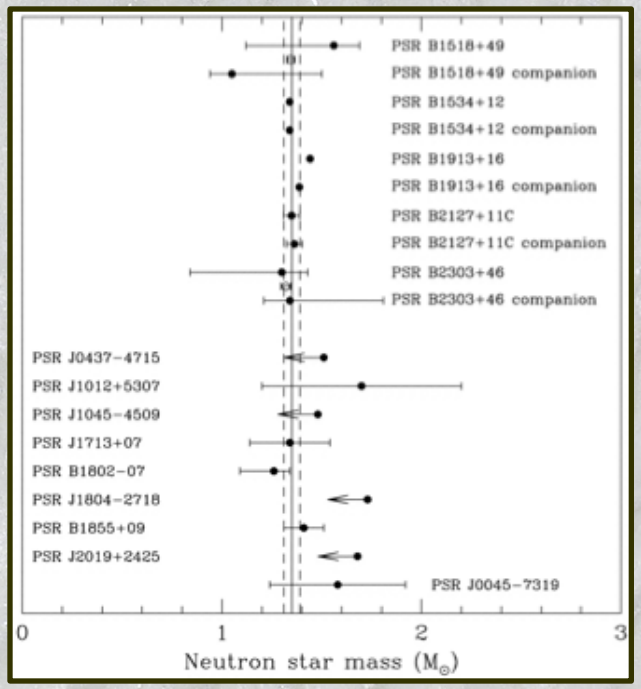
(F. Weber, 2000)

Some observables in compact stars

Mass-radius:

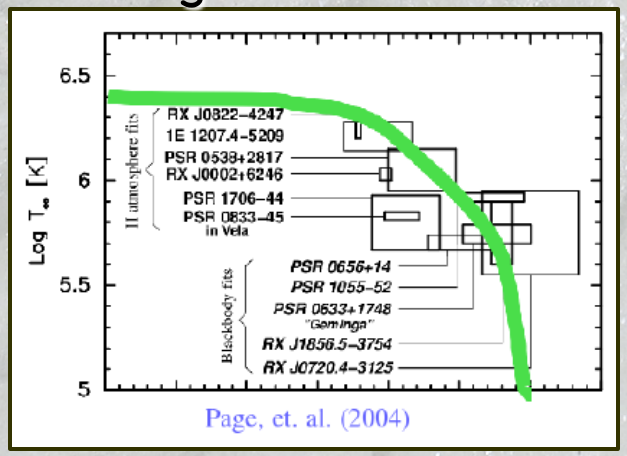


Mass:

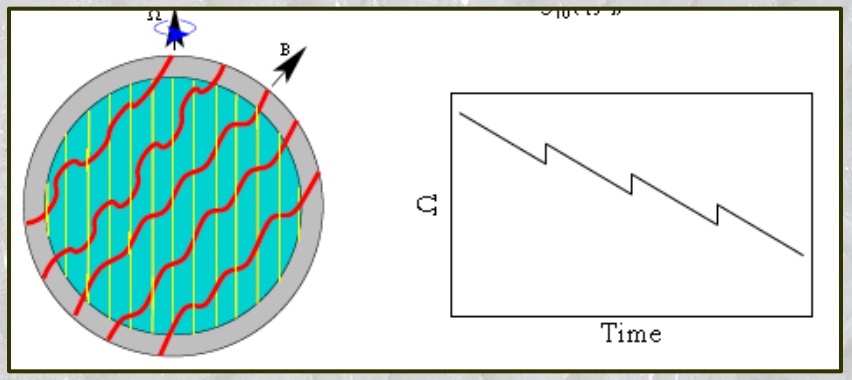


(Thorsett & Chakrabarty, 1999)

Cooling:



Glitches in Ω:



Quark star structure

Tolman-Oppenheimer-Volkov equations:

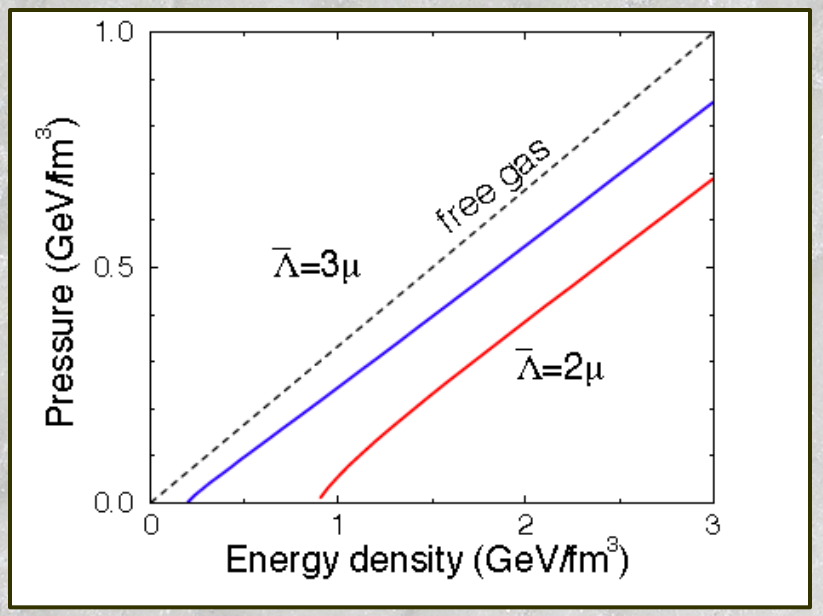
- Einstein's GR field equations
- Spherical symmetry
- Hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1-\frac{2G\mathcal{M}(r)}{r}\right]}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

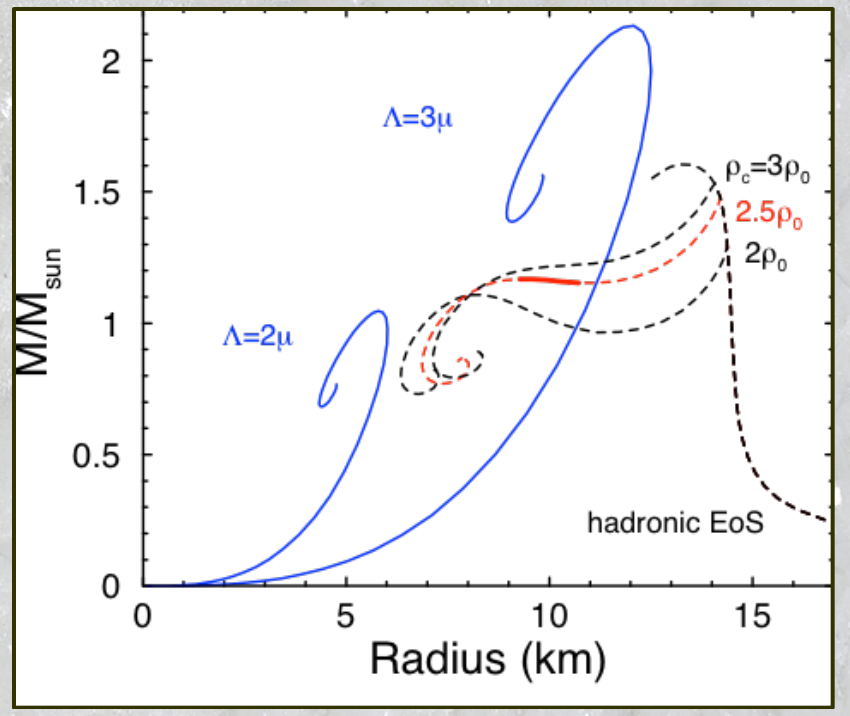
$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \epsilon(r) \;\; ; \;\; \mathcal{M}(R) = M$$

- Given the EoS p = p(ε), one can integrate the TOV equations from the origin until the pressure vanishes p(R) = 0
- Different EoS's define different types of stars: white dwarfs, neutron stars, quark stars, strange stars, ...

Results for massless quarks (pQCD) + hadronic mantle



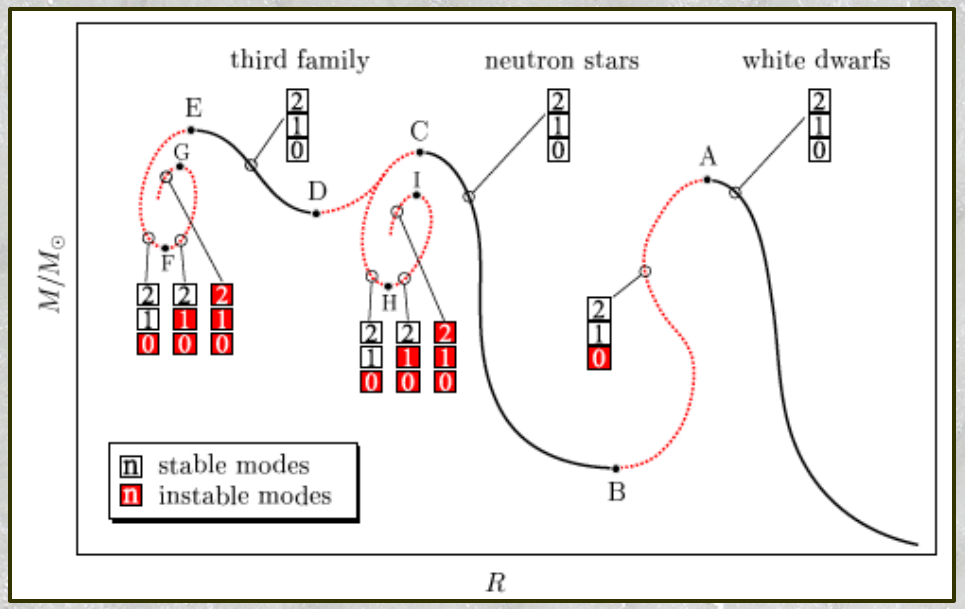
(finite-density pQCD)



- Interaction plays a role. One can still fit p=p(ε) with an effective bag model, but needs to modify not only B but also correct the free term

- Matching in the critical region is difficult because both calculations have problems for $\mu \sim \mu_c$ -> still: for a strongly 1st order transition, there can be a new class of compact stars (3rd family)

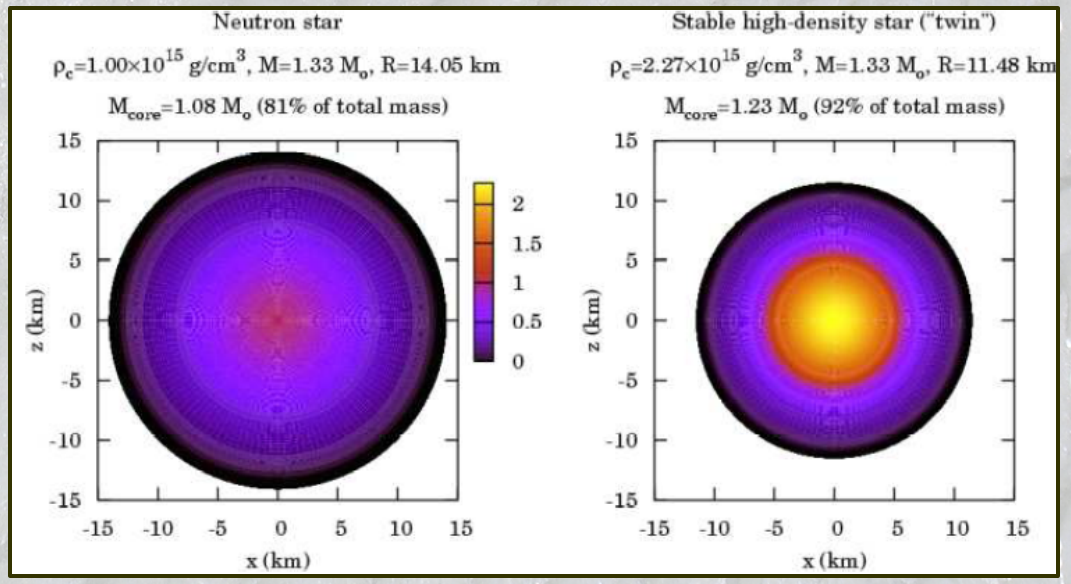
New class of stars and quark star twins



[Glendenning & Kettner, 2000; Schertler et al, 2000]

- Not so large differences in mass and radius
- Different density profile
- Not self-bound down to R=0 as strangelets

- Quark core with a hadronic mantle
- Smaller, denser companion (twin) to a hybrid (more hadronic) star



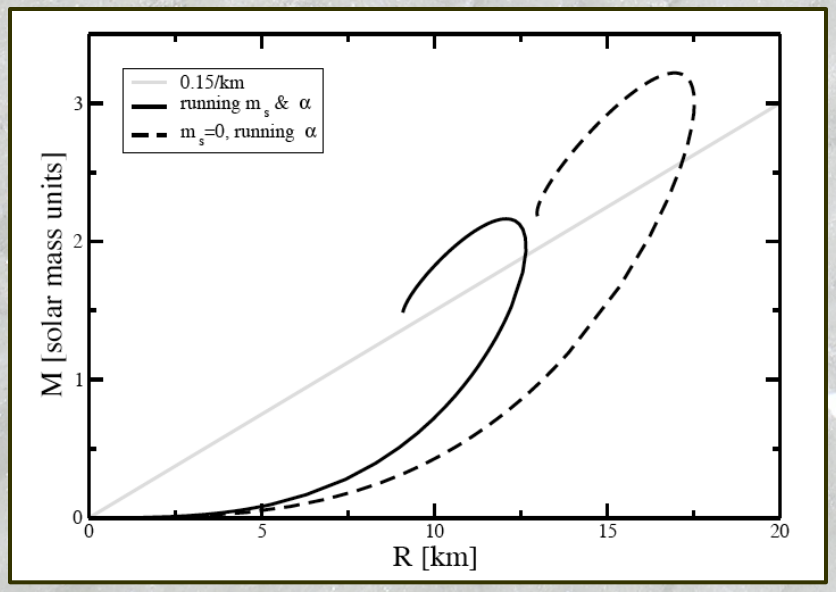
[Papasotiriou, 2006]

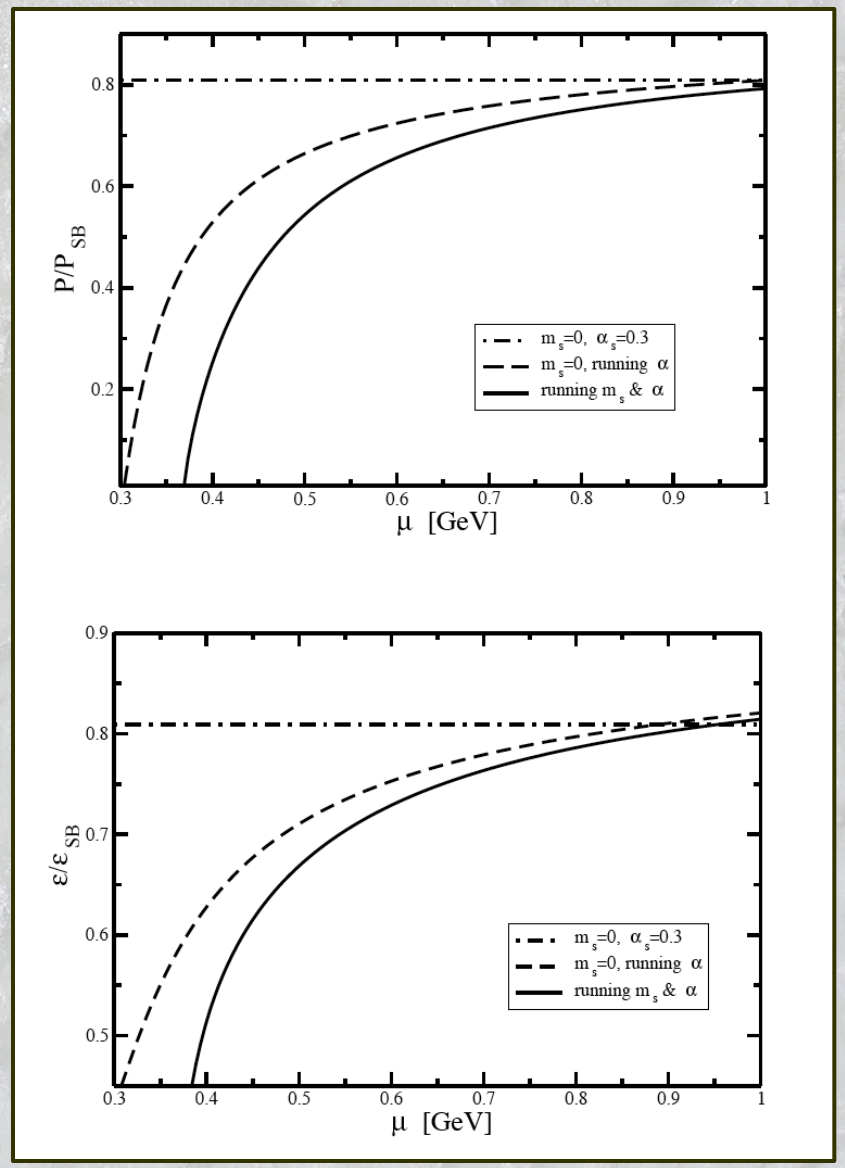
Results for pure massive quark matter + electrons ($m_s > 0$)

$$d \to u + e + \bar{\nu}_e, \quad u + e \to d + \nu_e,$$
$$s \to u + e + \bar{\nu}_e, \quad u + e \to s + \nu_e,$$
$$s + u \leftrightarrow d + u$$

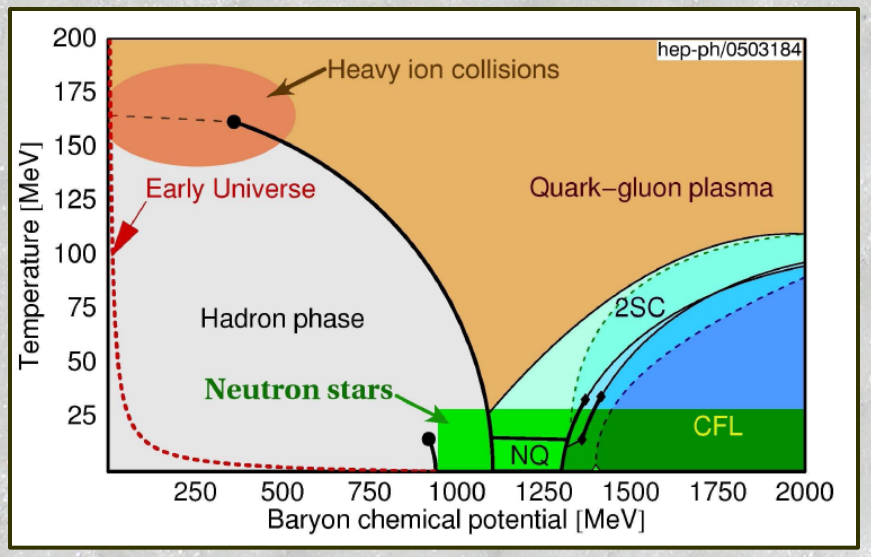
$$\mu_d = \mu_s = \mu, \quad \mu_u + \mu_e = \mu$$

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$$

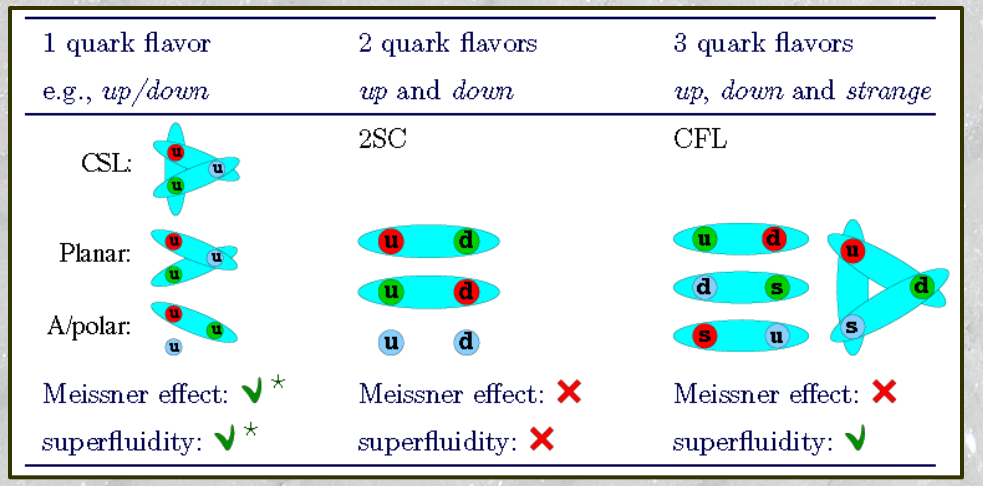


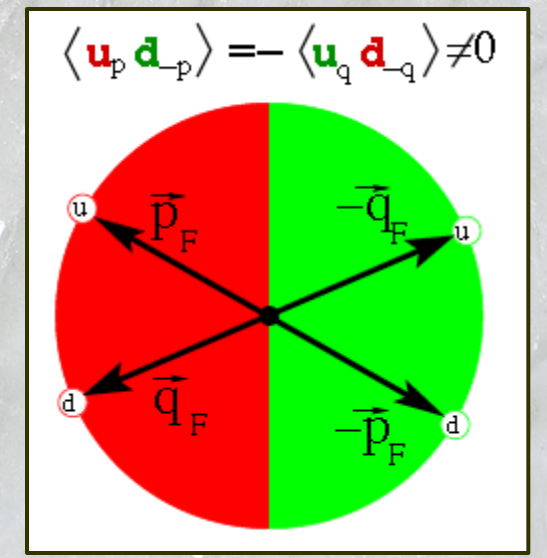


Color superconductivity



- Only quarks with different colors and flavors participate in Cooper pairing (attractive channel) -> diquark condensate





[From Shovkovy, SEWM 2005]

JAS 2009

Summary

• Lattice QCD is just starting to explore the finite density region, still far away from the high-density low-temperature sector.

• pQCD at finite density seems to provide sensible results, even for not so large values of μ. Mass and gap effects provide important contributions to the EoS near the critical region.

• The phase diagram can be very rich in the high-μ sector, with different possibilities for pairing and color superconductivity.

• Astrophysical measurements are becoming increasingly precise, and will start killing models soon. Some signatures (for strange, quark or hybrid neutron stars) are still very similar, though.

• The interior of compact stars is a very rich and intricate medium, which may contain all sorts of condensates as well as deconfined quark matter.

A few references:

QCD

- T. Muta, Foundations of Quantum Chromodynamics: An Introduction to Perturbative Methods in Gauge Theories (World Scientific, 1984)
- M.E. Peskin and D.V. Schroeder, An Introduction to Quantum Field Theory (Westview, 1995)
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Finite-temperature field theory

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Quark-gluon plasma in high-energy heavy ion collisions, compact stars and the early universe

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