# Interações Fortes em Condições Extremas

(fourth lecture)

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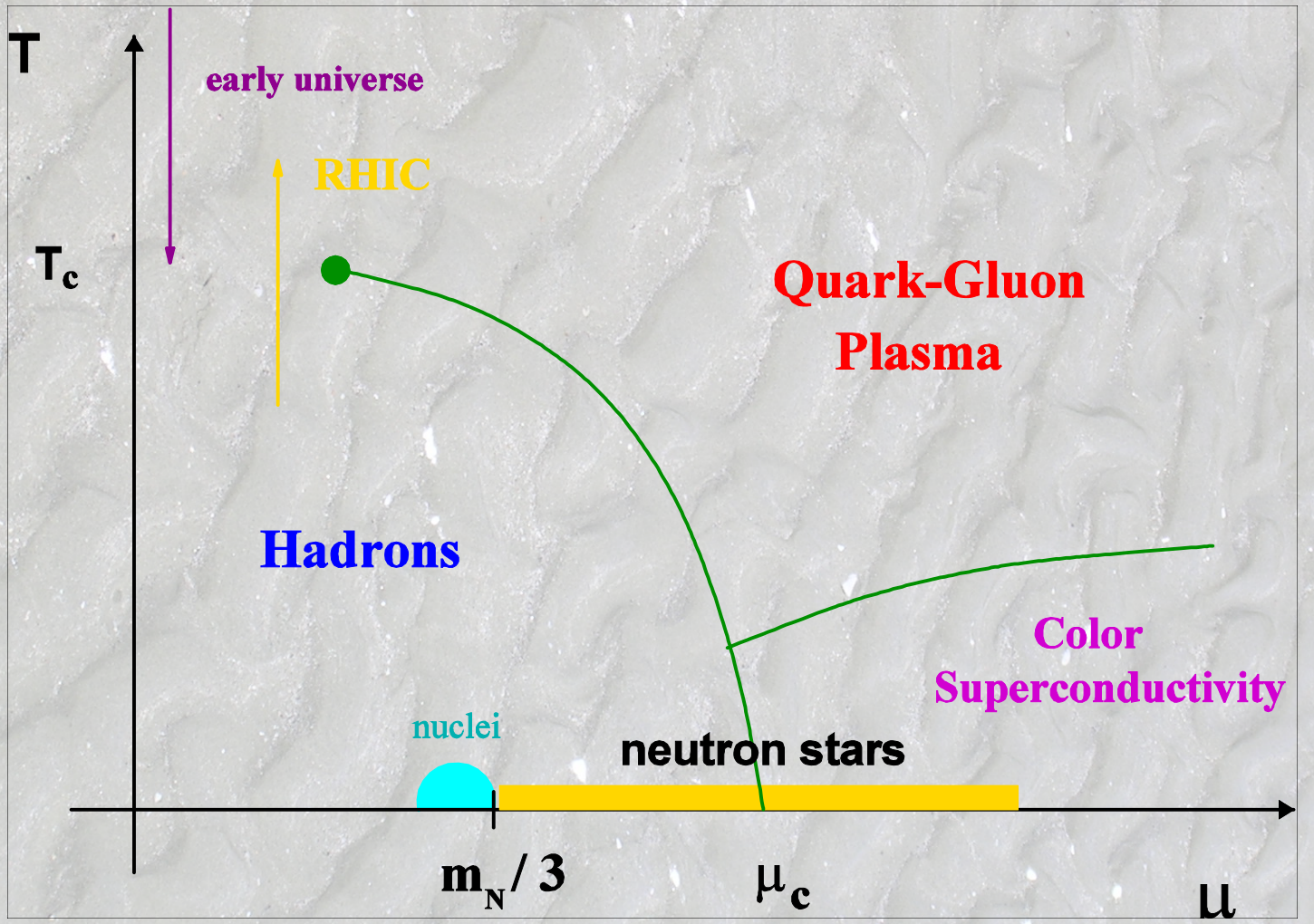

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# Fourth Lecture

- ★ Drawing the QCD phase diagrams
- ★ Phase transitions in QCD: how?
- ★ A very simple framework: the bag model
- ★ Symmetries in the QCD action: general picture
- ★ SU(Nc), Z(Nc) and Polyakov loops
- ★ Adding massless quarks (chiral symmetry)
- ★ Summary

# Drawing the QCD phase diagrams...



# Phase transitions in QCD
# How?

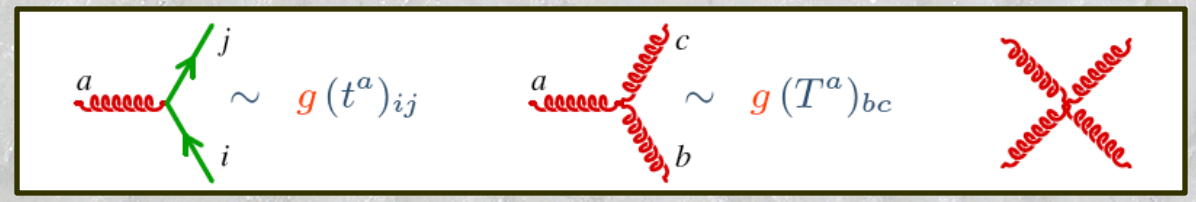
# So, we want to compute the EoS for QCD...

Then, all we need is...

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_f \overline{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A^a_\mu \tau^a)\psi_f$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

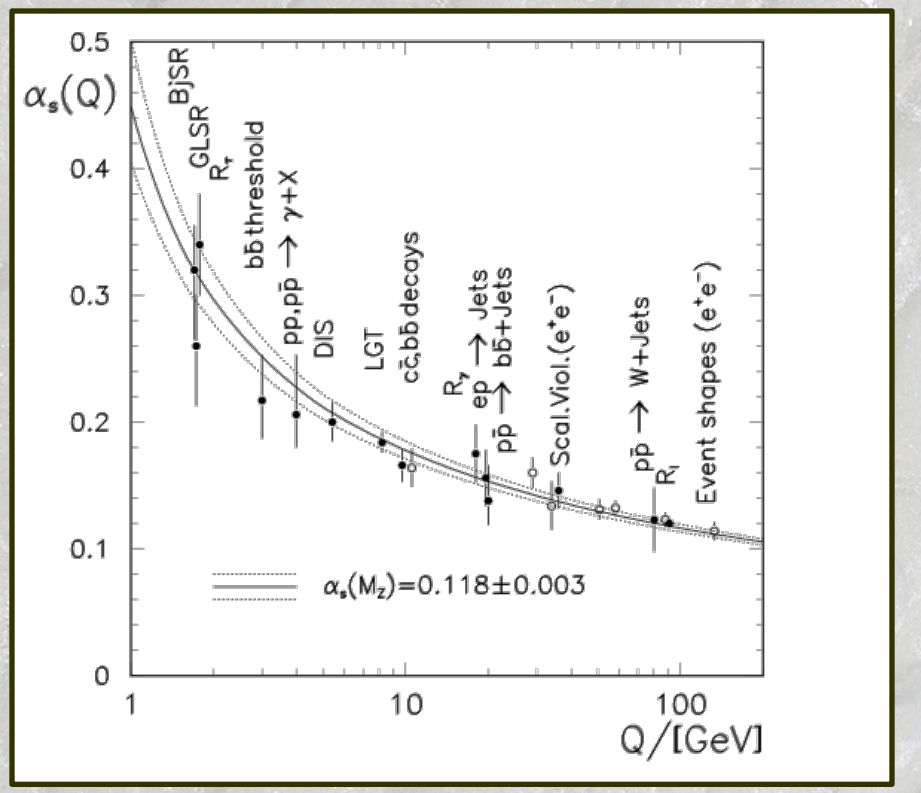
Dynamical fields: $N_f$ flavors of quarks in $N_c$ colors and ($N_c^2$-1) gluons; gauge symmetry given by SU($N_c$), etc.

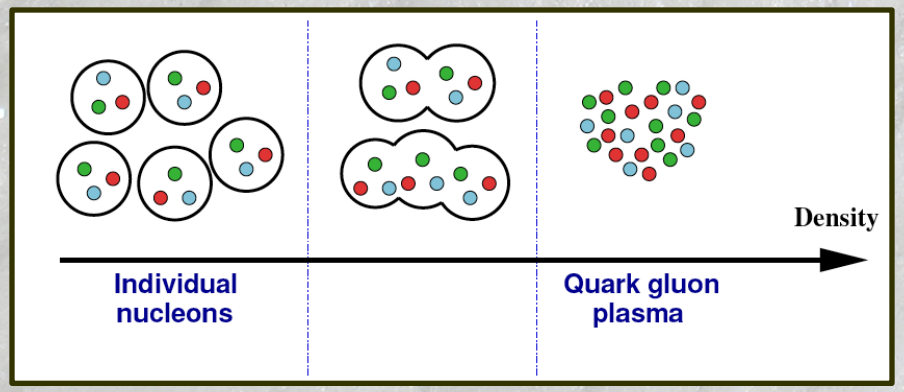



and compute the thermodynamic potential, blah blah...

Well, it's not so simple...

# Asymptotic freedom & the vacuum of QCD




- Matter becomes simpler at very high temperatures and densities (T and $\mu$ as energy scales in a plasma), but very complicated in the opposite limit...

- T and $\mu$ are not high enough in the interesting cases...

- Finite T pQCD is very sick...

## Finite T pQCD is very sick...

Informal discussion on the white board: qualitative points on the computation of the EoS, Matsubara frequencies, sum-integrals, infrared divergences, thermal mass, need of resummation, etc.

# Finite T pQCD is IR sick...

Nonlinear effects become important for: $\langle (\partial A)^2 \rangle \sim g^2 \langle A^4 \rangle \sim g^2 \langle A^2 \rangle^2$

Long-wavelength fluctuations: $\langle A^2 \rangle_\kappa \approx \int^\kappa \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{k} \frac{1}{\mathrm{e}^{k/T} - 1} \approx \int^\kappa \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{T}{k^2} \approx \kappa T$

Scale for PT breakdown: $\kappa \approx g^2 T$

- **Hard degrees of freedom: the plasma particles**

$$k \sim T \qquad \langle A^2 \rangle_T \sim T^2 \qquad \langle (\partial A)^2 \rangle_T \sim T^4 \qquad g^2 \langle A^2 \rangle_T^2 \sim g^2 T^4$$

- **Soft degrees of freedom, collective modes**

$$k \sim gT \qquad \langle A^2 \rangle_{gT} \sim gT^2 \qquad \langle (\partial A)^2 \rangle_{gT} \sim g^3 T^4 \qquad g^2 \langle A^2 \rangle_{gT}^2 \sim g^4 T^4$$

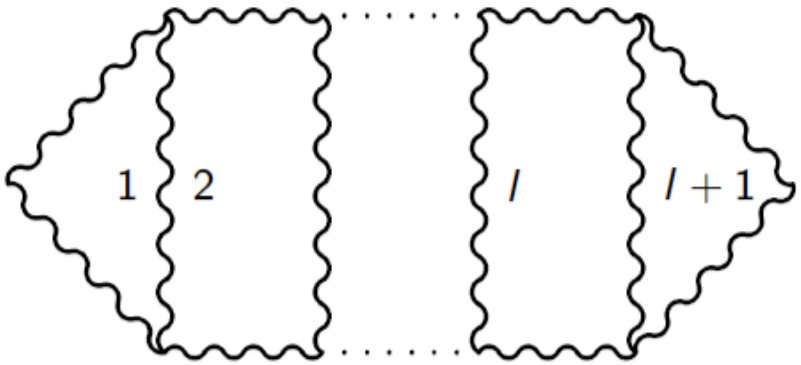
- **Ultrasoft degrees of freedom, unscreened magnetic fluctuations**

$$k \sim g^2 T \qquad \langle A^2 \rangle_{g^2 T} \sim g^2 T^2 \qquad \langle (\partial A)^2 \rangle_{g^2 T} \sim g^6 T^4 \qquad g^2 \langle A^2 \rangle_{g^2 T}^2 \sim g^6 T^4$$

[Blaizot (2004)]

# Even worse: Linde's problem [from Philipsen, Trento 2009]

- $(l+1)$-loop diagram: **contribution to pressure**




**contribution from Matsubara 0-mode:**

$$P \sim g^{2l} \left(T \int d^3p\right)^{l+1} p^{2l} (p^2 + m^2)^{-3l}$$

$$\begin{aligned} g^{2l} \quad & \text{for} \quad l = 1, 2 \\ g^6 T^4 \ln(T/m) \quad & \text{for} \quad l = 3 \\ g^6 T^4 (g^2 T/m)^{l-3} \quad & \text{for} \quad l > 3 \end{aligned}$$

magnetic mass $m_{mag} \sim g^2 T \Rightarrow$ **all loops** $(l > 3)$ contribute to $g^6$

even for weak coupling!

# Equation of state - naïve field map

hadrons
low temp.
and density

……

???

$T \sim T_c$ ; $n \sim n_c$

……

quarks
high temp.
and density

hadronic models,
nuclear field theory

pQCD at $T>0$ & $\mu>0$
asymptotic freedom

where all the things that matter happen...

there is no appropriate formalism yet!

## Ok, let's not desperate... there are many ways out!

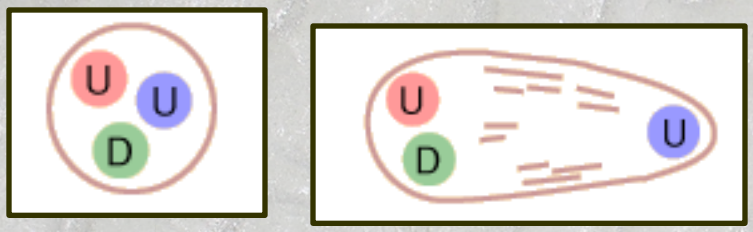
Some popular examples:

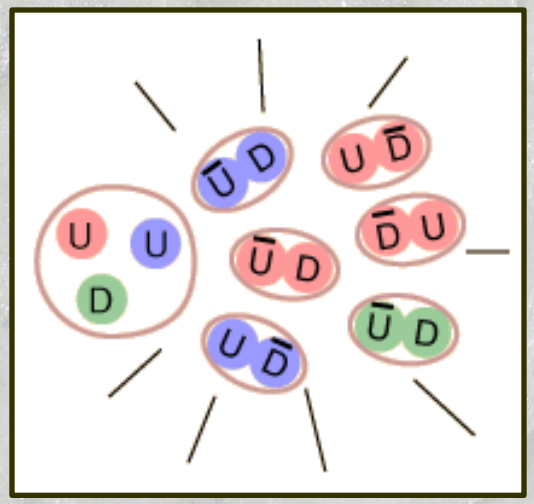
- Very intelligent and sophisticated brute force: lattice QCD
- Intensive use of symmetries: effective field theory models
- Redefining degrees of freedom: quasiparticle models
- "Moving down" from high-energy pQCD
- "Moving up" from hadronic low-energy (nuclear) models

We can also combine a bunch of them!

# But, before all that, let's do something REALLY simple...

## The MIT bag model (70's)




Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium.

+ eventual corrections $\sim \alpha_s$

- Asymptotic freedom: free quarks and gluons inside the color singlet bags
- Confinement: vector current vanishes on the boundary

- Confinement achieved by assuming a constant energy density for the vacuum (negative pressure) -> bag constant (B)

- B: phenomenological parameter, extracted from fits to masses
  (difference in energy density between the QCD and the pert. vacua)

- Hadron mass (spherical bag):

$E_h$ = "vacuum" + kinetic ~ $\frac{4}{3}\pi R^3 B + \frac{const}{R}$ $\Rightarrow$ $M_h = \frac{16}{3}\pi R_h^3 B$

- Hadron pressure: $p_h = -\frac{\partial E_h}{\partial V} = -B + \frac{const}{4\pi R^4} = 0$ (at equilibrium)

**Exercise:** show it in detail!

- Assuming a deconfining transition, the pressure in the QGP phase within this model is given by

$$p = \left(\nu_b + \frac{7}{4}\nu_f\right)\frac{\pi^2 T^4}{90} - B$$

whereas the pressure in the hadronic phase (pion gas) is

$$p_\pi = \nu_\pi \frac{\pi^2 T^4}{90}$$

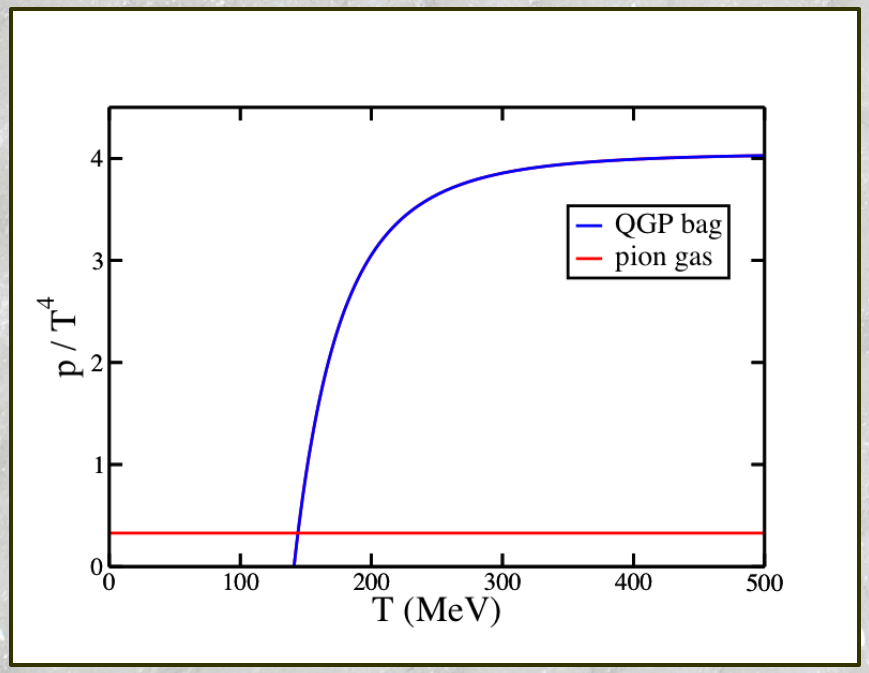
neglecting masses for simplicity. Here, we have the following numbers of d.o.f.'s: $\nu_\pi = 3$, $\nu_b = 2\,(N_c^2 - 1)$ and $\nu_f = 2\,N_c N_f$

For instance, for $N_c = 3$ , $N_f = 2$ and $B^{1/4} = 200$ MeV:

$$T_c = \left(\frac{45B}{17\pi^2}\right)^{1/4} \approx 144\ MeV$$

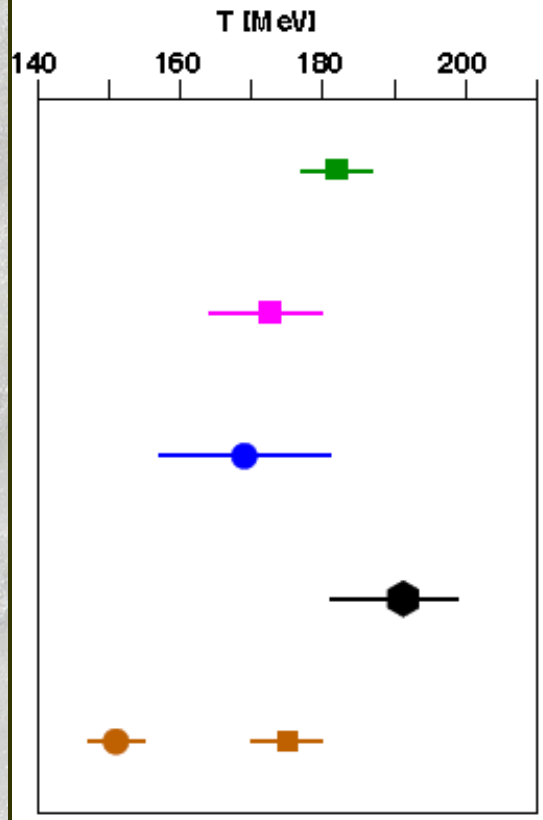
[not so bad as compared to lattice QCD results]

and a 1st order transition [differs from lattice QCD results]




Exercise: follow the same procedure in the case of finite density and T=0 to estimate the critical chemical potential for the quark-hadron transition.

# Summary of recent results on $T_c$




use T=0 scale: r0=0.469fm

Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)
(improved Wilson, Nt=8, 10; input: r0=0.5 fm)
(added Nt=12, Lattice'07) (rescaled to r0)

Y. Maezawa et al., hep-lat/0702005 (QM'2006)
(improved Wilson, Nt=4, 6; input: m-rho)
(no cont. exp. yet)

Nf=2+1:

C. Bernard et al., Phys.Rev. D71, 034504 (2005)
(improved staggered (asqtad), Nt=4,6,8, input r1)
(rescaled to r0)

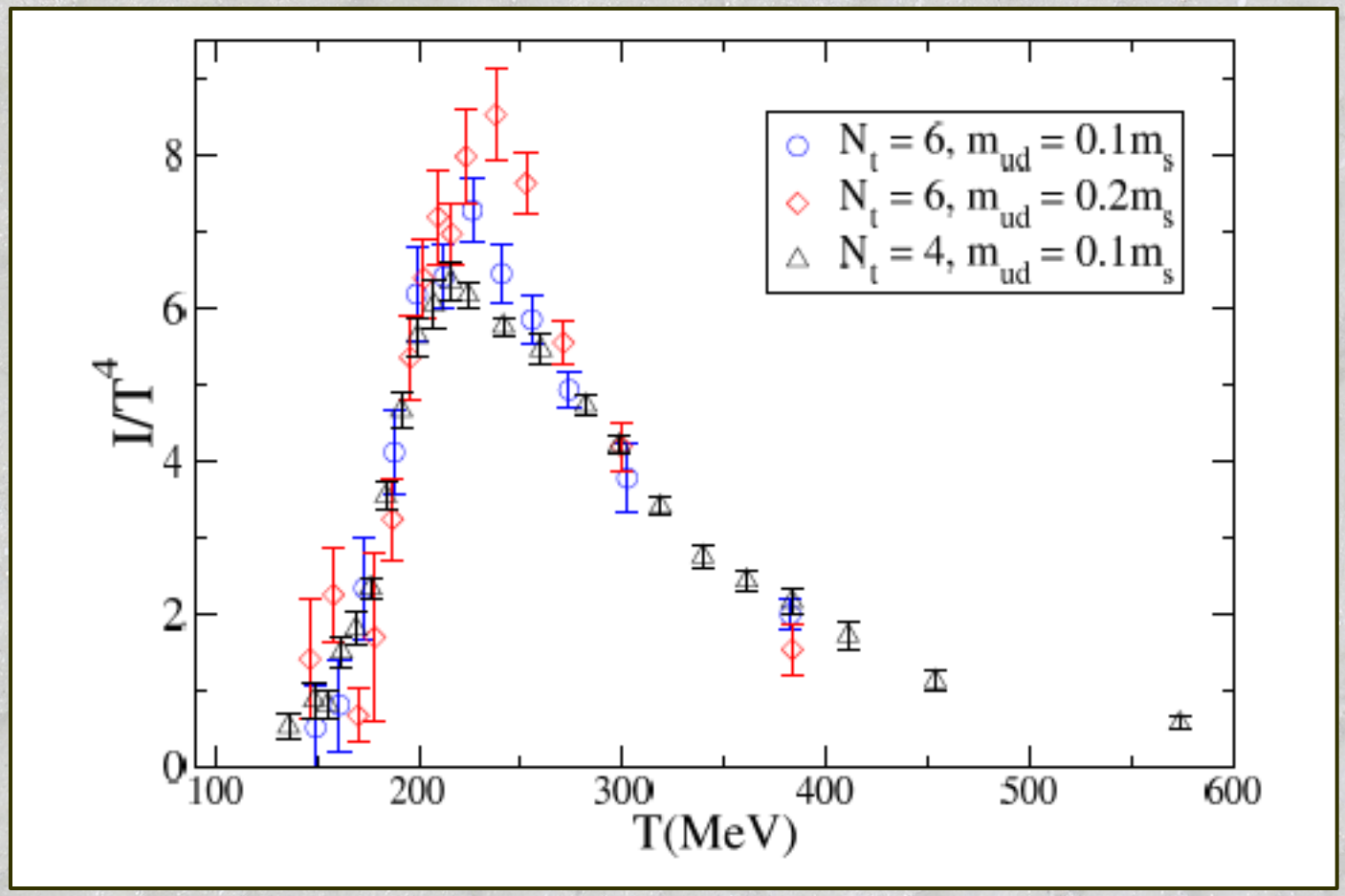
M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006)
(staggered (stout), Nt=4,6,8,10; input fK)
(converted to r0)

chiral deconfinement chiral+deconfinement

[From Karsch, Lattice 2007]

# Trace anomaly ("interaction measure") - $N_f$ = 2+1




[Bernard et al, 2006]

To go beyond in our study of the phases of QCD, we need to know its symmetries, and how they are broken spontaneously or explicitly. But QCD is very complicated:

- First, it is a non-abelian SU($N_c$) gauge theory, with gluons living in the adjoint representation
- Then, there are $N_f$ dynamical quarks (who live in the fund. rep.)
- On top of that, all these quarks have masses which are all different! Very annoying from the point of view of symmetries!

So, in studying the phases of QCD, we do it by parts, and consider many "cousin theories" which are very similar to QCD but simpler (more symmetric). We also study the dependence of physics on parameters which are fixed in nature.

The basic hierarchy is the following:

pure glue SU(N):

- Z(N) symmetry (SSB)
- order parameter: Polyakov loop L
- deconfining trans.: N=2 (2nd order), N=3 (weakly 1st order)

+ massless quarks:

- chiral symmetry (SSB)
- order parameter: chiral condensate $\sigma$
- Z(N) explicitly broken, but rise of L —> deconf.
- chiral trans.: N=3,2 ($N_f$=2) - 2nd order

+ massive quarks:

Z(N) and chiral explicitly broken
Yet vary remarkably and L <—> $\sigma$

# SU($N_c$), Z($N_c$) and the Polyakov loop

For the QCD Lagrangian (massless quarks)

$$\mathcal{L} = \frac{1}{2} tr G^2_{\mu\nu} + \bar{q} i \gamma^\mu D_\mu q$$

$$D_\mu = \partial_\mu - igA_\mu \quad , \quad G_{\mu\nu} = \frac{1}{-ig}[D_\mu, D_\nu]$$

we have invariance under local SU($N_c$). In particular, we have invariance under elements of the center Z($N_c$)

$$\Omega_c = e^{i\frac{2n\pi}{N_c}} \mathbf{1}$$

At finite temperature, one has also to impose the following boundary conditions:

$$A_\mu(\vec{x}, \beta) = +A_\mu(\vec{x}, 0) \quad , \quad q(\vec{x}, \beta) = -q(\vec{x}, 0)$$

Any gauge transf. that is periodic in $\tau$ will do it. However, 't Hooft noticed that the class of possible tranfs. is more general!

They are such that

$$\Omega(\vec{x},\beta) = \Omega_c \quad , \quad \Omega(\vec{x},0) = 1$$

$$A^{\Omega}(\vec{x},\beta) = \Omega_c^{\dagger} A_\mu(\vec{x},\beta)\Omega_c = +A_\mu(\vec{x},0)$$

$$q^{\Omega}(\vec{x},\beta) = \Omega_c^{\dagger} q(\vec{x},\beta) = e^{-i\phi} q(\vec{x},\beta) \neq -q(\vec{x},0)$$

keeping the gauge fields invariant but not the quarks!

For pure glue, this Z($N_c$) symmetry is exact, and we can define an order parameter from the trace of the Polyakov loop:

$$\ell(\vec{x}) = \frac{1}{N_c}\,\mathrm{Tr}\; L(\vec{x}) = \frac{1}{N_c}\,\mathrm{Tr}\;\mathcal{P}\exp\left[ig\int_0^\beta d\tau\;\tau^a A_0^a(\vec{x},\tau)\right]$$

$$L(\vec{x}) \mapsto \Omega_c\; L(\vec{x})\; \mathbf{1} = e^{i\frac{2n\pi}{N_c}}\, L(\vec{x})$$

Exercise: show it in detail!

N.B: $\ell$ is gauge invariant.

At very high T, g ~ 0, and $\beta$ -> 0, so that

$$\langle \ell \rangle = e^{i\frac{2n\pi}{N_c}} \ell_0 \,, \;\; \ell_0 \sim 1$$

and we have a N-fold degenerate vacuum, signaling SSB of global $Z(N_c)$. At T = 0, confinement implies that $\ell_0 = 0$ ['t Hooft (1978)].

Then, $\ell_0$ can be used as an order parameter for the deconfining transition:

$$\ell_0 = 0 \;\; , \;\; T < T_c \;\; ; \;\; \ell_0 > 0 \;\; , \;\; T > T_c$$

Usually the Polyakov loop is related to the free energy of an infinitely heavy test quark via [McLerran & Svetitsky (1981)]

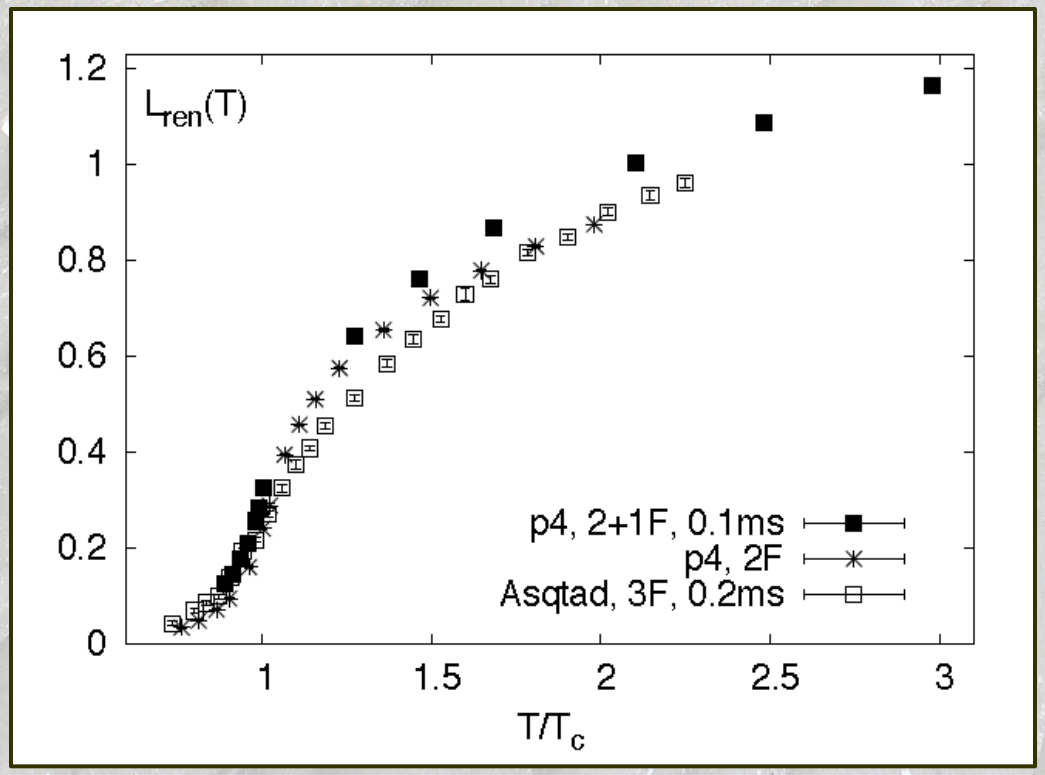
$$\langle \ell \rangle = e^{-F_{test}/T}$$

(confinement: no free quark)

Exercise: do you see any possible problem in the equation above? (Cf. 1st eqn in this slide!) If so, could we, instead, relate $< \ell >$ to the propagator for a test quark?

The analysis above is valid only for pure glue (no dynamical quarks). However, we can still ask: is Z(3) an approximate symmetry in QCD?

On the lattice ("full" QCD), one sees:




[Petrov, 2006]

- Remarkable variation of $\ell_0$ around $T_c$
- $\ell_0$ plays the role of an approximate order parameter
- N.B.: Z(3) is broken at high, not low T!

Exercise: compare this description to that of spin systems (Ising, Potts, etc.)

# Adding massless quarks (chiral symmetry)

In the limit of massless quarks, QCD is invariant under global chiral rotations $U(N_f)_L \times U(N_f)_R$ of the quark fields.

One can rewrite this symmetry in terms of vector (V = R + L) and axial (A = R - L) rotations

$$U(N_f)_L \times U(N_f)_R \sim U(N_f)_V \times U(N_f)_A$$

As U(N) ~ SU(N) x U(1), one finds

$$U(N_f)_L \times U(N_f)_R \sim SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

where we see the $U(1)_V$ from quark number conservation and the $U(1)_A$ broken by instantons.

Exercise: can we restore $U(1)_A$ in a hot and dense medium? Think of possible consequences and observables.

In QCD, the remaining $SU(N_f)_L \times SU(N_f)_R$ is explicitly broken by a nonzero mass term. Take, for simplicity, $N_f=2$:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}^f_L \gamma^\mu D_\mu \psi_L + \bar{\psi}^f_R \gamma^\mu D_\mu \psi^f_R - m_u\left(\bar{u}_L u_R + \bar{u}_R u_L\right) - m_d\left(\bar{d}_L d_R + \bar{d}_R d_L\right)$$

so that, for non-vanishing $m_u = m_d$, the only symmetry that remains is the vector isospin $SU(2)_V$. In the light quark sector of QCD, chiral symmetry is just approximate.

Then, for massless QCD, one should find parity doublets in the vacuum, which is not confirmed in the hadronic spectrum. Thus, chiral symmetry must be broken in the vacuum by the presence of a quark chiral condensate, so that

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

and the broken generators allow for the existence of pions, kaons, ...

Hence, for massless QCD, we can define an order parameter for the SSB of chiral symmetry in the vacuum - the chiral condensate:

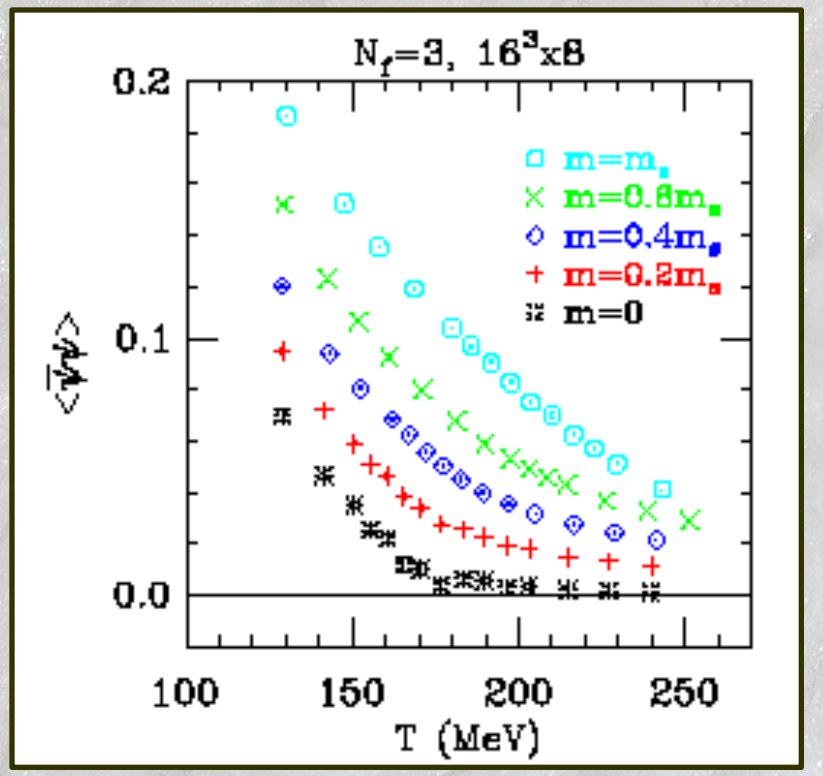
$$\langle 0|\bar{\psi}\psi|0\rangle = \langle 0|\bar{\psi}_L\psi_R|0\rangle + \langle 0|\bar{\psi}_R\psi_L|0\rangle$$

so that this vacuum expectation value couples together the L & R sectors, unless in the case it vanishes.

For very high temperatures or densities (low $\alpha_s$), one expects to restore chiral symmetry, melting the condensate that is a function of T and $\mu$ and plays the role of an order parameter for the chiral transition in QCD.

Again: the analysis above is valid only for massless quarks. However, we can still ask: is QCD approximately chiral in the light quark sector?

On the lattice ("full" massive QCD), one sees:




[MILC collab., 2003]

- Remarkable variation of the condensate around $T_c$
- The condensate plays the role of an approximate order parameter

## Summary

- Two relevant phase transitions in QCD associated with SSB mechanisms for different symmetries of the action

- Approximate $Z(N_c)$ symmetry and deconfinement [exact for pure gauge $SU(N_c)$]. Order parameter: (trace of the) Polyakov loop

- Approximate chiral symmetry and chiral transition [exact for massless quarks]. Order parameter: chiral condensate.

- Some good estimates within a very simple framework: the bag model. Very crude, disagrees with lattice QCD on the nature of the transition, but still used in several calculations (EoS for compact stars, hydro evolution of the QGP, etc.)

- Going beyond: effective models (based on symmetries of $S_{QCD}$)

# Fifth Lecture

- ★ Finite T x finite $\mu$: pQCD, lattice, sign problem, etc
- ★ Nuclear EoS: relativistic and non-relativistic (brief)
- ★ pQCD at nonzero T and $\mu$ (brief)
- ★ Cold pQCD at high density for massless quarks
- ★ Nonzero mass effects
- ★ Compact stars and QCD at high density
- ★ Summary