Interações Fortes em Condições Extremas

(third lecture)

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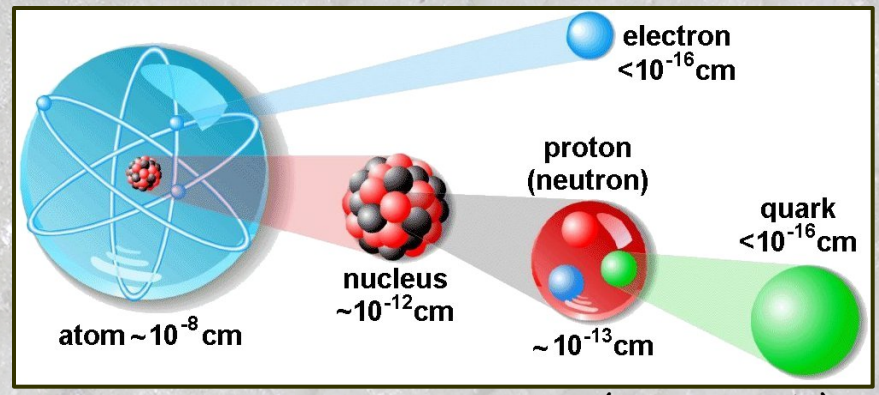
Third Lecture

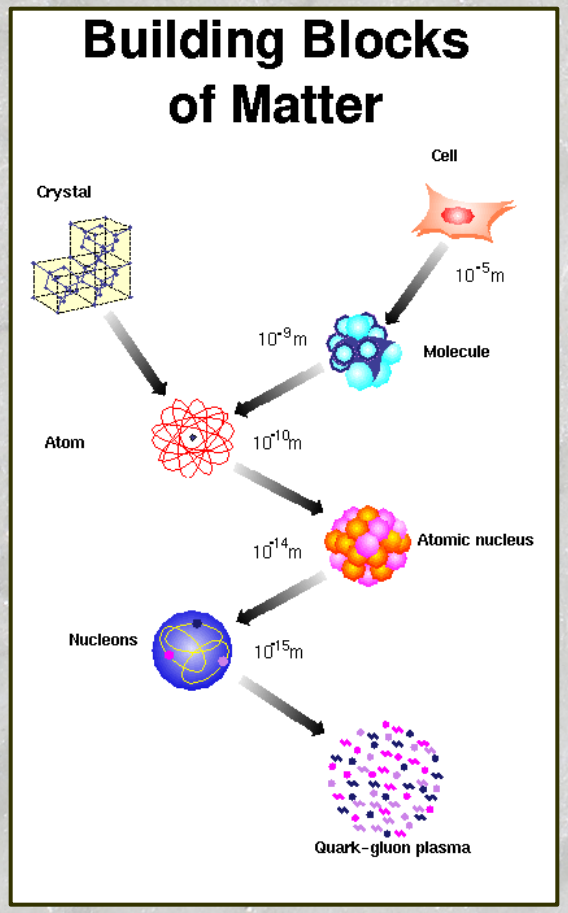
- ★ Introduction and motivation
- ★ Evidence for quarks, gluons & color
- ★ The QCD Lagrangian
- ★ Asymptotic freedom & the vacuum of QCD
- ★ Effective field theory basic recipe
- ★ Summary

Introduction and motivation

• Basic question: the structure of matter ("building blocks") and the nature of fundamental interactions.



(Univ. Toronto)



(M. Lahanas)

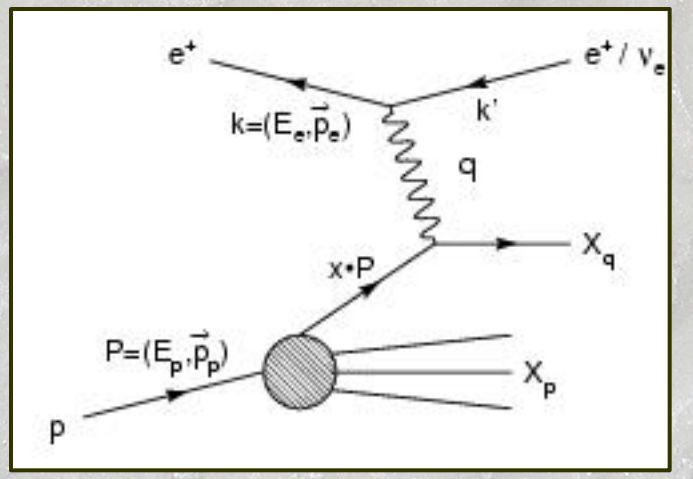
• New states of ("bulk", medium) matter - need of extreme conditions to create new kinds of "plasmas", "gases", "liquids", "crystals".

• Our focus: strong interactions and quark matter, QCD phase structure.

• Need of sophisticated field theory formulation & diverse complementary techniques. E.g.: effective models.

Evidence for quarks, gluons & color

Deep inelastic scattering



(DESY)

- Just like in Rutherford's experiments with (~7 MeV) α particles on gold sheet targets, one can use high-energy electrons (~20 GeV at SLAC) to probe the proton! 1968: 1st evidence for quarks.

$$Q^2 \equiv -q^2 \geq 0 \;\; ; \;\; x \equiv \frac{Q^2}{2P\cdot q} \;\; ; \;\; 0 < x < 1$$

Differential cross-section for γ^* absorption

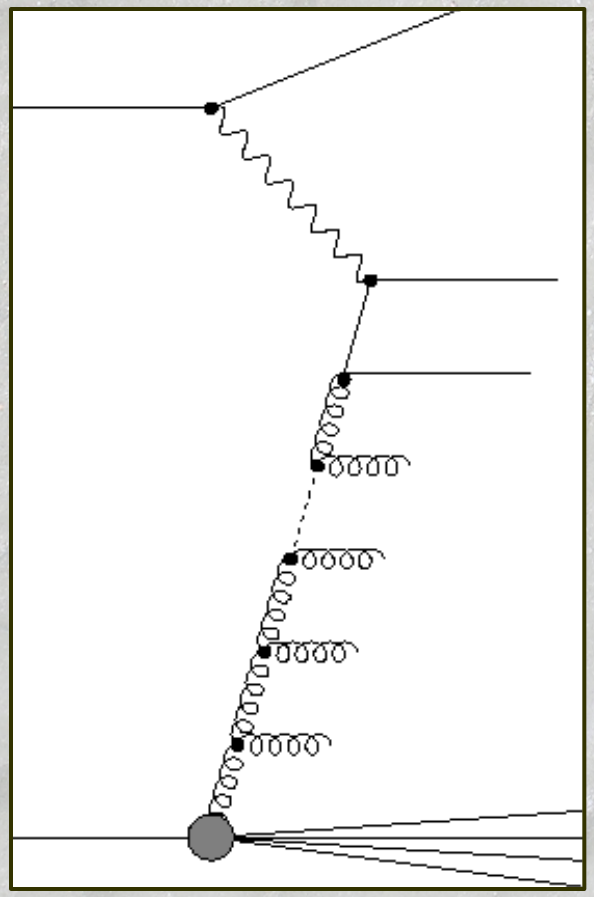
$$\sigma_{\gamma^* p}(x, Q^2) = \frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2)$$

F_2: proton structure function; measure of quark distribution

Bjorken scaling: For free independent pointlike partons, $F_2 = F_2(x)$

Exercise: Prove it!

But things are more complicated...



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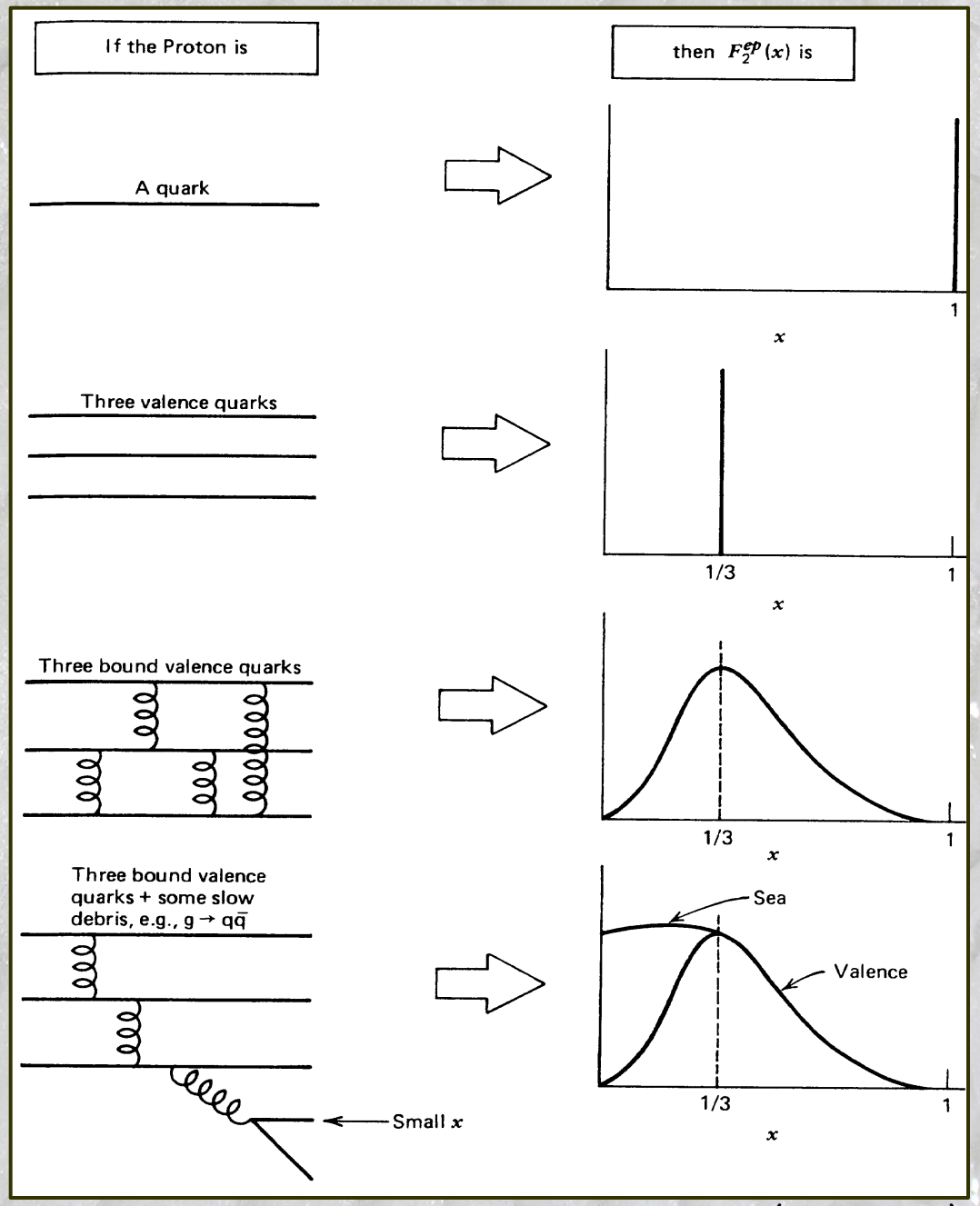
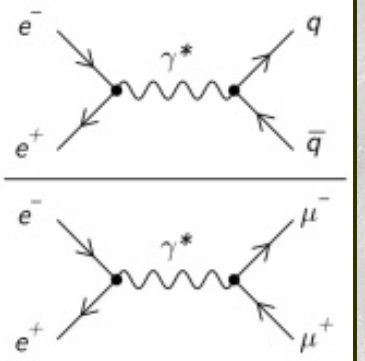


(Kass, 2003)

Pieces of evidence for color

The hadron-muon branching ratio [overall N_c factor]

$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} \approx$$

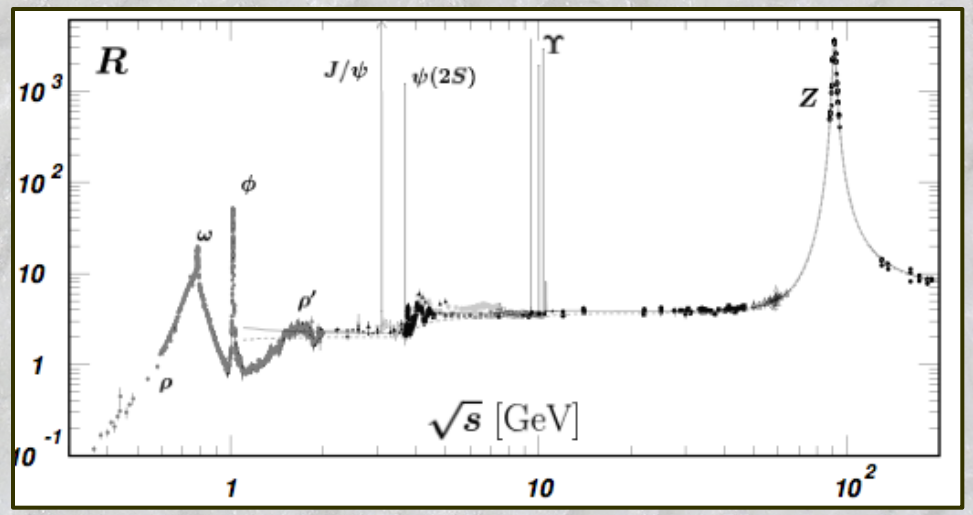


$$\sigma(e^+e^- \to hadrons) = \frac{4\pi\alpha^2}{3s} \left[N_c \sum_{f=1}^{N_f} Q_f^2 \right]$$

Flavors that contribute: $2m_f < \sqrt{s}$

Crossing the different quark thresholds, one has approximate steps:

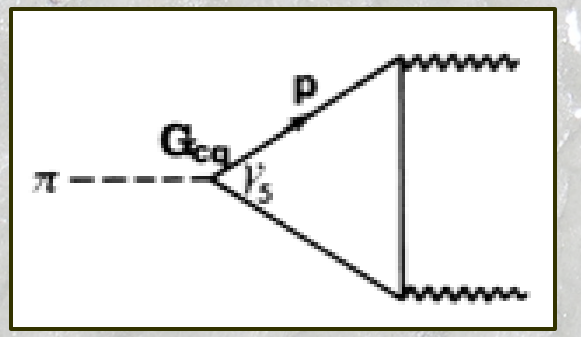
$$R(\sqrt{s} < 3 \text{ GeV}) = 2$$
$$R(4 < \sqrt{s} < 9 \text{ GeV}) = 10/3$$
$$\ldots$$



(PDG)

$\pi^0 \to \gamma\gamma$ [overall N_c^2 factor]

$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 \, (Q_u^2 - Q_d^2)^2 \, \frac{\alpha^2 m_{\pi^0}^3}{(4\pi)^3 f_\pi^2}$$



$$\Gamma^{(N_c=3)}(\pi^0 \to \gamma\gamma) = 7.6 \text{ eV}$$

$$\Gamma^{(exp)}(\pi^0 \to \gamma\gamma) = 7.7 \pm 0.5 \text{ eV}$$

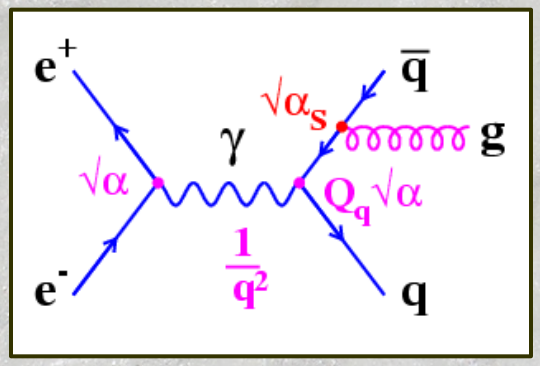
(PDG)

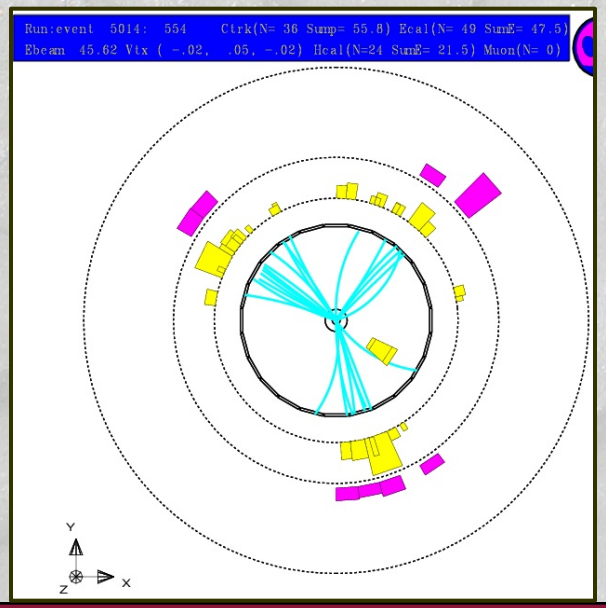
N_c=1, e.g., is totally off:

$$\Gamma^{(N_c=1)}(\pi^0 \to \gamma\gamma) = 0.84 \text{ eV}$$

Pieces of evidence for gluons and parton evolution

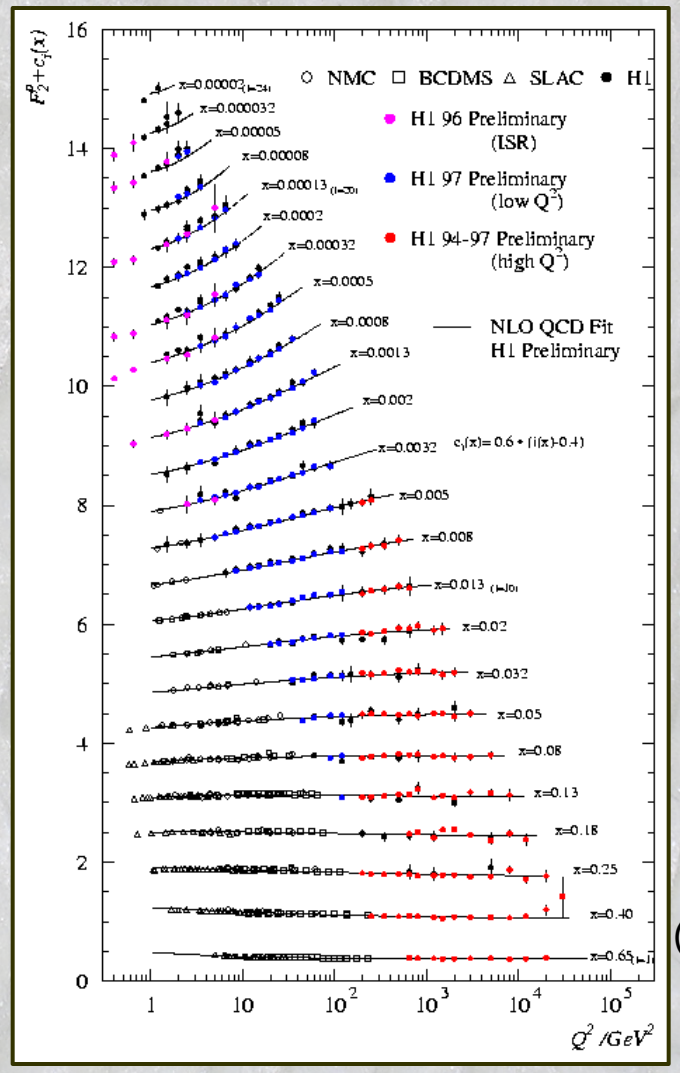
3-jet events
(gluon radiation)





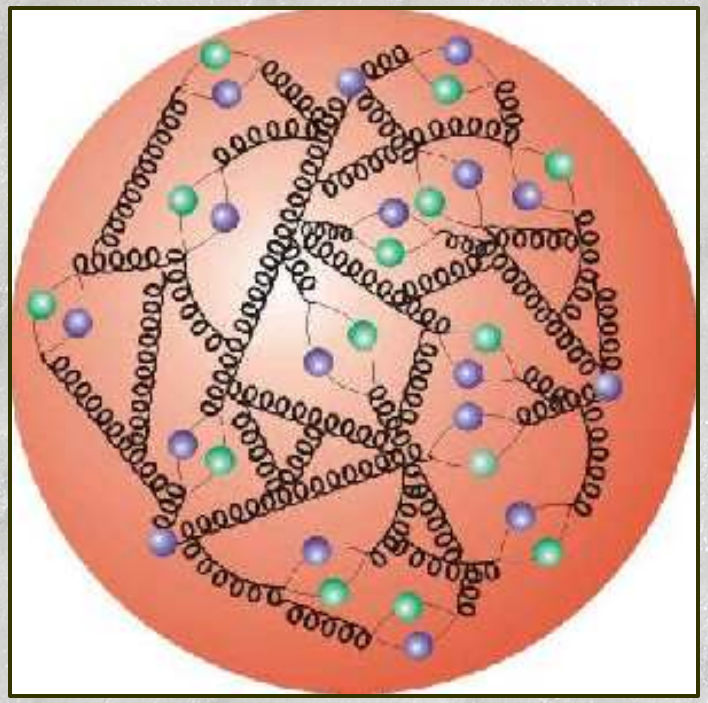
(LEP)

scaling violation
(deviations from the simplified quark model)



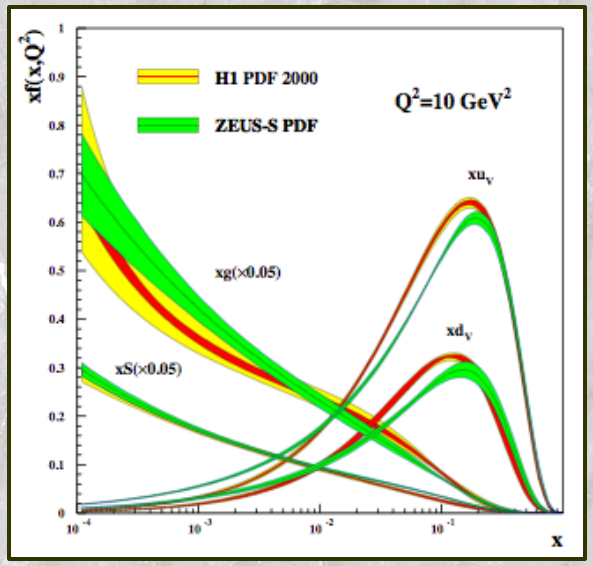
(HERA)

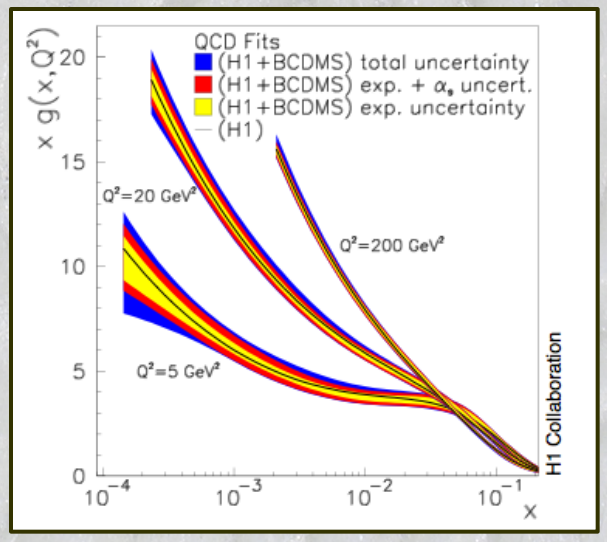
A "picture" of the proton (a real mess...)



(Univ. Oxford)

Parton distributions:





(HERA)

The QCD Lagrangian

- In the usual approach to QED, the theory is obtained by quantization of classical electrodynamics. Gauge invariance is, then, shown and used as a side feature.

- On the other hand, one could start with free electrons and assume local U(1) invariance to "give birth" to the electromagnetic field -> the gauge principle!

- To build the QCD Lagrangian, we can start with N_c spin-1/2 free fermions that live in the fundamental (vector) representation of $SU(N_c)$.

- New quantum number: color - in QCD, $N_c=3$.

Free quark Lagrangian (1 flavor):

$$\mathcal{L}_F = \overline{\psi}^{\alpha}(i\gamma^{\mu}\partial_{\mu} - m)\psi^{\alpha}$$

Global unitary SU(N_c) transformations:

$$\psi(x) \rightarrow \psi^{\Omega}(x) = \Omega\psi(x) \quad ; \quad \Omega \in SU(N_c)$$

Remarks:

- ψ^{α} behaves like a vector under SU(N_c)
- Ω are represented by unitary N_c x N_c matrices
- Usually ψ has many indices (color, flavor, spin, ...)

For infinitesimal transformations:

$$\psi(x) \to \psi^{\Omega}(x) = e^{i\omega^a T^a}\psi(x) = (\mathbf{1} + i\omega^a T^a)\psi(x)$$

ω^a: infinitesimal (constant) parameters

T^a: generators of $SU(N_c)$ $\quad [T^a, T^b] = f^{abc}\, T^c$

f^{abc}: structure constants of $SU(N_c)$

$a = 1, 2, 3, \ldots, (N_c^2 - 1)$

Generalizing to local transformations is simple:

$$\psi(x) \longrightarrow \psi^{\Omega}(x) = \Omega(x)\psi(x) \quad ; \quad \Omega(x) \in SU(N_c)$$

However, the free quark Lagrangian is no longer invariant:

$$\partial_\mu \psi(x) \mapsto \Omega(x)\partial_\mu \psi(x) + [\partial_\mu \Omega(x)]\psi(x)$$

So, we have to build a "covariant derivative" (transforms like ψ), which implies the introduction of a N_c x N_c matrix field:

$$A_\mu = A_\mu^a \, T^a$$ (gauge field)

which transforms under SU(N_c) in a way that suits the definition of the covariant derivative.

That means:

$$D_\mu(A)\psi(x) \equiv (\partial_\mu - igA_\mu)\psi(x)$$

$$D^\Omega_\mu(A)[\Omega(x)\psi(x)] = \Omega(x)[D_\mu(A)\psi(x)]$$

Exercise: imposing gauge invariance, show that the gauge field has to transform as

$$A^\Omega_\mu = \Omega A_\mu \Omega^{-1} + \frac{i}{g}\,\Omega\partial_\mu\Omega^{-1}$$

or, for infinitesimal transformations:

$$\delta A^a_\mu = f^{abc}\omega^b A^c_\mu + \frac{i}{g}\,\partial_\mu\omega^a$$

- If the field A_μ is to be identified with dynamical degrees of freedom (gluons), we must add a gauge-invariant kinetic term to the Lagrangian.

- Motivated by QED, we introduce

$$F_{\mu\nu} = \frac{i}{g}\,[D_\mu(A), D_\nu(A)] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

- By construction:

$$F^{\Omega}_{\mu\nu} = \Omega F_{\mu\nu} \Omega^{-1}$$

so that the trace of F^2 is an invariant scalar:

$$\left[\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\right]^{\Omega} = \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

The final form of the QCD Lagrangian is given by

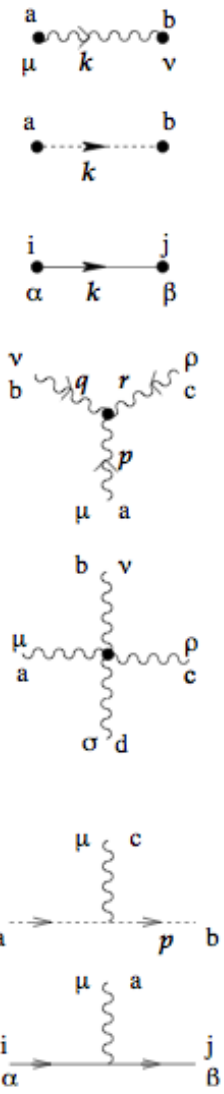
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_f \overline{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A^a_\mu \tau^a)\psi_f$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Remarks:

- Dynamical fields: N_f flavors of quarks in N_c colors & (N_c^2-1) gluons.
- In the functional (Fadeev-Popov) quantization procedure, there will be ghost fields, etc. which are omitted here.
- m_f is the quark mass matrix.

Feynman rules:



$$-\mathrm{i}\frac{\delta^{ab}}{k^2}\left(g_{\mu\nu}-(1-\alpha)\frac{k_\mu k_\nu}{k^2}\right)$$

$$\frac{\mathrm{i}}{k^2}\delta_{ab}$$

$$\left[\frac{\mathrm{i}\,\delta_{ij}}{\not{k}-m}\right]_{\alpha\beta}$$

$$gf^{abc}\left[(r-q)_\mu g_{\nu\rho}+(q-p)_\rho g_{\mu\nu}+(p-r)_\nu g_{\mu\rho}\right]$$

$$-\mathrm{i}\,g^2\left[f^{abe}f^{cde}(g_{\mu\rho}g_{\nu\sigma}-g_{\nu\rho}g_{\mu\sigma})+\ \text{2permutations}\right]$$

$$-gf^{abc}p_\mu$$

$$ig\,[\gamma_\mu]_{\alpha\beta}\,[t^a]_{ij}$$

Remarks:

- Compared to QED, QCD is a lot more complicated due to its non-Abelian nature.
- Gluons do interact with each other, and this brings new phenomena (like confinement).
- Multi-loop calculations become rapidly cumbersome.
- As always, perturbation theory is valid only in a particular domain in momentum.
- All this calls for efficient approximation schemes: one of them is effective field theory and effective model building.

Asymptotic freedom & the vacuum of QCD

- The behavior of the coupling with the momentum scale follows from the renormalization group (RG) flow equation:

$$\Lambda \frac{d}{d\Lambda} g(\Lambda) = \beta(g)$$

(Λ: mass scale)

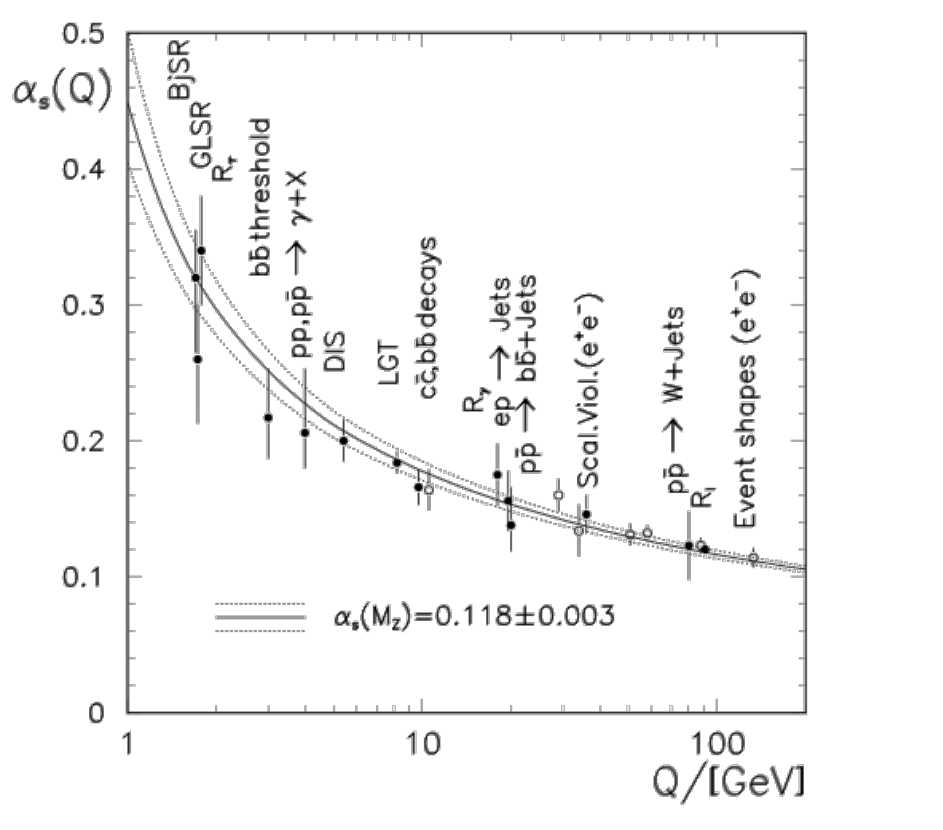
- The qualitative behavior is given by the sign of the beta function. To one loop:

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) + \mathcal{O}(g^5)$$

- β is negative for $N_f < (11/2)N_c = 16.5$ -> asymptotic freedom !

- This is the effect of anti-screening due to gluon self-interactions (opposite to the screening by fermionic fluctuations in QED).

As long as $N_f < 16.5$, gluons win !



- In line with the phenomenon of quark confinement into color singlets (hadrons)

- Matter becomes simpler at very high energies, but very complicated in the opposite limit...

- The vacuum (bound states: hadrons) and other low-energy phenomena cannot be described by perturbative QCD!

- Here, there is a clear need for effective models...

Effective field theory basic recipe

Basic recipe for building EFT models:

- Keep relevant symmetries
- Try to include in the effective action all terms allowed by the chosen symmetries
- Mimic of QCD (or other first-principle theory) at low energy using a simpler field theory
- Analytic results: estimates, qualitative behavior, etc. Often not too precise (except for χPT to higher orders)
- No need of renormalizability; natural cutoff (domain of validity)
- Just part of the story - combined with lattice QCD may provide good insight

Summary

- The building blocks of hadrons are quarks & gluons, although color is confined into singlet states.

- QCD is believed to be the fundamental theory of strong interactions. Its asymptotically free nature puts the vacuum out of reach for perturbation theory.

- The Lagrangian of QCD and the Feynman rules associated were built by using the Gauge Principle, starting from the quark matter fields and obtaining gluons as connections.

- A simpler, and sometimes necessary or complementary, approach is provided by effective field theories or effective models, especially when one has to deal with the nonperturbative sector of the theory.

Fourth Lecture

- ★ Drawing the QCD phase diagrams
- ★ Phase transitions in QCD: how?
- ★ A very simple framework: the bag model
- ★ Symmetries in the QCD action: general picture
- ★ SU(Nc), Z(Nc) and Polyakov loops
- ★ Adding massless quarks (chiral symmetry)
- ★ Summary