Distinguishing noise from chaos: an Information Theory approach

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Distinguishing noise from chaos

Although being of a quite different physical origin, time series arising from chaotic systems share with those generated by stochastic processes several properties that make them almost undistinguishable:

* a wide-band power spectrum

* power spectrum of type f^{-k} $(k \geq 0)$

* a delta-like autocorrelation function

* an irregular behavior of the measured signals

Our objective: distinguish chaos from noise using Information Theory quantifiers.

Chaos & Information

Chaotic systems display ''sensitivity to initial conditions'' which manifests instability everywhere in the phase space and leads to non-periodic motion (chaotic time series). They display long-term unpredictability despite the deterministic character of the temporal trajectory.
In a system undergoing chaotic motion, two neighboring points in the phase space move away exponentially rapidly. Let $x_1(t)$ and $x_2(t)$ be two such points, located within a ball of radius R at time t. Further, assume that these two points cannot be resolved within the ball due to poor instrumental resolution. At some later time t' the distance between the points will typically grow to

$$\left|x_1(t') - x_2(t')\right| \approx |x_1(t) - x_2(t)| \; exp\left(\lambda \left|t' - t\right| \right)$$

with $\lambda > 0$ for a chaotic dynamics and λ the biggest Lyapunov exponent. When this distance at time t' exceeds R, the points become experimentally distinguishable.

This implies that instability reveals some information about the phase space population that was not available at earlier times.

We can use Information Theory based quantifiers to characterize chaotic systems

Information Theory Quantifiers

Given a time series $X = \{x_n \; ; \; n = 1, \ldots, M\}$ and an associated PDF $P = \{p_i \; ; i = 1, \ldots, N\}$ with $\sum_{i=1}^{N} p_i = 1$, we define:

Normalized Shannon Entropy:

$$H[P] = \mathrm{S}\,[P] \, / \, \mathrm{S}_{\max} = \left(- \textstyle\sum_{\mathrm{i}=1}^{N} \mathrm{p_i} \;\log \mathrm{p_i}\right) / \mathrm{S}_{\max} \quad \text{with} \quad \mathrm{S}_{\max} = \mathrm{S}\,[P_{\mathrm{e}}] = \log N$$

Statistical Complexity:

$$C[\mathrm{P}] = Q[P, P_e] \, . \, H[P]$$

$$Q[P, P_e] = Q_0 \, J[P, P_e] = Q_0 \left\{ \mathrm{S}\left[\frac{P + P_{\mathrm{e}}}{2}\right] - \tfrac{1}{2}\mathrm{S}\,[P] - \tfrac{1}{2}\mathrm{S}\,[P_{\mathrm{e}}] \right\}$$

Fisher Information Measure:

$$F[P] = F_0 \sum_{i=1}^{N-1} \{ (p_{i+1})^{1/2} - (p_i)^{1/2} \}$$

- *Distinguishing Noise from Chaos.*
  O. A. Rosso, H. A. Larrondo, M. T. Martin, A. Plastino, M. A. Fuentes.
  Phys. Rev. Lett. 99 (2007) 154102
- *Contrasting chaos with noise via local versus global informatio quantifiers.*
  F. Olivares, A. Plastino, O. A. Rosso.
  Physics Letters A 376 (2012) 1577–1583

PDF Selection

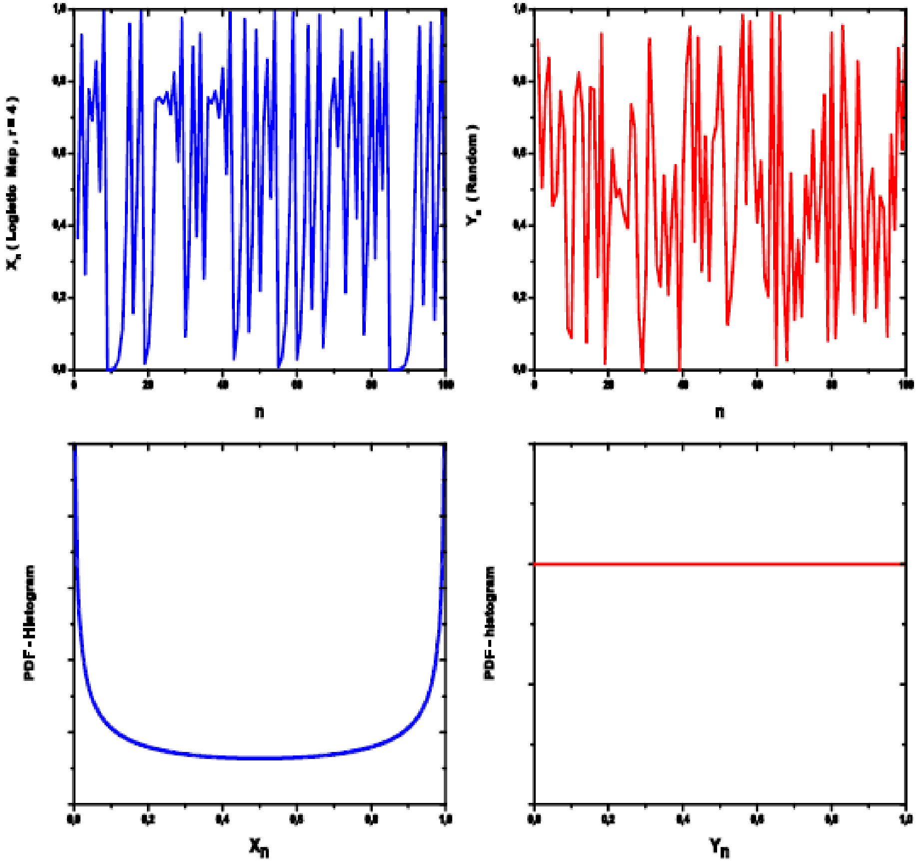
Given a time series

$$X = \{x_j,\ j = 1, \cdots, N\} \in \mathbb{R}^N$$

we can define the associate probability distribution function based on

- *Frequency counting*
- *Histogram of amplitudes*
- *Binary distribution*
- *Frequency representation (Fourier Transform)*
- *Frequency bands representation (Wavelet Transform)*
- *Ordinal patterns (Bandt-Pompe methodology)*

PDF selection



$S\left[P^{(hist)}\right] \cong 1$

$C\left[P^{(hist)}\right] \cong 0$

$S\left[P^{(hist)}\right] = 1$

$C\left[P^{(hist)}\right] = 0$

Bandt-Pompe PDF

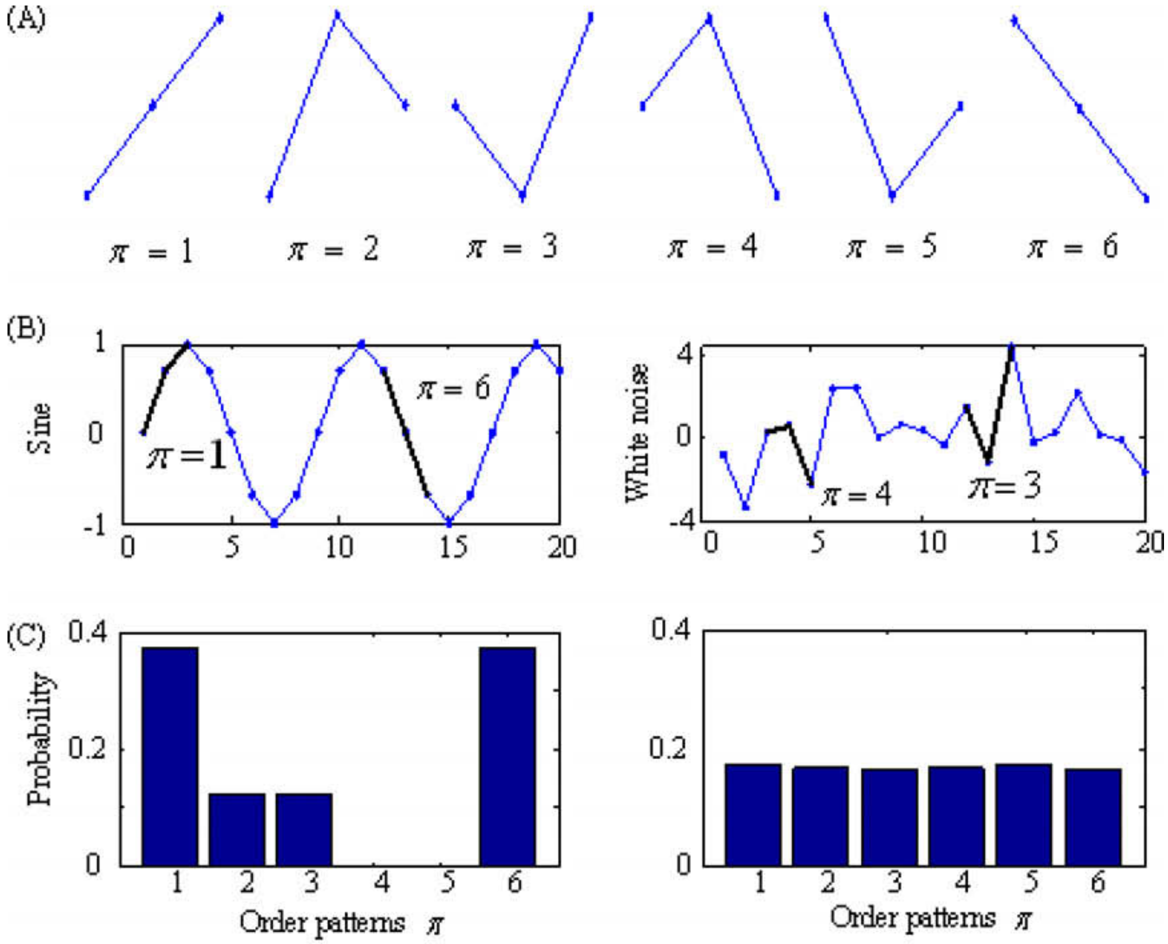
BANDT AND POMPE SIMBOLIZATION TECHNIQUE: the pertinent symbolic data are
* created by ranking the values of the series and
* defined by reordering the embedded data in ascending order,
which is tantamount to a phase space reconstruction with embedding dimension (pattern-length) D and time τ. In this way it is possible to quantify the diversity of the ordering symbols (patterns) derived from a scalar time series.

Characteristics of BP-PDF:

- no model-based assumptions are needed
- “partitions” are devised by comparing the order of neighboring relative values rather than by apportioning amplitudes according to different levels
- time causality is naturaly incorporate
- give information about temporal correlation
- few parameters: the pattern-length/embedding dimension and the embedding delay
- the extremely fast nature of the pertinent calculation-process

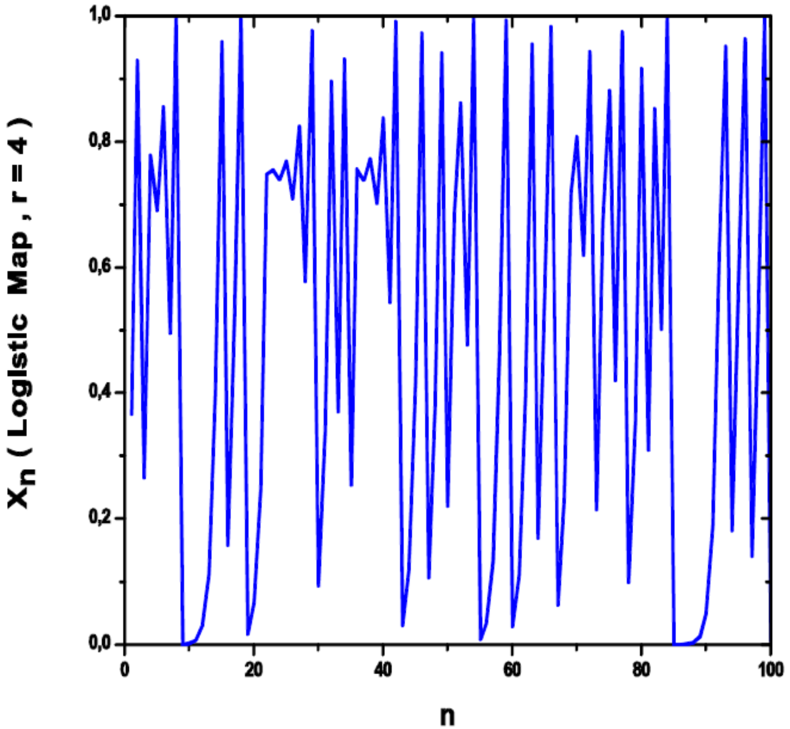
- Permutation entropy: a natural complexity measure for time series. C. Bandt, B. Pompe. Phys. Rev. Lett. 88 (2002) 174102.

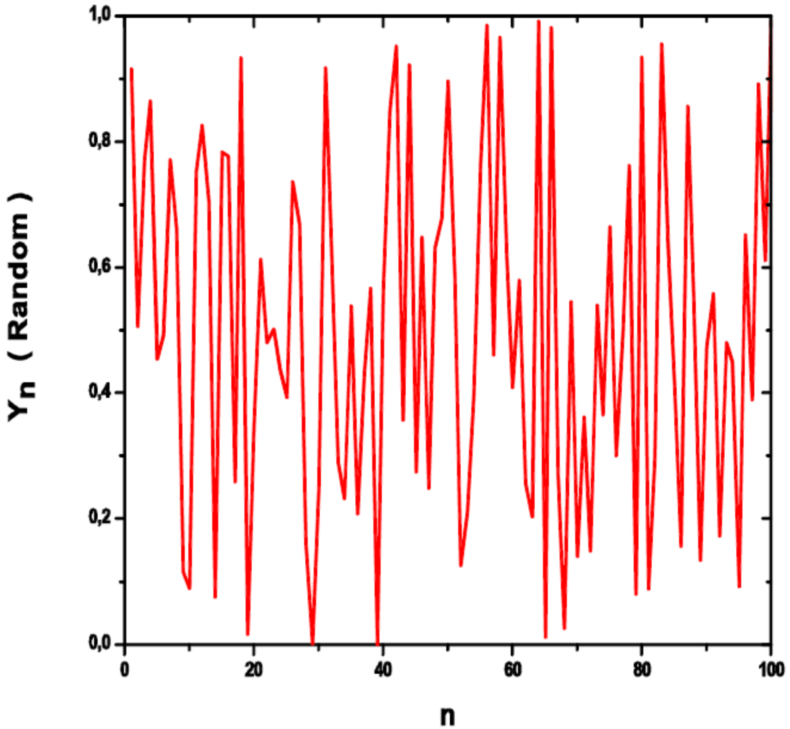
Bandt-Pompe PDF



Ordinal patterns in a simple time series. (A) Ordinal patterns at the dimension d = 3. (B) Illustration of the ordinal procedure for d = 3 for sine and white noise time series. (C) Probability distribution of ordinal patterns π.

Bandt-Pompe PDF





$$S\left[P^{(BP)}\right] = 0.629$$
$$C\left[P^{(BP)}\right] = 0.484$$

$$S\left[P^{(BP)}\right] = 0.99994$$
$$C\left[P^{(BP)}\right] = 0.00013$$

$$D = 6 \text{ and } \tau = 1$$

Bandt-Pompe PDF

Bandt and Pompe – PDF:

- ***Local characteristics*** ➡ ***forbidden /missing ordinal patterns***
- ***Global characteristics*** ➡ ***causality entropy-complexity plane***

  causality Shannon-Fisher plane

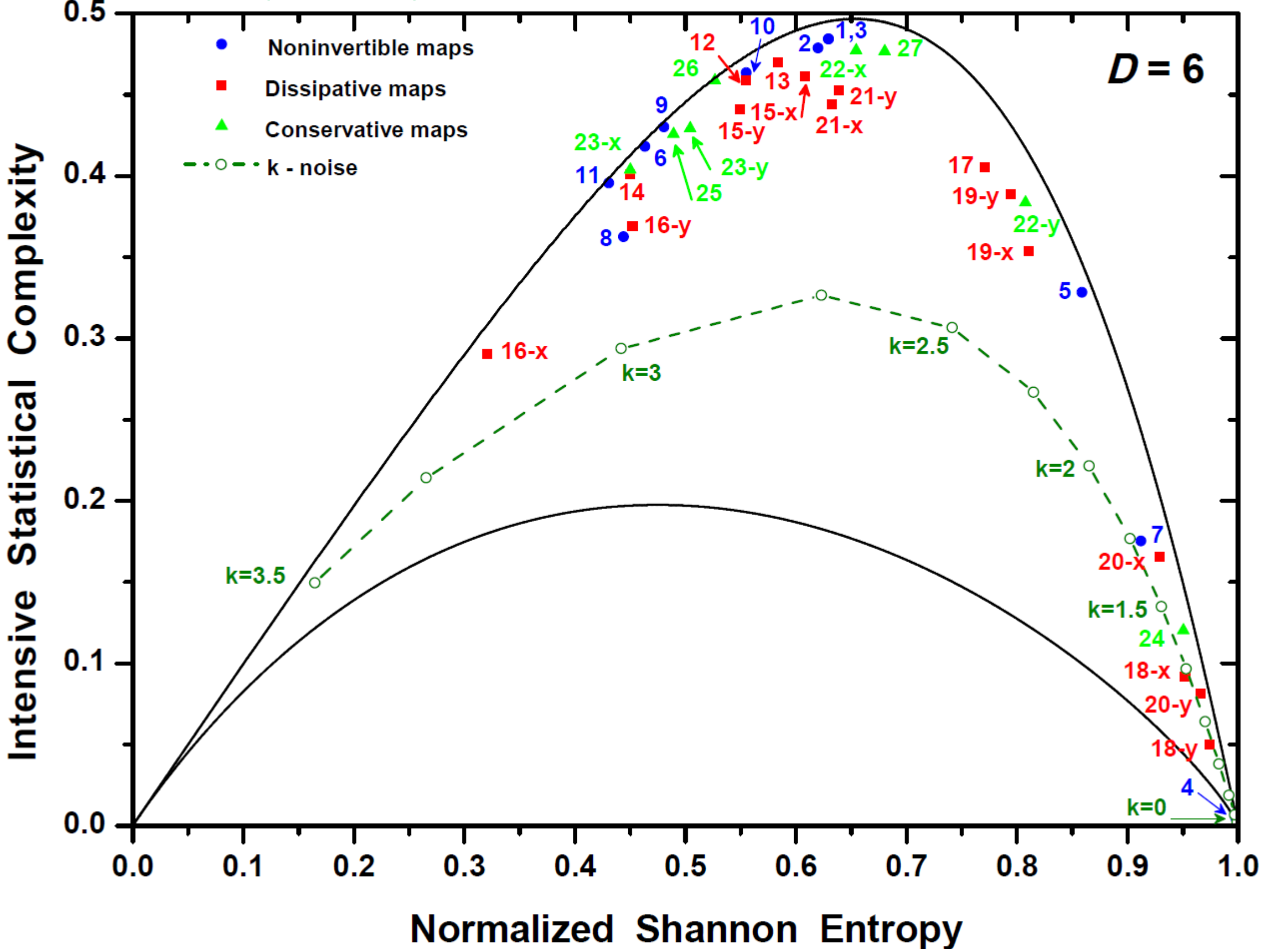
Chaotic maps

Noninvertible maps	Dissipative maps	Conservative maps
1. Logistic map	*12. Henon map*	*22. Chirikov standard map*
2. Sine map	*13. Lozi map*	*23. Henon area-preseving*
3. Tent map	*14. Delayed logistic map*	*24. Arnold's cat map*
4. Linear Congruential generator	*15. Tinkerbell map*	*25. Gingerbreadman map*
5.Cubic map	*16. Bugers' map*	*26. Chaotic web map*
6. Ricker's population model	*17. Holmes cubic map*	*27. Lorenz 3D chaotic map*
7. Gauss map	*18. Dissipative standard map*	
8. Cusp map	*19. Ikeda map*	
9. Pinchers map	*20. Sinai map*	
10. Spence map	*21. Discrete predator-prey map*	
11. Sine-circle map		

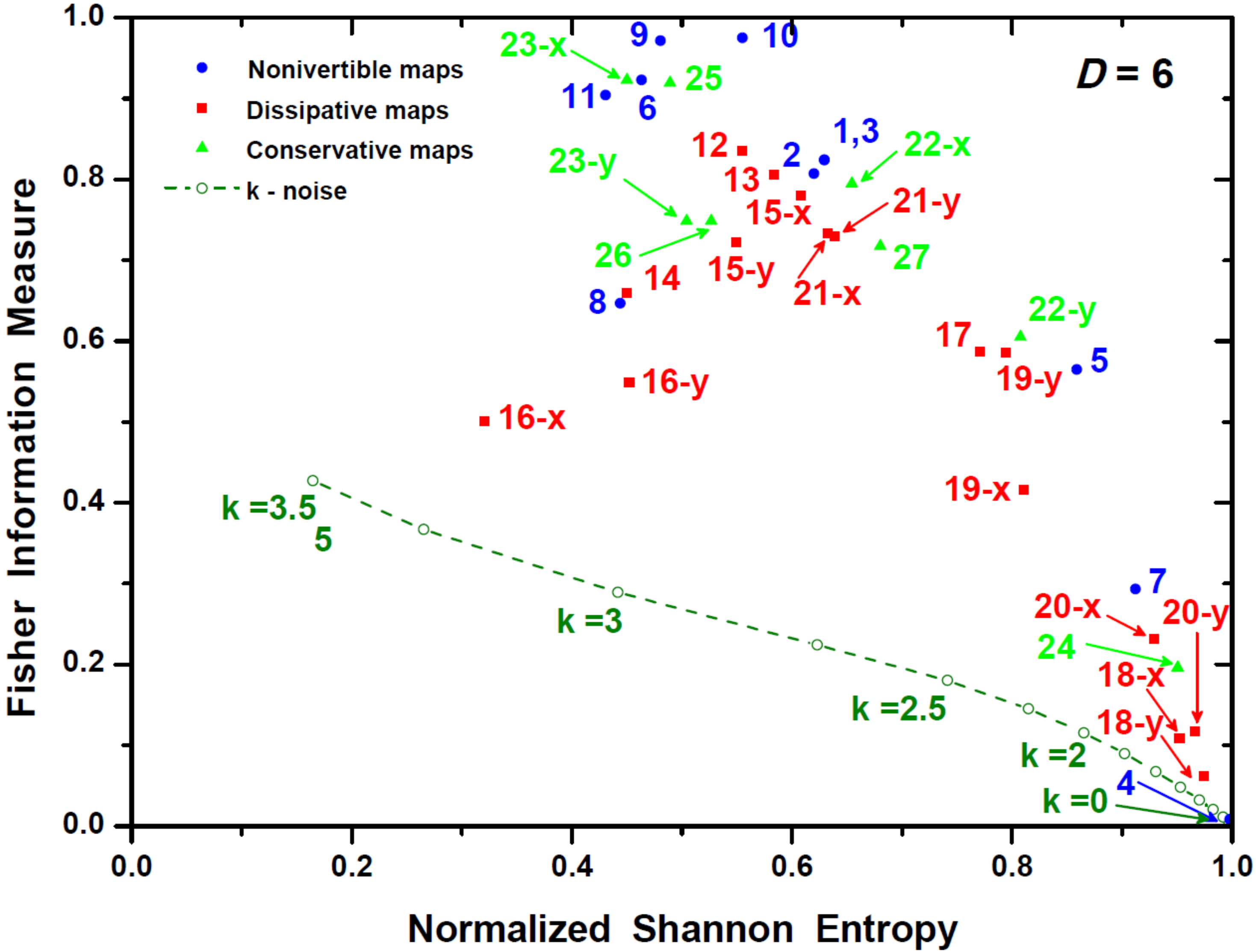
- *Chaos and time-series analysis (Appendix)*
  J. C. Sprott (Oxford University Press, New York, 2003).

Causality Entropy-Complexity plane

O. A. Rosso et al. The European Physics Journal B 86 (2013) 116 – 128



Causality Shannon-Fisher plane



Chaos + Noise

Logistic Map + Observational Noise (Correlated Noise)

$$Y_n = X_n + A\,\eta_n^{(k)}$$

$$X_{n+1} = R\,X_n(1 - X_n)$$

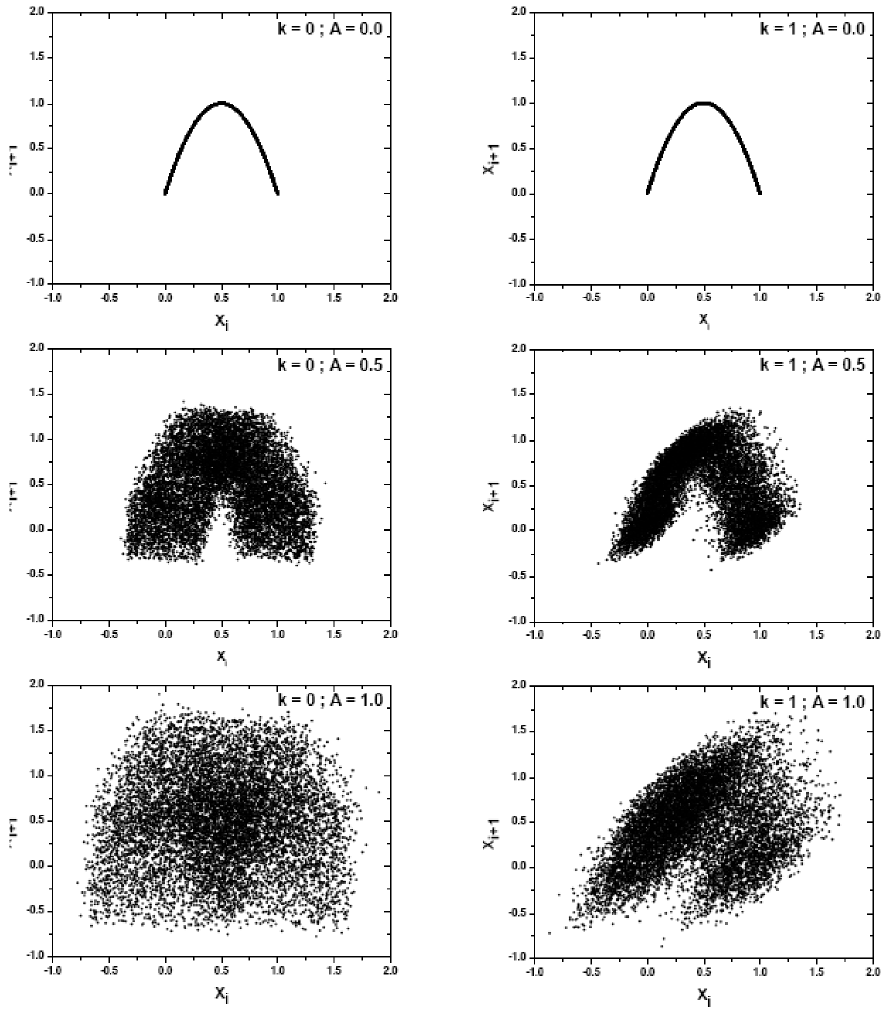
$$R = 4$$

$\eta_n^{(k)}$: Colored Noise with

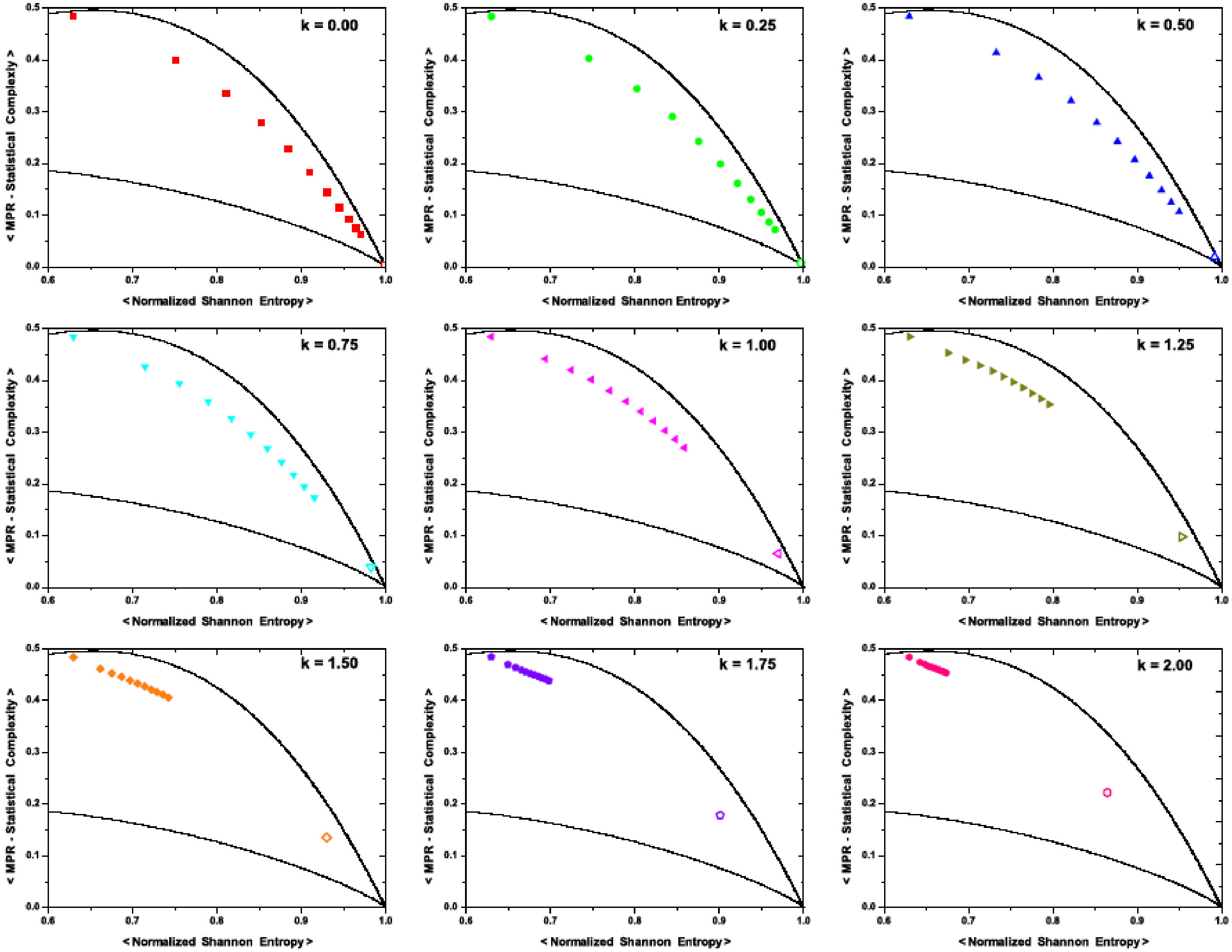
Power Spectrum f^{-k}

$$0 \le k \le 2$$

$A \ge 0$ Noise Amplitude

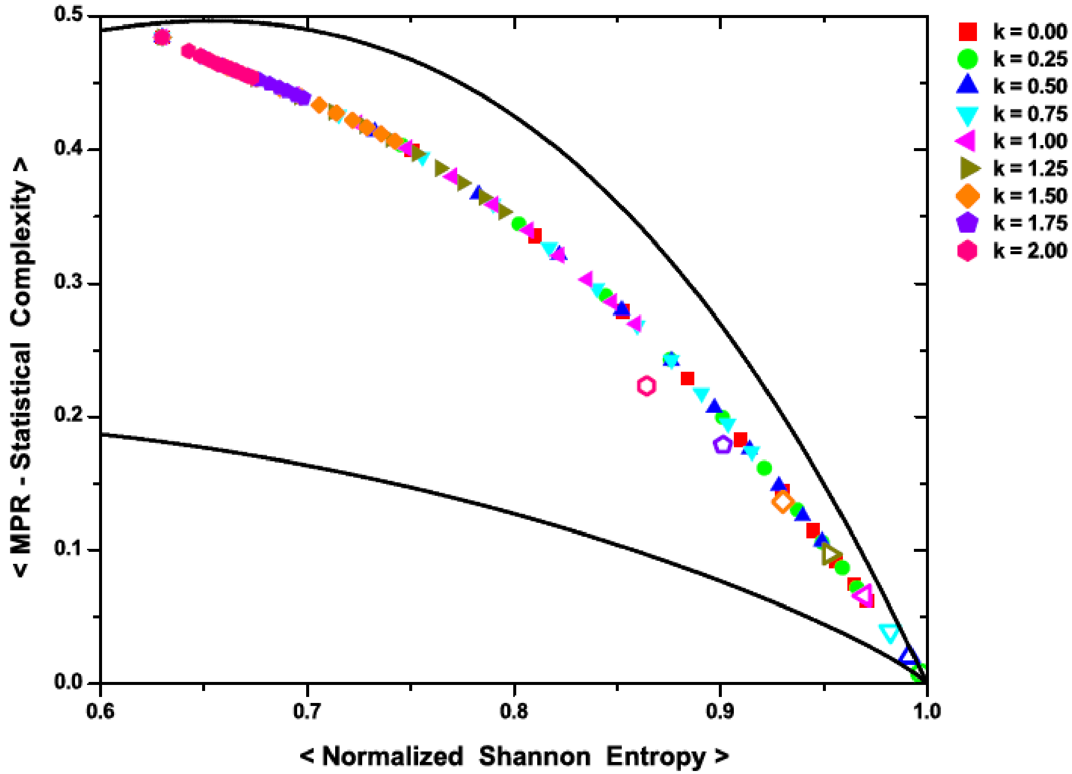


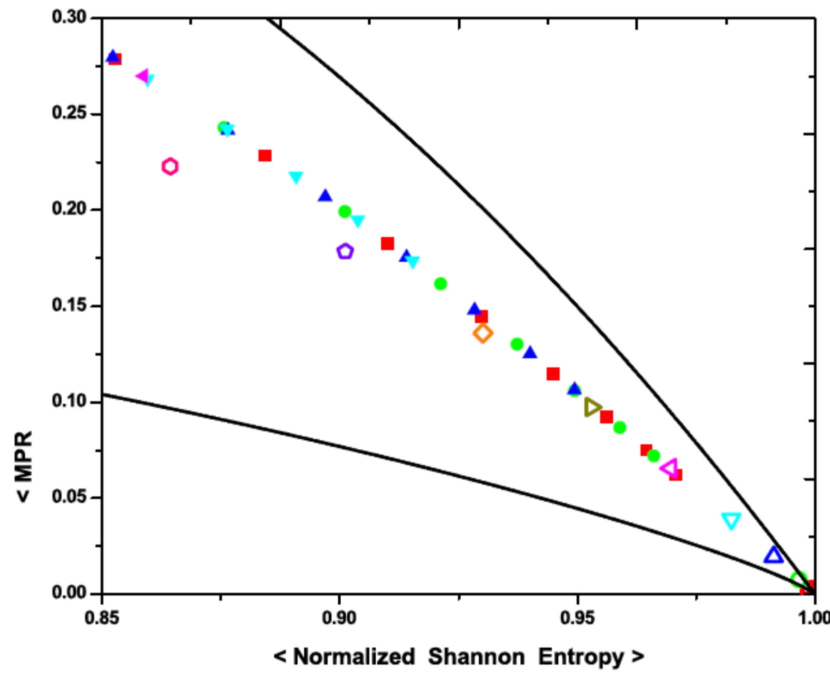
Chaos + Noise



k = 0.00
k = 0.25
k = 0.50
k = 0.75
k = 1.00
k = 1.25
k = 1.50
k = 1.75
k = 2.00
< MPR - Statistical Complexity >
< Normalized Shannon Entropy >

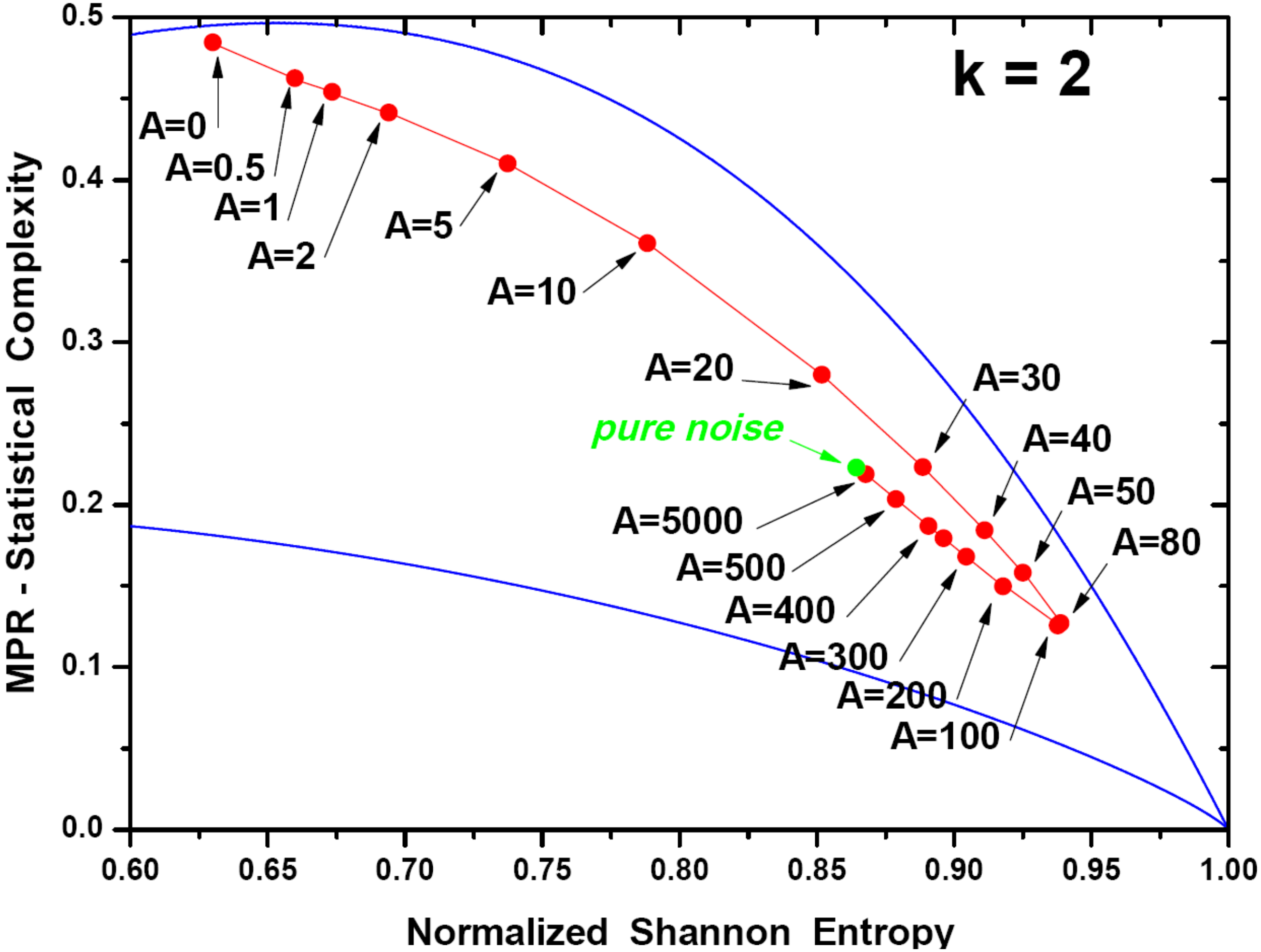
Chaos + Noise





O. A. Rosso, L. C. Carpi, P. M. Saco, M. Gómez Ravetti, A. Plastino, H. A. Larrondo
Causality and the Entropy-Complexity Plane: Robustness and Missing Ordinal Patterns
Physica A 391 (2012) 42 – 55

Chaos + Noise



N = 10**5 ;
D = 6

O. A. Rosso, L. C. Carpi, P. M. Saco, M. Gómez Ravetti, H. A. Larrondo, A. Plastino
The Amigó paradigm of forbidden/missing patterns: a detailed analysis
The European Physics Journal B 85 (2012) 419 – 430

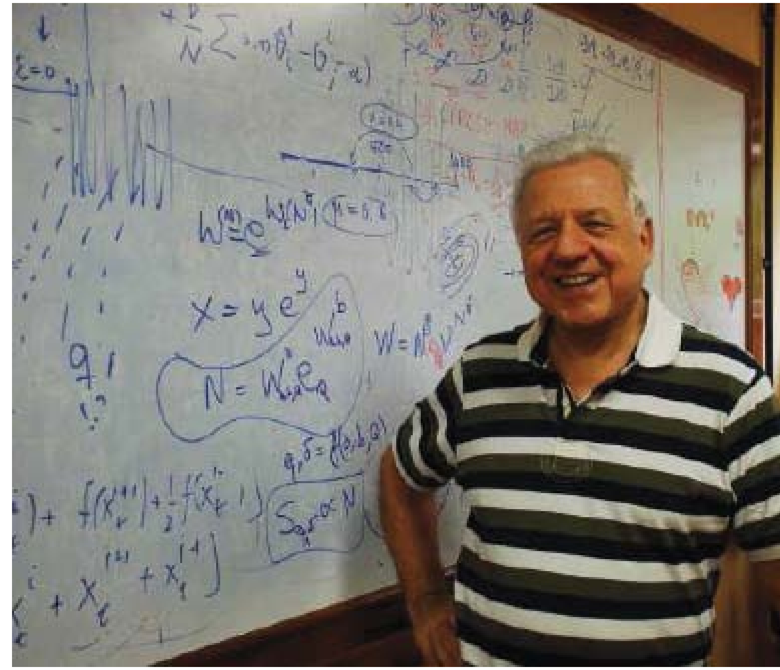
Conclusions

In the case of quantifiers evaluated making use of the whole BP-PDF, for fixed values of pattern length (embedding dimension) D and time lag τ a specific behavior is observed for the case of chaotic dynamics.

Localization in the entropy-complexity plane for the case of chaotic maps, closely approaches the limiting curve of maximum statistical complexity.

Similar behavior is still observed when chaotic maps' time series contaminated with additive uncorrelated or correlated noise is analyzed [O. A. Rosso et al., Physica A 391 (2012) 42-55].

A more "robust" distinction between deterministic and stochastic dynamics is given via the present time series treatment, that takes into account the whole of the permutation Bandt-Pompe probability distribution function (global quantifier), and not just part of it.



HAPPY BIRTHDAY

CONSTANTINO !!!!