A Non-Phenomenological Model to Explain Population Growth Behaviors

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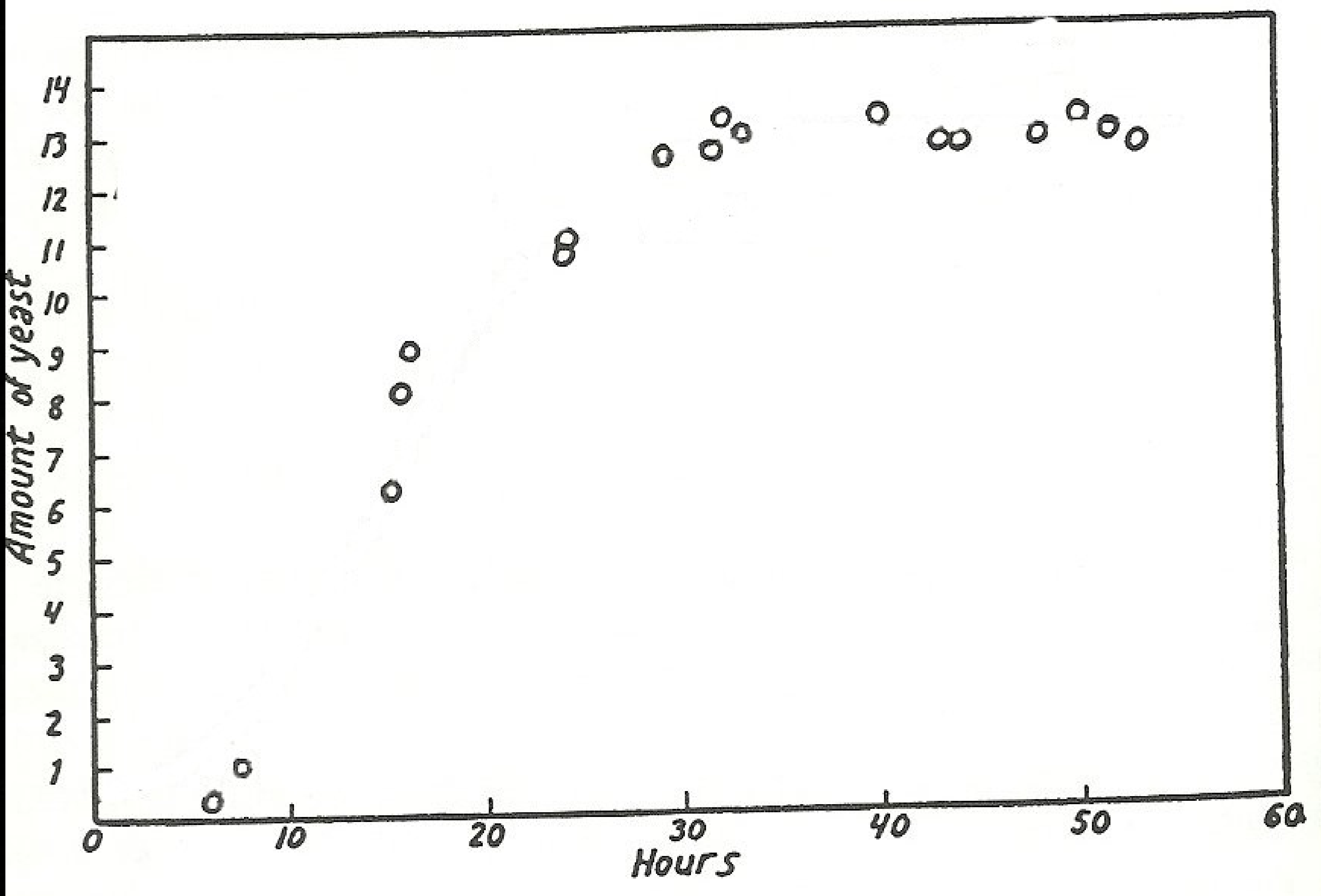
Outline

- Proposal: Modeling Population Growth considering microscopic interaction;
- Well-know phenomenological population growth models as special case;
- Comparison with empirical data;

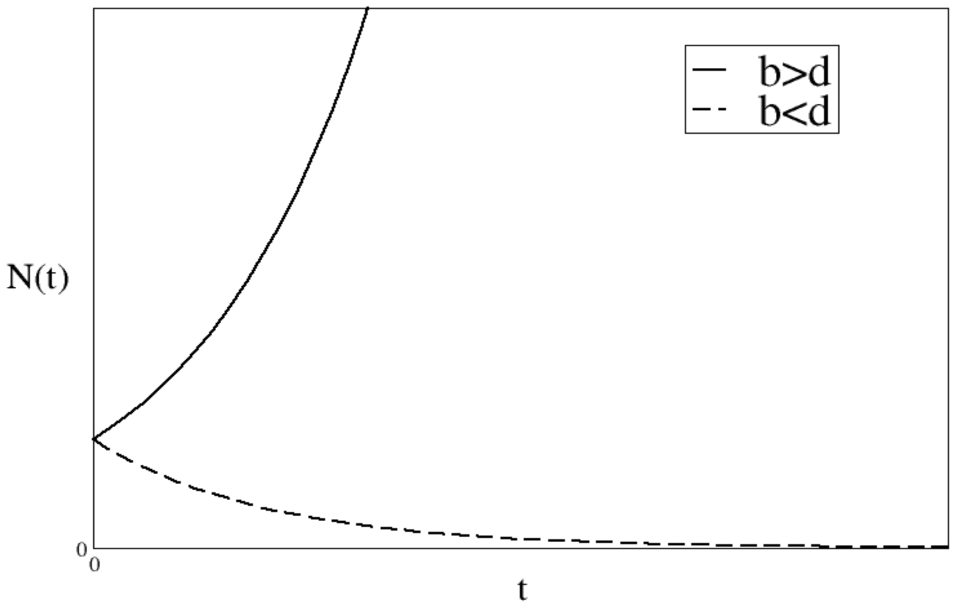

Amount of yeast
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
10
20
30
40
50
60
Hours

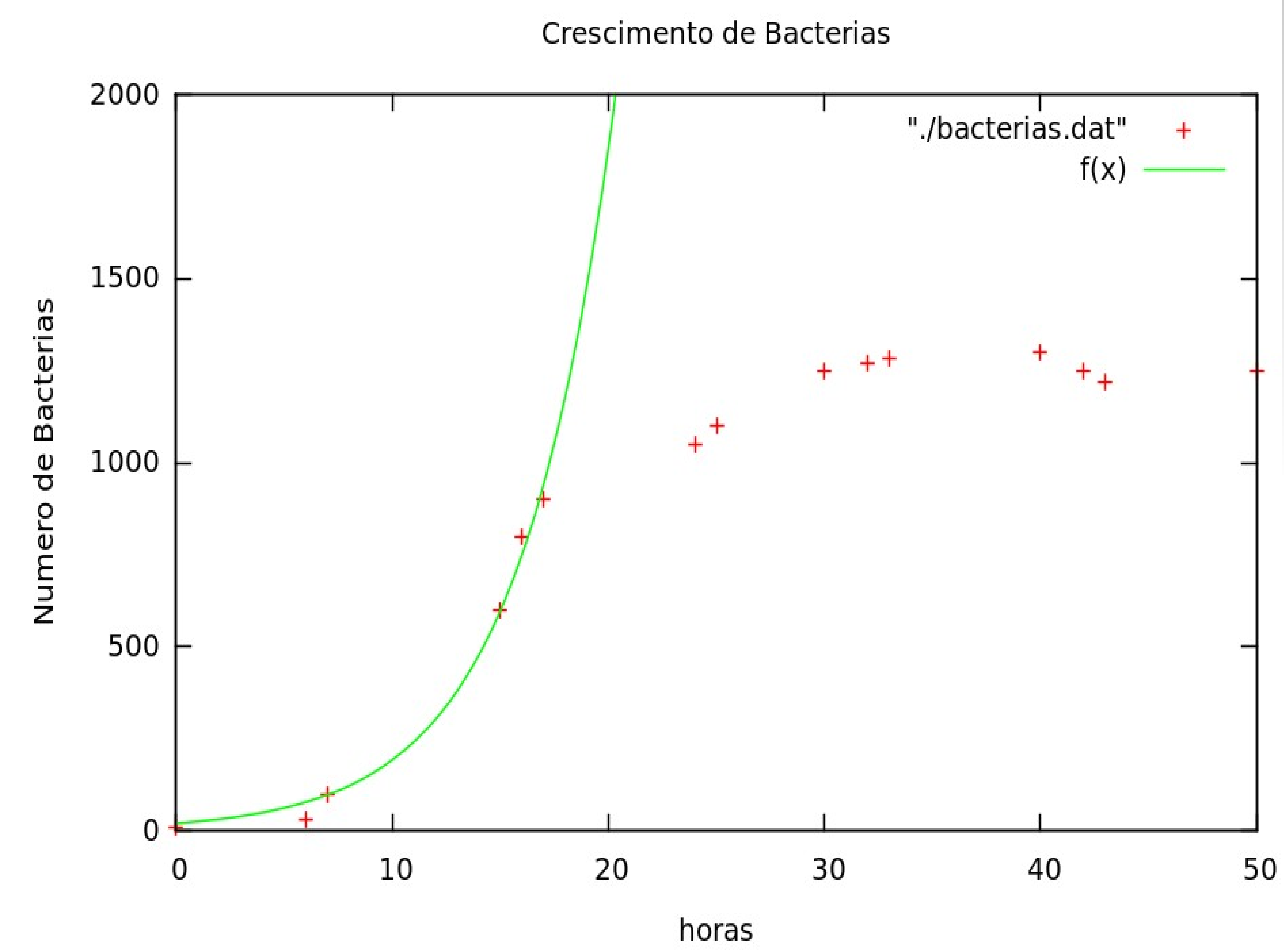
$$\Delta N(t) = \text{births} - \text{deaths}$$

Malthus' Model:

$$\frac{dN}{dt} = bN - dN = (b-d)N$$

$$N(t) = N_0 e^{(b-d)t}$$

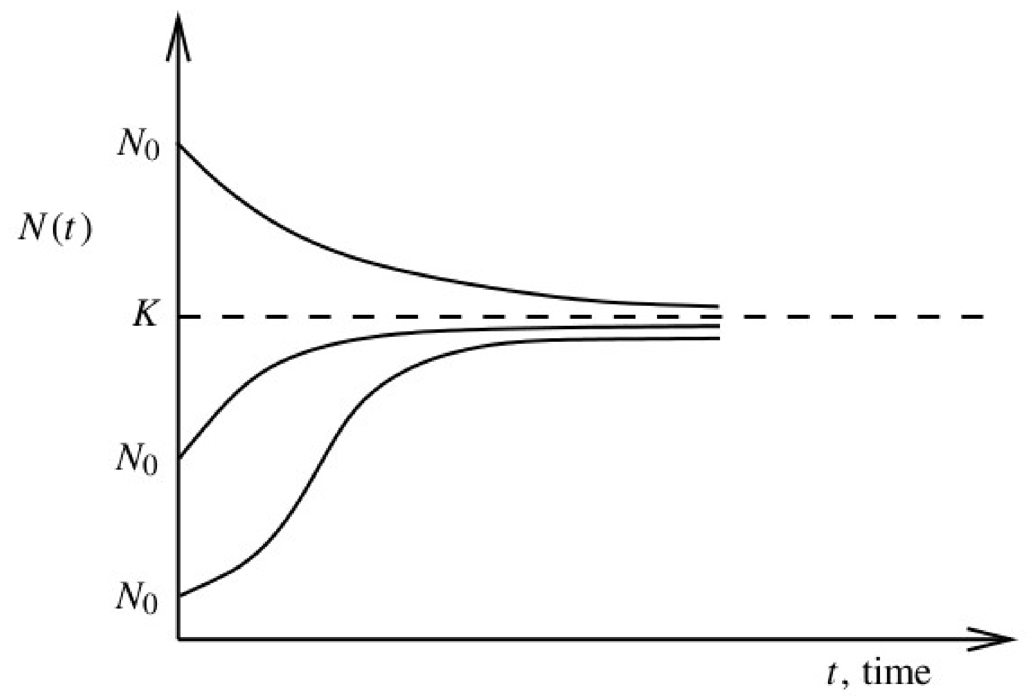




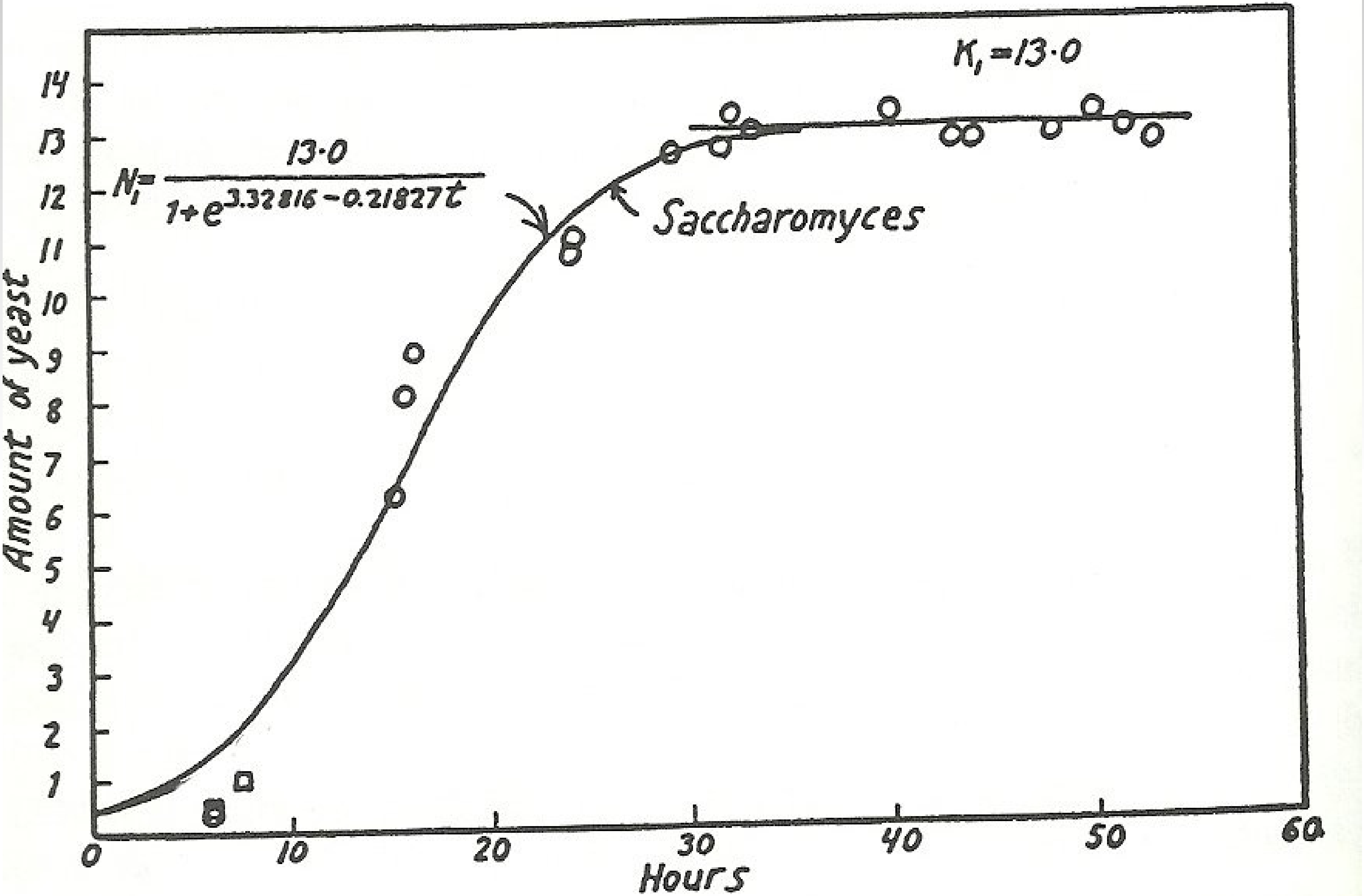
Crescimento de Bacterias
"./bacterias.dat"
f(x)
Numero de Bacterias
2000
1500
1000
500
0
0
10
20
30
40
50
horas

Verhulst Model

$$\frac{dN}{dt} = rN(1 - N/K),$$

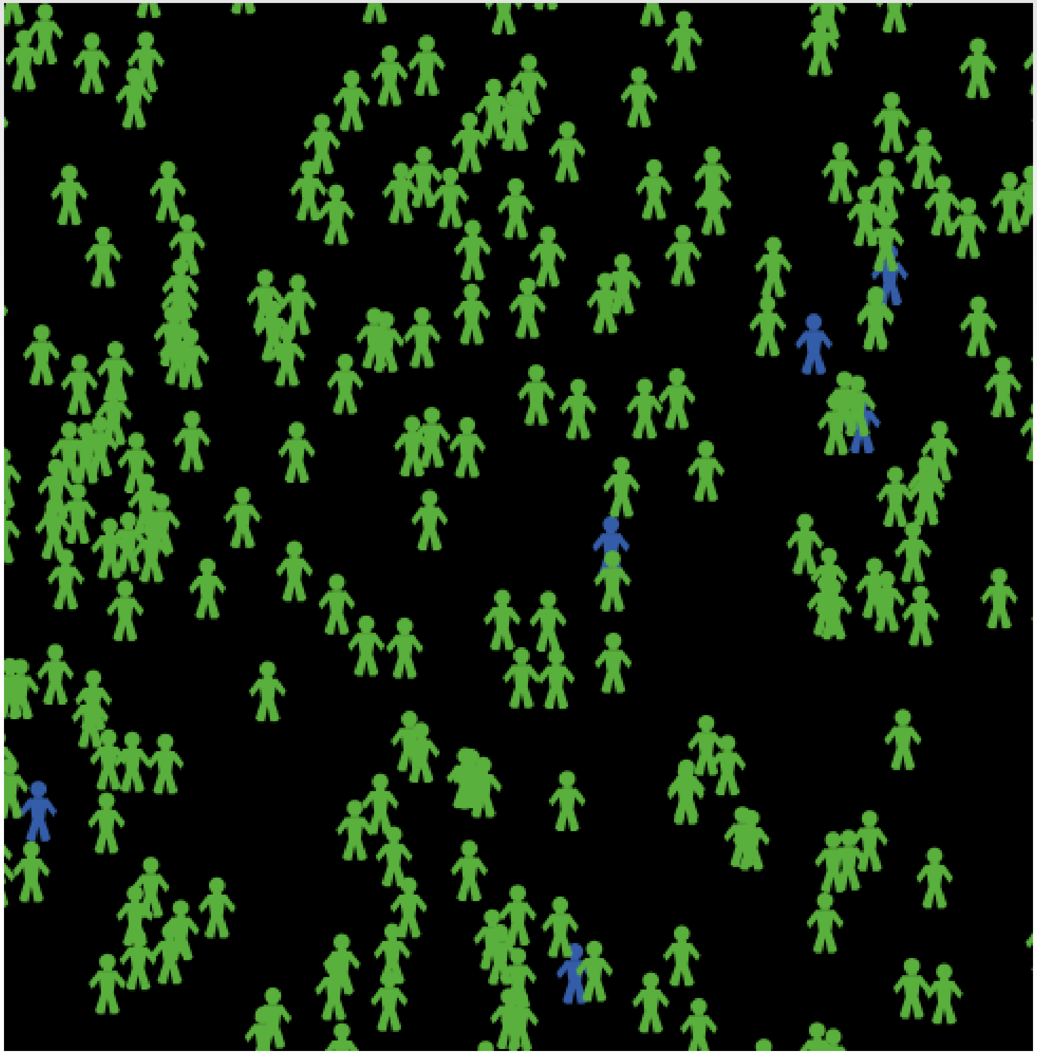


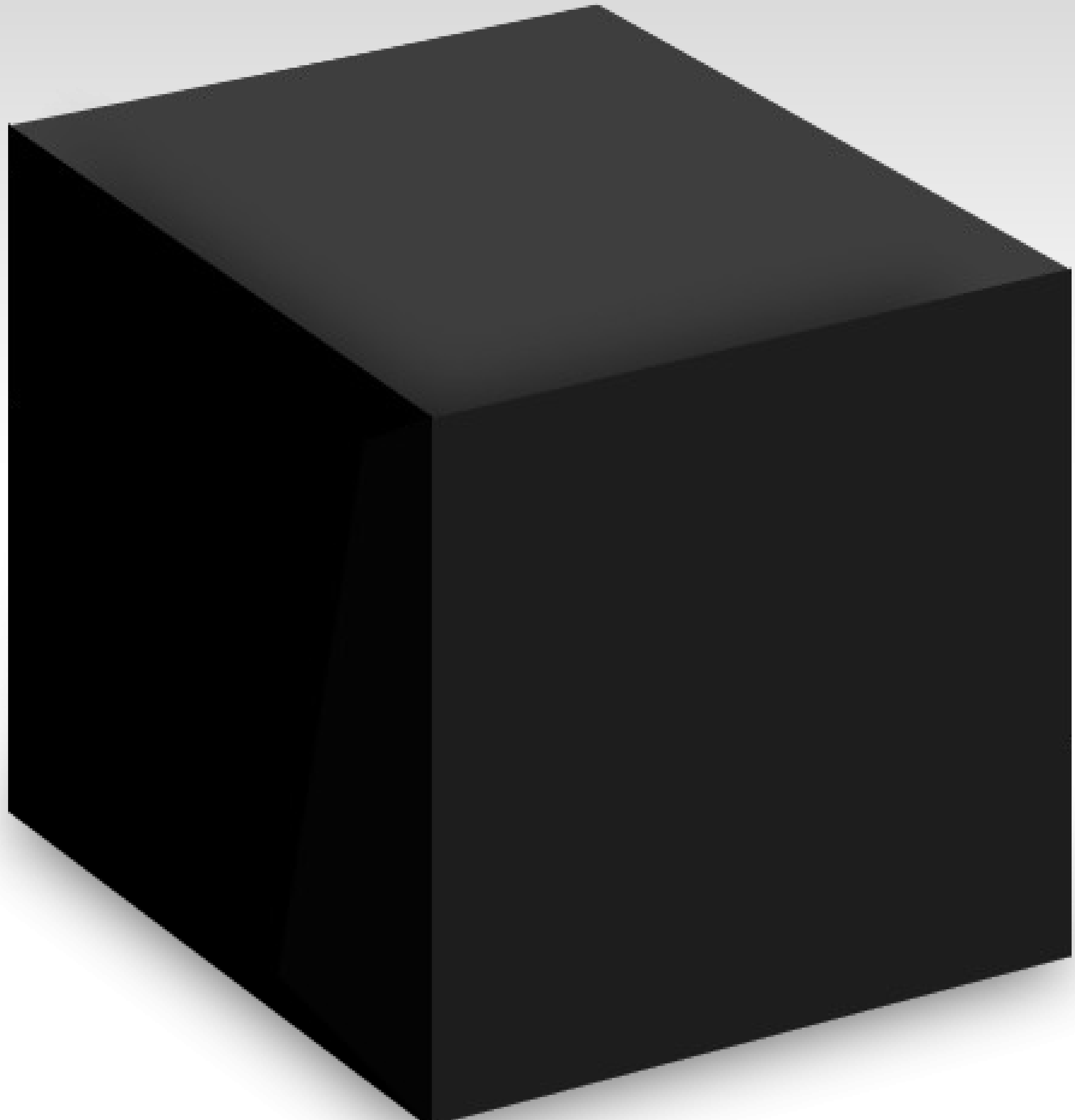
$$N(t) = \frac{N_0 K e^{rt}}{[K + N_0 (e^{rt} - 1)]} \quad \rightarrow K \quad \text{as} \quad t \rightarrow \infty,$$


$N_1 = \frac{13.0}{1+e^{3.32816-0.21827t}}$
Saccharomyces
$K_1 = 13.0$
Amount of yeast
Hours
0
10
20
30
40
50
60
1
2
3
4
5
6
7
8
9
10
11
12
13
14

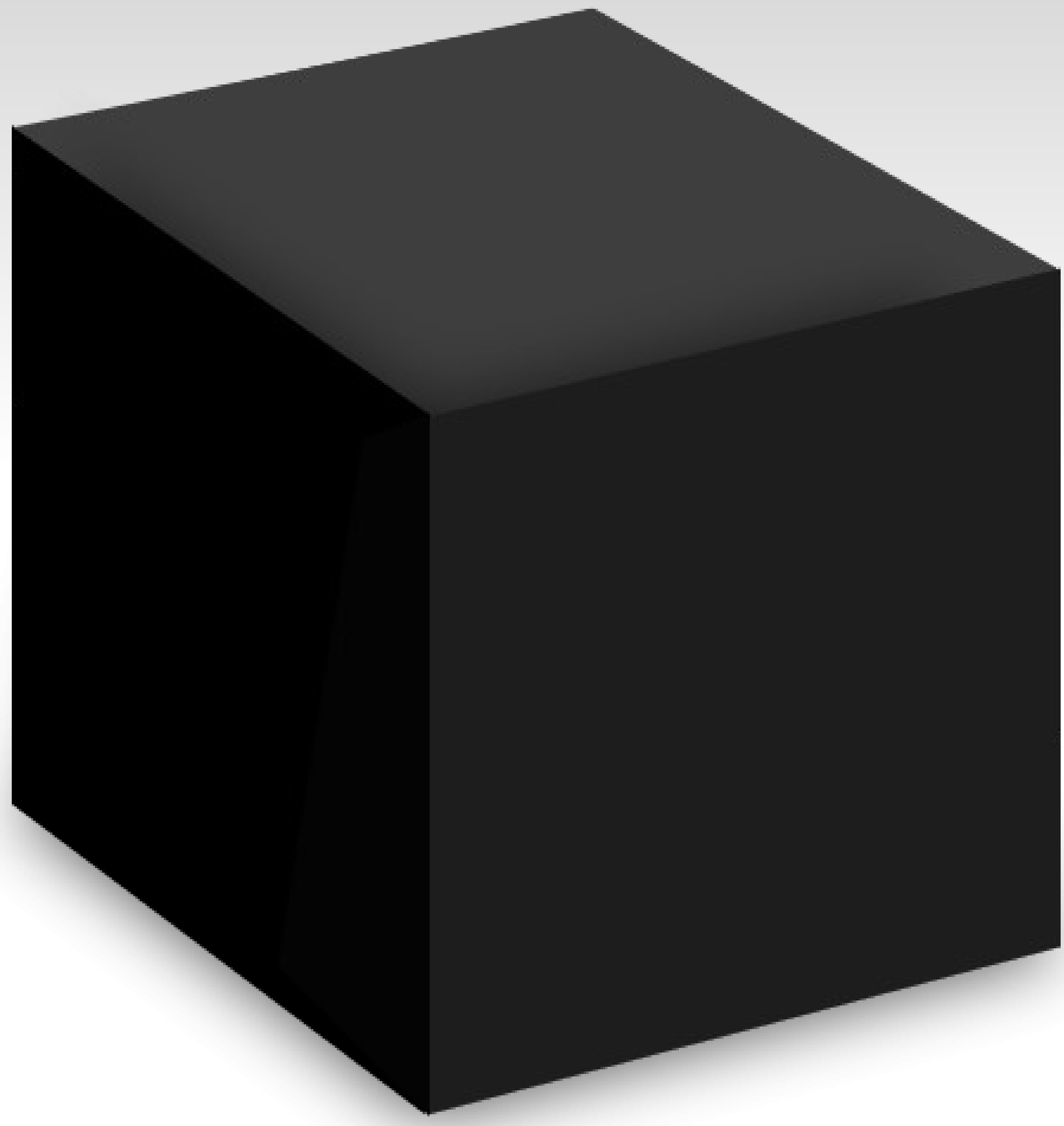
Examples of Phenomenological Models

- **Malthus;**
- **Verhust;**
- **Theta-Logistic;**
- **Gompertz;**
- **Richard;**
- **...**

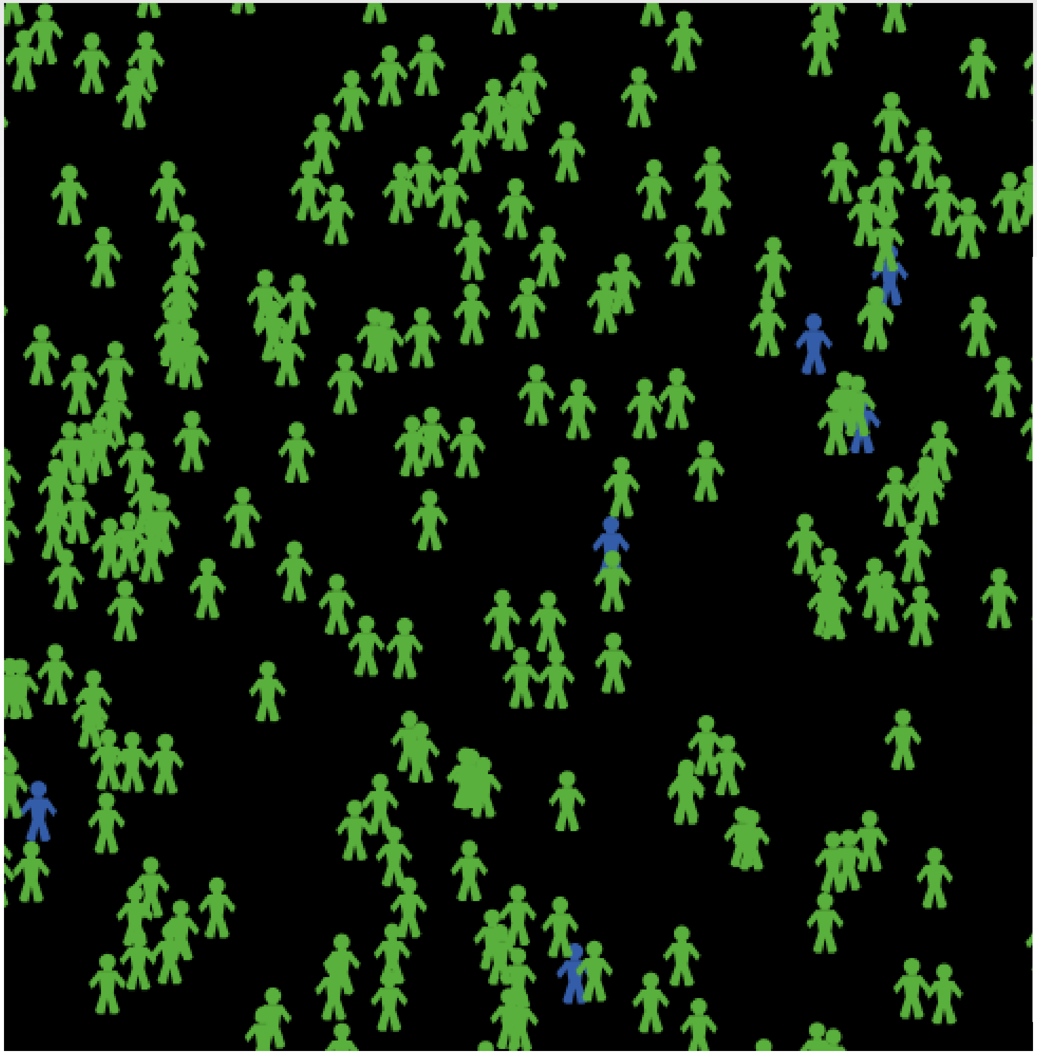




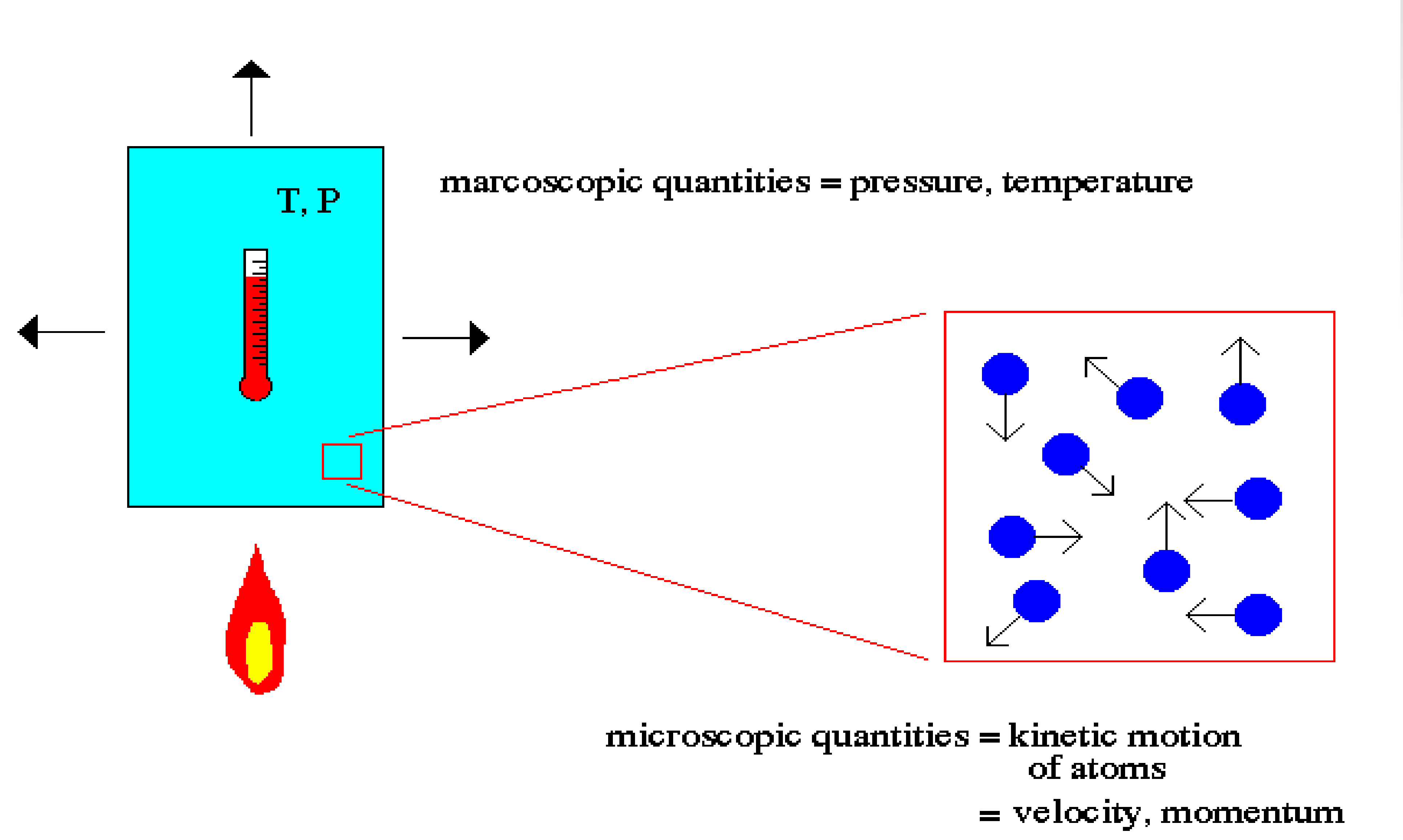
Phenomenological Models



Non-Phenomenological Model: Individual Interactions



Termodynamic and Statistical Physics



The Model

- Replication rate of the i-th agent:

$$R = [\text{Self-stimulated replication}] - [\text{competition from field}]$$

Mombach et Al, Europhys. Lett., **59** (2002)

The Model

- Replication rate of the i-th agent:

$$R = [\text{Self-stimulated replication}] - [\text{competition from field}] + \\ +[\text{cooperation from field}].$$

Fabiano Ribeiro, submitted to Journal of Theoretical Biology (2013)

The Model

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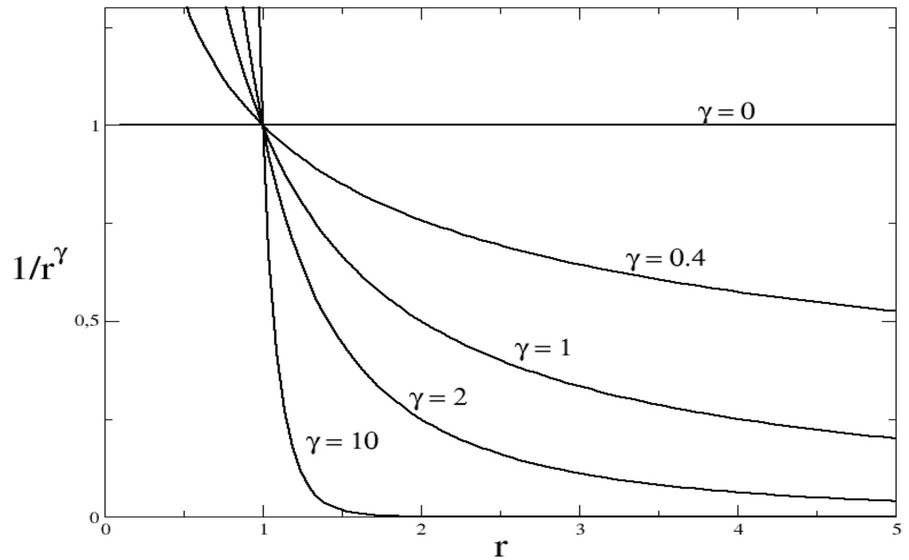
The Model

- Replication rate of the i-th agent:

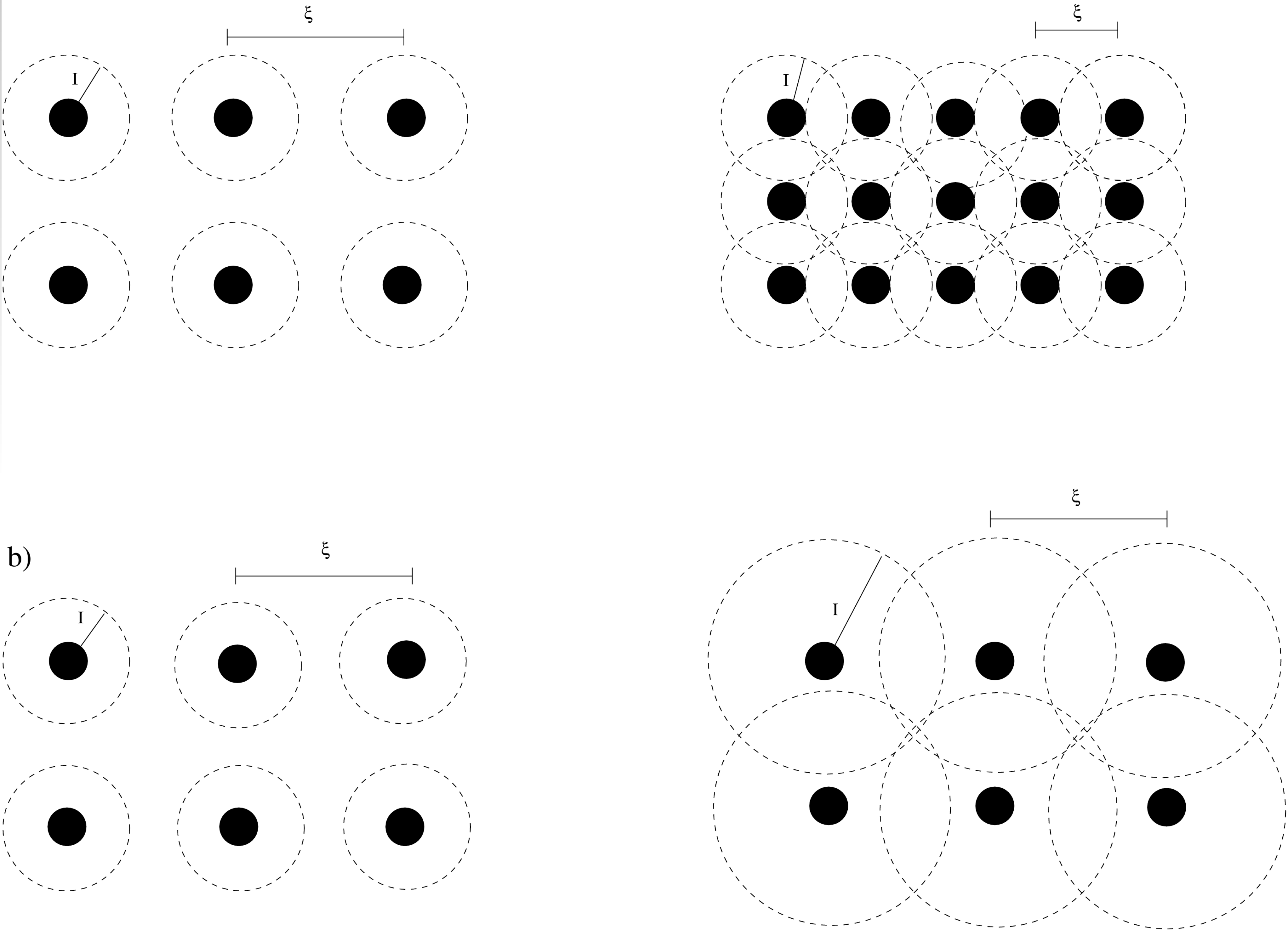
$$R = [\text{Self-stimulated replication}] - [\text{competition from field}] + \\ +[\text{cooperation from field}].$$

- Interaction strength decays with distance by: $1/r^{\gamma}$



a)

b)



$$R_i = k_i - J_1 \sum_{j \neq i} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^\gamma} + J_2 \sum_{j \neq i} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^\gamma}$$

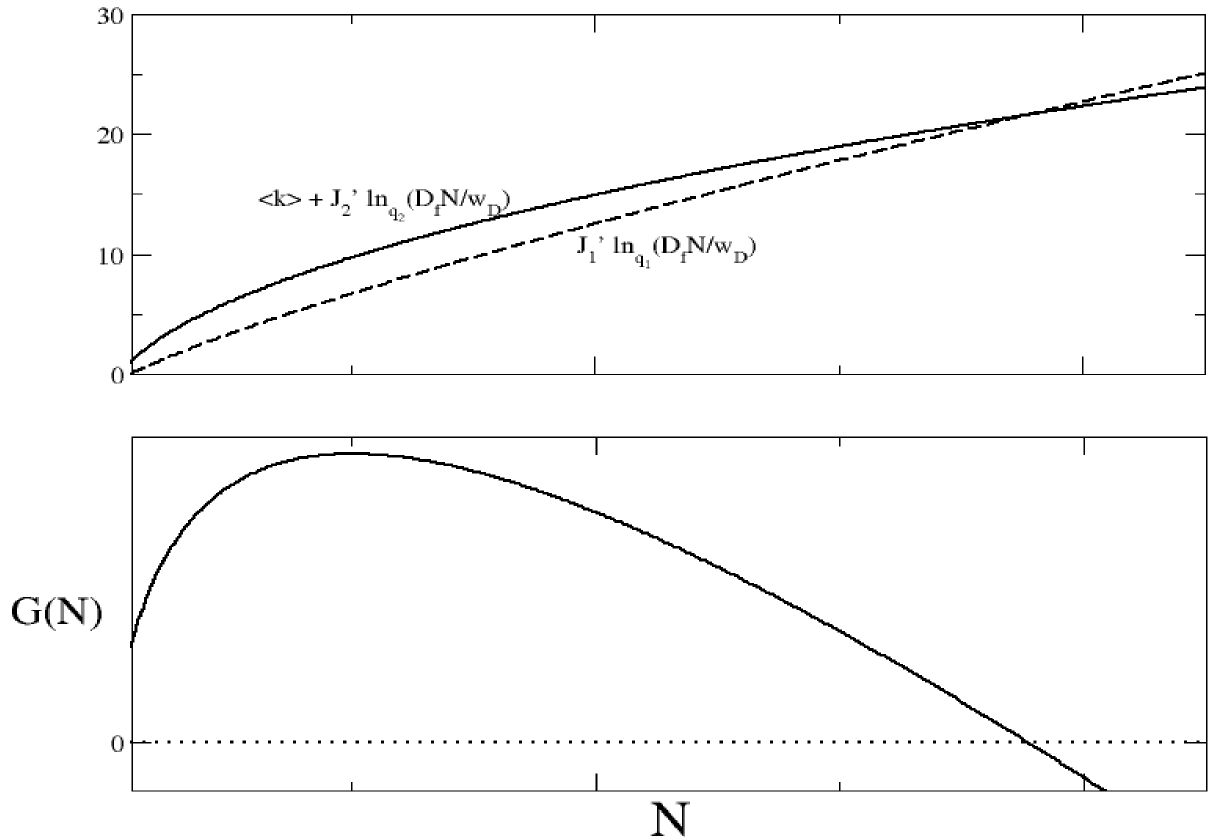
$$\frac{d}{dt} N = \sum_{i=1}^{N} R_i$$

$$\frac{d}{dt} N = N \left[\langle k \rangle - J_1 I(N) + J_2 I(N) \right].$$

$$I(N) = \frac{\omega_D}{D_f (1 - \frac{\gamma}{D_f})} \left[\left(\frac{D_f}{\omega_D} N \right)^{1 - \frac{\gamma}{D_f}} - 1 \right].$$

$$\frac{d}{dt}N = N\left[\langle k\rangle - J_1 I(N) + J_2 I(N)\right].$$

$$I(N) = \frac{\omega_D}{D_f(1-\frac{\gamma}{D_f})}\left[\left(\frac{D_f}{\omega_D}N\right)^{1-\frac{\gamma}{D_f}} - 1\right].$$



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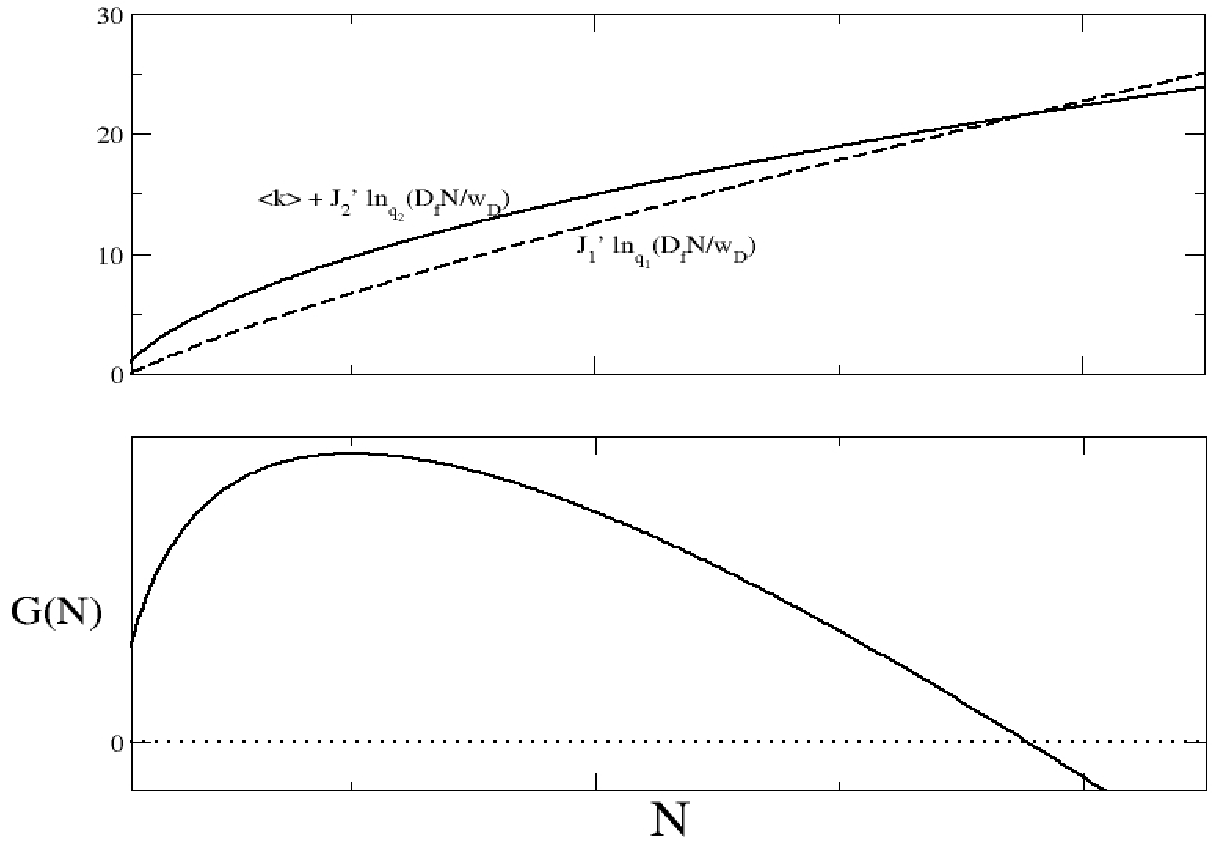
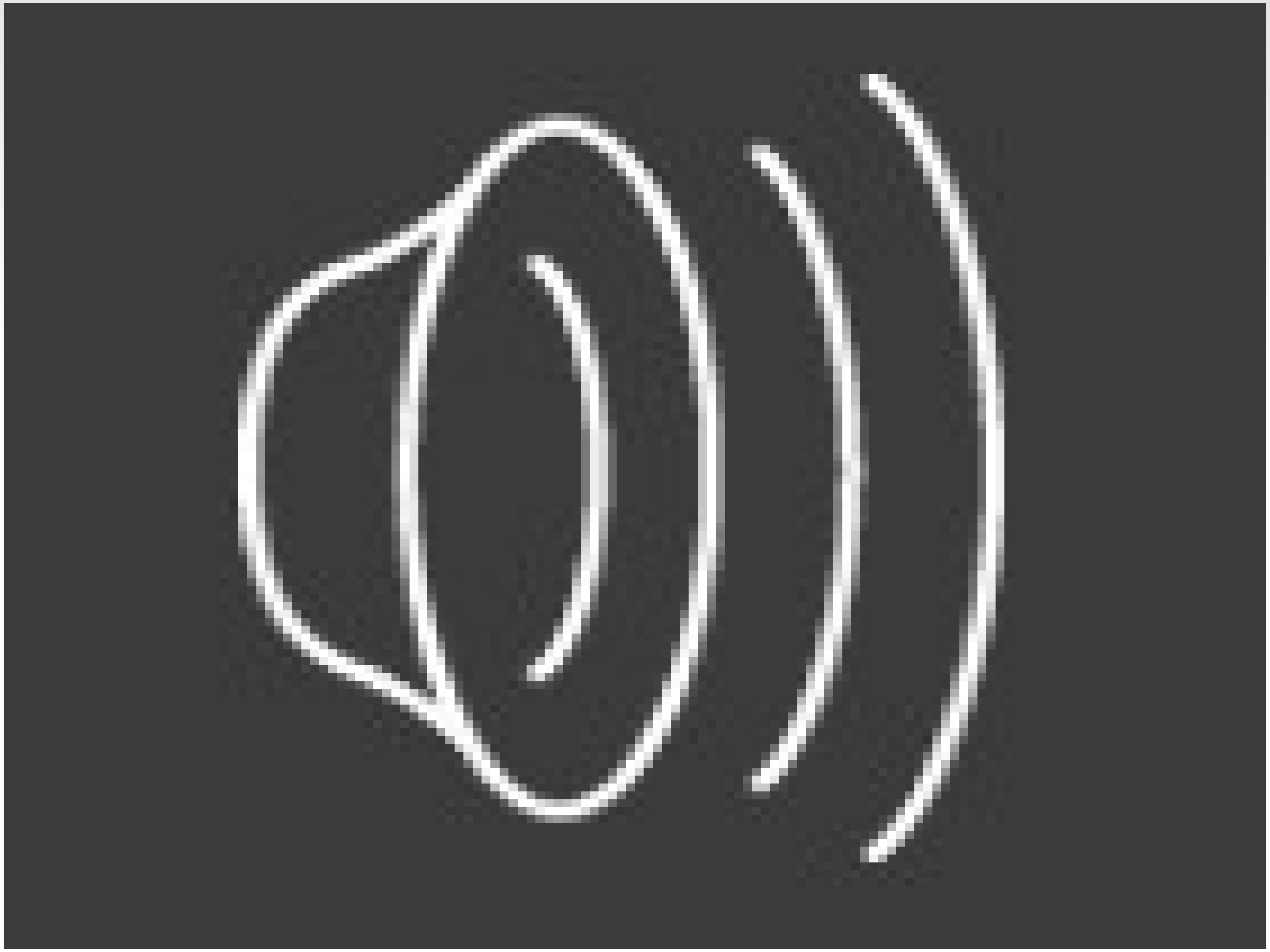
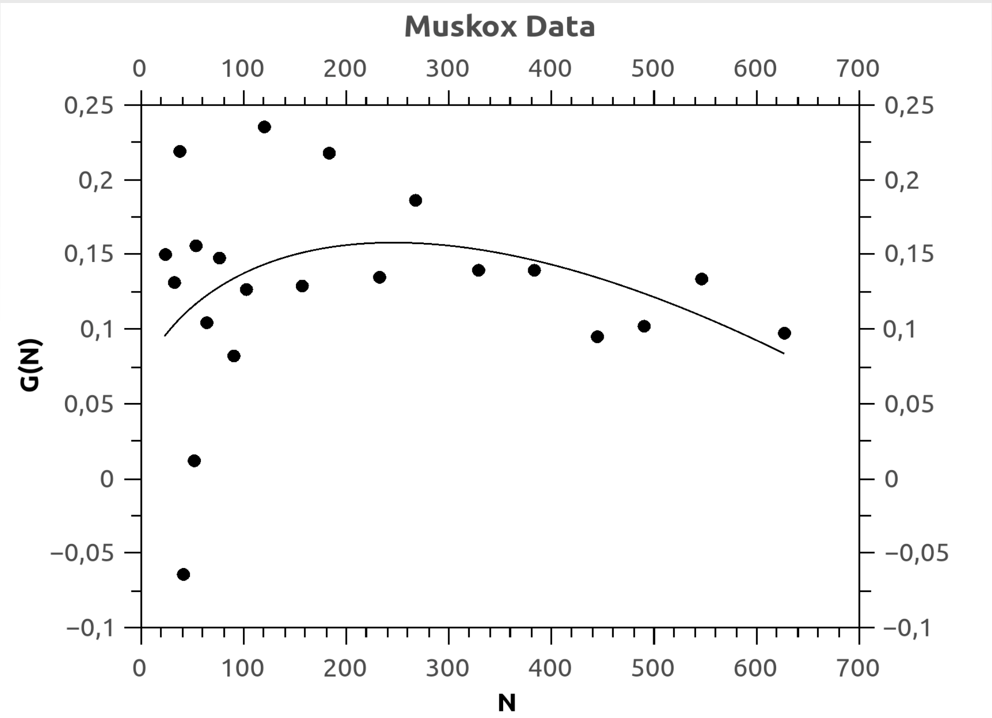


Allee Effect!

The Mukox

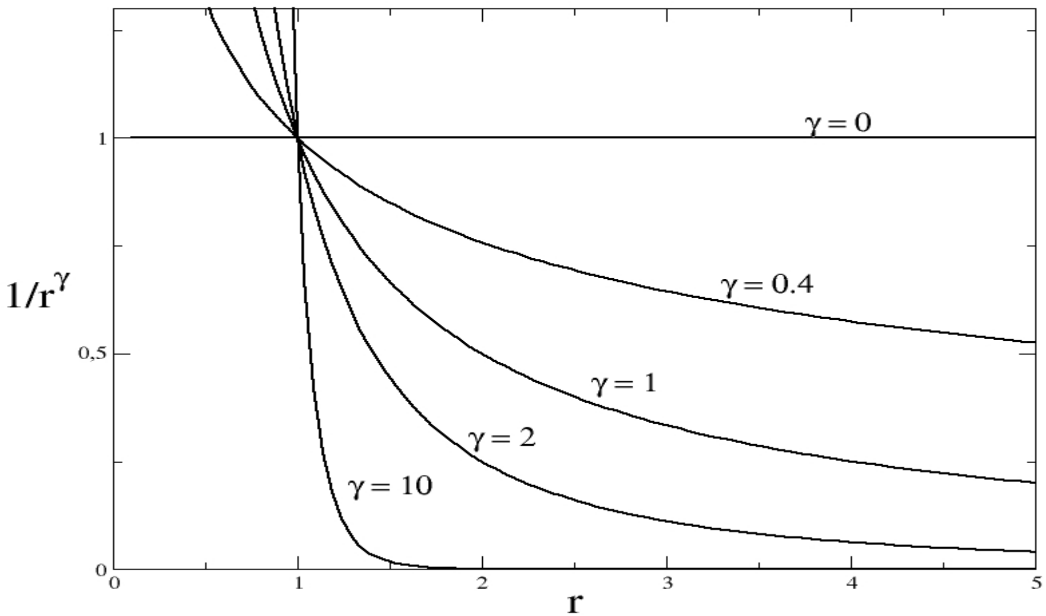


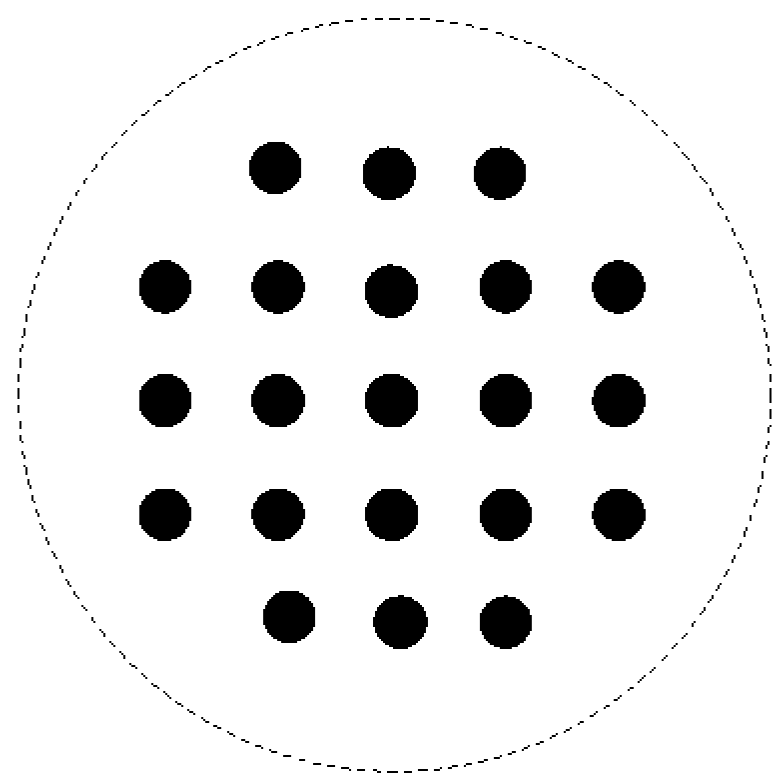
The Mukox


Muskox Data
G(N)
N
0
100
200
300
400
500
600
700
0,25
0,2
0,15
0,1
0,05
0
−0,05
−0,1

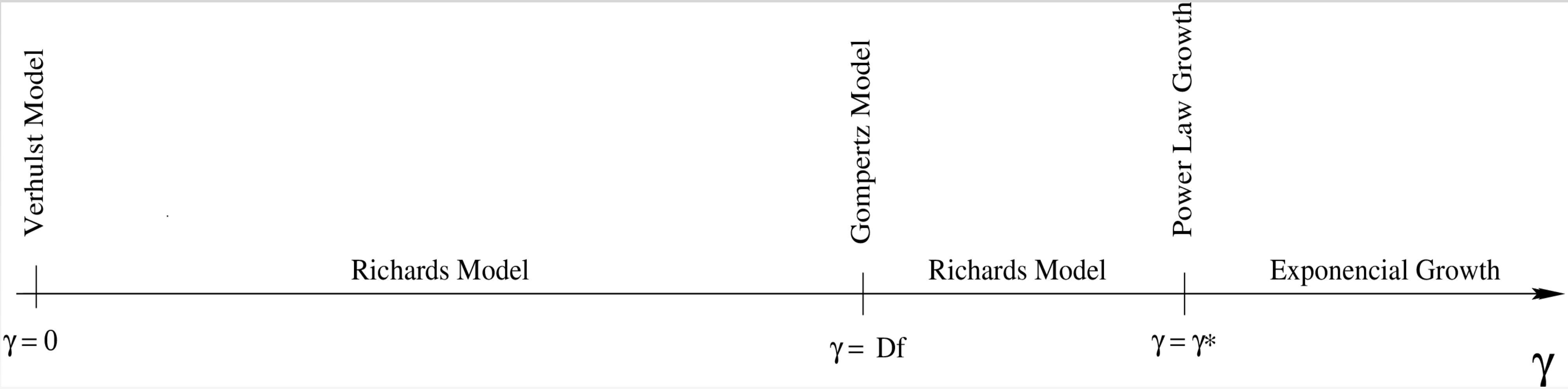
- $\gamma \ll Df$: Modelo de Verhulst

$$\frac{dN}{dt} = rN(1 - N/K),$$



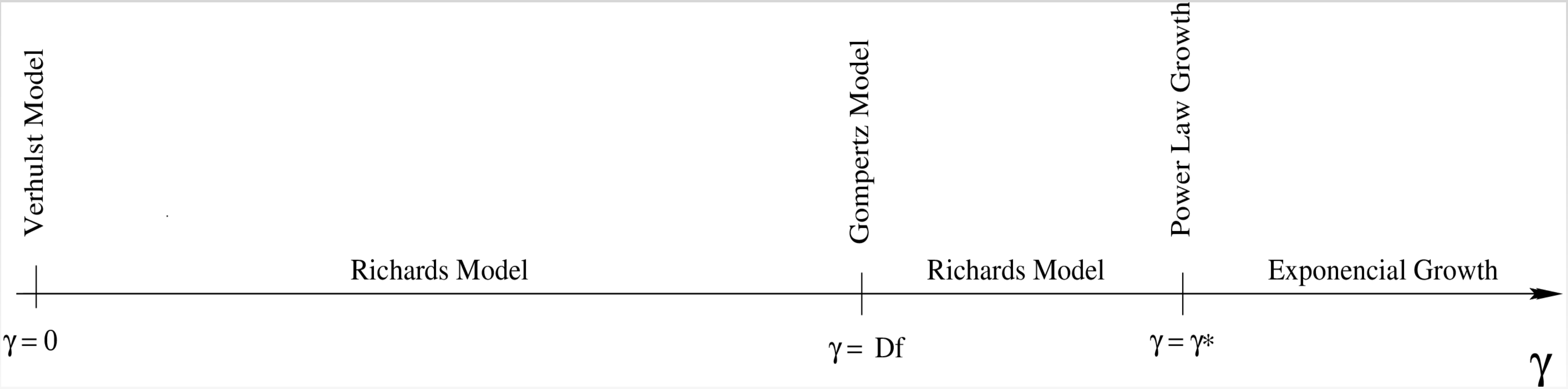


- When **Competition** Predominates :

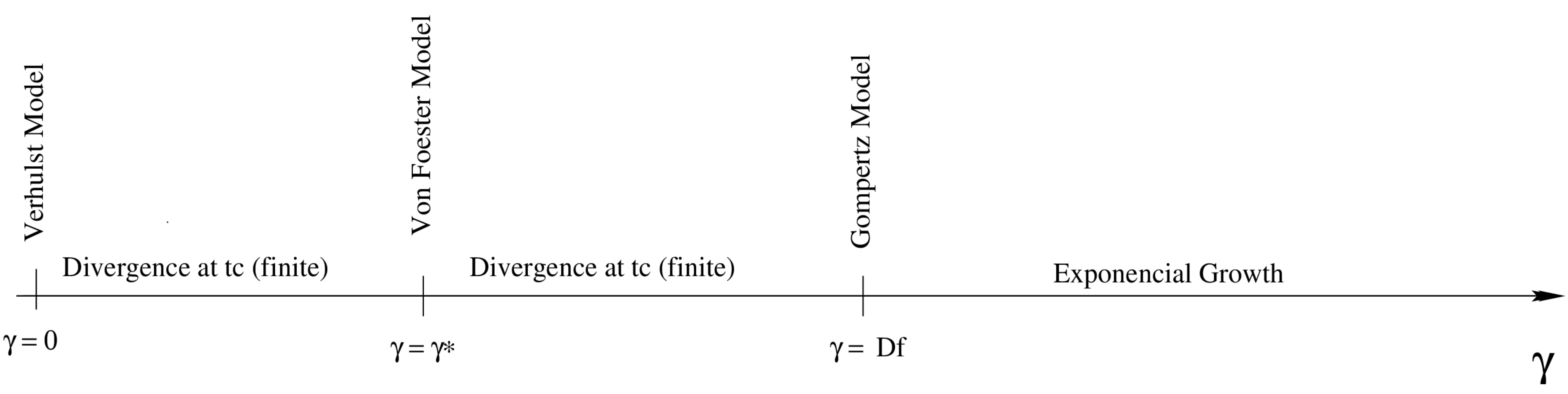


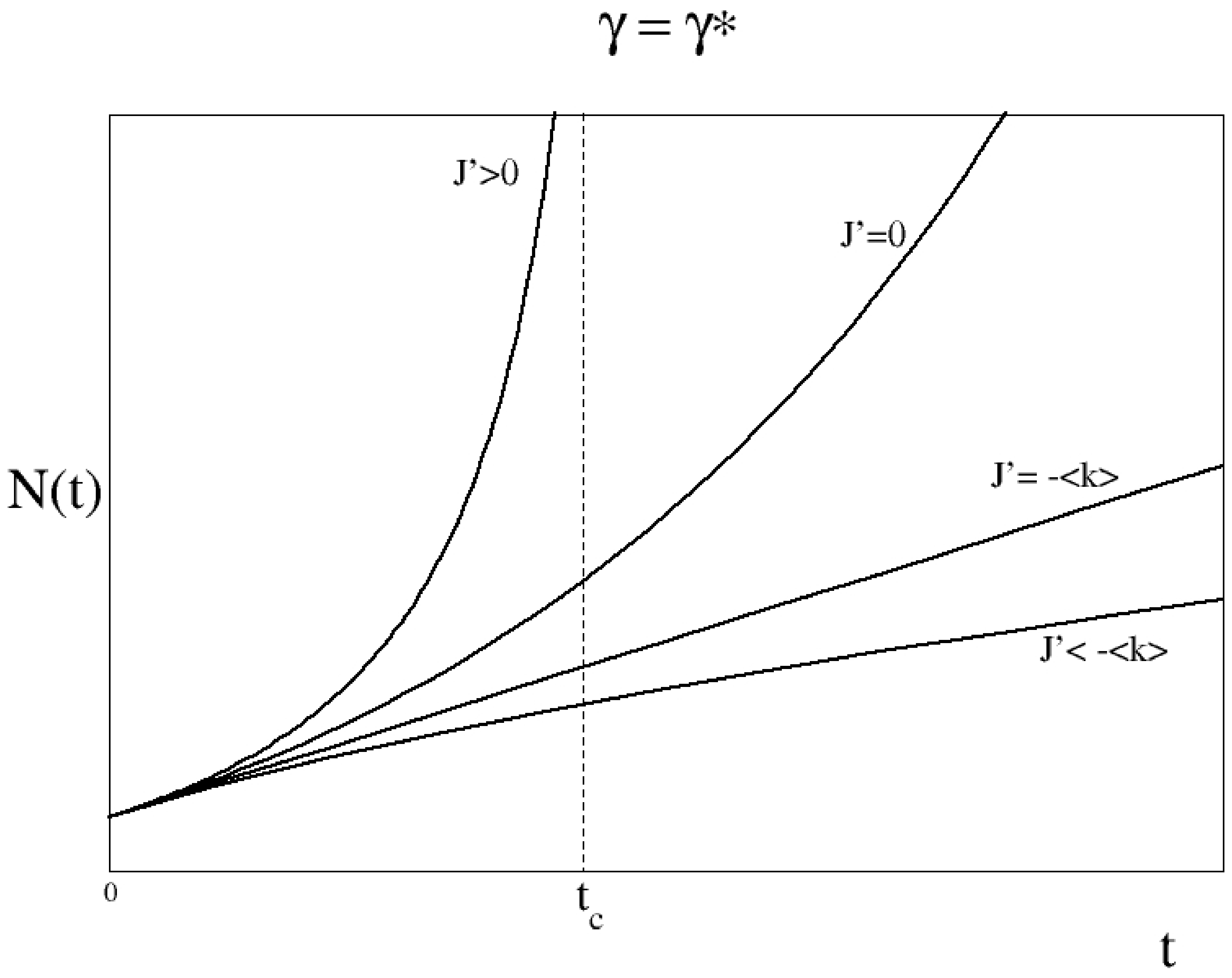
Mombach et Al, Europhys. Lett., **59** (2002)

- When **Competition** Predominates :



- When **Cooperation** predominates:





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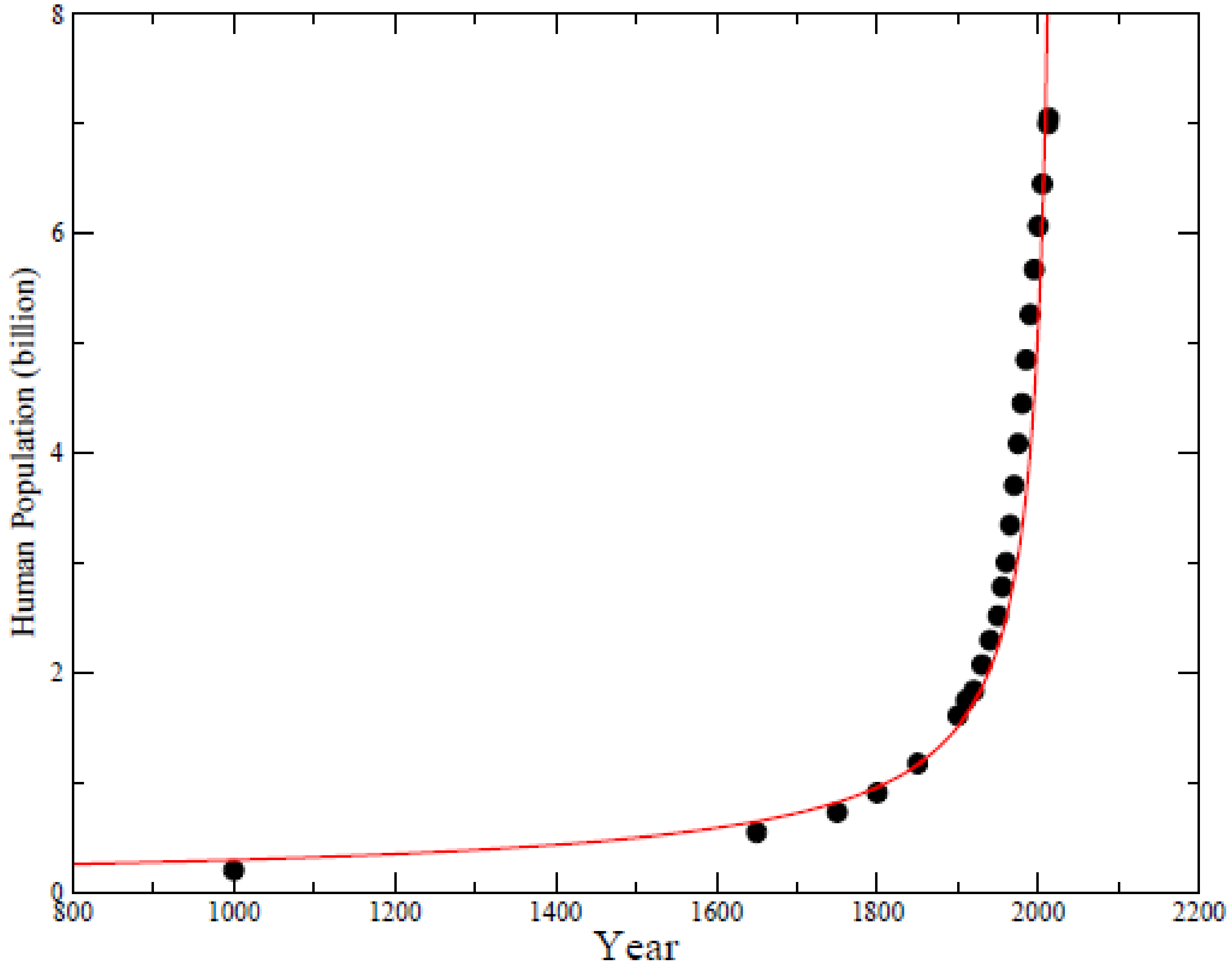

Figure 7: The human population as a function of time since the Middle Ages. The data were obtained from [3] and from the U.S. Census Bureau [34]. The curve is a plot of the equation $N(t) = 65,6(2026 - t)^{-0,78}$ (from Eq. (25)), whose parameters values were obtained via a data fit.

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Doomsday: Friday, 13 November, A.D. 2026

At this date human population will approach infinity if it grows as it has grown in the last two millenia.

Heinz von Foerster, Patricia M. Mora, Lawrence W. Amiot

Among the many different aspects which may be of interest in the study of biological populations (*1*) is the one in which attempts are made to estimate the past and the future of such a population in terms of the number of its elements, if the behavior of this population is observable over a reasonable period of time.

All such attempts make use of two fundamental facts concerning an individual element of a closed biological population—namely, (i) that each element comes into existence by a sexual or asexual process performed by another element of this population ("birth"), and (ii) that after a finite time each element will cease to be a distinguishable member of this population and has to be excluded from the population count ("death").

Under conditions which come close to being paradise—that is, no environmental hazards, unlimited food supply, and no detrimental interaction between elements—the fate of a biological population as a whole is completely determined at all times by reference to the two fundamental properties of an individual element: its fertility and its mortality. Assume, for simplicity, a fictitious population in which all elements behave identically (equivivant population, *2*) displaying a fertility of γ_0 offspring per element per unit time and having a mortality $\theta_0 = 1/t_m$, derived from the life span for an individual element of t_m units of time. Clearly, the elements in the population, is given by

$$\frac{dN}{dt} = \gamma_0 N - \theta_0 N = a_0 N \qquad (1)$$

where $a_0 = \gamma_0 - \theta_0$ may be called the productivity of the individual element. Depending upon whether $a_0 \gtreqqless 0$, integration of Eq. 1 gives the well-known exponential growth or decay of such a population with a time constant of $1/a_0$.

In reality, alas, the situation is not that simple, inasmuch as the two parameters describing fertility and mortality may vary from element to element and, moreover, fertility may have different values, depending on the age of a particular element.

To derive these distribution functions from observations of the behavior of a population as a whole involves the use of statistical machinery of considerable sophistication (*3, 4*).

However, so long as the elements live in our hypothetical paradise, it is in principle possible, by straightforward mathematical methods, to extract the desired distribution functions, and the fate of the population as a whole, with all its ups and downs, is again determined by properties exclusively attributable to individual elements. If one foregoes the opportunity to describe the behavior of a population in all its temporal details and is satisfied

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The authors are members of the staff of the department of electrical engineering, University of Illinois, Urbana.

Conclusion

- A Microscopic Population Model was proposed;
- Some phenomenological models are reached as special cases;
- Comparassion with empirical data;
- Insights about universal growth behaviours.