Generalized entropic measures of quantum correlations

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in collaboration with

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and N. Gigena, J.M. Matera, L. Rebón

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Complex Systems / TSALLIS'70 q-mpleaños @ Rio 2013

CONICET

Comisión de Investigaciones Científicas (C.I.C.)
Gobierno de la Provincia de Buenos Aires

Correlations in pure quantum states

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Bipartite quantum system $A + B$

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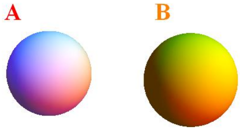

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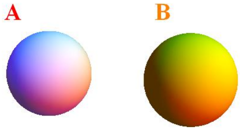

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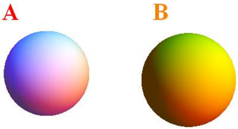

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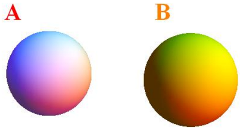

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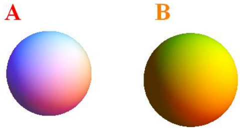

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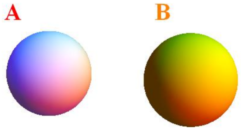

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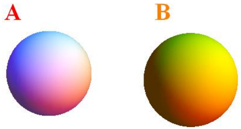

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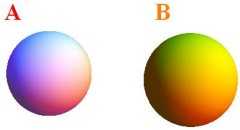

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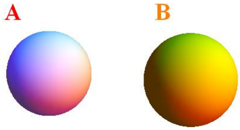

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- Entangled states **cannot** be created by local operations and classical communication (LOCC)

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- Entanglement entropy defined as the "impossible" entropy:

$$E(A,B)=S(A)=S(B)\quad (\text{when } S(A,B)=0)$$

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- Quantum-like correlations still present in states $\in$ II.c !

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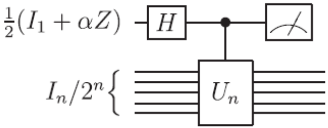

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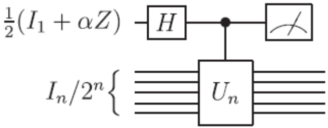

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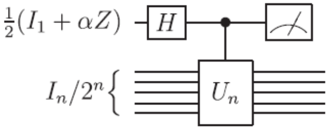

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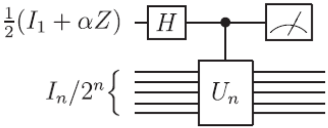

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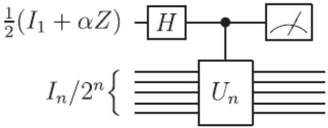

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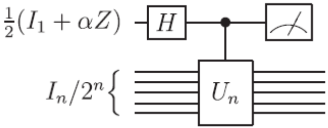

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- In contrast, entanglement needs **finite** threshold: ρ_{AB} entangled for $x > \frac{1}{d-1}$

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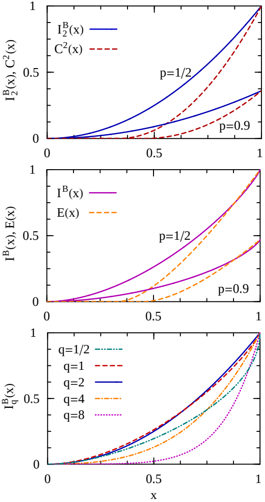

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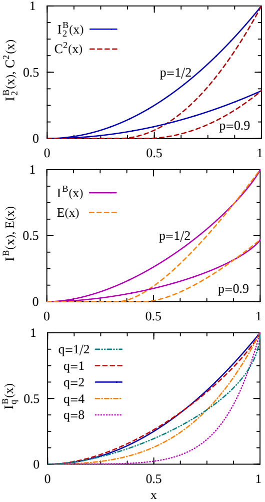

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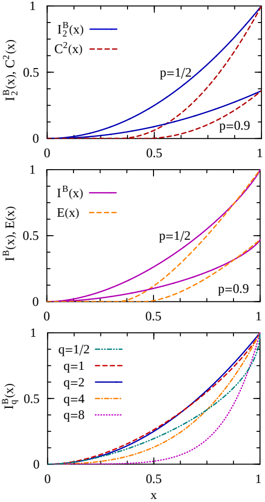

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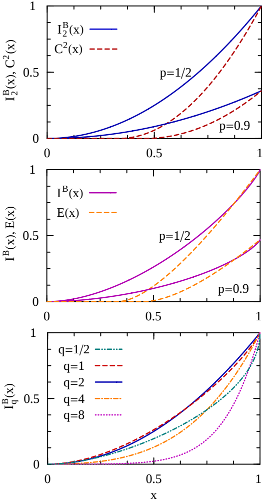

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RR NC LC PRA 2010

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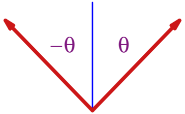

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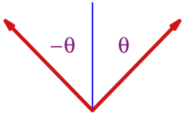

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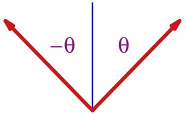

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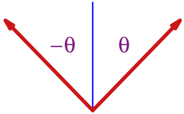

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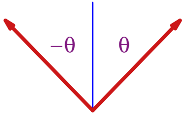

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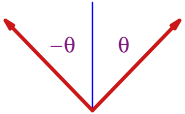

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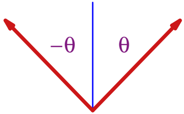

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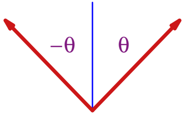

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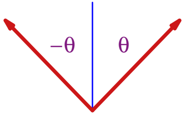

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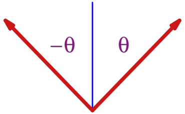

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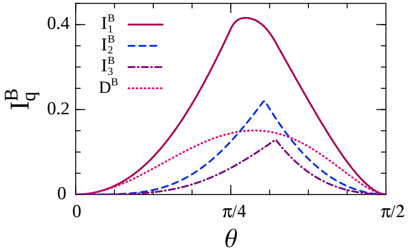

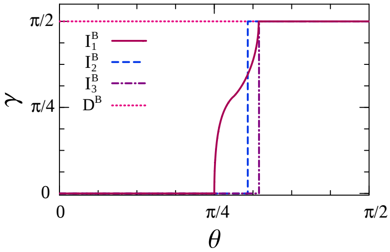

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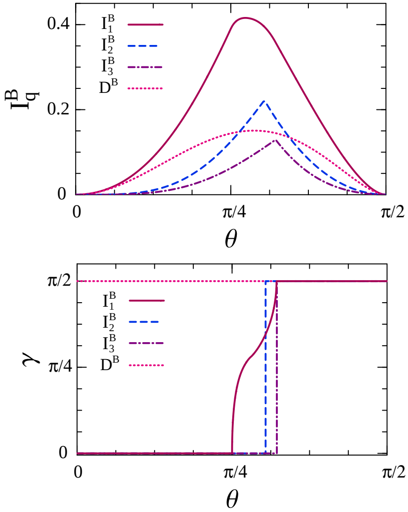

Standard (D), Geometric (I_2), Cubic (I_3) and Information Deficit (I_1) vs. θ

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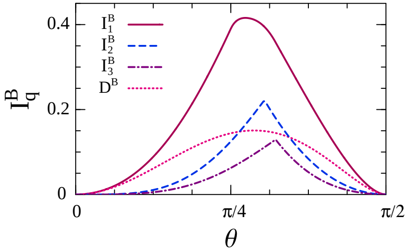

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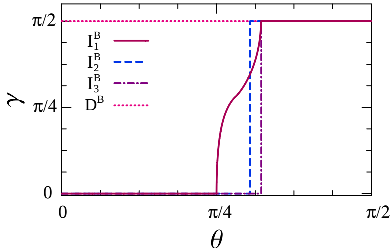

Minimizing measurement angle

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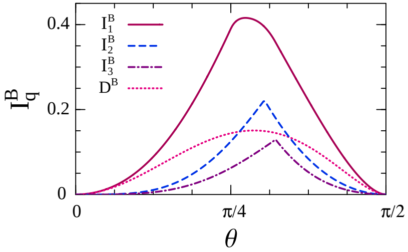

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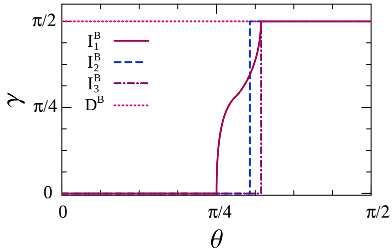

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$z \to x$ transition in all I_f

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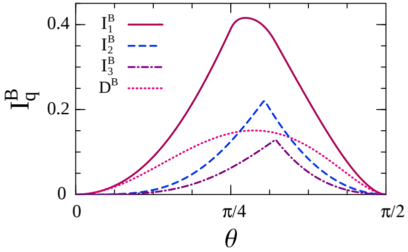

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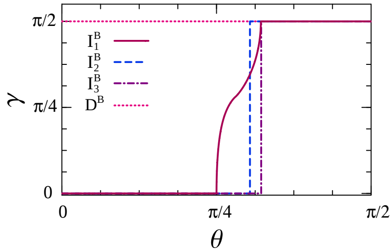

Minimizing measurement angle
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Absent in QD

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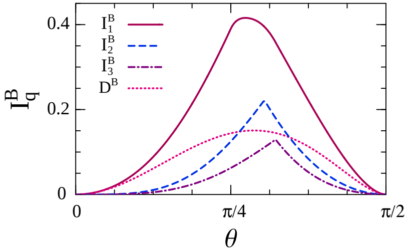

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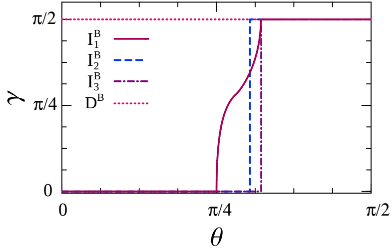

Minimizing measurement angle
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(RR NC LC PRA 2011)

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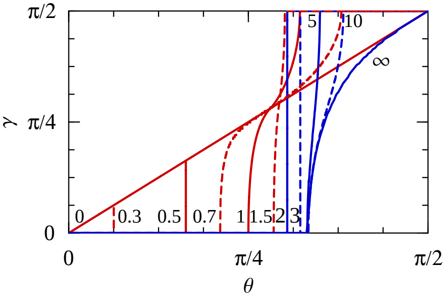

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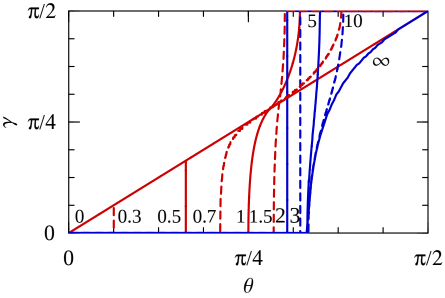

Least disturbing measurement angle γ vs. θ for different q

RR NC LC PRA 2011

Results

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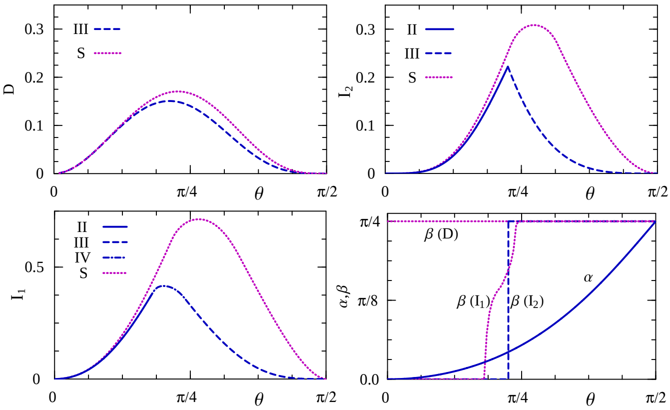

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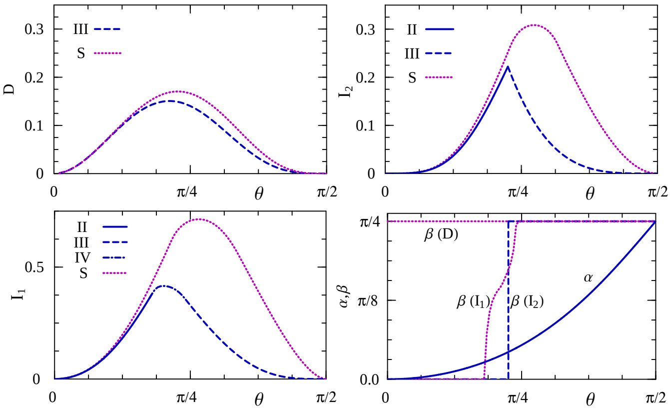
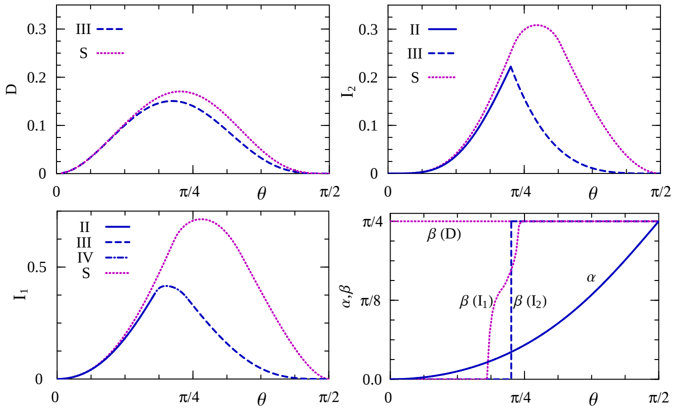


Quantum Discord, Geometric Discord I_2, Information Deficit I_1
and minimizing measurement angles in a mixture of **spin** 1 aligned states

Results



Quantum Discord, Geometric Discord I_2, Information Deficit I_1
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Measurement transition present in all I_f and absent in QD

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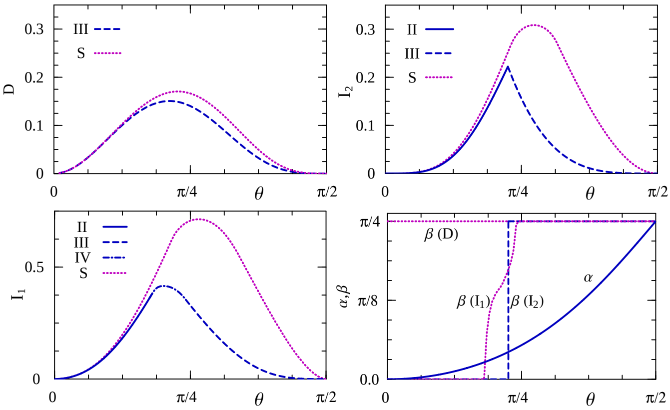

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Measurements in spin 1 systems

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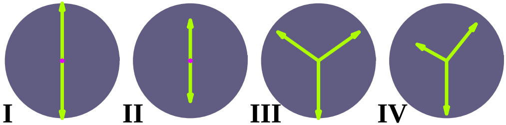

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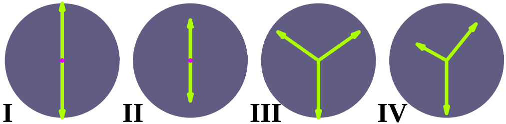

Spin averages in basis states

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Entanglement and Discord of spin pairs in a spin chain

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- Finite cyclic spin $1/2$ chain with
  XY couplings in a transverse field

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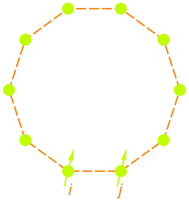

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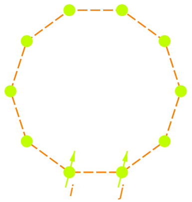

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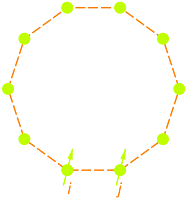

- Hamiltonian

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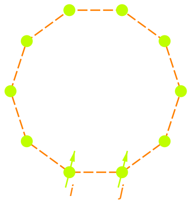

- Hamiltonian

$$H = b\sum_i s_i^z - \tfrac{1}{2}\sum_{i\neq j}(v_x^{ij}s_i^x s_j^x + v_y^{ij}s_i^y s_j^y)$$
$$= b\sum_i s_i^z - \tfrac{1}{2}\sum_{i\neq j}[v_+^{ij}s_i^+ s_j^- + \tfrac{1}{2}v_-^{ij}(s_i^+ s_j^+ + h.c.)]$$

Results

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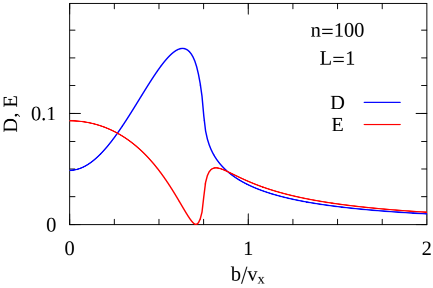

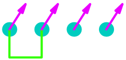

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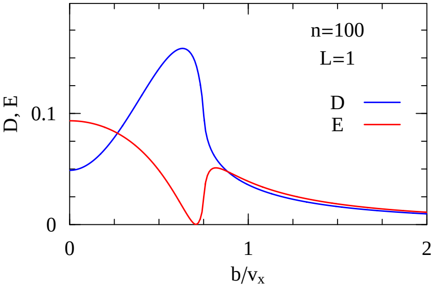

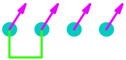

Quantum Discord and entanglement of contiguous pairs vs. transverse field for first neighbor *XY* couplings with anisotropy $v_y/v_x = 1/2$ and $n = 100$ spins

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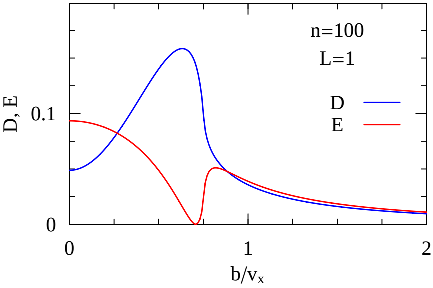

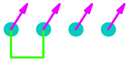

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L. Ciliberti, N. Canosa, R. Rossignoli, PRA 82 2010

Results

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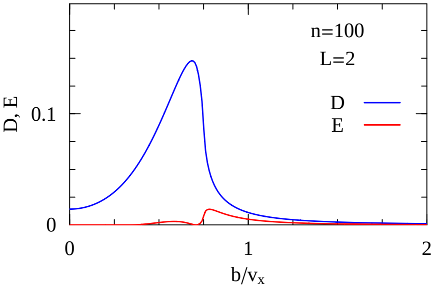

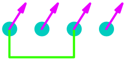

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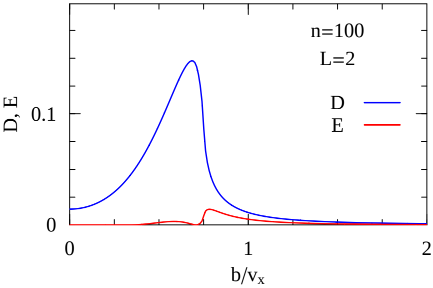

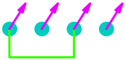

Quantum Discord and entanglement of second neighbors in the same chain

Results

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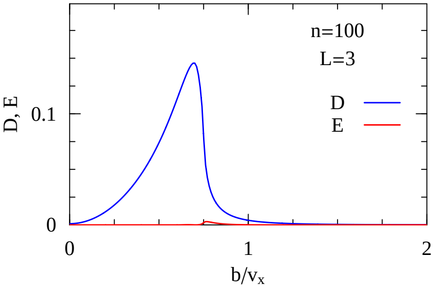

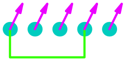

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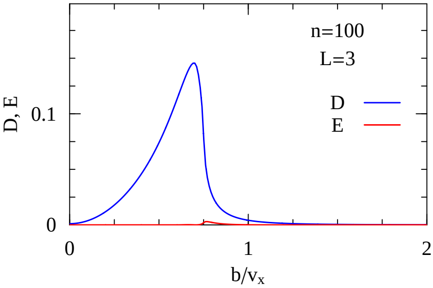

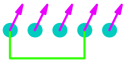

Quantum Discord and entanglement of third neighbors in the same chain

Results

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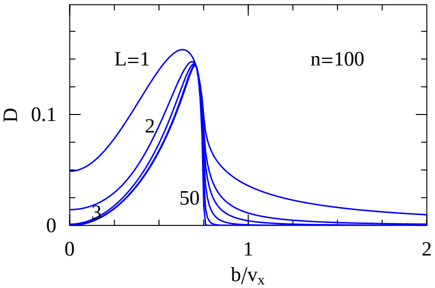

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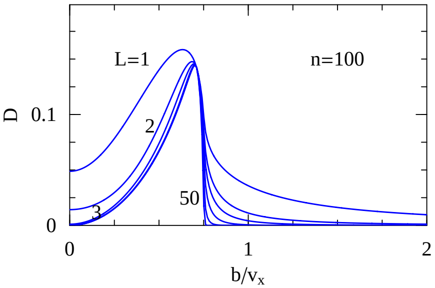

Discord of spin pairs for separations $L = 1, \ldots, 50$ in the same chain

Results

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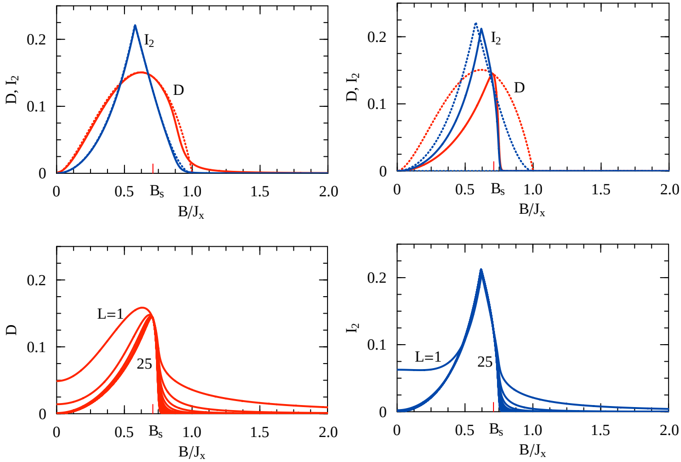
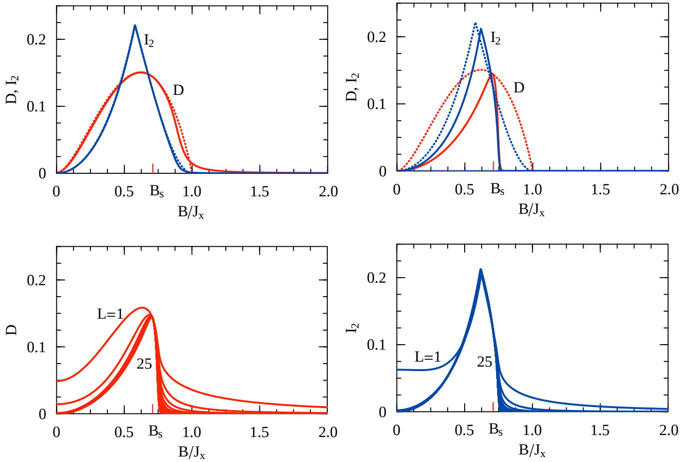
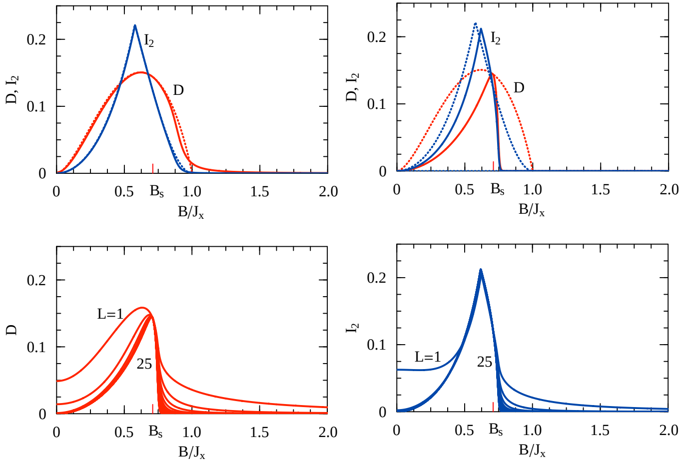


Results



Stdandard vs. Geometric Discord in spin chains

Results



Stdandard vs. Geometric Discord in spin chains

(NC LC RR, IJMPB 2012)

Results

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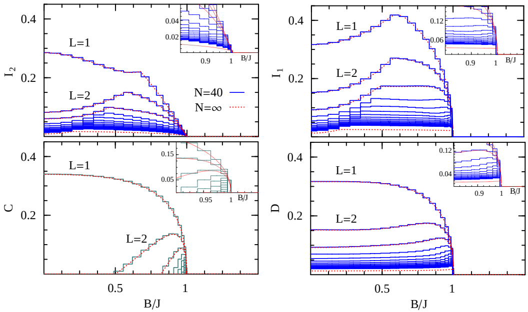

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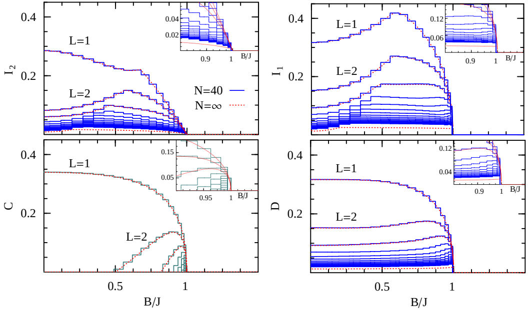

Quantum Discord, Geometric Discord I_2 and Information Deficit I_1 of spin pairs for separations $L = 1, \ldots, N/2$ in an XX chain

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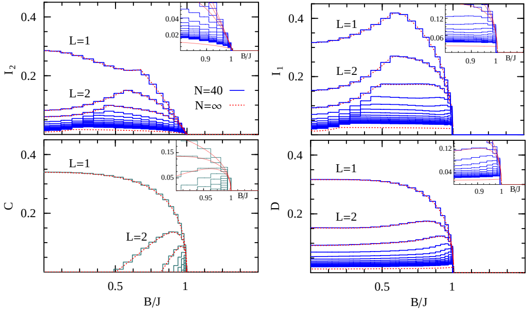

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LC RR NC PRA 2013

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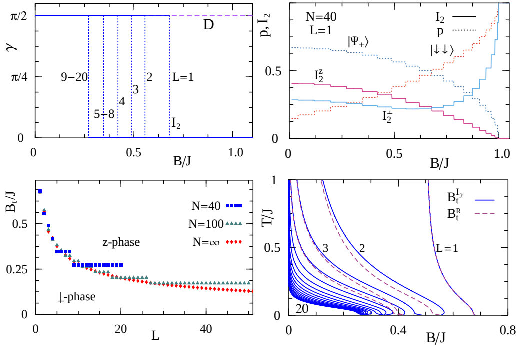
π/2
π/4
0
γ
D
9−20
5−8
4
3
2
L=1
I2
0
0.5
1.0
B/J
N=40
L=1
|Ψ+⟩
|↓↓⟩
I2
p
p, I2
0.5
0
B/J
1
0.5
0
T/J
0
0.4
0.8
B/J
3
2
L=1
20
Bt/J
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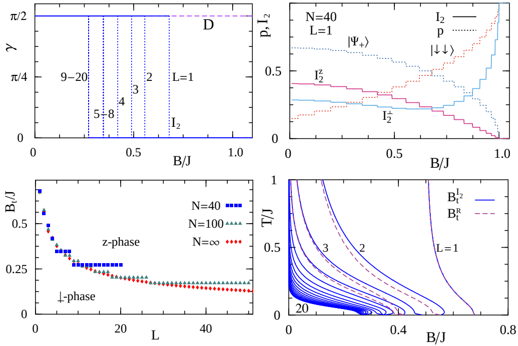
0
N=40
N=100
N=∞
z-phase
⊥-phase
0
20
40
L

Results



Phases of Geometric Discord I_2 in the XX chain

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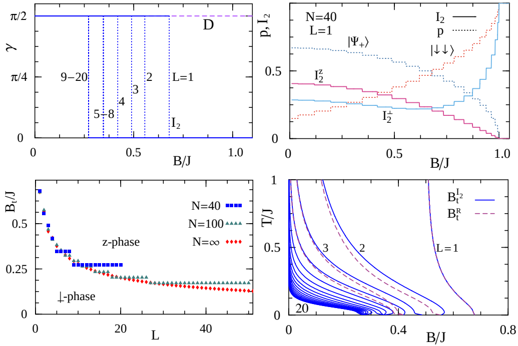

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Than-q !

 

References

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(A.Kowalski, R.Rossignoli, E.M.F. Curado eds. Bentham 2013)