THEORETICAL PHYSICS:

CROSSROADS OF TRUTH AND BEAUTY

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SANTA FE INSTITUTE

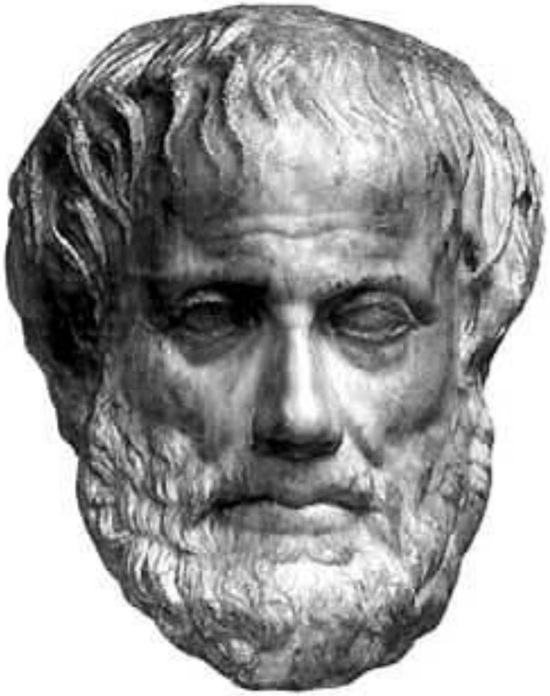
Rio de Janeiro, 31 October 2013

Dedicated to

Emmanuel and Cleopatra
Demetrio and Thalia
Maria Cristina, Alexandra and Adrian, Flora and Enzo
Maria Aparecida and Emmanuel
Marisa

Beauty is truth, truth beauty,
—that is all
Ye know on earth,
and all ye need to know.

- John Keats



Poetry is more elevated and more philosophical than history; for poetry expresses the universal, and history only the particular. History tells us the events as they happened, whereas poetry tells them as they could or should have happened.

Aristotle

(~ 1978 - ~ 1991)

FIDELITY, PERCOLATION, POTTS MODEL,

TRANSMISSIVITY, BREAK-COLLAPSE METHOD,

RANDOM RESISTORS, RANDOM MAGNETISM,

RENORMALIZATION GROUP, AND ALL THAT JAZZ

VOLUME 47, NUMBER 13 PHYSICAL REVIEW LETTERS 28 SEPTEMBER 1981

Simple Method to Calculate Percolation, Ising, and Potts Clusters: Renormalization-Group Applications

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(Received 26 May 1981)

ELSEVIER

Physics Reports 268 (1996) 305–430

PHYSICS REPORTS

Pure and random Potts-like models: real-space renormalization-group approach

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PROBABILITIES:

AND: $p = p_1 p_2 \qquad p_s = p_1 p_2$ (series)

OR:

$$p = p_1 p_2 + p_1(1 - p_2) + p_2(1 - p_1)$$

$$= p_1 + p_2 - p_1 p_2$$

$$p_p = p_1 + p_2 - p_1 p_2$$ (parallel)

$$\textit{hence } 1 - p_p = (1 - p_1)(1 - p_2)$$

$$p^D \equiv 1 - p$$ (definition)

$$\textit{hence } p_p{}^D = p_1{}^D p_2{}^D$$ (parallel)

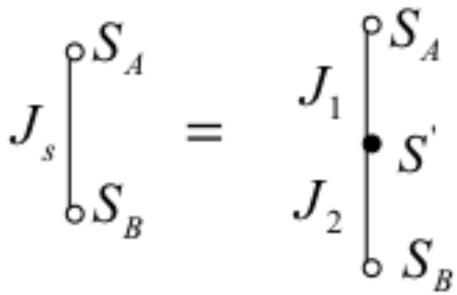
Application: Bond percolation in square lattice

$$p_c = p_c{}^D \Rightarrow p_c = 1 - p_c \Rightarrow p_c = 1/2$$ exact!

Q-STATE POTTS MODEL:

$$H = -QJ\delta_{S_A,S_B} \qquad (S_A, S_B = 1, 2, \ldots, Q)$$

AND:



$$\sum_{S'=1}^{Q} e^{-\beta Q(J_1 \delta_{S_A,S'} + J_2 \delta_{S_B,S'})} \propto e^{-\beta Q J_s \delta_{S_A,S_B}}$$

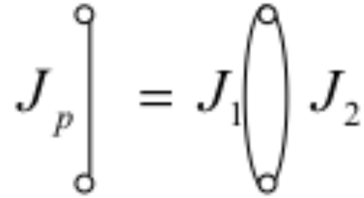
hence $\quad t_s = t_1 t_2 \quad (\forall Q)$ (series)

with $\quad t \equiv \dfrac{1 - e^{-Q\beta J}}{1 + (Q-1)e^{-Q\beta J}}$ (transmissivity)

$Q = 1 \Rightarrow t = 1 - e^{-\beta J}$ (Kasteleyn and Fortuin)

$Q = 2 \Rightarrow t = th\beta J$ (Ising)

OR:



$$J_p = J_1 + J_2$$

$$t_p = \frac{t_1 + t_2 + (Q-2)t_1 t_2}{1 + (Q-1)t_1 t_2} \qquad \text{(parallel)}$$

Remark: $J_2 \to \infty \Rightarrow t_2 = 1 \Rightarrow t_p = 1 \quad (\forall Q, \forall t_1$

$$\frac{1-t_p}{1+(Q-1)t_p} = \frac{1-t_1}{1+(Q-1)t_1} \; \frac{1-t_2}{1+(Q-1)t_2}$$

$$t^D \equiv \frac{1-t}{1+(Q-1)t} \qquad \text{(duality)}$$

i.e., $t_p{}^D = t_1{}^D t_2{}^D$ (parallel)

Application: Square-lattice Q-state Potts ferromagnet

$$t_c = t_c{}^D \Rightarrow t_c = \frac{1-t_c}{1+(Q-1)t_c} \Rightarrow t_c = \frac{1}{\sqrt{Q}+1} \qquad \text{exact!}$$

FURTHER COMPACTIFICATION:

$$p^D = 1 - p$$

$$t^D = \frac{1-t}{1+(Q-1)t}$$

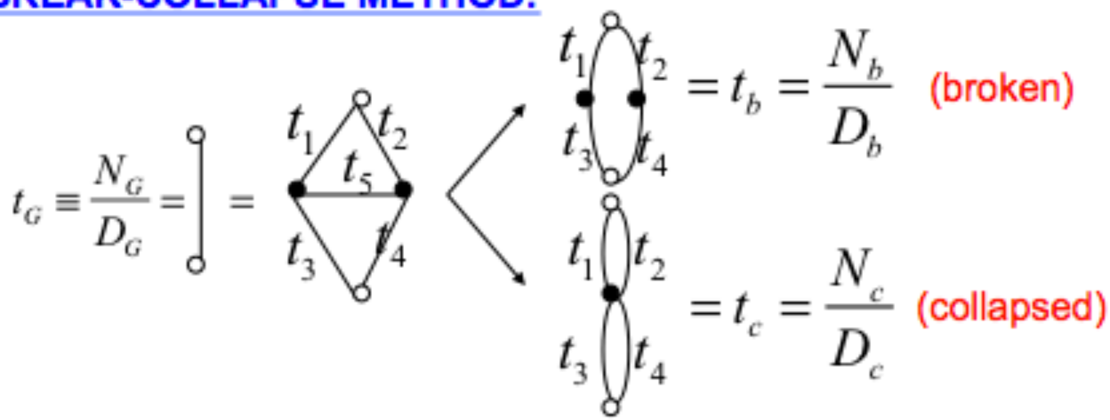
Define $$s \equiv \frac{\ln[1+(Q-1)t]}{\ln Q}$$

hence $$s^D(t) \equiv s(t^D) = 1 - s(t) \quad (\forall Q)$$

Application: Square-lattice Q-state Potts ferromagnet

$$s_c = 1/2 \quad (\forall Q)$$ exact!

BREAK-COLLAPSE METHOD:



$$N_G = (1-t_5)N_b + t_5 N_c$$

$$D_G = (1-t_5)D_b + t_5 D_c$$

(~ 1986 - ...)

NONADDITIVE ENTROPIES, NONEXTENSIVE STATISTICAL MECHANICS, WEAK CHAOS, LONG-RANGE INTERACTIONS, GENERALIZED THERMODYNAMICS, *q*-TRIPLET, CENTRAL LIMIT THEOREMS, NONLINEAR FOKKER-PLANCK, BLACK HOLES, AND ALL THAT BOSSA NOVA

Ludwig BOLTZMANN

Vorlesungen uber Gastheorie (Leipzig, 1896)
Lectures on Gas Theory, transl. S. Brush
(Univ. California Press, Berkeley, 1964), page 13

The forces that two molecules impose one onto the other during an interaction can be completely arbitrary, only assuming that their sphere of action is very small compared to their mean free path.

Ettore MAJORANA

The value of statistical laws in physics and social sciences.
Original manuscript in Italian published by G. Gentile Jr. in *Scientia* **36**, 58 (1942); translated into English by R. Mantegna (2005).

This is mainly because entropy is an additive quantity as the other ones. In other words, the entropy of a system composed of several independent parts is equal to the sum of entropy of each single part. [...]
Therefore one considers all possible internal determinations as equally probable. This is indeed a new hypothesis because the universe, which is far from being in the same state indefinitively, is subjected to continuous transformations. We will therefore admit as an extremely plausible working hypothesis, whose far consequences could sometime not be verified, that all the internal states of a system are a priori equally probable in specific physical conditions. Under this hypothesis, the statistical ensemble associated to each macroscopic state A turns out to be completely defined.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy (*q* = *1*)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (*q real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J. Stat. Phys. **52**, 479 (1988)

DEFINITIONS: *q-logarithm:* $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-exponential: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

1985 Ciudad de Mexico + Rio de Janeiro

1987 Maceio + Rio de Janeiro

1987 Notas de Física - CBPF

1988 Journal of Statistical Physics → 3081 citations/ISI Web of Knowledge

1991 Journal of Physics A (with E.M.F. Curado) → 460 citations/ISI Web of Knowledge

1998 Physica A (with R.S. Mendes and A.R. Plastino) → 728 citations/ISI Web of Knowledge

$p_i^q > p$ if $q < 1$ (*aristocratic*: minority controls)

$p_i^q < p$ if $q > 1$ (*demagogic*: majority controls)

$p_i^q = p$ if $q = 1$ (*democratic*: equal opportunities) Boltzmann-Gibbs

$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in R)$$

A and B probabilistically independent (i.e., $p_{i,j}^{A+B} = p_i^A p_j^B$, i.e., series)

$$\Rightarrow \qquad \frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} \qquad \text{(nonadditive)}$$

$$\Rightarrow \qquad S_q(A+B) = S_q(A) + S_q(B) + \frac{1-q}{k}S_q(A)S_q(B)$$

$$\Rightarrow \quad \frac{\ln\left[1+(1-q)S_q(A+B)/k\right]}{1-q} = \frac{\ln\left[1+(1-q)S_q(A)/k\right]}{1-q} + \frac{\ln\left[1+(1-q)S_q(B)/k\right]}{1-q}$$

$$\Rightarrow \qquad S_q^{'}(A+B) = S_q^{'}(A) + S_q^{'}(B)$$

with $$S_q^{'} \equiv \frac{\ln\left[1+(1-q)S_q/k\right]}{1-q} = \frac{\ln\sum_{i=1}^{W} p_i^q}{1-q}$$

$$S_q = k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in R)$$

A and B probabilistically independent (i.e., $p_{i,j}^{A+B} = p_i^A p_j^B$, i.e., series)

$$\Rightarrow \qquad \frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} \qquad \text{(nonadditive)}$$

$$\Rightarrow \qquad S_q(A+B) = S_q(A) + S_q(B) + \frac{1-q}{k} S_q(A) S_q(B)$$

$$\Rightarrow \qquad \frac{\ln\left[1+(1-q)S_q(A+B)/k\right]}{1-q} = \frac{\ln\left[1+(1-q)S_q(A)/k\right]}{1-q} + \frac{\ln\left[1+(1-q)S_q(B)/k\right]}{1-q}$$

$$\Rightarrow \qquad S_q^{'}(A+B) = S_q^{'}(A) + S_q^{'}(B)$$

with $$S_q^{'} \equiv \frac{\ln\left[1+(1-q)S_q/k\right]}{1-q} = \frac{\ln \sum_{i=1}^{W} p_i^q}{1-q}$$

compare with $s \equiv \frac{\ln\left[1+(Q-1)t\right]}{\ln Q} \sim \frac{\ln\left[1+(Q-1)t\right]}{Q-1}$ from graph approach!

TYPICAL SIMPLE SYSTEMS:

e.g., $W(N) \propto \mu^N \ (\mu > 1)$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

e.g., $W(N) \propto N^\rho \ (\rho > 0)$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ **Entropy S*q* (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

Therefore, since

$$S_q(A+B)=S_q(A)+S_q(B)+(1-q)\,S_q(A)\,S_q(B)\ ,$$

S_{BG} and $S_q^{Renyi}\,(\forall q)$ are additive, and $S_q\ (\forall q\neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma\equiv A_1+A_2+...+A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0<\lim_{N\to\infty}\frac{S(N)}{N}<\infty\ ,\ i.e.,\ S(N)\propto N\ \ (N\to\infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

If $W(N) \sim \mu^{N} \quad (\mu > 1)$

$$\Rightarrow \; S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^{\rho} \quad (\rho > 0)$

$$\Rightarrow \; S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow \; S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^{\gamma}} \quad (\nu > 1;\; 0 < \gamma < 1)$

$$\Rightarrow \; S_{\delta}(N) = k_B \left[\ln W(N)\right]^{\delta} \propto N^{\gamma\,\delta}$$

$$\Rightarrow \; S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

IMPORTANT: $\mu^{N} >> \nu^{N^{\gamma}} >> N^{\rho}$ if $N >> 1$

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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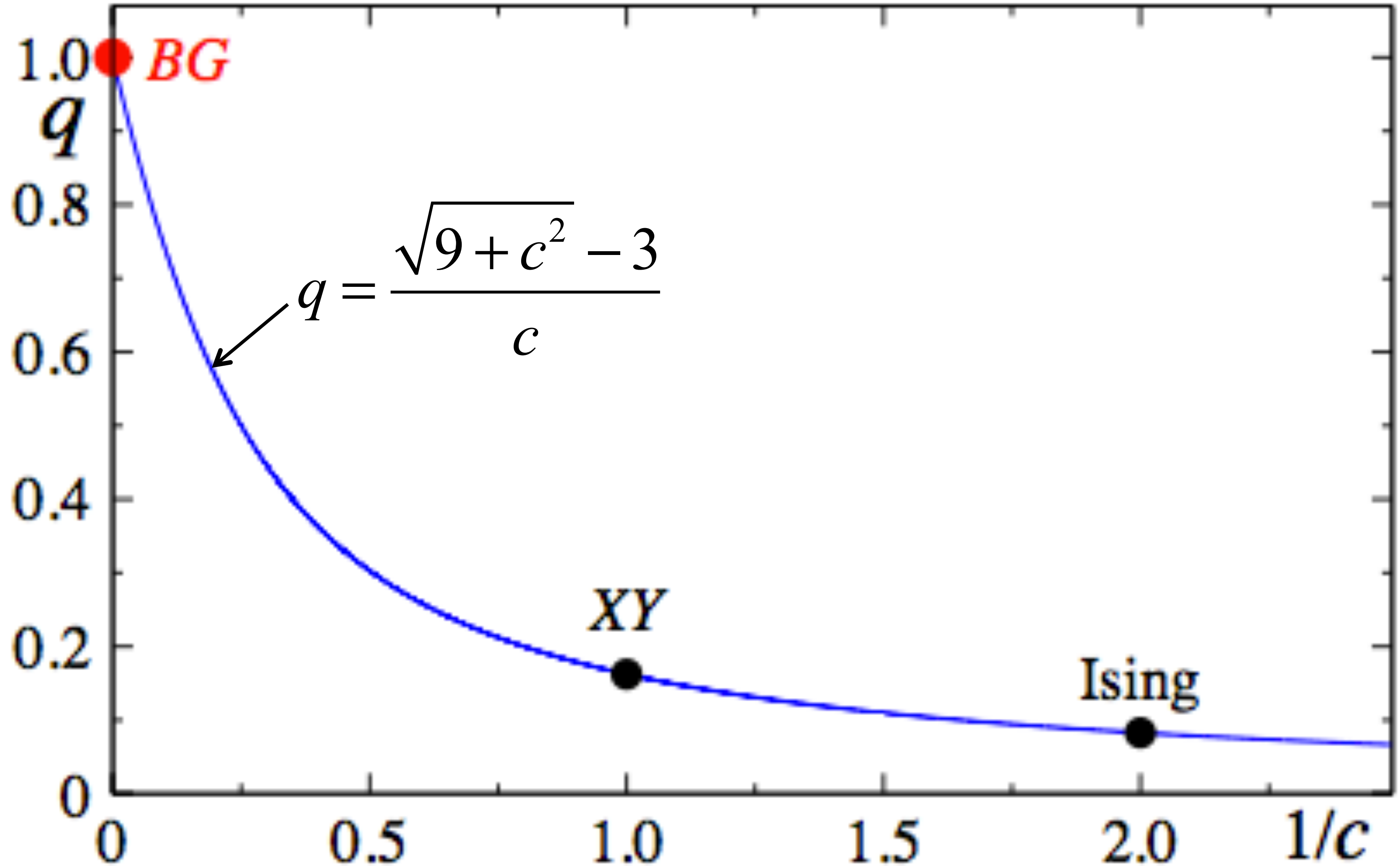
The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.



1.0
q
0.8
0.6
0.4
0.2
0
BG
$q = \frac{\sqrt{9+c^2}-3}{c}$
XY
Ising
0
0.5
1.0
1.5
2.0
1/c

EDGE OF CHAOS OF THE LOGISTIC MAP:

(Using result in http://pi.lacim.uqam.ca/piDATA/feigenbaum.txt)

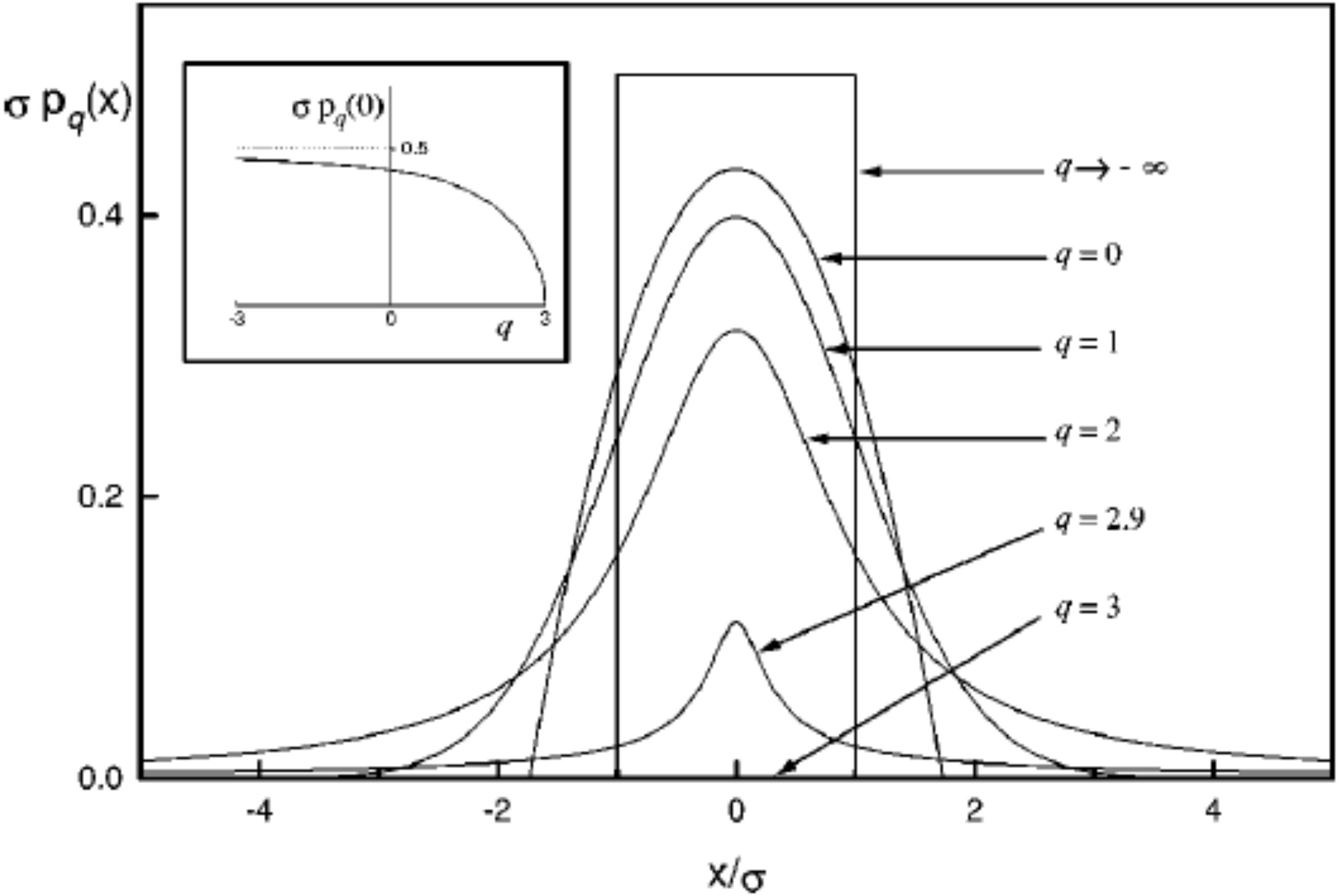
$$q = 1 - \frac{\ln 2}{\ln \alpha_F} =$$

0.2444877013412820661987704234046804052344469354900576736703650
986327749672766558665755156226857540706288349640382728306063600
193730331818964551341081277809792194386027083194490052465813521
503174534952074940448165460949087448334056723622466488083333072
142318987145872992681548496774607864821834569063370205946820461
899021675321457546117438305008496860408846969491704367478991506
016646491060217834827889993818382522554582338038113118031805448
236757944990397074395466146340815553168788535030113821491411266
246328940130370152354936571471269917921021622688833029675405780
630706822368810432015790352123740735444602970006055250423142028
089193578811239731977974844235152456040926446709579570304658614
129566479666687743683240492022757393004750895311855179558720483
992696896827555852445024436526825609423780128033094877954403542
524859043379761802711830004573585550738941136758784400629135630
421674541694092135698603207859088199859359007319336801069967496
707904456092418632112054130547393985795544410347612222592136846
219346009360…

(1018 meaningful digits)

q-GAUSSIANS:

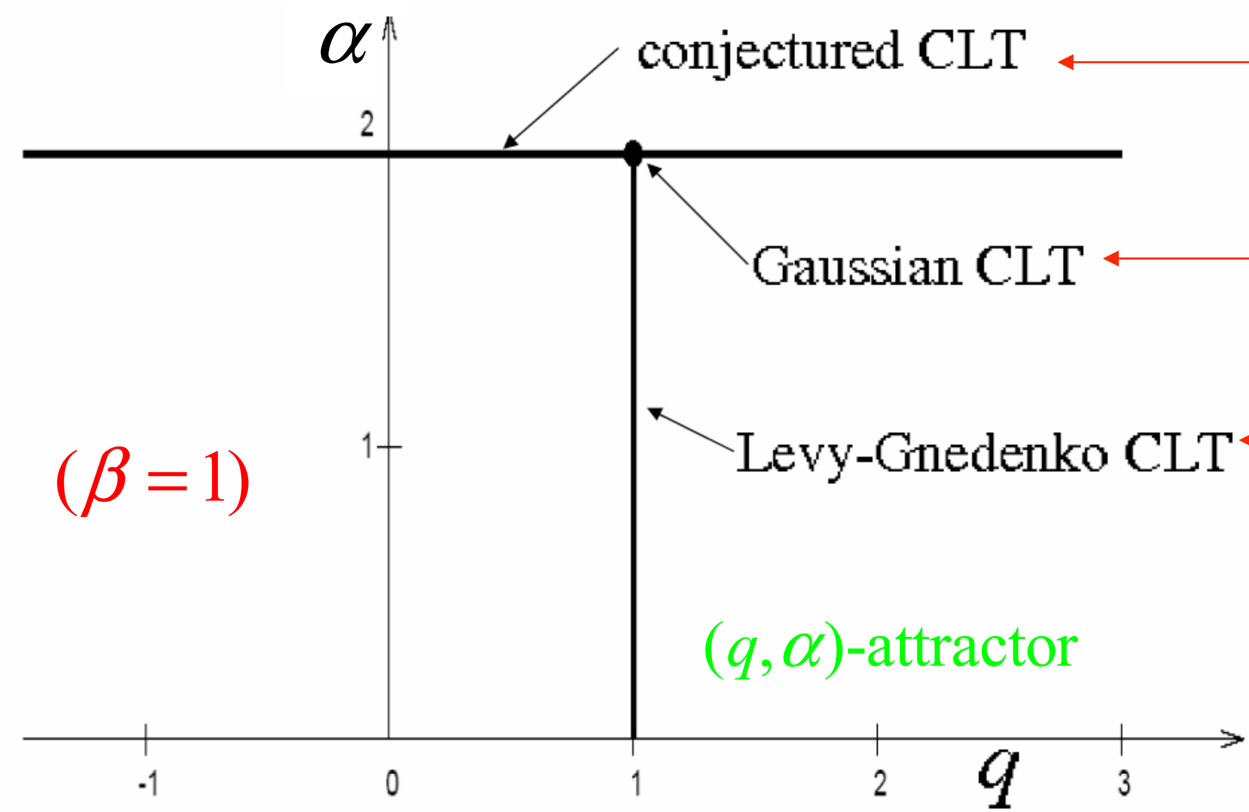
$$p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}} \qquad (q<3)$$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

LOOKING FOR A q-GENERALIZED CENTRAL LIMIT THEOREM:

$$\frac{\partial^\beta p(x,t)}{\partial t^\beta} = D \frac{\partial^\alpha [p(x,t)]^{2-q}}{\partial |x|^\alpha} \quad (0 < \alpha \le 2;\ q < 3;\ t \ge 0)$$



globally correlated variables
finite q-variance
q-Gaussian attractor

independent variables
finite variance
Gaussian attractor

independent variables
divergent variance
Lévy attractor

M. Bologna, C. T. and P. Grigolini, Phys. Rev. E **62**, 2213 (2000)
C. T., Milan J. Math. **73**, 145 (2005)

Milan j. math. 76 (2008), 307–328

1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
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See also: H.J. Hilhorst, JSTAT P10023 (2010)

M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)

M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)

A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)

A. Plastino and M.C. Rocca (2013)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ - *scaled attractor* $\mathrm{F}(x)$ *when summing* $N \to \infty$ q-*independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

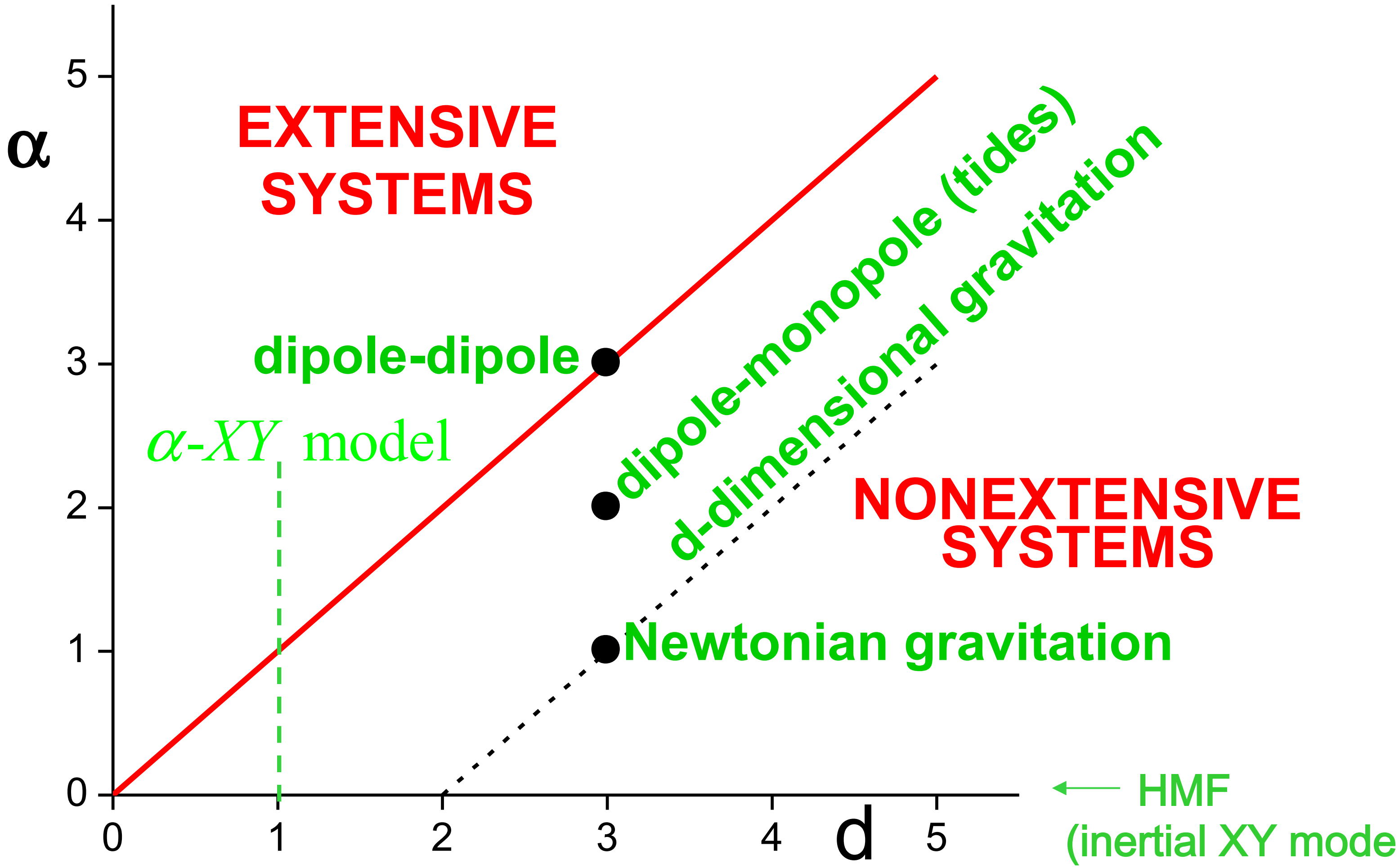
	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathrm{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathrm{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathrm{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $\mathrm{L}_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathrm{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\,\alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \text{(intermediate regime)} \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\,2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \text{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0, \;\; \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*



Physica A 393 (2014) 286–296

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Influence of the interaction range on the thermostatistics of a classical many-body system

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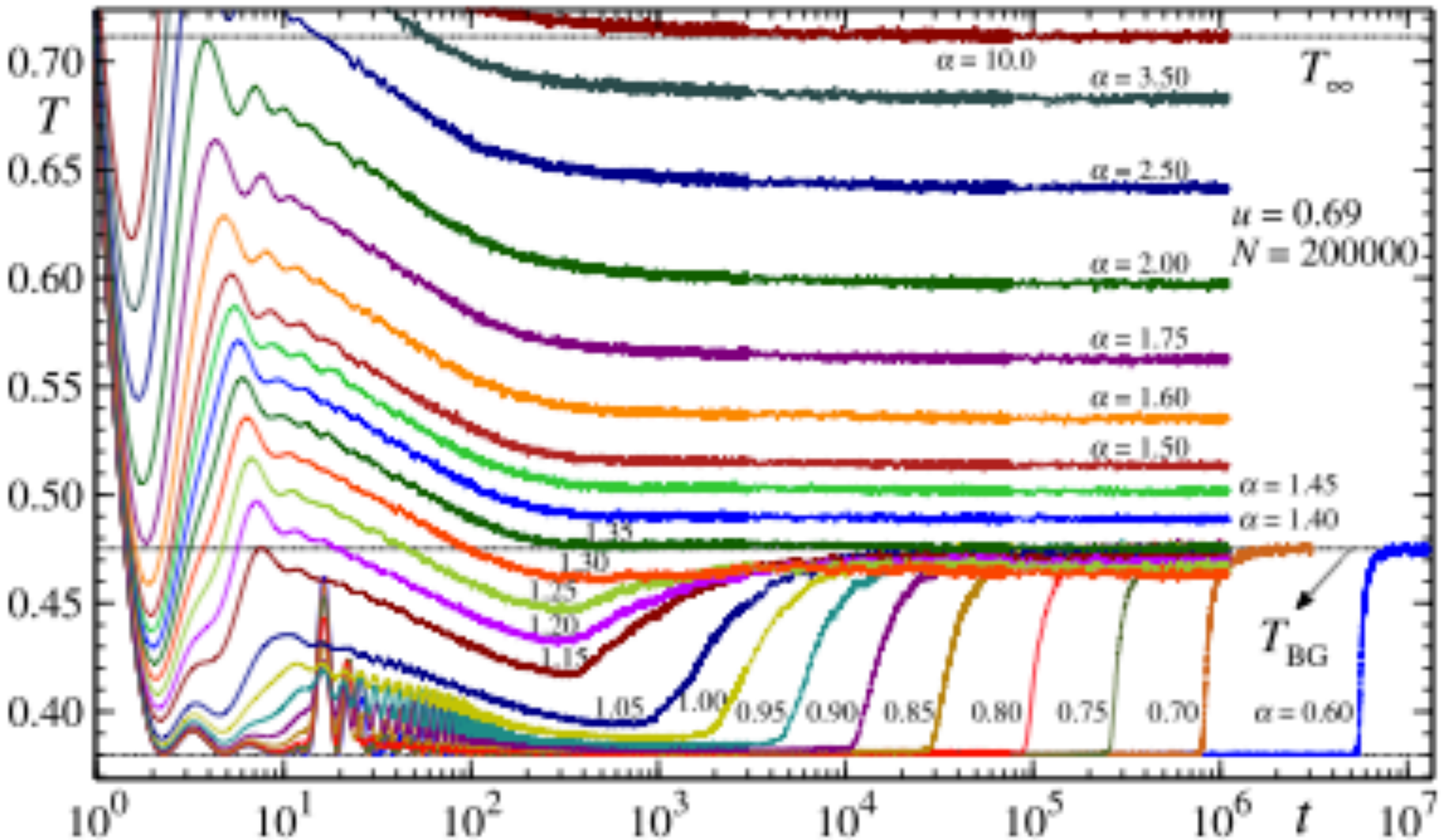
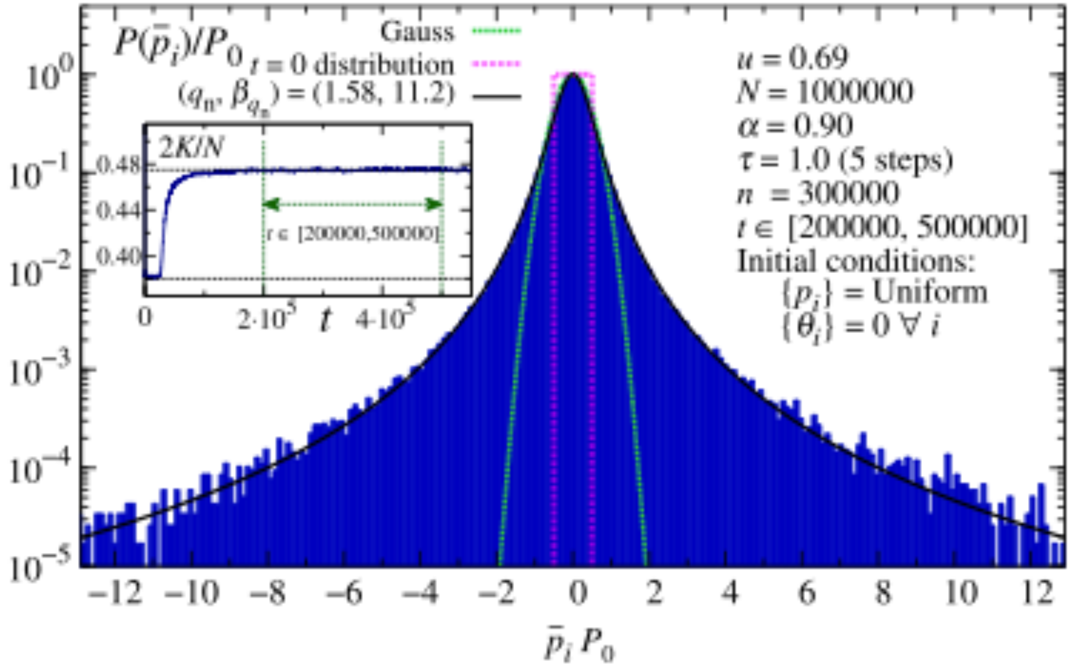
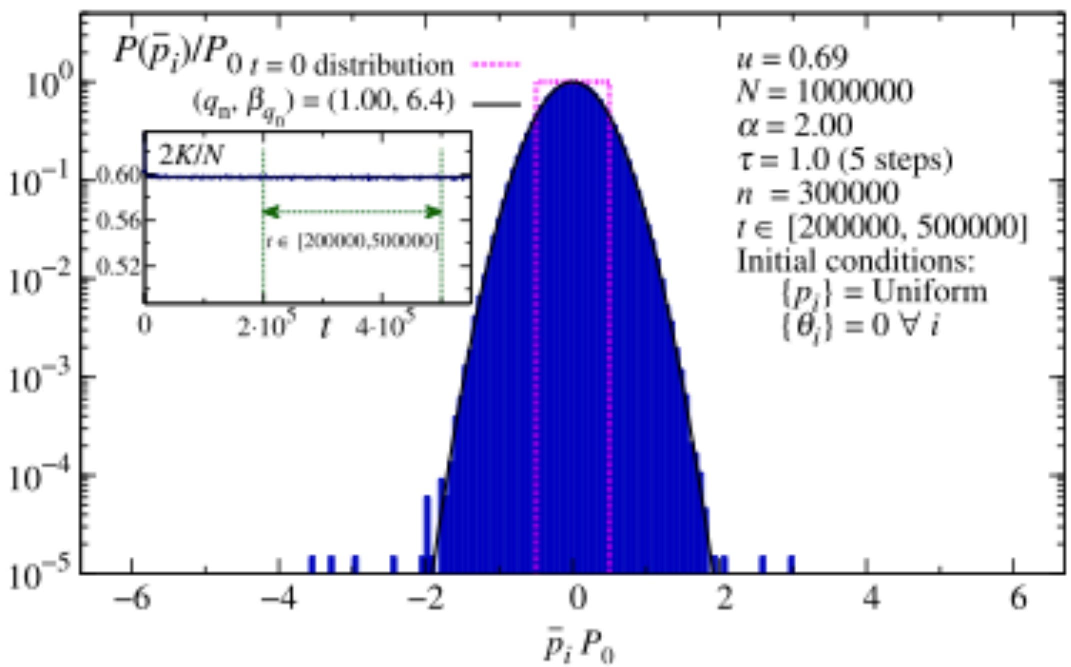


L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)



$\alpha = 0.9$

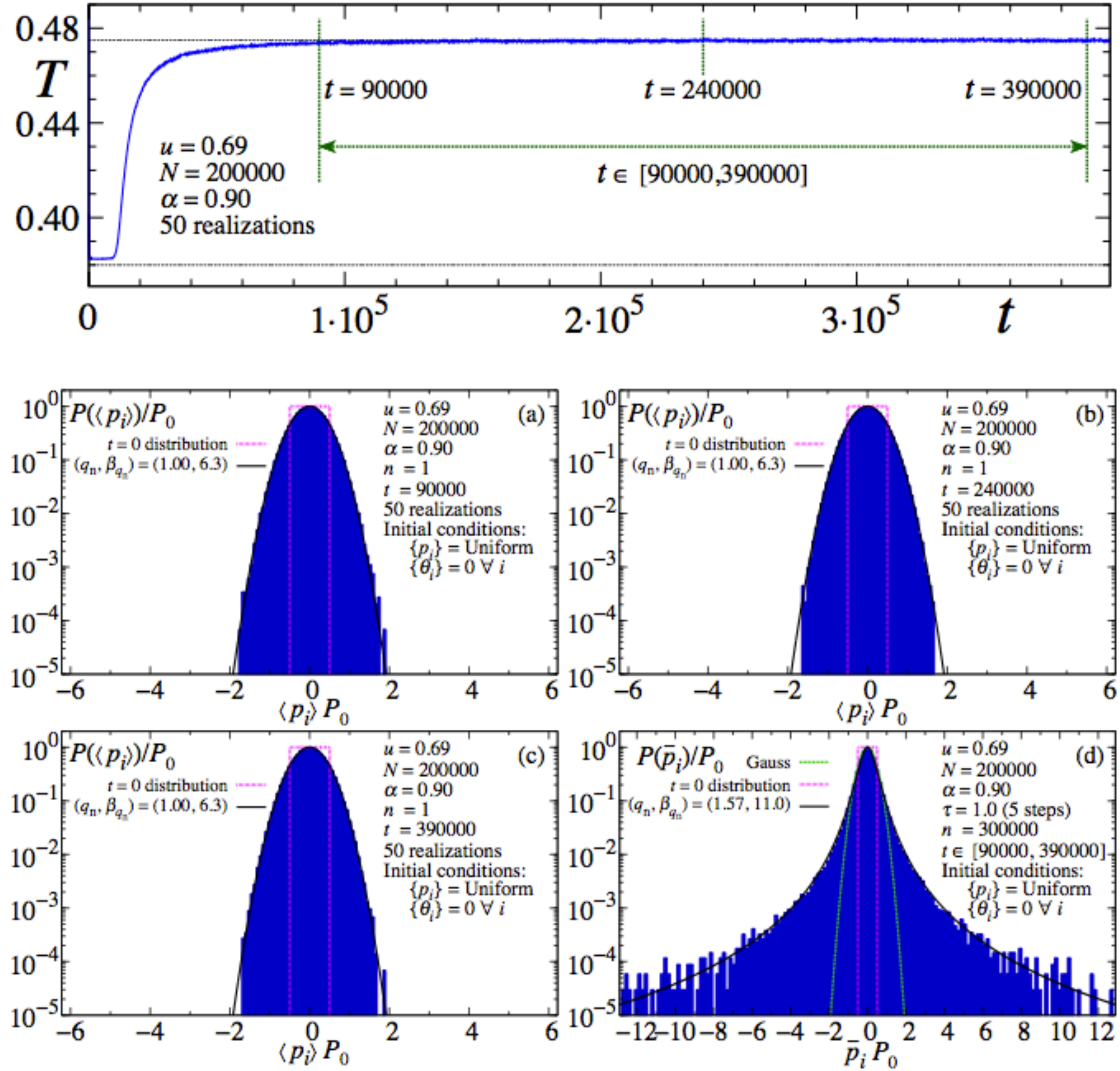
$q = 1.58$



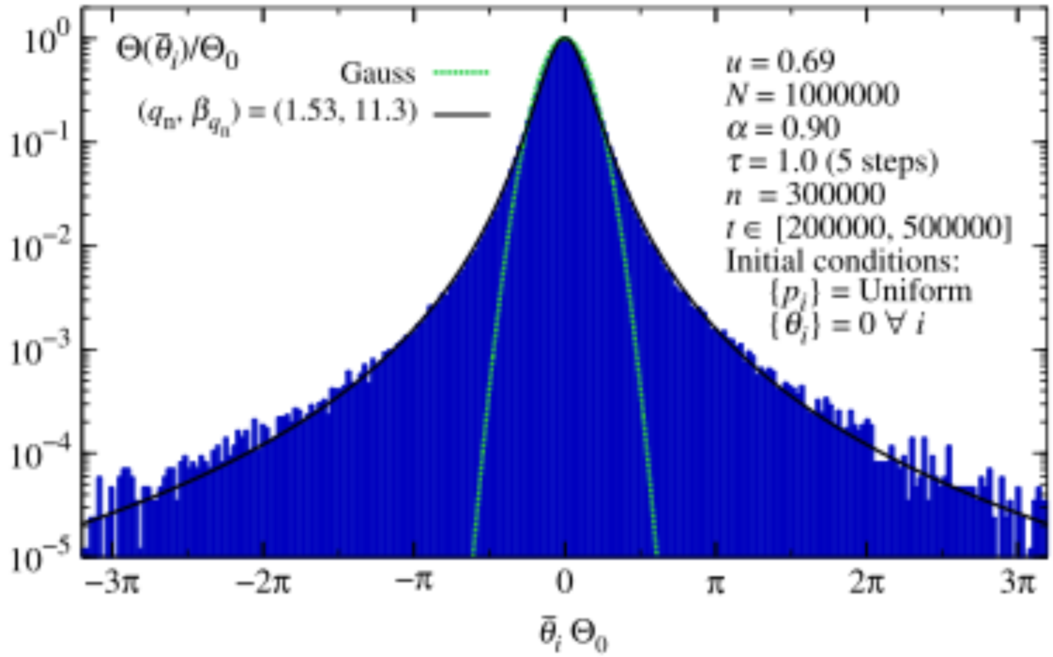
$\alpha = 2$

$q = 1$

L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)



L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)



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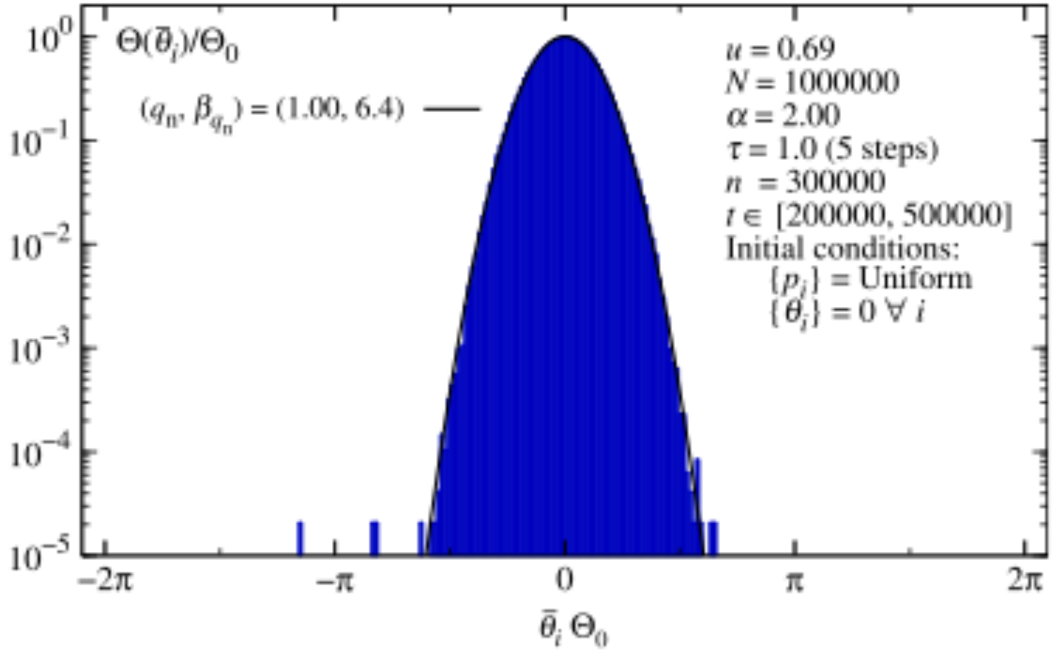
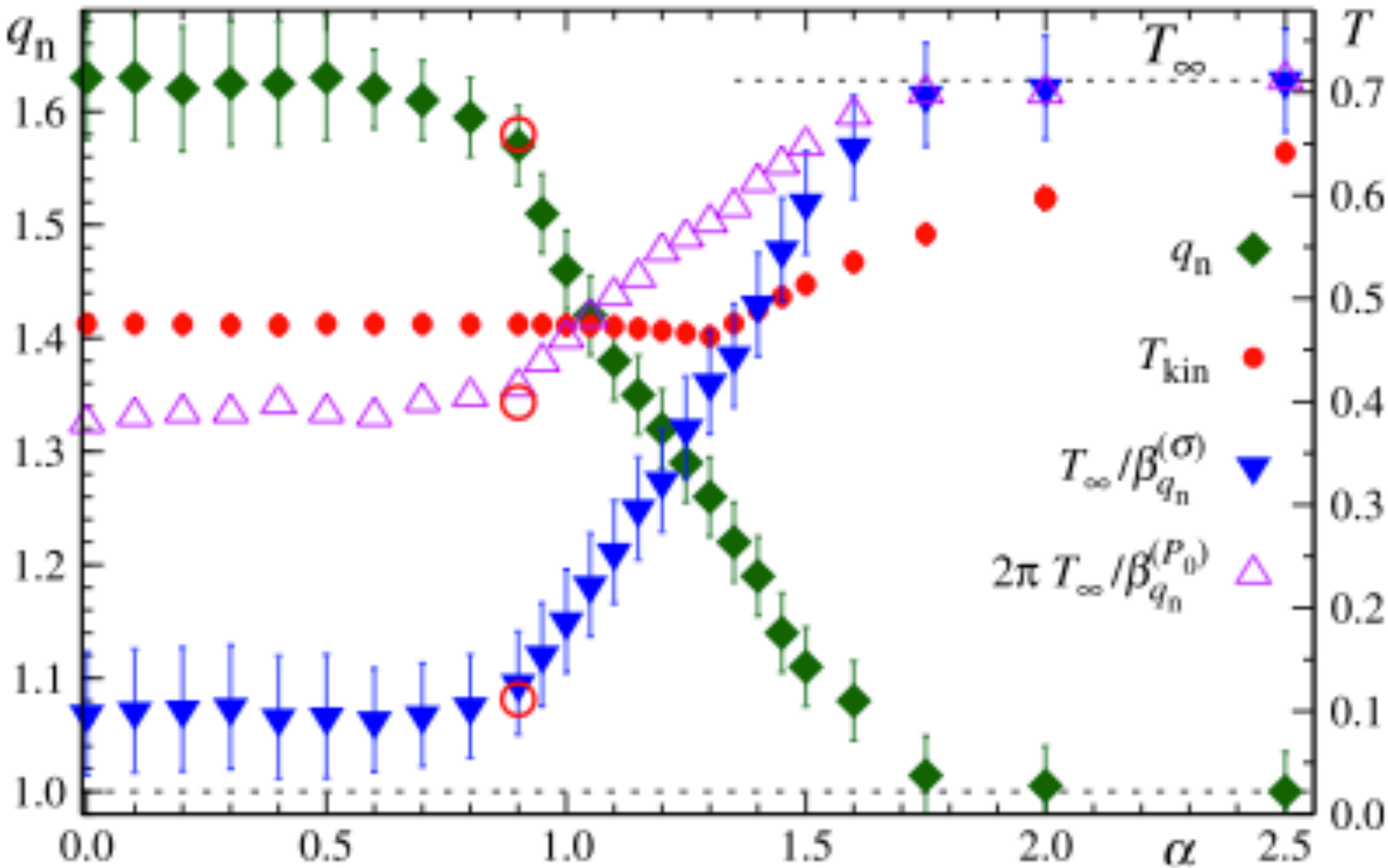


$\alpha = 2$

$q = 1$

L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)

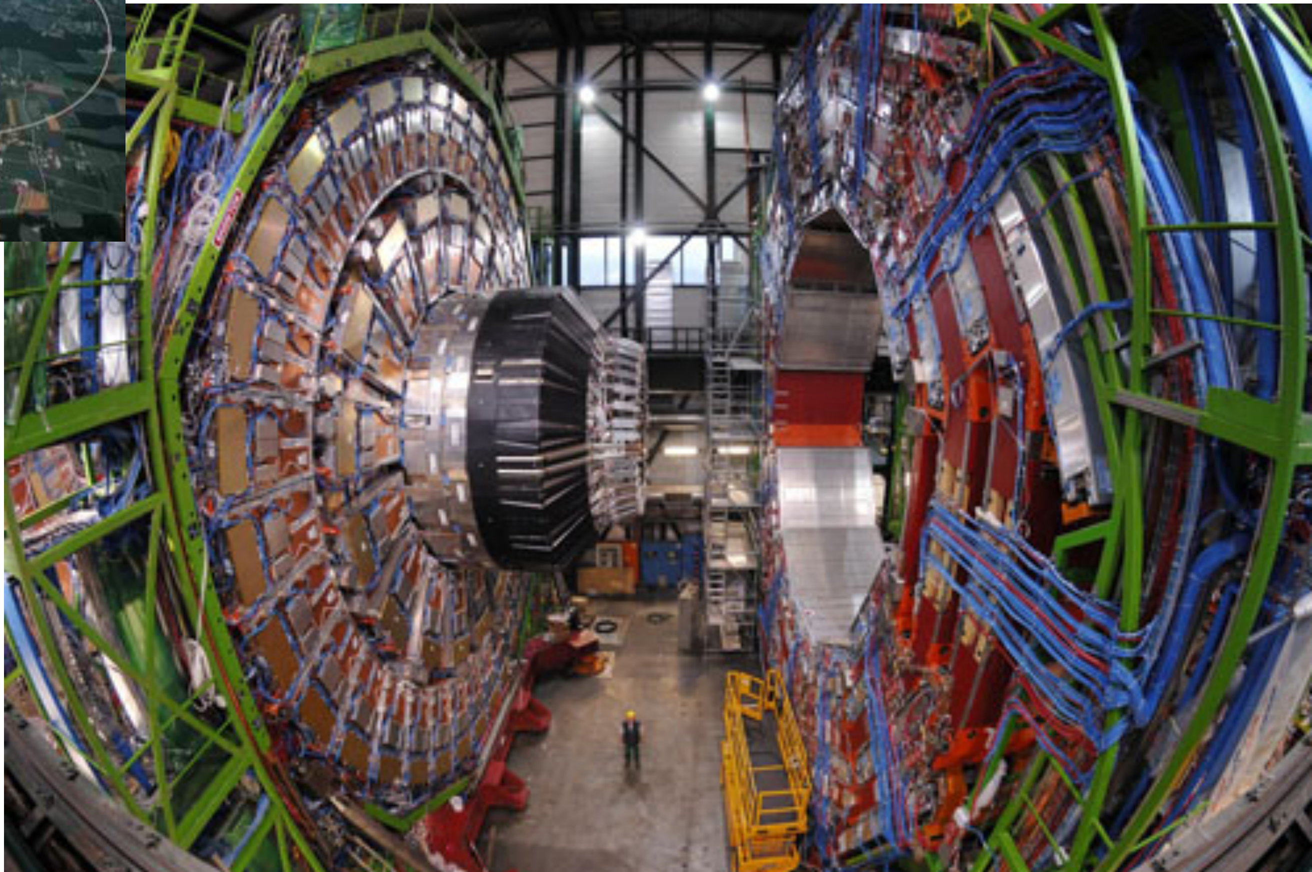


L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)

LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

~ 2500 scientists/engineers from 183 institutions of 38 countries



PHYSICAL REVIEW D **87,** 114007 (2013)

Tsallis fits to p_T spectra and multiple hard scattering in pp collisions at the LHC

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Grzegorz Wilk
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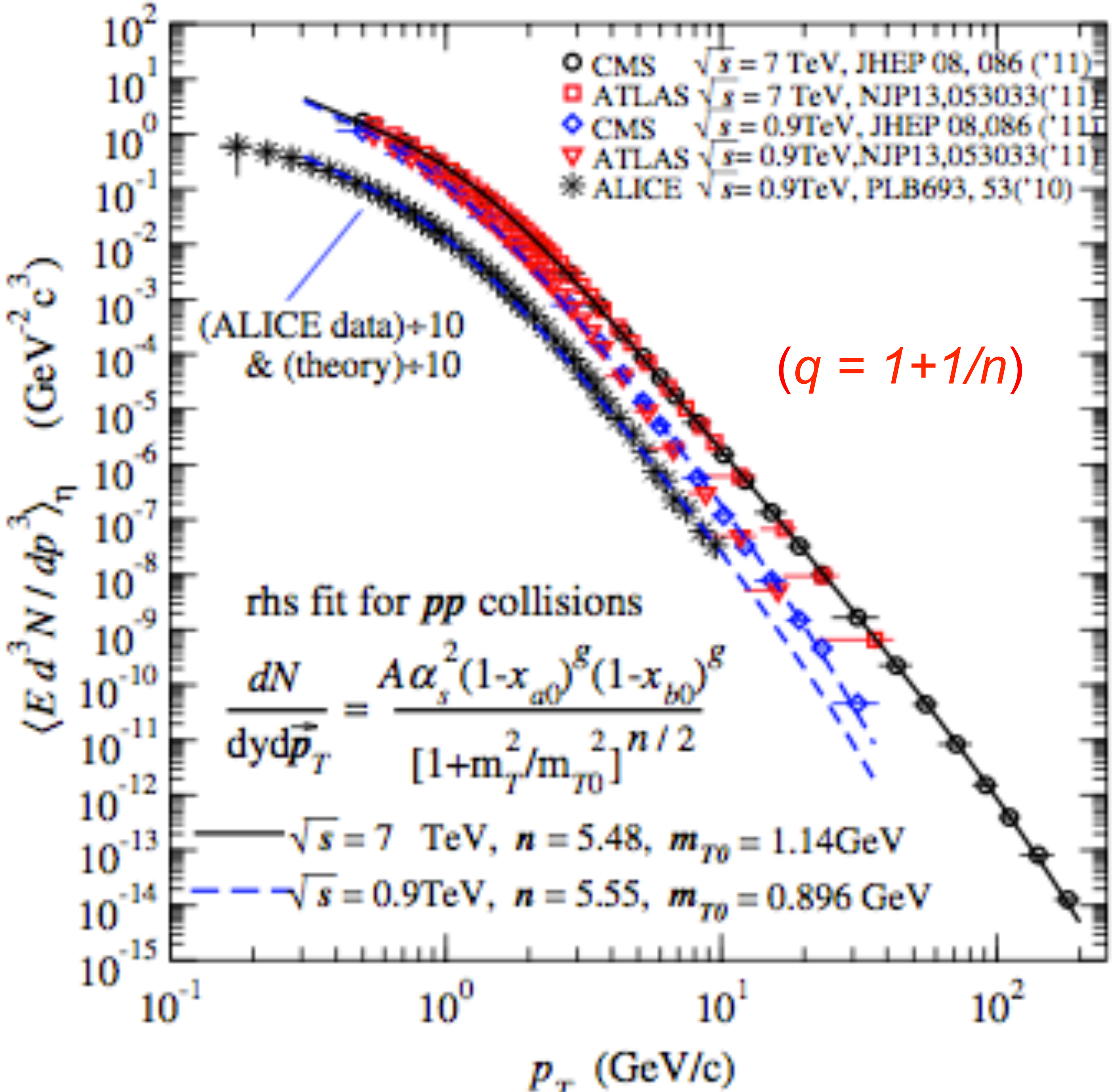
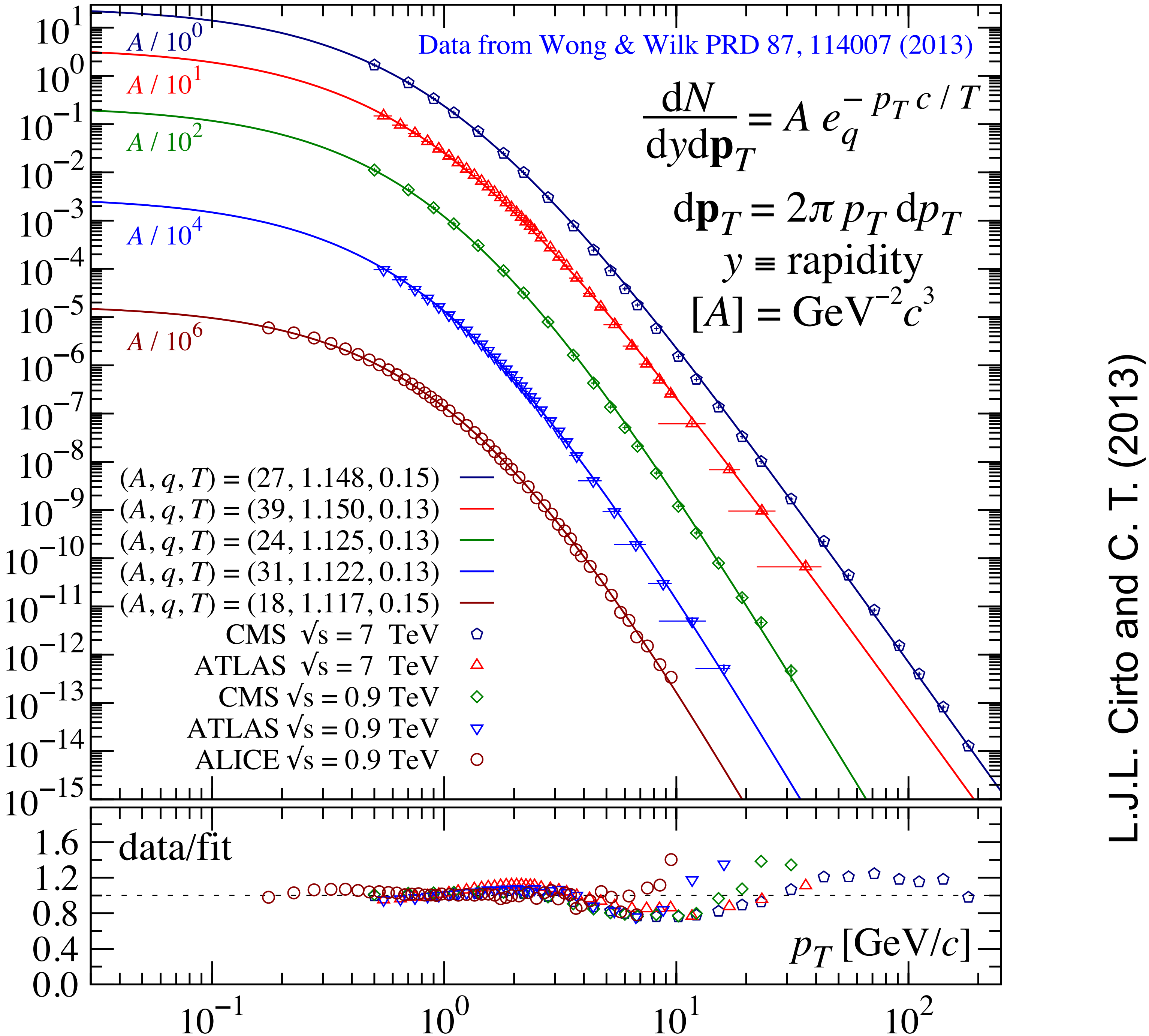
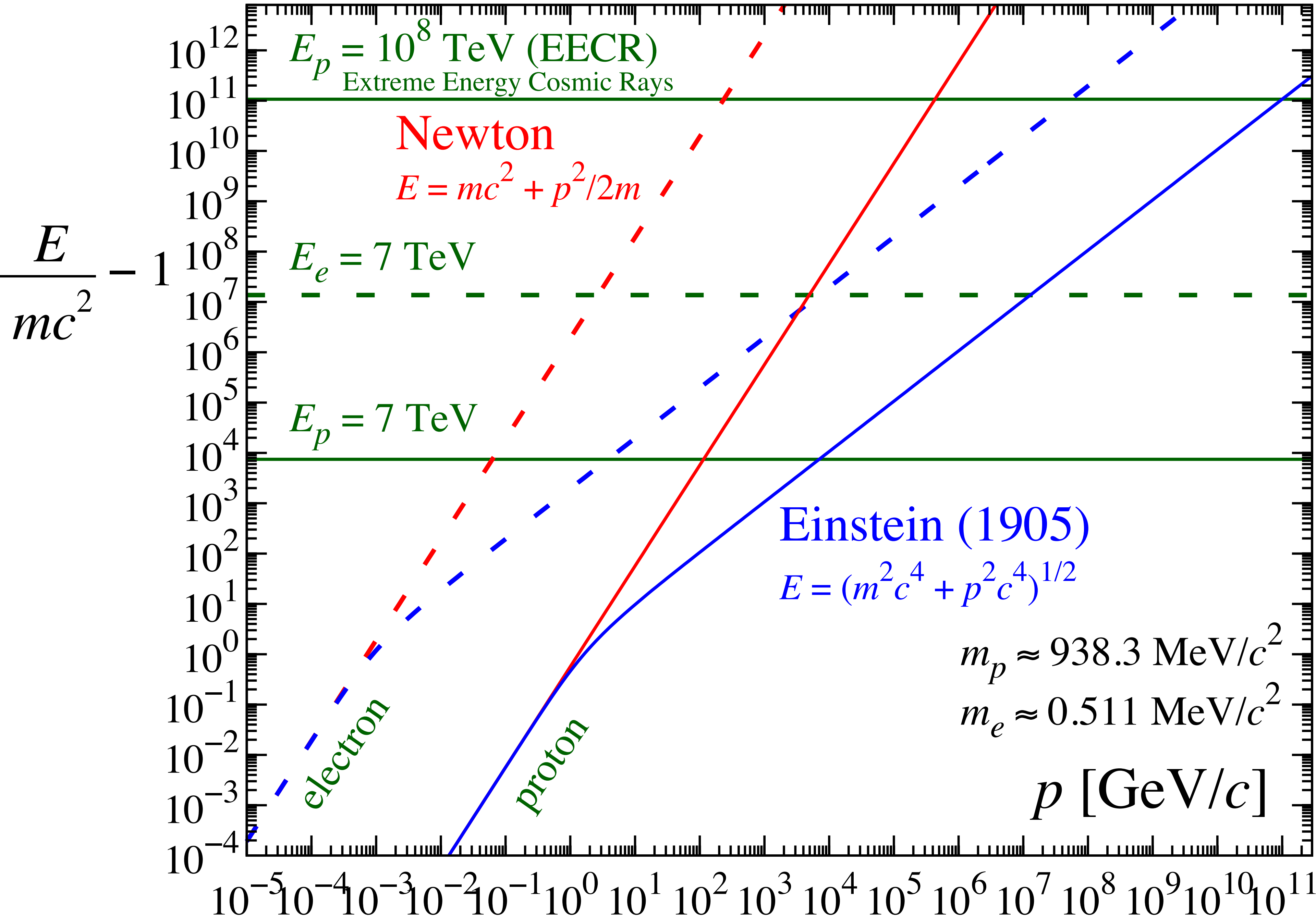
(Received 12 May 2013; published 5 June 2013)

Phenomenological Tsallis fits to the CMS, ATLAS, and ALICE transverse momentum spectra of hadrons for pp collisions at LHC were recently found to extend over a large range of the transverse momentum. We investigate whether the few degrees of freedom in the Tsallis parametrization may arise from the relativistic parton-parton hard-scattering and related processes. The effects of the multiple hard-scattering and parton showering processes on the power law are discussed. We find empirically that whereas the transverse spectra of both hadrons and jets exhibit power-law behavior of $1/p_T^n$ at high p_T, the power indices n for hadrons are systematically greater than those for jets, for which $n \sim 4$–5.



CMS √s = 7 TeV, JHEP 08, 086 ('11)
ATLAS √s = 7 TeV, NJP13,053033('11)
CMS √s = 0.9TeV, JHEP 08,086 ('11)
ATLAS √s= 0.9TeV,NJP13,053033('11)
ALICE √s= 0.9TeV, PLB693, 53('10)
(ALICE data)÷10
& (theory)÷10
(q = 1+1/n)
rhs fit for pp collisions
$\frac{dN}{dyd\vec{p}_T} = \frac{A\alpha_s^2(1-x_{a0})^g(1-x_{b0})^g}{[1+m_T^2/m_{T0}^2]^{n/2}}$
$\sqrt{s} = 7$ TeV, $n = 5.48$, $m_{T0} = 1.14$GeV
$\sqrt{s} = 0.9$TeV, $n = 5.55$, $m_{T0} = 0.896$ GeV
$\langle E\, d^3N/dp^3 \rangle_\eta$ (GeV^{-2}c^3)
p_T (GeV/c)


Data from Wong & Wilk PRD 87, 114007 (2013)
dN/dydp_T = A e_q^{− p_T c / T}
dp_T = 2π p_T dp_T
y ≡ rapidity
[A] = GeV^{-2} c^3
A / 10^0
A / 10^1
A / 10^2
A / 10^4
A / 10^6
(A, q, T) = (27, 1.148, 0.15)
(A, q, T) = (39, 1.150, 0.13)
(A, q, T) = (24, 1.125, 0.13)
(A, q, T) = (31, 1.122, 0.13)
(A, q, T) = (18, 1.117, 0.15)
CMS √s = 7 TeV
ATLAS √s = 7 TeV
CMS √s = 0.9 TeV
ATLAS √s = 0.9 TeV
ALICE √s = 0.9 TeV
data/fit
p_T [GeV/c]
L.J.L. Cirto and C. T. (2013)



$E_p = 10^8$ TeV (EECR)
Extreme Energy Cosmic Rays
Newton
$E = mc^2 + p^2/2m$
$\frac{E}{mc^2} - 1$
$E_e = 7$ TeV
$E_p = 7$ TeV
Einstein (1905)
$E = (m^2c^4 + p^2c^4)^{1/2}$
$m_p \approx 938.3$ MeV/c^2
$m_e \approx 0.511$ MeV/c^2
electron
proton
p [GeV/c]
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12}
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11}

PHYSICAL REVIEW D VOLUME 13, NUMBER 2 15 JANUARY 1976

Black holes and thermodynamics*

S. W. Hawking †
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(Received 30 June 1975)

A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

When entropy does not seem extensive

Earlier speculations about the entropy of black holes has prompted an ingenious calculation suggesting that entropy may (in special circumstances) be the same inside and outside an arbitrary boundary.

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. That, of course, is why the entropy of some substance will be quoted as so much per gram, or mole. If you then take two grams, or two moles, of the same material under the same conditions, the entropy will be twice as much. And there should be no confusion about the units; the simple Carnot definition of a change of entropy in a reversible process is the heat transfer divided by the absolute temperature, so that the units of entropy are simply those of energy divided by temperature, joules per degree (kelvin) in the SI system. The definitions of the Gibbs and Helmholtz free energies would be dimensionally discordant for that reason were it not that entropy (S) always turns up multiplied by temperature T. So much will readily be agreed.

Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? By the number of ways in which the constituents of some material (the atoms and molecules) can be rearranged without changing its properties and without energetic consequences. But now there comes a snag.

Like any extensive property, the combined entropy of two separate chunks of material should be the *sum* of the two entropies, but the number of rearrangements of the combined system must be the *product* of the numbers of ways in which the two parts separately can be rearranged. How to reconcile that with extensivity? By supposing entropy is proportional not to the number of rearrangements (technically called 'complexions'), but with the logarithm thereof. And because entropy decreases as disorder increases, the constant of proportionality must be a negative (real) number.

From that it follows that $S = S_0 - K\log N$, where K is a positive constant with the dimensions of entropy, N is a number (without dimensions) measuring disorder and S_0 is an arbitrary constant entropy. All that is simply a precis of the standard introductory chapter in statistical mechanics textbooks, most of which go on to show how to calculate the properties of assemblages of, say, diatomic molecules from a knowledge of their individual behaviour. Because the number of complexions of a particular state of an assemblage is invariably a function of the number (n) of molecules it contains, usually in the form of $n!$, because n is usually large and because $\log(n!)$ can then be approximated by $n\log n$, the extensive property of entropy then follows simply from the appearance of the leading factor n: entropy is proportional to the number of molecules.

That is what the textbooks say. It also makes sense of what is known of the thermodynamics of the real world. In a sample of a diatomic gas, for example, there are vibrations (one) and rotations (two) as well as three rectilinear degrees of freedom. But the problem is to tell how the energy available is distributed among the different degrees of freedom. The arithmetic simplifies marvellously because (in this case) each molecule and each of its degrees of freedom is independent. The best measure of disorder works out at $N = Z^n$, where n is the number of molecules, and where Z, which must be a […]

A bit of quantum mechanics goes into the argument as well, notably the notion of the density matrix — an artificially constructed operator (on quantum states) that is well suited to the discussion of systems in which one part (say the black hole) is singled out for attention while the remainder (the Universe outside it) is dealt with in less detail, perhaps because some averaging process is appropriate, or because the whole problem may not be calculable at all. (In Dirac's notation, the density matrix corresponding to some state of the whole Universe would be represented as $|1><1|$, where "1" is simply the name for a particular state of the Universe.) What matters, where entropy is concerned, is that the density matrix, like all matrices, has eigenvalues from which the entropy can be calculated.

So imagine that the Universe is partitioned into two parts by means of a closed boundary of some kind and filled with a […]

[…] dealt with explicitly, as other entropy calculations are made. And that could be exceedingly important.

John Maddox

Tackled by
- Jacob D. Bekenstein
- Stephen W. Hawking
- Gary W. Gibbons
- Gerard 't Hooft
- Leonard Susskind
- Michael J. Duff
- Juan M. Maldacena
- Thanu Padmanabhan
- Robert M. Wald

and many others

When entropy does not seem extensive
John Maddox, Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

RAPID COMM

PHYSICAL REVIEW D **73,** 121701(R) (2006)

How robust is the entanglement entropy-area relation?

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(Received 30 November 2005; revised manuscript received 24 May 2006; published 28 June 2006)

We revisit the problem of finding the entanglement entropy of a scalar field on a lattice by tracing over its degrees of freedom inside a sphere. It is known that this entropy satisfies the area law—entropy proportional to the area of the sphere—when the field is assumed to be in its ground state. We show that the area law continues to hold when the scalar field degrees of freedom are in generic coherent states and a class of squeezed states. However, when excited states are considered, the entropy scales as a lower power of the area. This suggests that, for large horizons, the ground state entropy dominates, whereas entropy due to excited states gives power-law corrections. We discuss possible implications of this result to black hole entropy.

The area (as opposed to volume) proportionality of BH entropy has been an intriguing issue for decades.

PHYSICAL REVIEW D **83,** 064034 (2011)

Ideal gas in a strong gravitational field: Area dependence of entropy

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(Received 24 January 2011; published 24 March 2011)

We study the thermodynamic parameters like entropy, energy etc. of a box of gas made up of indistinguishable particles when the box is kept in various static background spacetimes having a horizon. We compute the thermodynamic variables using both statistical mechanics as well as by solving the hydrodynamical equations for the system. When the box is far away from the horizon, the entropy of the gas depends on the volume of the box except for small corrections due to background geometry. As the box is moved closer to the horizon with one (leading) edge of the box at about Planck length (L_p) away from the horizon, the entropy shows an area dependence rather than a volume dependence. More precisely, it depends on a small volume $A_\perp L_p/2$ of the box, up to an order $\mathcal{O}(L_p/K)^2$ where $A_\perp$ is the transverse area of the box and K is the (proper) longitudinal size of the box related to the distance between leading and trailing edge in the vertical direction (i.e. in the direction of the gravitational field). Thus the contribution to the entropy comes from only a fraction $\mathcal{O}(L_p/K)$ of the matter degrees of freedom and the rest are suppressed when the box approaches the horizon. Near the horizon all the thermodynamical quantities behave as though the box of gas has a volume $A_\perp L_p/2$ and is kept in a Minkowski spacetime. These effects are: (i) purely kinematic in their origin and are independent of the spacetime curvature (in the sense that the Rindler approximation of the metric near the horizon can reproduce the results) and (ii) observer dependent. When the equilibrium temperature of the gas is taken to be equal to the horizon temperature, we get the familiar $A_\perp/L_p^2$ dependence in the expression for entropy. All these results hold in a $D+1$ dimensional spherically symmetric spacetime. The analysis based on methods of statistical mechanics and the one

lead to the same result

Thus the extensive property of entropy no longer holds and one can check that it does not hold even in the weak field limit discussed above when $L \gg \lambda$ that is, when gravitational effects subdue the thermal effects along the direction of the gravitational field.

SINCE THE PIONEERING BEKENSTEIN-HAWKING RESULTS, PHYSICALLY MEANINGFUL EVIDENCE HAS ACCUMULATED (e.g., HOLOGRAPHIC PRINCIPLE) WHICH MANDATES THAT

$$\ln W_{black\ hole} \propto \text{AREA}$$

THIS IS PERFECTLY ADMISSIBLE AND MOST PROBABLY CORRECT.

HOWEVER,

IS THIS QUANTITY THE THERMODYNAMICAL ENTROPY???

Eur. Phys. J. C (2013) 73:2487
DOI 10.1140/epjc/s10052-013-2487-6

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Black hole thermodynamical entropy

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[2]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

Abstract As early as 1902, Gibbs pointed out that systems whose partition function diverges, e.g. gravitation, lie outside the validity of the Boltzmann–Gibbs (BG) theory. Consistently, since the pioneering Bekenstein–Hawking results, physically meaningful evidence (e.g., the holographic principle) has accumulated that the BG entropy S_{BG} of a $(3+1)$ black hole is proportional to its area L^2 (L being a characteristic linear length), and not to its volume L^3. Similarly it exists the *area law*, so named because, for a wide class of strongly quantum-entangled d-dimensional systems, S_{BG} is proportional to $\ln L$ if $d=1$, and to L^{d-1} if $d>1$, instead of being proportional to L^d $(d \geq 1)$. These results violate the extensivity of the thermodynamical entropy of a d-dimensional system. This thermodynamical inconsistency disappears if we realize that the thermodynamical entropy of such nonstandard systems is *not* to be identified with the BG *additive* entropy but with appropriately generalized *nonadditive* entropies. Indeed, the celebrated usefulness of the BG entropy is founded on hypothesis such as relatively weak probabilistic correlations (and their connections to ergodicity, which by no means can be assumed as a general rule of nature). Here we introduce a generalized entropy which, for the Schwarzschild black hole and the area law, can solve the thermodynamic puzzle.

ENTROPIES

$$S_{BG} = k_B \sum_{i=1}^{W} p_i \ln \frac{1}{p_i} \quad \rightarrow \textit{additive}$$

$$S_q = k_B \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i} \quad (S_1 = S_{BG}) \quad \rightarrow \textit{nonadditive} \text{ if } q \neq 1$$

C. T. (1988)

$$S_\delta = k_B \sum_{i=1}^{W} p_i \left(\ln \frac{1}{p_i} \right)^\delta \quad (S_1 = S_{BG}) \quad \rightarrow \textit{nonadditive} \text{ if } \delta \neq 1$$

C. T. (2009)

$$S_{q,\delta} = k_B \sum_{i=1}^{W} p_i \left(\ln_q \frac{1}{p_i} \right)^\delta \quad (S_{q,1} = S_q;\ S_{1,\delta} = S_\delta;\ S_{1,1} = S_{BG})$$

C. T. (2011)

$$\rightarrow \textit{nonadditive} \text{ if } (q,\delta) \neq (1,1)$$

C. T. and L.J.L. Cirto, Eur Phys J C 73, 2487 (2013)

Various arguments (phenomenological, holographic principle, string theory, area law, etc) yield

$$S_{BG}(L) \equiv k_B \ln W(L) \propto L^{d-1} \quad (d>1)$$

hence

$$W(L) \propto \Phi(L)\, \nu^{L^{d-1}} \quad \left(\text{with } \lim_{L\to\infty} \frac{\ln \Phi(L)}{L^{d-1}} = 0; \text{ e.g., } \Phi(L) \propto L^{\rho} \right)$$

hence, for $d>1$, the entropy which is extensive is S_δ with $\delta = \dfrac{d}{d-1}$

i.e.,

$$S_{\delta=d/(d-1)}(L) = k_B \sum_{i=1}^{W(L)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{d}{d-1}} \propto L^d \quad (d>1)$$

Consequently

$$S_{\delta=3/2}^{black\ hole}(L) = k_B \sum_{i=1}^{W(N)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{3}{2}} \propto L^3 \quad !!!$$

C.T. and L.J.L. Cirto, Eur. Phys. J. C **73**, 2487 (2013)

CONSEQUENCE (for the equal-probability case):

$$\frac{S_{\delta=3/2}^{black\ hole}}{k_B} \propto \left(\frac{S_{Bekenstein-Hawking}}{k_B} \right)^{3/2}$$

with

$$S_{Bekenstein-Hawking} = \frac{k_B}{4} \frac{A_H}{G\hbar / c^3} = \frac{k_B \pi}{2} \frac{A_H}{(L_{Planck})^2}$$

$A_H \equiv$ event horizon area

$L_{Planck} \equiv$ Planck length

C.T. and L.J.L. Cirto, Eur. Phys. J. C **73**, 2487 (2013)

SYSTEMS $W(N)$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$\sim \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$\sim N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE	NONEXTENSIVE
$\sim \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE

King Thutmosis I
18th Dynasty
circa 1500 BC

Entropic cosmology for a generalized black-hole entropy

Nobuyoshi KOMATSU[1*] and Shigeo KIMURA[2]
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[2] *The Institute of Nature and Environmental Technology, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan*
(Dated: July 24, 2013)

An entopic-force scenario, i.e., entropic cosmology, assumes that the horizon of the universe has an entropy and a temperature. In the present study, in order to examine entropic cosmology, we derive entropic-force terms not only from the Bekenstein entropy but also from a generalized black-hole entropy proposed by C. Tsallis and L.J.L. Cirto [Eur. Phys. J. C **73**, 2487 (2013)]. Unlike the Bekenstein entropy, which is proportional to area, the generalized entropy is proportional to volume because of appropriate nonadditive generalizations. The entropic-force term derived from the generalized entropy is found to behave as if it were an extra driving term for bulk viscous cosmology, in which a bulk viscosity of cosmological fluids is assumed. Using an effective description similar to bulk viscous cosmology, we formulate the modified Friedmann, acceleration, and continuity equations for entropic cosmology. Based on this formulation, we propose two entropic-force models derived from the Bekenstein and generalized entropies. In order to examine the properties of the two models, we consider a homogeneous, isotropic, and spatially flat universe, focusing on a single-fluid-dominated universe. The two entropic-force models agree well with the observed supernova data. Interestingly, the entropic-force model derived from the generalized entropy predicts a decelerating and accelerating universe, as for a fine-tuned standard ΛCDM (lambda cold dark matter) model, whereas the entropic-force model derived from the Bekenstein entropy predicts a uniformly accelerating universe.

arXiv:1307.5949v1 [astro-ph.CO] 23 Jul 2013

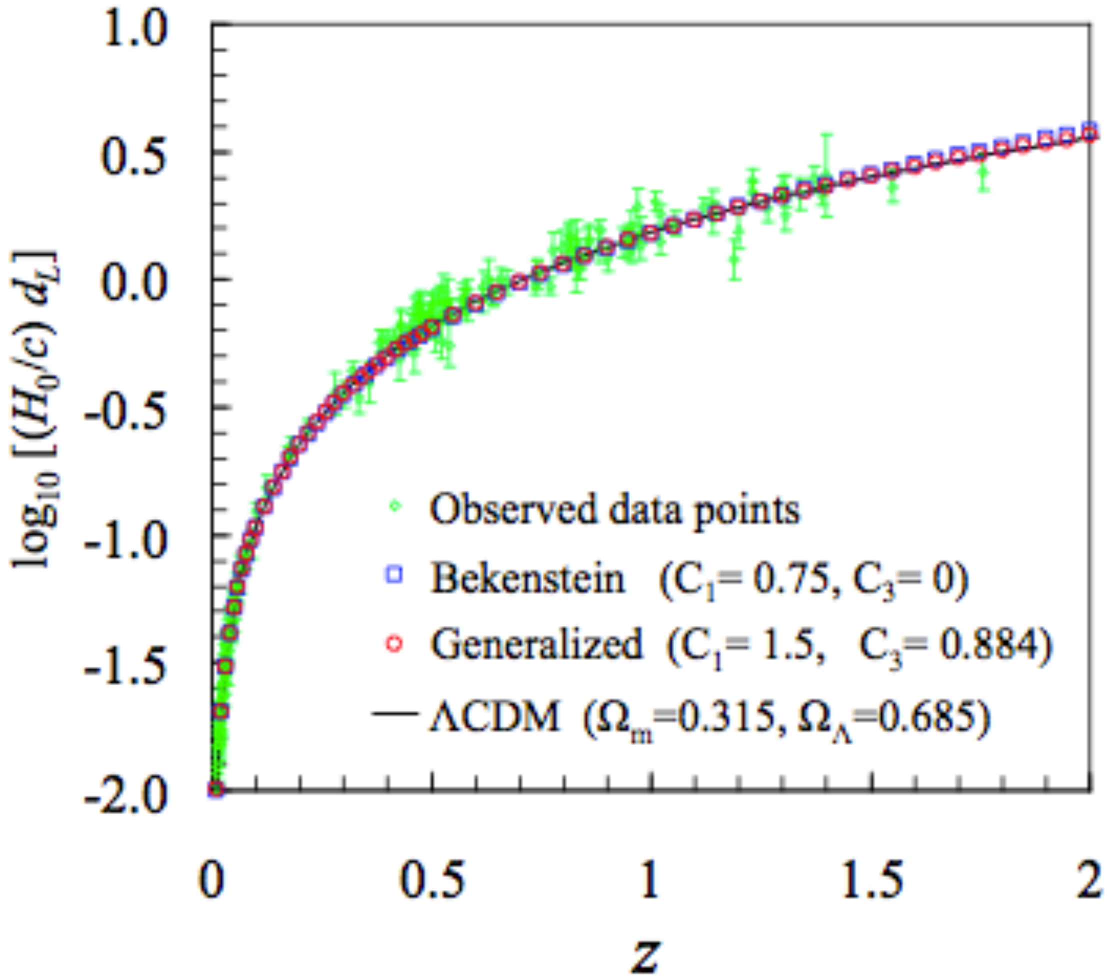


FIG. 1: (Color online). Dependence of luminosity distance d_L on redshift z. Here, Bekenstein and Generalized indicate the information for the entropic-force models derived from the Bekenstein and generalized entropies, respectively. The open diamonds with error bars are supernova data points taken from Ref. [3]. For supernova data points, H_0 is set to be 67.3 km/s/Mpc based on the Planck 2013 results [6].

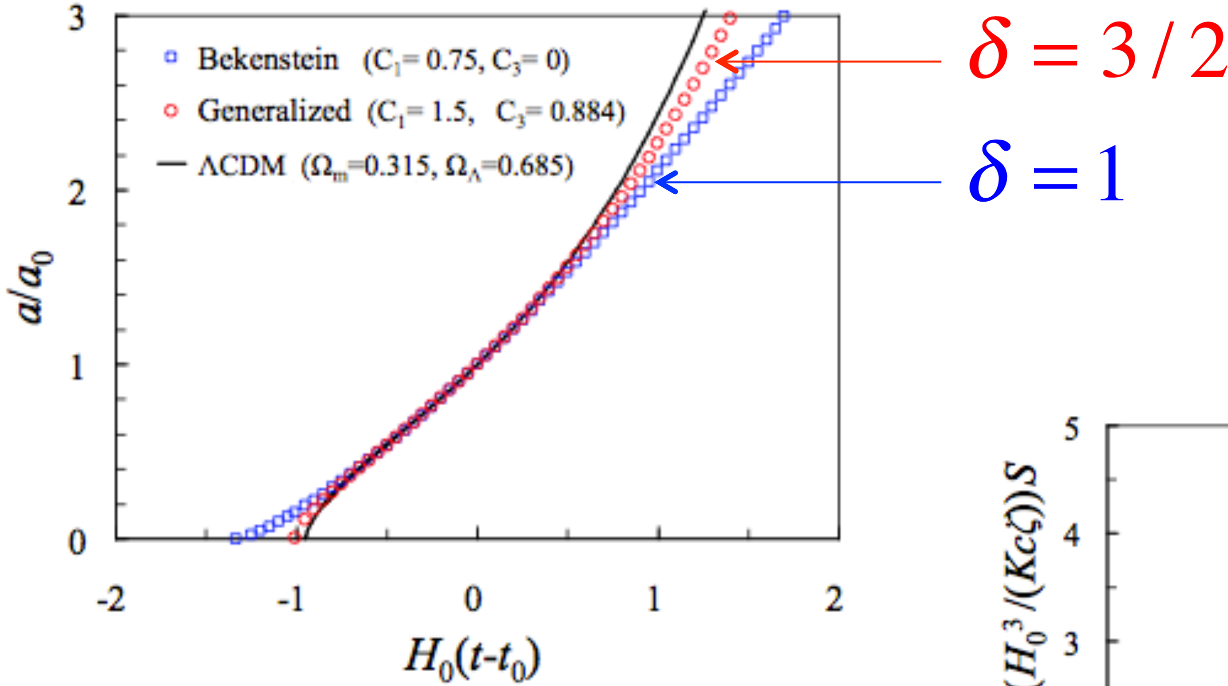


FIG. 2: (Color online). Time evolution of normalized scale factor a/a_0. The horizontal axis is normalized as $H_0(t-t_0)$. Here, Bekenstein and Generalized indicate the information for the entropic-force models derived from the Bekenstein and generalized entropies, respectively.

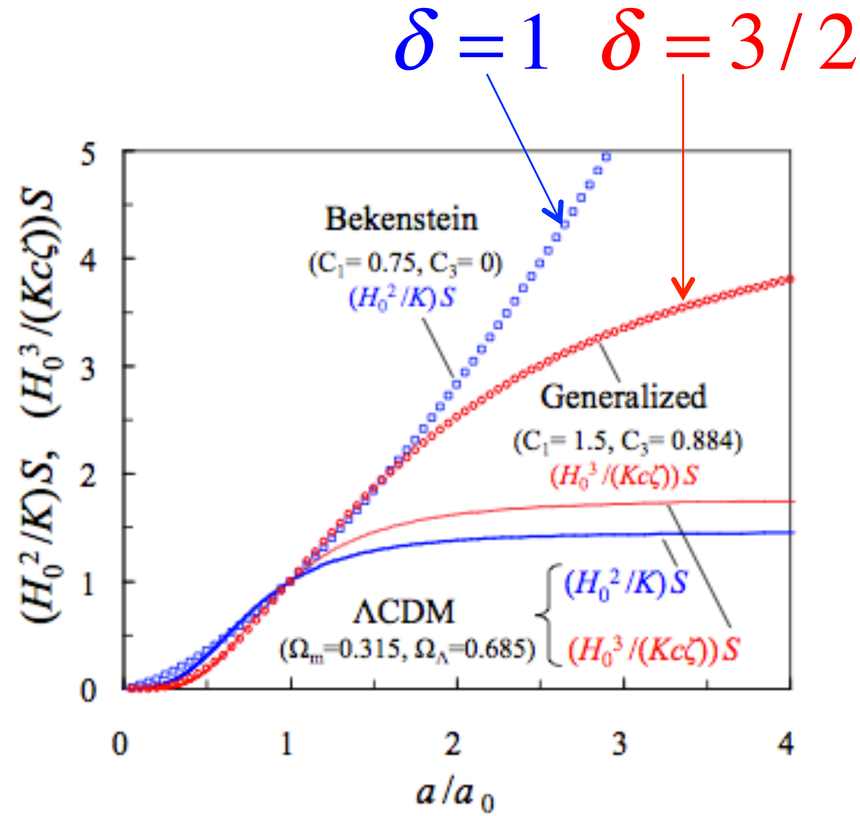
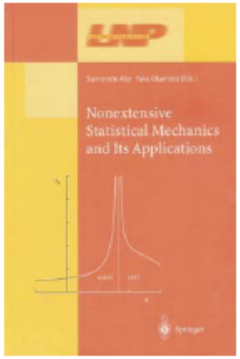


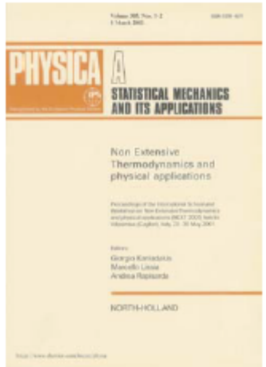
FIG. 3: (Color online). Evolutions of the Bekenstein and generalized entropies. The vertical axis represents $(H_0^2/K)S$ and $(H_0^3/(Kc\zeta))S$, for the Bekenstein and generalized entropic-force models, respectively. The solid lines represent $(H_0^2/K)S$ and $(H_0^3/(Kc\zeta))S$ for the fine-tuned standard ΛCDM model and are numerically calculated from $(H/H_0)^{-2}$ and $(H/H_0)^{-3}$.

BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

1999

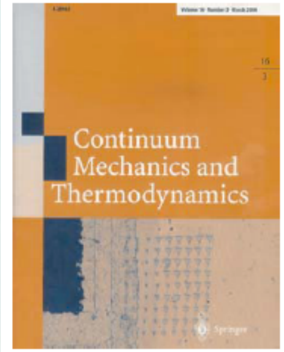


2001



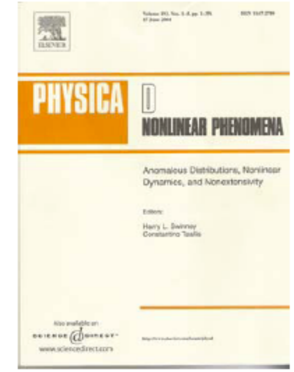
2002

2002



2004

2004



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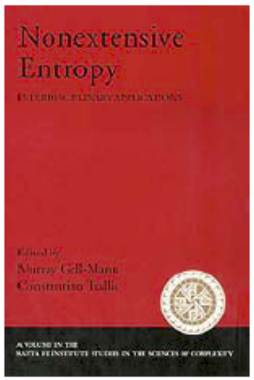

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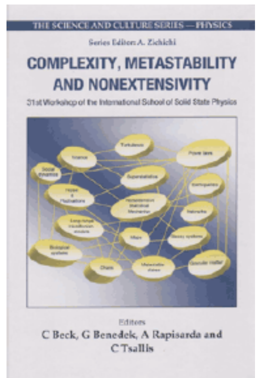

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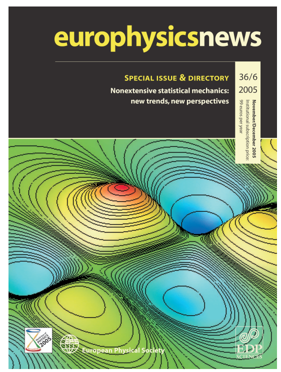

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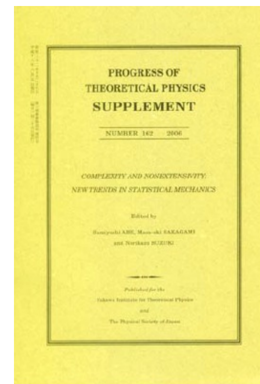

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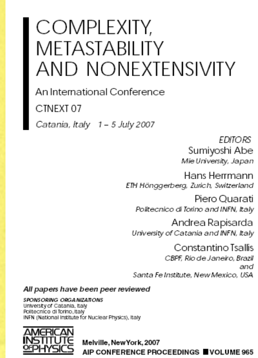

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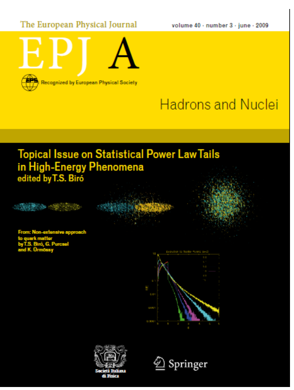

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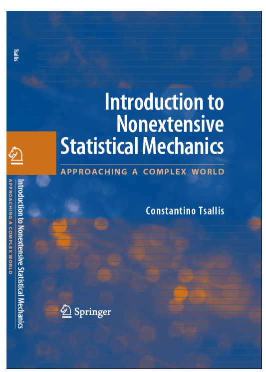

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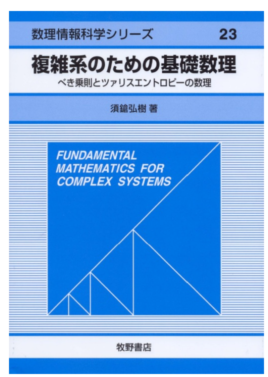

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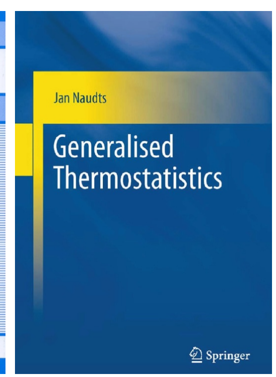
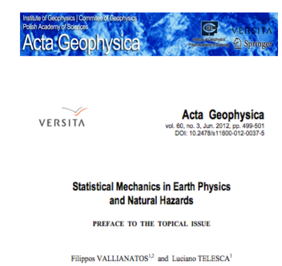


2011

2011



2012

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

4319 articles by 6137 scientists from 76 countries

[29 October 2013]

CONTRIBUTORS **(4319 MANUSCRIPTS)** [Updated 29 October 2013]

USA	1377	PORTUGAL	45	CYPRUS	11	BELARUS	3
BRAZIL	613	TAIWAN	43	NORWAY	11	PERU	3
ITALY	436	UKRAINE	41	VENEZUELA	11	KAZAKSTAN	2
CHINA	398	AUSTRALIA	40	SERBIA	11	MOLDOVA	2
FRANCE	324	MEXICO	40	PUERTO RICO	10	PHILIPINES	2
JAPAN	301	CZECK	37	SWEDEN	10	UNITED ARAB EMIRATES	2
GERMANY	300	ISRAEL	35	SLOVAK	8		
RUSSIA	216	FINLAND	24	BANGLADESH	7	ECUADOR	1
UNIT. KINGDOM	213	BELGIUM	22	CHILE	7	GEORGIA	1
SPAIN	206	NETHERLANDS	20	ESTONIA	7	INDONESIA	1
INDIA	204	BULGARIA	19	IRELAND	7	JORDAN	1
SWITZERLAND	189	PAKISTAN	17	NEW ZEALAND	7	QATAR	1
TURKEY	103	CROATIA	16	SAUDI ARABIA	7	SRI LANKA	1
ARGENTINA	101	ALGERIA	14	SINGAPORE	7	UZBEKISTAN	1
GREECE	96	COLOMBIA	14	ICELAND	5		
POLAND	90	EGYPT	14	ARMENIA	4		
SOUTH KOREA	85	SOUTH AFRICA	14	BOLIVIA	4		
CANADA	70	CUBA	12	MALAYSIA	4		
HUNGARY	70	DENMARK	12	SLOVENIA	4		
AUSTRIA	56	LITHUANIA	12	THAILAND	4		
IRAN	56	ROMENIA	12	URUGUAY	4		

76 COUNTRIES **6137 SCIENTISTS**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From the sandwiched quantum relative Tsallis entropy to its conditional form: Separability criterion beyond local and global spectra

A. K. Rajagopal,[1,2] Sudha,[3,1] Anantha S Nayak,[3] and A. R. Usha Devi[4,1]

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(Dated: September 27, 2013)

The quantum relative Rényi entropy of two density matrices was recently extended when the two do not commute, from which a conditional entropy is identified. This is here extended to the corresponding Tsallis relative entropy and to its conditional form. This new expression of Tsallis conditional entropy is shown to witness entanglement beyond the method based on global and local spectra of composite quantum states. When the reduced density matrix happens to be a maximally mixed state, this conditional entropy coincides with the expression in terms of Tsallis entropies derived earlier by Abe and Rajagopal (Physica A **289**, 157 (2001)). Separability range in one parameter family of W and GHZ states with 3 and 4 qubits is explored here and it is shown that the results inferred from negative Tsallis conditional entropy matches with that obtained through Peres' partial transpose criteria for one-parameter family of W states, in one of its partitions. The criteria is shown to be non-spectral through its usefulness in identifying entanglement in isospectral density matrices.

PACS numbers: 03.65.Ud, 03.67.-a

I. INTRODUCTION

earlier version of Tsallis conditional entropy, introdu

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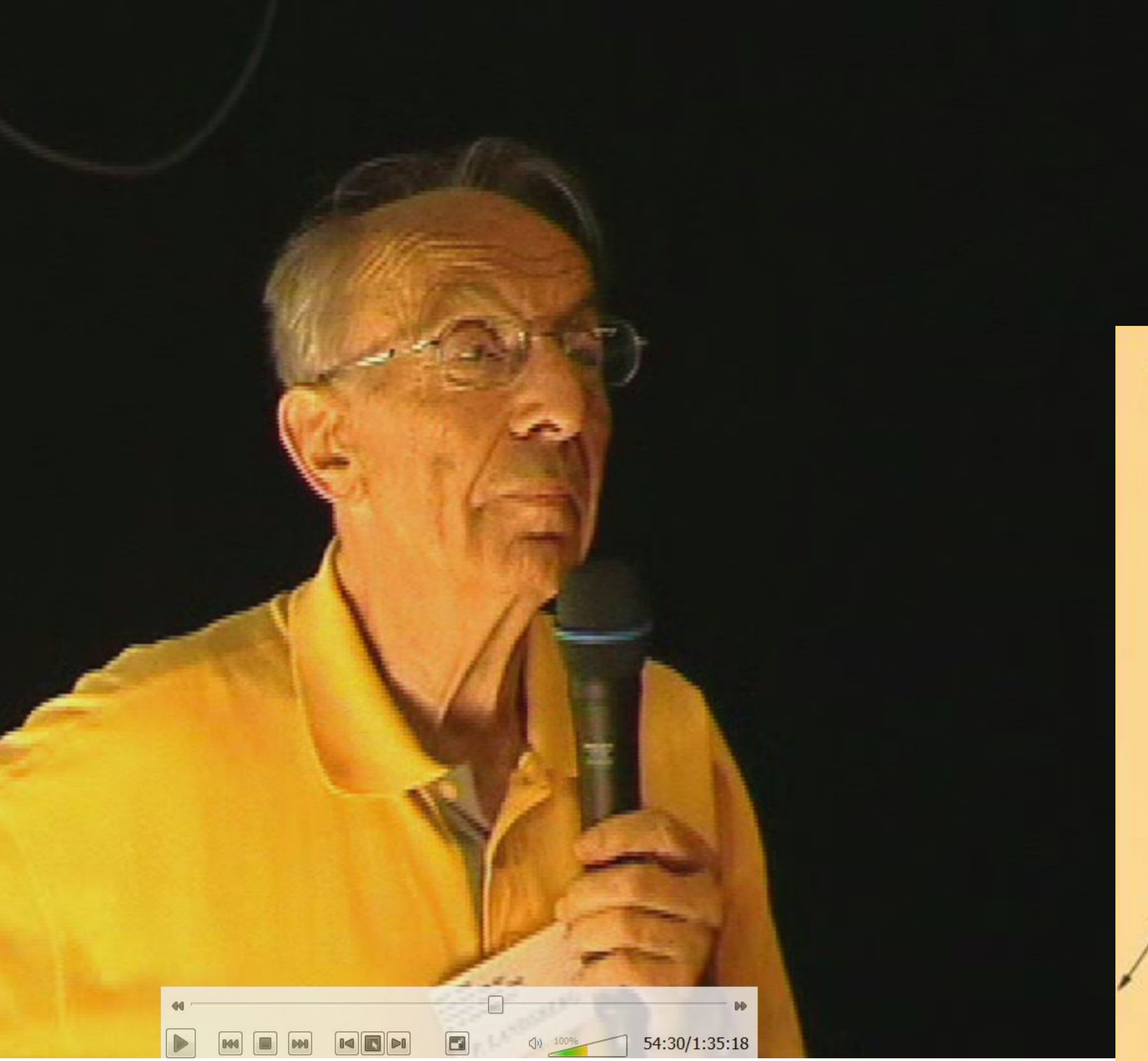

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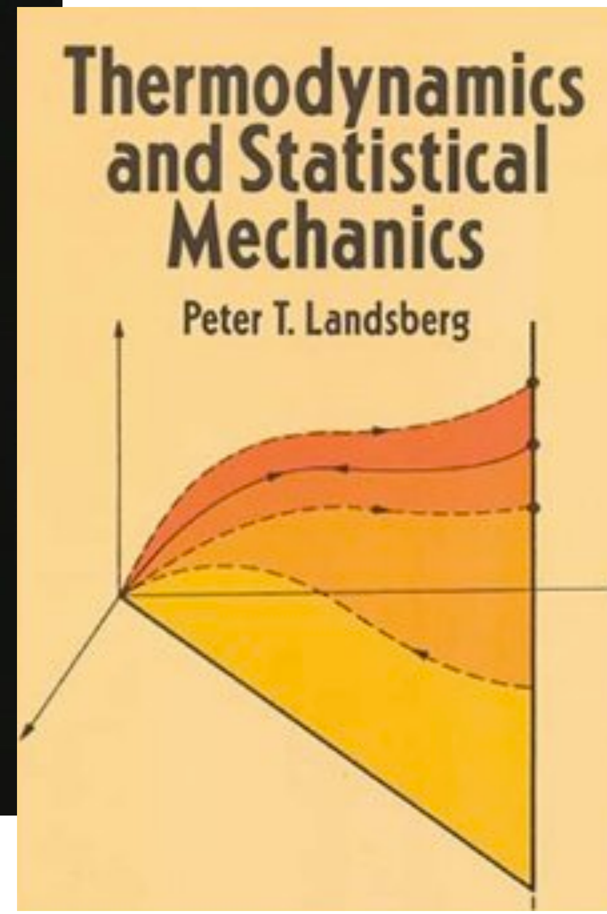
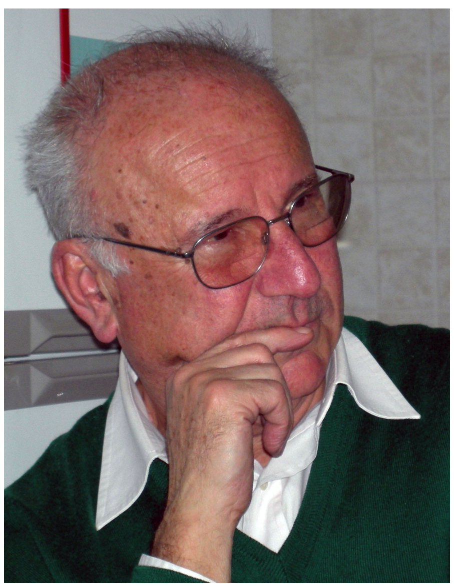


"The presence of long-range forces causes important amendments to thermodynamics, some of which are not fully investigated as yet." (1978)

Jan Naudts

Hans Herrmann

Roger Maynard



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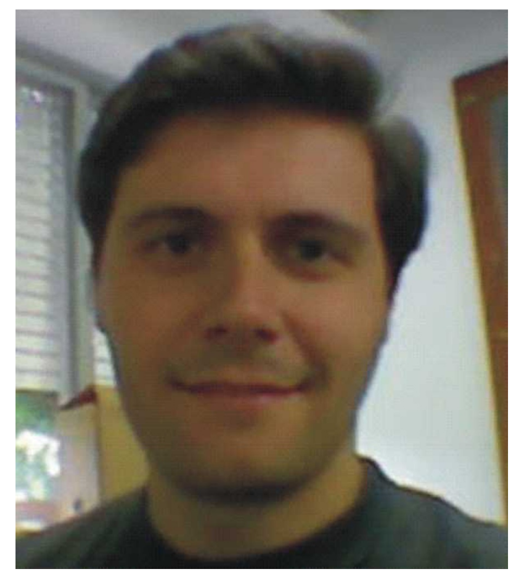

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Murray Gell-Mann, Santa Fe Institute (2005)

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Important remarks

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Edouard Brezin
Robin Stinchcombe
Albert W. Overhauser
....
....

Si l’action n’a quelque splendeur de liberté, elle n’a point de grâce ni d’honneur.

Montaigne

Tout le monde savait que c’ était impossible.

Il y avait un qui ne le savait pas.

Alors il est allé et il l’a fait.

Jean Cocteau (Marcel Pagnol, Winston Churchill, Mark Twain ...)

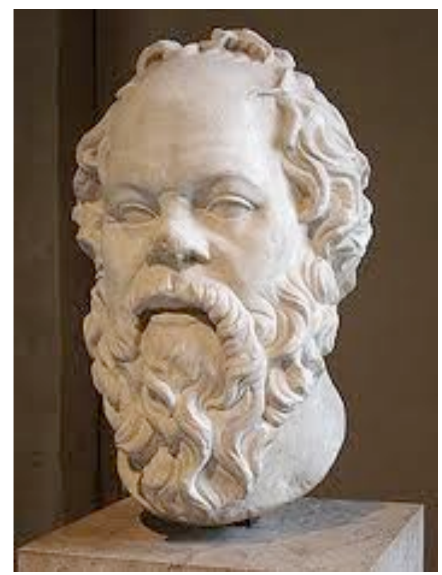
[Freeman Dyson] ***est convaincu que les vérités scientifiques sont si profondement enfouies que la seule certitude que nous puissions avoir, c'est que la plupart de choses que nous pensons se révéleront fausses.*** Courrier International **974**, 36 (2 Juillet 2009)

[Extraits du *New York Times*]

ἓν οἶδα ὅτι οὐδὲν οἶδα

sólo sé que no sé nada

Sócrates



Obrigado!