Aspects of the Non-Linear NRT Schroedinger Equation

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We explore some features of the recently proposed Nobre-RegoMonteiro-Tsallis (NRT) non-linear Schroedinger-like equation, inspired on the nonextensive thermostatitical formalism, that admits q-exponential (q-plane wave) analytical exact solutions, and reduces to the standard linear Schroedinger equation in a limit case. We examine some symmetry properties exhibited by this equation and discuss a more general family of exact, q-Gaussian time dependent wave-packet solutions.

Tsallis Entropy

$$S_q = \frac{1 - \sum_i p_i^q}{q - 1}$$

$$\lim_{q \to 1} S_q = -\sum_i p_i \ln p_i$$

Non-Additivity: For $p_{ij}^{(AB)} = p_i^{(A)} p_j^{(B)} \longrightarrow$
$S_q(AB) = S_q(A) + S_q(B) + (1-q) S_q(A) S_q(B)$

- C. Tsallis, J. Stat. Phys. **52**, 479 (1988).

- C. Tsallis, *Introduction to Nonextensive Statistical Mechanics* (Springer, New York, 2009).

- M. Gell-Mann and C. Tsallis, Eds. *Nonextensive Entropy: Interdisciplinary applications*, Oxford University Press, Oxford, 2004.

Tsallis MaxEnt Distributions

Constraints:$\langle A^{(k)} \rangle$, $k = 1, \ldots, M$; $\sum_i p_i = 1$.

$$p_i = \frac{1}{Z_q} \left[1 - (1-q) \sum_{k=1}^{M} \lambda_k A_i^{(k)} \right]^{\frac{1}{1-q}}$$

Lagrange Multipliers: λ_k, $k = 1, \ldots, M$

Part. Func.: $Z_q = \sum_i \left[1 - (1-q) \sum_{k=1}^{M} \lambda_k A_i^{(k)} \right]^{\frac{1}{1-q}}$

q-Exponential Function:

$\exp_q(u) = \frac{1}{Z_q} \left[1 + (1-q)u \right]^{\frac{1}{1-q}}$; $\exp_1(u) \equiv \exp(u)$.

$$p_i = \frac{1}{Z_q} \exp_q \left[- \sum_{k=1}^{M} \lambda_k A_i^{(k)} \right]$$

q-Gaussians

$$f(x) = \frac{1}{Z_q}\left[1 - \beta(1-q)x^2\right]^{\frac{1}{1-q}}$$

- $q < 1 \longrightarrow$ cut-off: $x_c = \pm\sqrt{\frac{1}{(1-q)\beta}}$

- $q = 1 \longrightarrow p_1 = \frac{1}{Z}\exp\left[-bx^2\right]$

- $q > 1 \longrightarrow$ power-law decay $x^{\frac{2}{1-q}}$ (normalizable for $1 < q < 3$).

Some Historical Examples of q-Gaussians

- J.C. Maxwell, "*On Boltzmann's Theorem on the Average Distribution of Energy in a System of Material Points*", Eq. (49) (Cambridge Phil. Soc. Trans., **12**, 547 (1879)].

- J.H. Jeans, *The Dynamical Theory of Gases* (2nd. ed., Cambridge U.P., 1916) eq. (269), pag. 99, Chapter V.

- H.C. Plummer, Monthly Notices of the Royal Astronomical Society, **71** (1911) 460.

Polytropic Models of Self-Gravitating Systems; Galactic Dark Matter Halos

V.F. Cardone, M.P. Leubner and A. Del Popolo, Mon. Not. Roy. Astr. Soc. **414** (2011) 2265.

C. Vignat, A. Plastino and A.R. Plastino, Physica A **390** (2011) 2491.

M.P. Leubner, Astroph. Journ. **632** (2005) L1.

Exact Time-Dependent Solutions of Non-Linear FP Equations and Related Reaction-Diffusion Equations.

$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2}{\partial x^2}\left[\rho^{2-q}\right] + \frac{\partial}{\partial x}\left[V\rho\right]$$

A.R. Plastino and A. Plastino, Physica A **222** (1995) 347.

C. Tsallis and Buckman PRE 54 (1996) R2197.

Several works by the Mendes-Lenzi group (Maringa).

T.D. Frank, *Nonlinear Fokker-Planck Equations: Fundamentals and Applications* (Springer, Berlin, 2005).

P. Troncoso, O. Fierro, S. Curilef, and A.R. Plastino, Physica A **375** (2007) 457.

One More Example of q-Gaussian

C. Vignat, A. Plastino, A.R. Plastino, and J.S. Dehesa, *Physica A* **391** (2012) 1068.

The probability density in momentum space associated with the ground state of the Coulomb potential (in D dimensions) is a q-Gaussian.

Table 1
Forms of the potential function $V_\nu(r)$ and corresponding values of the parameter q, as a function of the space dimension D, for different half-integer values of the parameter ν.

ν	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
q	$\frac{D}{D+1}$	$\frac{D+2}{D+3}$	$\frac{D+4}{D+5}$	$\frac{D+6}{D+7}$
$V_\nu(r)$	$-\frac{1}{r}$	$-\frac{1}{2}\left[\frac{1+D}{1+r}\right]$	$-\frac{1}{2}\left[\frac{(3+D)(r+1)}{r^2+3r+3}\right]$	$-\frac{1}{2}\left[\frac{(5+D)(r^2+3r+3)}{r^3+6r^2+15r+15}\right]$

ICARUS 46, 293–327 (1981)

Galactic Civilizations: Population Dynamics and Interstellar Diffusion

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The interstellar diffusion of galactic civilizations is reexamined by potential theory; both numerical and analytical solutions are derived for the nonlinear partial differential and difference equations which specify a range of relevant models, drawn from blast wave physics, soil science, and, especially, population biology. An essential feature of these models is that, for all civilizations, population growth must be limited by the carrying capacity of the planetary environments. Dispersal is fundamentally a diffusion process; a directed density-dependent diffusivity describes interstellar emigration. We concentrate on two models, the first describing zero population growth (ZPG) and the second which also includes local growth and saturation of a planetary population, and for which we find an asymptotic travelling wave solution. For both models the colonization wavefront expands slowly and uniformly, but only the frontier worlds are sources of further expansion. For nonlinear diffusion with growth and saturation, the colonization wavefront from the nearest independently arisen galactic civilization can have reached the Earth only if its lifetime exceeds 2.6×10^6 years. If discretization can be neglected, the critical lifetime is 2.0×10^7 years. For ZPG the corresponding number is 1.3×10^{10} years. These numerical results depend on our choices for the specific emigration rate, the distribution of colonizable worlds, and, in the second model, the population growth rate; but the dependence on these parameters is entrancingly weak. We conclude that the Earth is uncolonized not because interstellar spacefaring societies are rare, but because there are too many worlds to be colonized in the plausible lifetime of the colonization phase of nearby galactic civilizations. This phase is, we contend, eventually outgrown. We also conclude that, except possibly early in the history of the Galaxy, there are no very old galactic civilizations with a consistent policy of conquest of inhabited worlds; there is no Galactic Empire. There may, however, be abundant groups of $\sim 10^5$ to 10^6 worlds linked by a common colonial heritage. The radar and television announcement of an emerging technical society on Earth may induce a rapid response by nearby civilizations, thus newly motivated to reach our system directly rather than by diffusion.

Alexander wept when he heard from Anaxarchus that there was an infinite number of worlds; and his friends asking him if any accident had befallen him, he returned this answer: "Do you not think it a matter worthy of lamentation that when there is such a vast multitude of them, we have not yet conquered one?"—Plutarch, *On the Tranquility of the Mind.*

tion only of the radius (expressed as a fraction of the thermal wavefront radius). To illustrate, consider the one-dimensional analogue of (24),

$$\partial T/\partial t = (\partial/\partial x)\,\{aT^N(\partial T/\partial x)\}. \quad (25)$$

From this equation, we see that the quantity

$$Q = \int_{-\infty}^{\infty} T(x, t)dx \quad (26)$$

is conserved (Q is proportional to the total thermal energy). There is only one dimensionless combination of the coordinate x and the time t that can be obtained in terms of a and Q using (25) and (26):

$$\xi = x/(aQ^N t)^{1/(N+2)}. \quad (27)$$

The quantity $(Q^2/at)^{1/(N+2)}$ has the dimensions of temperature, and a solution to (25) which preserves its shape is

$$T(x, t) = (Q^2/at)^{1/(N+2)} f(\xi). \quad (28)$$

The solution for $f(\xi)$ (see Zel'dovich and Raizer, 1967, for details) is

$$f(\xi) = \{N\xi_0^2/[2(N+2)]\}^{1/N}[1 - (\xi/\xi_0)^2]^{1/N}, \qquad |\xi| < \xi_0,$$
$$= 0 \qquad |\xi| > \xi_0, \quad (29)$$

where

$$\xi_0 = [(N+2)^{1+N}2^{1-N}/N\pi^{N/2}]^{1/(N+2)} \times [\Gamma(\tfrac{1}{2} + 1/N)/\Gamma(1/N)]^{N/(N+2)} \quad (30)$$

and Γ is the gamma function.

For the case $N = 0$ [i.e., Eq. (10) taking $D \equiv a$],

$$f(\xi) = (4\pi)^{1/2}e^{-\xi^2/4} \quad (31)$$

The normalization employed provides

$$\int_{-\infty}^{\infty} f(\xi)d\xi = 1 \quad (32)$$

and, for $N > 0$, the position of the thermal wavefront, using (27), is just

$$x_f = \pm\xi_0(aQ^N t)^{1/(N+2)}. \quad (33)$$

In Fig. 1, $f(\xi)$ is shown for $N = 0$, 1, and 2. From Eqs. (29) and (31), we see that:

(a) if $N = 0$, the distribution is unconfined;

(b) if $0 < N < 1$, the distribution has a finite cutoff, where the temperature gradient vanishes;

(c) if $N = 1$, the temperature distribution has a finite cutoff with a finite, nonvanishing temperature gradient;
and

(d) if $N > 1$, the temperature distribution has a finite cutoff with an infinite temperature gradient.

The particular curves given in Fig. 1 correspond to a Gaussian, a parabola, and an ellipse, respectively. Apart from a differing normalization factor, the $f(\xi)$ profiles for

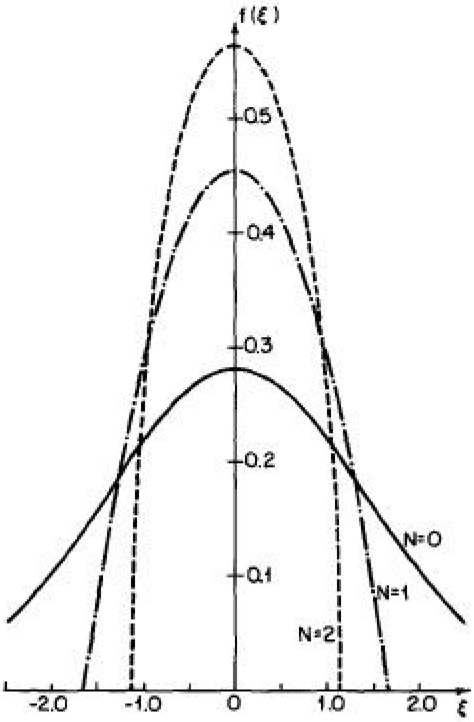

FIG. 1. Thermal wave profiles for diffusion coefficients with a power law density dependence with exponent N.

From real q-Gaussians to Complex q-Gaussians

evol. eq.	linear	non-linear
real diff. coeff.	standard diff.	pm eq.
imaginary coeff.	Schr. eq.	??

real Gaussians	real q-Gaussians
complex Gaussian wp	??

Three of the four possibilities in the above Tables are known to be mathematically interesting and physically relevant. This constitutes a motivation to explore the fourth possibility.

The NRT Non-Linear Schroedinger Equation

F.D. Nobre, M.A. Rego-Monteiro and C. Tsallis, Phys. Rev. Lett. **106**, 140601 (2011).

$$i\hbar\frac{\partial}{\partial t}\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right] = -\frac{1}{2-q}\,\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right]^{2-q} \quad (q \geq 1)$$

Related to:

- Nonlinear Diffusion Eq.: $\frac{\partial\rho}{\partial t} = D\nabla^2\left[\rho^{2-q}\right]$
- Linear Diffusion Eq: $\frac{\partial\rho}{\partial t} = D\nabla^2\rho$
- Free Particle Schr. Eq.: $i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi$

Possible Motivations for Introducing Non-Linear Schroedinger Equations

(1) Formulate effective, single-particle wave-function descriptions of interacting many-body quantum systems (Example: the Gross-Pitaievskii equation).

(2) Use non-linear Schroedinger equarions as classical field equations to describe diverse non-linear phenomena (such as water waves).

At a more fundamental (and speculative) level:

(3) Explore the possibility of formulating a non-linear version of quantum mechanics. There are scenarios that seem to lead to non-linear extensions of quantum mechanics, for instance, composite quantum systems in the presence of closed time-like curves (CTC).

D. Deutsch, Phys. Rev. D **44** (1991) 3197.

C. Zander and A.R. Plastino, in *A Century of Relativity Physics; AIP Conference Proceedings* **841** (2006) 570.

(4) Implications of an eventual non-linear quantum mechanics for quantum information processing. Information-theoretical answer to the question: why is Q.M. linear ??

q-Plane Wave Solutions

The NRT equation admits the q-plane wave solutions

$$\Phi = \Phi_0 \left[1 - i(1-q)(\omega t - kx)\right]^{\frac{1}{1-q}},$$

provided that the "wave number" k and the "frequency" ω comply with,

$$\omega = \frac{\hbar k}{2m}.$$

Therefore, this family of solutions are compatible with the de Broglie relations,

$$E = \hbar\omega,$$

$$p = \hbar k,$$

leading to the correct energy-momentum relation,

$$E = \frac{p^2}{2m}.$$

NRT also considered relativistic versions of the nonlinear wave equation (Klein-Gordon and Dirac) and obtained q-plane wave solutions compatible with the relativistic energy-momentum relation.

q-plane wave: $\Psi(x,t) = \Psi_0 \; \exp_q\left[i(kx - \omega t)\right].$

q-exponential function $\exp_q(iu)$, $u \in \mathcal{R}$ is defined as the principal value of

$\exp_q(iu) = [1 + (1-q)iu\,]^{\frac{1}{1-q}}$; $\exp_1(iu) \equiv \exp(iu)$.

The above function satisfies,

$$\begin{aligned}
\exp_q(\pm iu) &= \cos_q(u) \pm i \sin_q(u) \;, \\
\cos_q(u) &= \rho_q(u) \cos\left\{\frac{1}{q-1}\arctan[(q-1)u]\right\} \;, \\
\sin_q(u) &= \rho_q(u) \sin\left\{\frac{1}{q-1}\arctan[(q-1)u]\right\} \;, \\
\rho_q(u) &= \left[1 + (1-q)^2 u^2\right]^{1/[2(1-q)]} \;,
\end{aligned}$$

$$\begin{aligned}
\exp_q(iu)\exp_q(-iu) &= [\rho_q(u)]^2 = \exp_q(-(q-1)u^2), \\
\exp_q(iu_1)\exp_q(iu_2) &\neq \exp_q\left[i(u_1+u_2)\right], \; (q \neq 1)
\end{aligned}$$

A q-exponential with a pure imaginary argument, $\exp_q(iu)$, presents an oscillatory behavior with a u-dependent amplitude $\rho_q(u)$. The function $\exp_q(iu)$ is square integrable for $1 < q < 3$, whereas the concomitant integral diverges in both limits $q \to 1$ and $q \to 3$ and also for $q < 1$.

NRT Equation and Galilean Transformation

Original inertial frame (x', t'),

$$\Psi(x', t') = \Psi_0 \, \exp_q \left[i(kx' - \omega t') \right].$$

Let us consider now a Galilean transformation to the new inertial frame (x, t),

$$t = t'; \quad x = x' - vt'$$

Just re-expressing the "old" solution in terms of the new variables (x, t)),

$$\begin{aligned} \Psi(x', t') \rightarrow \Phi(x, t) &= \Psi(x + vt, t) \\ &= \Psi_0 \exp_q \left[i(k(x + vt) - \omega t) \right] \end{aligned}$$

does not lead to a solution of the NRT equation in the new frame. In order to obtain a valid solution it is necessary to add an extra term to the argument of the q-exponential. Indeed,

$$\Phi_0 \left[1 - i(1-q) \left\{ \omega t - k(x + vt) + \frac{1}{\hbar} \left(mvx + \frac{1}{2} mv^2 t \right) \right\} \right]^{\frac{1}{1-q}}$$

does satisfy the nonlinear Schroedinger equation.

Galilean transformed solution,

$$\Phi_0 \left[1 - i(1-q)\left\{\omega t - k(x+vt) + \frac{1}{\hbar}\left(mvx + \frac{1}{2}mv^2 t\right)\right\}\right]^{\frac{1}{1-q}}.$$

The extra term $\frac{1}{\hbar}\left(mvx+\frac{1}{2}mv^2t\right)$ appearing in the argument of the power-law admits a clear physical interpretation. Recasting the new, transformed solution under the guise,

$$\Phi_0 \left[1 - i(1-q)\left\{\left(\omega - kv + \frac{mv^2}{2\hbar}\right) t - \left(k - \frac{mv}{\hbar}\right) x\right\}\right]^{\frac{1}{1-q}}$$

is plain that it has the form of a q-plane wave with frequency $\tilde{\omega}$ and wave number $\tilde{k}$ respectively given by

$$\tilde{\omega} = \omega - kv + \frac{mv^2}{2\hbar}; \qquad \tilde{k} = k - \frac{mv}{\hbar}.$$

de Broglie relations we obtain

$$\tilde{E} = E - pv + \frac{mv^2}{2}, \text{ and } \tilde{p} = p - mv,$$

which are the correct Galilean transformations for the kinetic energy and momentum of a particle of mass m obeying the (non-relativistic) energy-momentum relation $E = p^2/2m$.

Taking the limit $q \to 1$ of the transformed solution,

$$\Phi_0 \left[1 - i(1-q)\left\{\omega t - k(x+vt) + \frac{1}{\hbar}\left(mvx + \frac{1}{2}mv^2 t\right)\right\}\right]^{\frac{1}{1-q}},$$

on sees that the relation between the original solution $\Psi(x', t')$ and the transformed one $\Phi(x,t)$ becomes,

$$\begin{aligned}\Psi(x', t') \;&\to\; \Phi(x,t) \\ &= \exp\left[-\frac{i}{\hbar}\left(\frac{mv^2}{2}t + mvx\right)\right]\Psi(x+vt, t),\end{aligned}$$

thus recovering the transformation rule corresponding to the linear Schroedinger equation.

Let us now consider a uniformly accelerated reference frame. The corresponding spatio-temporal coordinates (x,t) are

$$t = t'; \quad x = x' - \frac{1}{2}at'^2 = x' - \frac{1}{2}\frac{F}{m}t'^2 \,,$$

where (x',t') are the variables associated with an inertial frame, a is the constant acceleration of reference frame (x,t), and $a = \frac{F}{m}$. As in the previous discussion, we assume that the nonlinear Schroedinger equation holds in the inertial frame (x',t'), and also that in this frame our system is described by the q-plane wave solution.

Again, simply re-writting the q-plane wave solution in terms of the new variables (x,t) does not yield a solution of the nonlinear Schroedinger equation. As in the above Galilean transformation case, new terms are needed in the argument of the q-exponential to obtain a valid solution.

"Accelerated" q-Plane Waves

Let us consider the ansatz,

$$\frac{\Phi}{\Phi_0} = \left[1 - i(1-q)\left\{\omega t - k\left(x + \frac{Ft^2}{2m}\right) + \frac{F}{\hbar}\left(xt + \frac{Ft^3}{6m}\right)\right\}\right]^{\frac{1}{1-q}}$$

It satisfies the nonlinear equation,

$$i\hbar\frac{\partial}{\partial t}\left(\frac{\Phi}{\Phi_0}\right) = -\frac{1}{2-q}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\left(\frac{\Phi}{\Phi_0}\right)^{2-q}\right] + V(x)\left(\frac{\Phi}{\Phi_0}\right)^q,$$

where $V(x) = Fx$.

The nonlinear equation,

$$i\hbar\frac{\partial}{\partial t}\left(\frac{\Phi}{\Phi_0}\right)=-\frac{1}{2-q}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\left(\frac{\Phi}{\Phi_0}\right)^{2-q}\right]+V(x)\left(\frac{\Phi}{\Phi_0}\right)^q,$$

can be interpreted as describing the motion of a particle of mass m under a constant force $-F$ (with the associated potential function $V = Fx$). This is consistent with the well-known fact that the behavior of a free particle with respect to a uniformly accelerated reference frame is equivalent to the the behavior of a particle in an inertial reference frame moving under the effect of a constant force.

An interesting feature of the nonlinear equation is that the potential V couples to Φ^q, instead of coupling to Φ, as happens in the standard linear case ($q = 1$).

Consistently with the equation

$$i\hbar\frac{\partial}{\partial t}\left(\frac{\Phi}{\Phi_0}\right)=-\frac{1}{2-q}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\left(\frac{\Phi}{\Phi_0}\right)^{2-q}\right]+V(x)\left(\frac{\Phi}{\Phi_0}\right)^q,$$

the q-plane wave $\Phi(x,t) = \Phi_0 \exp_q[i(kx - \omega t)]$ is not only a solution of the free-particle nonlinear Schroedinger equation (when $\hbar\omega = \frac{\hbar^2 k^2}{2m}$), but also of the nonlinear equation

$$i\hbar\frac{\partial}{\partial t}\left(\frac{\Phi}{\Phi_0}\right)=-\frac{1}{2-q}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\left(\frac{\Phi}{\Phi_0}\right)^{2-q}\right]+V_0\left(\frac{\Phi}{\Phi_0}\right)^q,$$

with a constant potential V_0, provided that

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V_0,$$

which, using the Planck and de Broglie relations, becomes $E = \frac{p^2}{2m} + V_0$, as expected.

“Accelerated" q-Plane Waves

Considering now the limit $q \to 1$ of the transformed solution,

$$\frac{\Phi}{\Phi_0} = \left[1 - i(1-q)\left\{\omega t - k\left(x + \frac{Ft^2}{2m}\right) + \frac{F}{\hbar}\left(xt + \frac{Ft^3}{6m}\right)\right\}\right]^{\frac{1}{1-q}}$$

we verify that the original (non-accelerated) solution $\Psi(x', t')$ and the transformed one $\Phi(x,t)$ are linked through

$$\begin{aligned}\Psi(x', t') &\to \Phi(x,t) \\ &= \exp\left[-\frac{i}{\hbar}\left(Fxt + \frac{F^2t^3}{6m}\right)\right]\Psi\left(x + \frac{Ft^2}{2m}, t\right),\end{aligned}$$

thus recovering the (non-relativistic) transformation rule for accelerated corresponding to the linear Schroedinger equation.

q-Gaussian Time dependent Wave-Packet Solutions

We now consider solutions to the NRT equation based upon the q-Gaussian wave packet ansatz,

$$\Phi(x,t) = \Phi_0 \left[1-(1-q)(a(t)x^2+b(t)x+c(t))\right]^{\frac{1}{1-q}},$$

where a, b, and c are appropriate (complex) time dependent coefficients.

The above ansatz constitutes a solution of the NRT equation provided that the coefficients a, b, and c comply with the set of coupled ordinary differential equations,

$$\begin{aligned}
i\dot{a}(t) &= \frac{\hbar}{m}(3-q)a(t)^2 \\
i\dot{b}(t) &= \frac{\hbar}{m}(3-q)a(t)b(t) \\
i\dot{c}(t) &= \frac{\hbar}{m}\left((1-q)a(t)c(t) - a(t) + \frac{b(t)^2}{2}\right).
\end{aligned}$$

Harmonic Oscillator

The q-Gaussian ansatz also yields exact analytical time dependent solutions for the NRT equation corresponding to a harmonic potential,

$$i\hbar\frac{\partial}{\partial t}\left[\frac{\Phi}{\Phi_0}\right] = -\frac{1}{2-q}\,\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi}{\Phi_0}\right]^{2-q} + V(x)\left[\frac{\Phi}{\Phi_0}\right]^{q},$$

with $V(x) = kx^2/2$.

Quasi-Stationary Solution: A particular q-Gaussian solution of the above equation corresponds, in the $q \to 1$ limit to the ground state of the standard, linear harmonic oscillator. Other q-Gaussian solutions correspond, in the same limit, to Schroedinger's time dependent Gaussian wave packet solutions of the h.o.

More on quasi-Stationary Solutions: I.V. Toranzo, A.R. Plastino, J.S. Dehesa, A. Plastino, Physica A **392** (2013) 3945.

Summary:

- Behavior of q-plane wave solutions under Galilean Transformations. Compatibility with the de Broglie Relations.

- “Accelerated" q-plane wave solutions $\longrightarrow$ Extension of the NRT equation to the case of a particle moving in a potential $V(x)$.

- Time dependent q-Gaussian wave packet solutions. Harmonic Oscillator.

- Quasi-Stationary Solutins of the NRT equation.

Some Open Questions that May Shed Light on Possible Applications

(1) Is the NRT dynamics (or an appropriate restriction of it) “conservative"?

The NRT can be derived from a variational principle (NRT, EPL **97**, 41001 (2012).)

The NRT admits a “time reversal symmetry": $\Psi(x,t) \longrightarrow \Psi^*(x,-t)$.

(2) Full characterization of the set of norm-preserving solutions of the NRT equation.

The non-preservation of the norm is related to the fact that the set of solutions to the NRT eq. is not closed under global phase changes ($\Psi(x,t) \longrightarrow e^{i\alpha}\Psi(x,t)$). In the linear case this symmetry leads, via Noether's Theorem, to the continuity eq. for de prob. density and the preservation of the norm. The NRT equation admits a “discrete" version of this symmetry, given by $(1-q)\alpha_n = 2\pi n$.

(3) Information-related aspects of the NRT eq.

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I.V. Toranzo, A.R. Plastino, J.S. Dehesa, A. Plastino, Physica A **392** (2013) 3945.