Nonextensive thermodynamics of hadronic medium

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Compsyst - Rio de Janeiro, October, 2013

Hagedorn's theory

A hadronic system is considered as an ideal gas of hadrons a a temperature T. The partition function is

$$ln[1 + Z(V_0, T)] = \frac{V_0 T}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \int_0^{\infty} \rho(m; n) m^2 K_2 (\beta\, n\, m)\, dm.$$

The bootstrap idea is:
Fireball is "$*$the static equillibrium of a system composed by *fireballs*, which on their turn are ... (goto $*$)"

The partition function can also be written as

$$Z(V_0, T) = \int_0^E \sigma(E')Z_0(E')dE'$$

According to the bootstrap principle, both forms of partition function must be asymptotically identical with

$$\ln[\sigma(E')] = \ln[\rho(m)]$$

Tsallis statistics I

Tsallis entropy can be written in the form (J. Stat. Phys. 52 (1988) 479.)

$$S_q = -k \sum_{i=1}^{W} p_i \, ln_q \, p_i , \tag{1}$$

where we defined the q-logarithm function

$$ln_q x = \frac{x^{1-q} - 1}{1-q} \quad (q \in \Re). \tag{2}$$

The q-exponential function is defined as the inverse of the q-logarithm

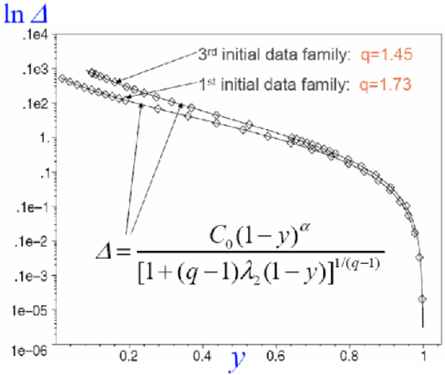
$$e_q^x \equiv [1 + (1-q)x]^{1/1-q} \quad (q \in \Re), \tag{3}$$

GRAVITATIONAL EMISSION FROM A BLACK HOLE

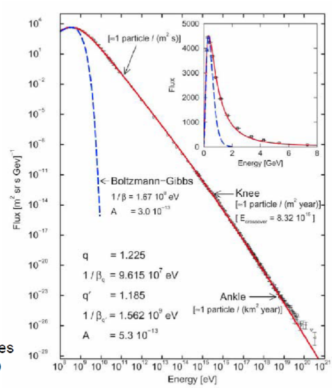
[Oliveira and Damiao Soares, Phys. Rev. D **70**, 084041(2004)]

$$\Delta \equiv fraction\ of\ mass\ extracted \equiv \frac{M_{init} - M_{\infty}}{M_{init}}$$

$$y \equiv dimensionless\ initial\ mass \propto M_{init}$$



FLUX OF COSMIC RAYS:



C. T, J. Anjos and E.P. Borges
Phys. Lett. A **310**, 372 (2003)

IMAGE THRESHOLDING:

M.P. de Albuquerque, I.A. Esquef, A.R.G. Mello and M.P. de Albuquerque
Pattern Recognition Letters **25**, 1059 (2004)

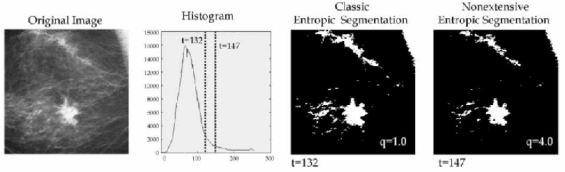
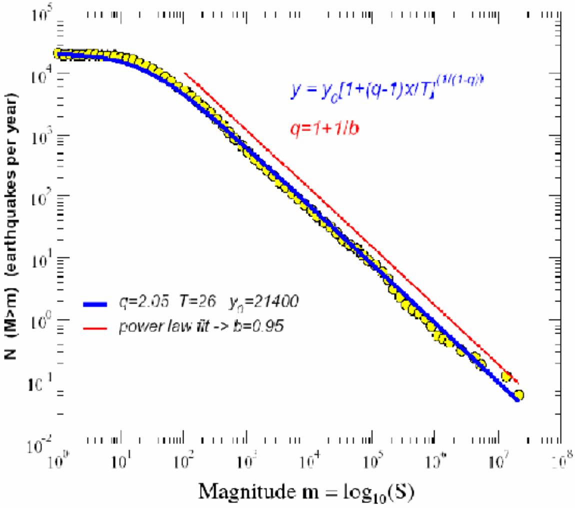


Fig. 3. Example of entropic segmentation for mammography image with an inhomogeneous spatial noise. Two image segmentation results are presented for $q = 1.0$ (classic entropic segmentation) and $q = 4.0$.

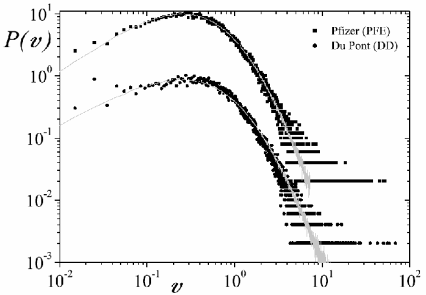
Earthquakes

Data from

P.Bak, K. Christensen, L. Danon and T. Scanlon, Phys Rev Lett **88**, 178501 (2002)



STOCK VOLUMES:



$$p(u) = \frac{1}{Z}\left(\frac{v}{\theta}\right)^{-\alpha-2} \exp_q\left[-\frac{\theta}{v}\right]$$

J de Souza, LG Moyano and SMD Queiros, Eur Phys J B **50**, 165 (2006)

Self-consistency in the nonextensive thermodynamics

$$Z_q(V_o,T) = \int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}} dE$$

e

$$\ln[1+Z_q(V_o,T)] = \frac{V_o}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^\infty dm \int_0^\infty dp\, p^2\rho(n;m) \times [1+(q-1)\beta\sqrt{p^2+m^2}]^{-\frac{nq}{(q-1)}},$$

The bootstrap principle

$$Z_q(V_o,T) = \int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}} dE = \exp\left\{\frac{V_o}{2\pi^2\beta^{3/2}}\int_0^\infty dm\, m^{3/2}\rho(m)[1+(q-1)\beta m]^{-\frac{1}{q-1}}\right\} - 1$$

At the same time we must have

$$\ln[\sigma(E)] = \ln[\rho(m)]$$

Mass spectrum and density of states

The self-consistency principle is satisfied if

$$m^{3/2}\rho(m) = \frac{\gamma}{m}\left[1 + (q_o - 1)\beta_o m\right]^{\frac{1}{q_o-1}} = \frac{\gamma}{m}[1 + (q_o' - 1)m]^{\frac{\beta_o}{q_o'-1}}$$

and

$$\sigma(E) = bE^a\left[1 + (q_o' - 1)E\right]^{\frac{\beta_o}{q_o'-1}}$$

Using properties of $\Gamma(z)$ function it results that for $(q_o' - 1) \to 0$,

$$Z_q(V_o, T) \to b\Gamma(a+1)\left(\frac{1}{\beta - \beta_o}\right)^{a+1}$$

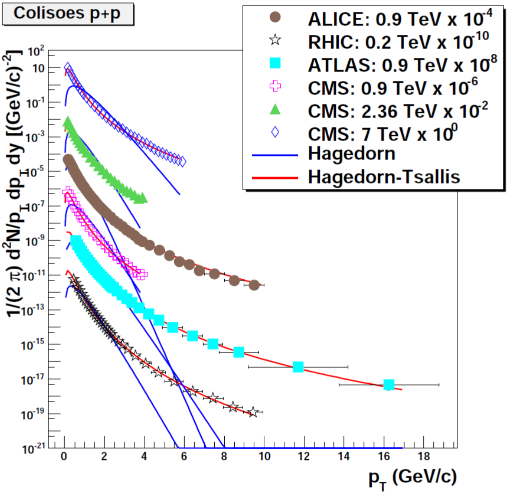
Then both expression for the partition function Z_q converge if

$$a + 1 = \alpha = \frac{\gamma V_o}{2\pi^2\beta^{3/2}}$$

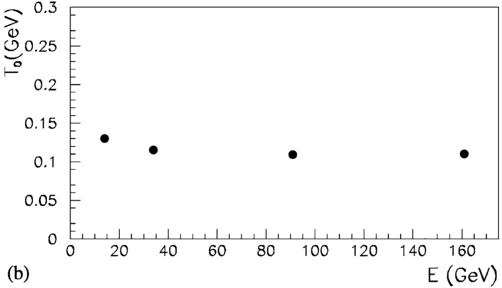
Limiting temperature: β_o and entropic index: q_o.

A. Deppman, Physica A 391 (2012) 6380–6385.

Comparison with experiment

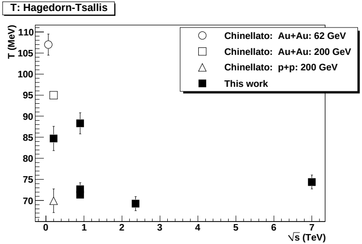


Evidences from e^+e^- collisions

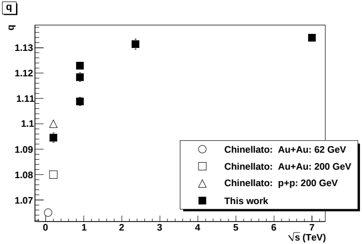


Bediaga (2000).

Evidences from pp collisions



(a)

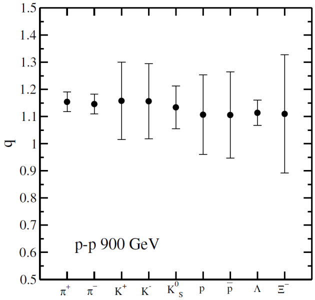


(b)

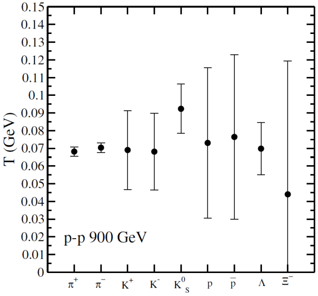
I. Sena and AD Eur. Phys. J. A 49 (2013) 17

q and T as a function of m

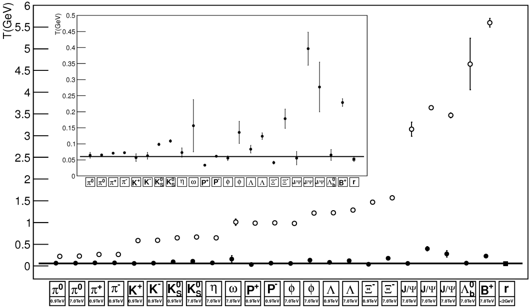
J Cleymans e D Worku: J. Phys. G: Nucl. Part. Phys. 39 (2012) 025006.

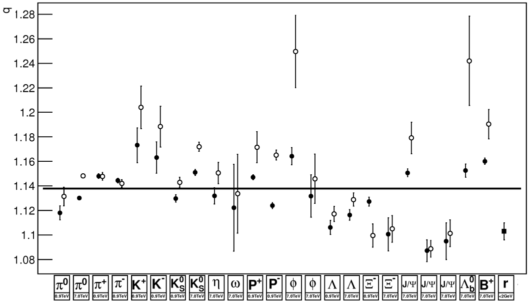


(c)



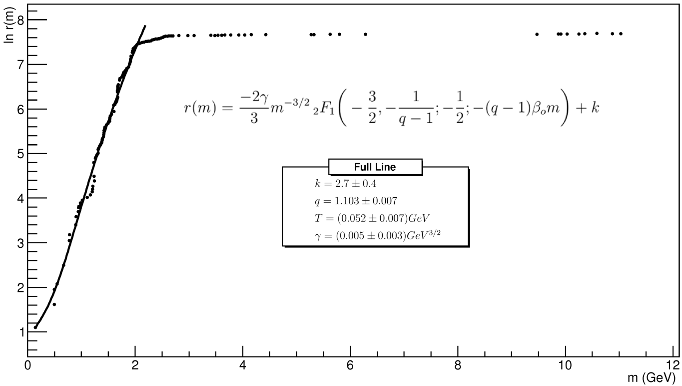
(d)





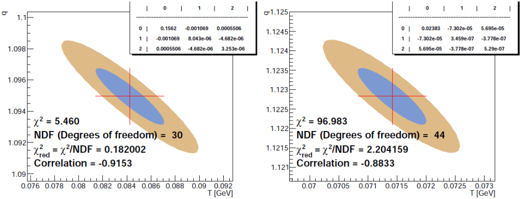
$T_o = (60\pm7)$ MeV $\quad q_o = 1.103\pm0.007$

Hadron mass spectrum

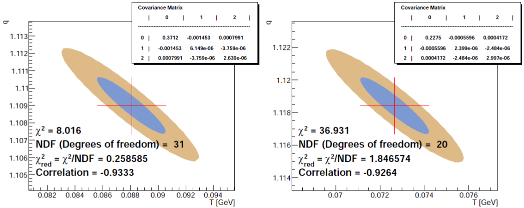


L. Marques, E. Andrade and AD, Phys. Rev. D 87, 114022 (2013)

Correlation between T and q



(a) RHIC: 200 GeV (b) ALICE: 900 GeV



Wilk e Wlodarczyk: Cent. Eur. J. Phys. 10 (2012) 568-575

$$T_{eff} = T_o - (q-1)c$$

T_H = (192±15) MeV c =-(950±10) MeV

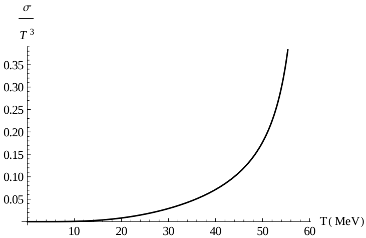
Thermodynamical functions

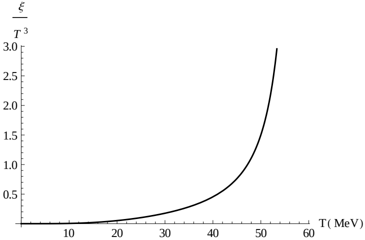
$$\frac{1}{V}\ln[Z(V,\beta)] = \frac{\gamma(q-1)}{2\pi^2\beta^{3/2}}\left(\frac{1}{(\beta-\beta_o)M(q-1)}\right)^{\frac{1}{q-1}} \times {}_2F_1\left[\frac{1}{q-1},\frac{1}{q-1},\frac{q}{q-1},\frac{-1}{(q-1)(\beta-\beta_o)M}\right]$$

pressure: $p = \frac{T}{V}\ln[Z(V,\beta)]$ entropy density: $s = \frac{\partial p}{\partial T}$

energy density: $\varepsilon = \frac{T^2}{V}\frac{\partial}{\partial T}\ln[Z(V,\beta)]$

trace anomaly: $a(T) = \frac{\varepsilon - 3p}{T^2} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right)$





Map between non-extensive and extensive quantities

Mapa entre T e τ:

$\tau(T_o) = \tau_o$

$\tau(0) = 0$

$\tau(T_1 + T_2) = \tau(T_1) + \tau(T_2)$

The function satisfying this conditions is:

$$\tau(T) = kT = \frac{\tau_o}{T_o} T$$

From here we get the following relations:

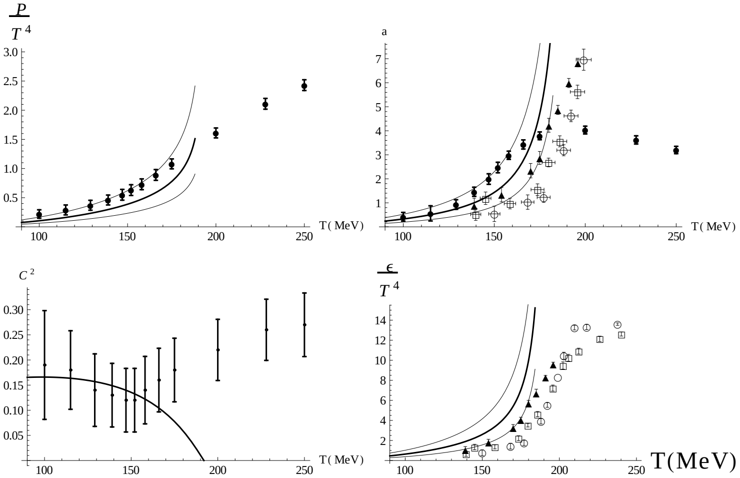
$$\frac{p}{T^4} = k^{-3} \frac{k^{-1} p}{(k^{-1} T)^4} = k^{-3} \frac{\pi}{\tau^4}$$

$$s = \sigma$$

$$\varepsilon = k^{-1} \epsilon$$

$$a = k^{-3} \alpha \,,$$

Comparison to lattice-QCD



Finite chemical potential

In collaboration with E. Megías and D. P. Menezes

Defining the partition function as

$$\log(Z_q(V,\tau,\mu)) = \begin{cases} V\sum_i g_i\xi_i \int \frac{d\varepsilon_i}{(2\pi)^3} \log_q^{(+)}\left(\frac{e_q^{(-)}(x)-\xi_i}{e_q^{(-)}(x)}\right), & x_i < 0 \\ V\sum_i g_i\xi_i \int \frac{d\varepsilon_i}{(2\pi)^3} \log_q^{(-)}\left(\frac{e_q^{(+)}(x)+1}{e_q^{(+)}(x)}\right), & x_i > 0 \end{cases} \quad (4)$$

where $\xi = 1$ for bosons and $\xi = -1$ for fermions.
The occupation number is obtained by

$$n_i = -\tau \frac{\partial}{\partial \mu_i} \log[Z_q(V,\tau,\mu)] \quad (5)$$

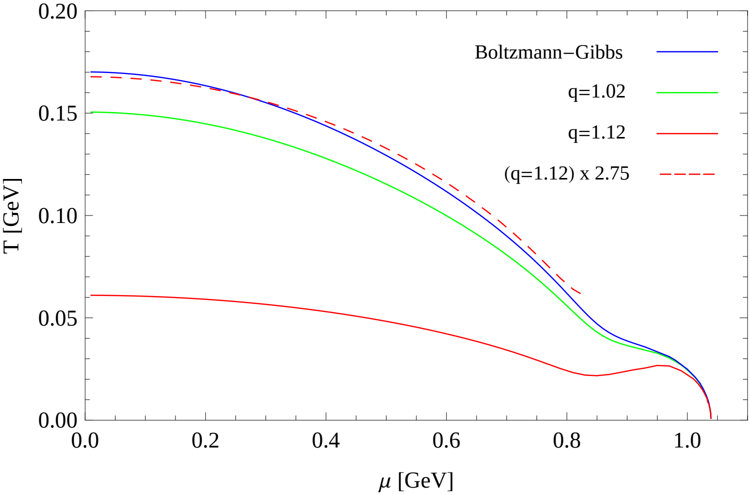
resulting

$$n_i(V,\tau,\mu) = \begin{cases} \int \frac{d\varepsilon_i}{(2\pi)^3} \left[-\xi_i + e_q\Big(\beta(\varepsilon_i-\mu_i)\Big)\right]^{-q}, & x_i > 0 \\ \int \frac{d\varepsilon_i}{(2\pi)^3} \left[-\xi_i + e_q\Big(\beta(\varepsilon_i-\mu_i)\Big)\right]^{q-2}, & x_i < 0 \end{cases} \quad (6)$$

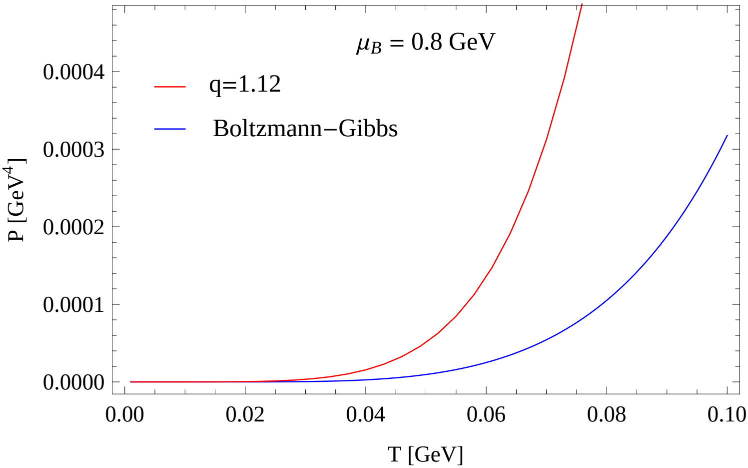
Entropy results to be the same proposed by Conroy, Miller and Plastino (PLA 374 (2010) 4581).

Phase transition to deconfined state - Preliminary

The condition for phase transition is $\langle E \rangle / \langle N \rangle = 1$ GeV (Cleymans and Redlich - PRC60, 054908 - 1999)

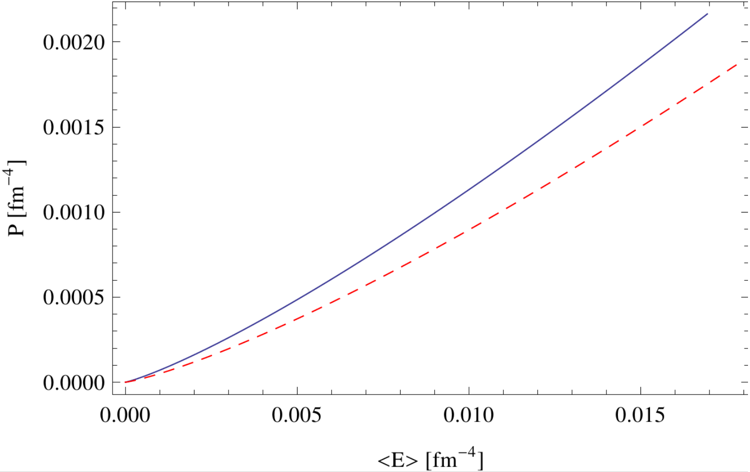


Pressure as a function of temperature - Preliminary



Neutron stars stability? - Preliminary

Relation between pressure and energy



Conclusions

- It is possible to obtain a self-consistent theory for fireballs in the non-extensive thermodynamics.
- Self-consistency leads to a limiting effective temperature, T_o, and a limiting entropic index, q_o.
- Experimental data for p_T-distributions give support for the existence of T_o and q_o.
- The mass-spectrum formula describes very well the known hadronic states (mesons and barions).
- It is possible to find a connection between extensive and non-extensive thermodynamics functions.
- Thermodynamics functions resulting from the non-extensive self-consistent theory are in agreement with lattice-QCD results.
- The nonextensive thermodynamics has been extended to $\mu \neq 0$, and the phase transition line has been found.
- There are indications that the neutron star stability can be achieved in Tsallis statistics.

Thank you!