

Testing the Goodwin growth-cycle macroeconomic dynamics in Brazil



Marcelo B. Ribeiro

Physics Institute

Federal University of Rio de Janeiro – UFRJ

N.J. Moura Jr.

Brazilian Institute for Geography and Statistics - IBGE

Macroeconomic dynamics of capitalist systems

- *Karl Marx* (*Capital*, vol 1, book 1; 1867):
 - capitalist production grows on cycles of booms and busts;
 - during a boom, profits increase and unemployment decreases (shortage of manpower);
 - a boom is followed by a bust: less unemployment reduces profits, causing then higher unemployment;
 - smaller salaries increase profit margin, renewed investment and a new boom starts...
 - ...followed by another bust, and so on...
- A **century** later *Richard Goodwin* (1967) proposed a mathematical model that tried to capture the essence of this dynamics.

The Goodwin model

- Marx's qualitative dynamics is represented by a modified Lotka-Volterra predator-prey system of 1st order ODEs;
- The number of predators and preys are replaced by two variables, u and v :
 - $u \rightarrow$ worker's share of total production \leftrightarrow capitalist's profit margin
 - $v \rightarrow$ employment rate \leftrightarrow total share of those marginalized by the production, the unemployed
- To build the model, Goodwin advanced a series of economic hypotheses linking capital, output, total labor, output labor ratio, population, average and total wage, employment rate, profit level and investment;
- The model translated these hypotheses into an ODE system with various parameters whose signs are also fixed.

The Goodwin model (part 2)

Goodwin model is a Lotka-Volterra predator-prey like system of 2 ODEs:

$$\frac{\dot{u}}{u} = -a_1 + b_1 v$$
$$\frac{\dot{v}}{v} = a_2 - b_2 u$$

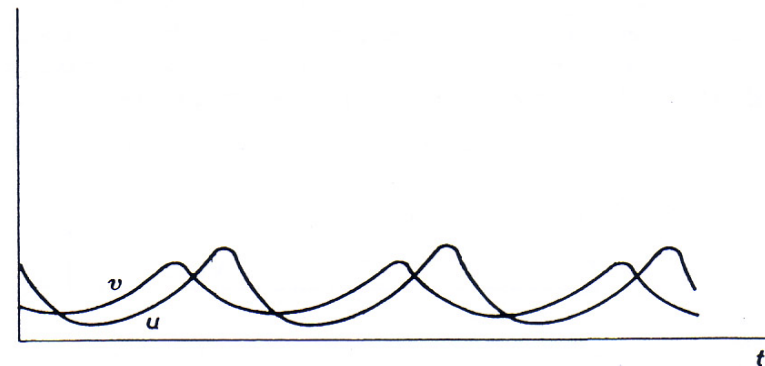
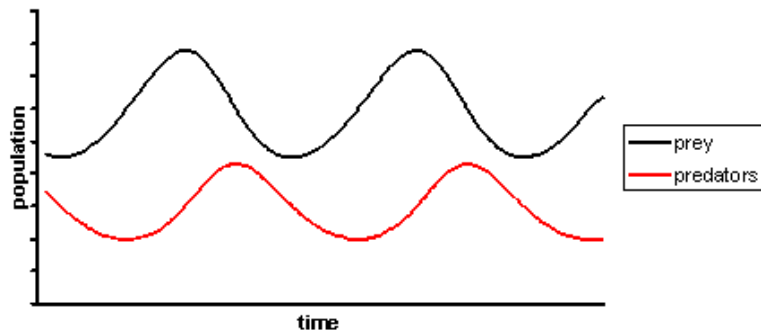
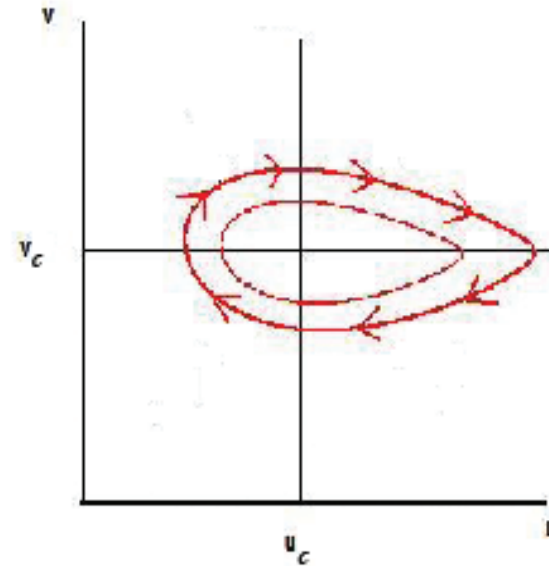
$$a_1 > 0,$$
$$b_1 > 0,$$
$$a_2 > 0$$
$$b_2 > 0.$$

- 1st equation: positive slope
- 2nd equation: negative slope
- The model also has a fixed center (u_c, v_c)

$$u_c = c a_2, \quad c > 0; \quad v_c = \frac{a_1}{h}, \quad h > 0$$

The Goodwin model (part 3)

- Goodwin model has clockwise orbits with an unique center in the u - v phase plane;
- Variables have a predator-prey like time evolution;
- Model is unstable to a change in its parameters, but the single center remains.



Empirical evidence

- Since its proposal, several *theoretical* developments were advanced by economists;
- The Goodwin model gained a dedicated group of supporters;
- However, after 46 years very few *empirical* studies were carried out trying to **test** its validity with real data;
- The limited empirical results range from partial qualitative acceptance to total quantitative rejection;
- Partial qualitative acceptance motivated this study;
- It is based on a different approach to analyze data, inspired by recent efforts made by econophysicists on the problem of characterizing income distribution;
- Income distribution functions are used to characterize the model's variables u and v .

Testing the Goodwin model with Brazilian data

- Individual income distribution can be modeled by the Gompertz-Pareto distribution (GPD);
- Gompertz curve (double exponential) + Pareto power law
- Complementary cumulative distribution for average income x

$$F(x) = \begin{cases} G(x) = e^{e^{(A-Bx)}}, & (0 \leq x < x_t), & \text{(Gompertz)} \\ P(x) = (x_t)^\alpha e^{e^{(A-Bx_t)}} x^{-\alpha}, & (x_t \leq x \leq \infty), & \text{(Pareto)} \end{cases}$$

- x_t – transition income value
- α – Pareto index
- B – Gompertz parameter
- A – Gompertz boundary condition $A = \ln(\ln 100) = 1.5272.$
- Availability of Brazilian income data and previous studies with that database made this work possible.

The Gompertz-Pareto distribution (GPD) and Brazilian individual income data

- Essential results stemming from recent studies:

Eur.Phys.J. B **67** (2009) 101 - arXiv:0812.2664

Physica A **390** (2011) 689 – arXiv:1010.1994

1% are “the rich” (Pareto) and 99% are “the rest” (Gompertz);

- GPD is a good approximation for highly polarized (high Gini coefficient) income distributions, like the Brazilian one:
- “Middle class” is represented by the exponential approximation of the upper part of the Gompertzian component (consistent with Dragulescu and Yakovenko 2001);

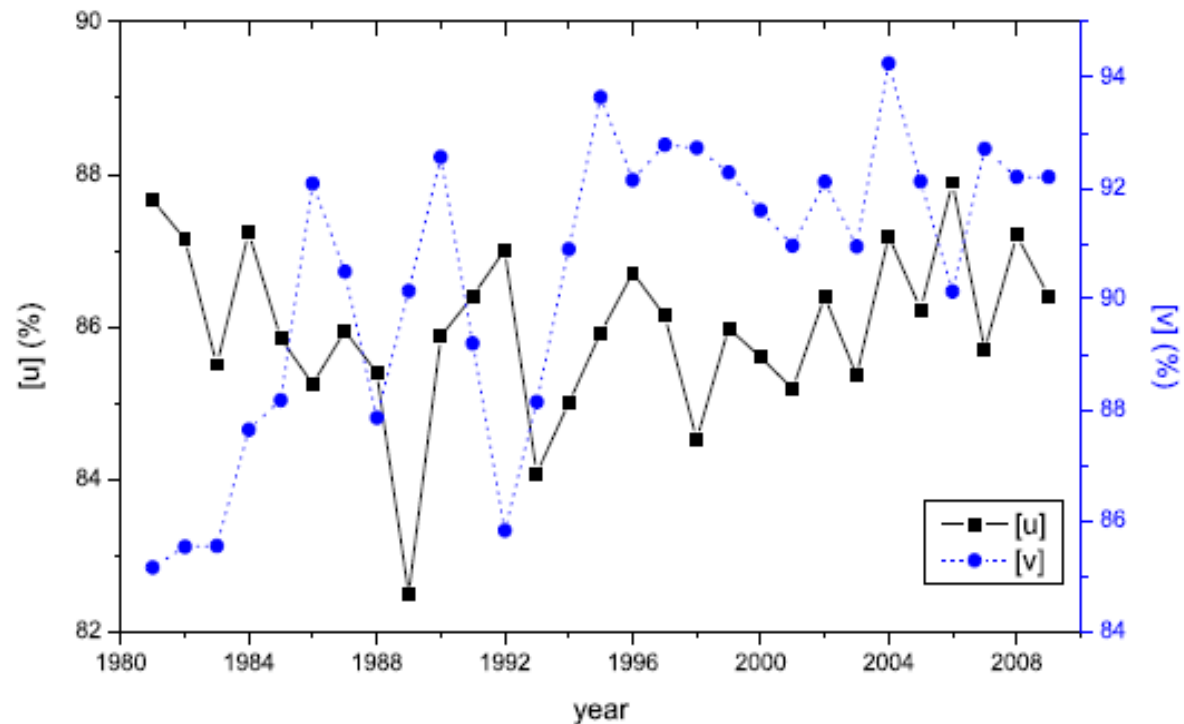
Detailed characterization of “middle class” is work in progress;

- GPD is a tool to partition the income distribution in segments capable of characterizing the Goodwin variables u and v ;

GPD and the Goodwin model

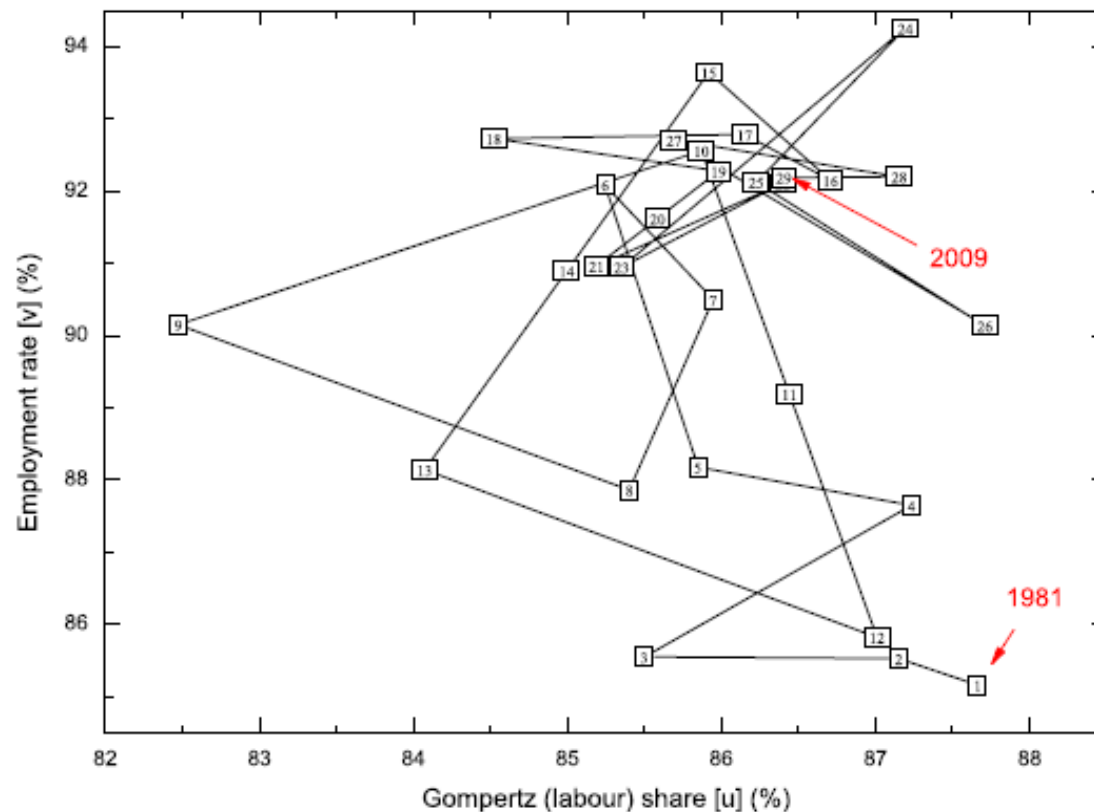
- Gompertzian segment characterizes u ;
- Unemployment is characterized as a lower limit income threshold value;
- Unemployment share is obtained from the income distribution, and not from official unemployment statistics (drop long term unemployment), but percentages are in general agreement for the last 15 years;
- Employment rate $v = (100\% - \text{unemployment share})$;
- Once u and v are yearly defined, their time derivatives, \dot{u} and \dot{v} can be obtained numerically and straight line fitting is used to ascertain the validity of the economic hypotheses of the model;
- Time evolution of these variables can also be studied.

Time evolution of the Goodwin variables in Brazil 1981-2009



- Approximate cycling behavior with 4-year periods;
- Variables have phase difference of about 2 years;
- Short term cycles.

Brazilian u - v phase plane: 1981-2009

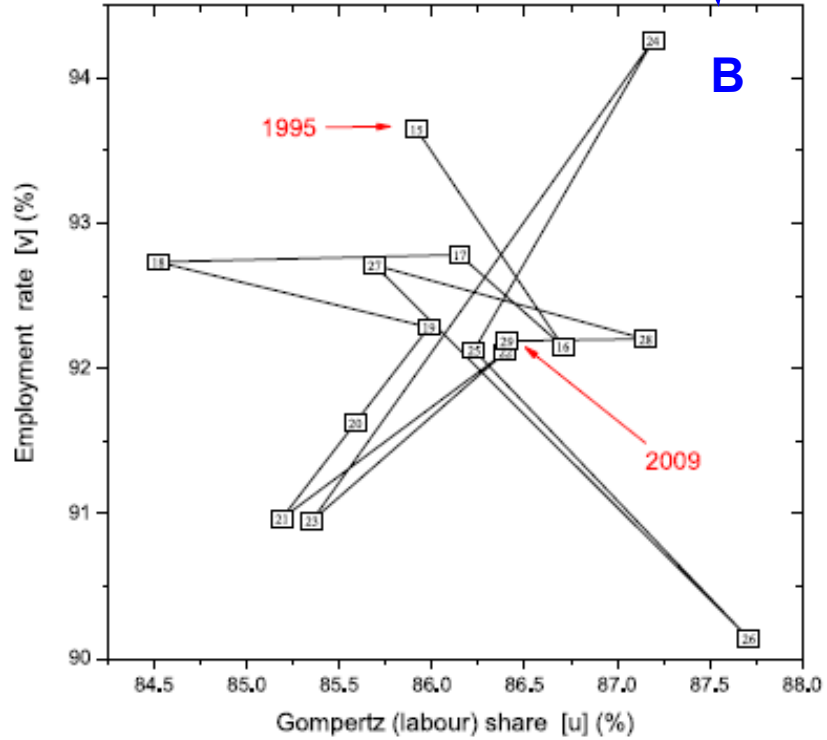
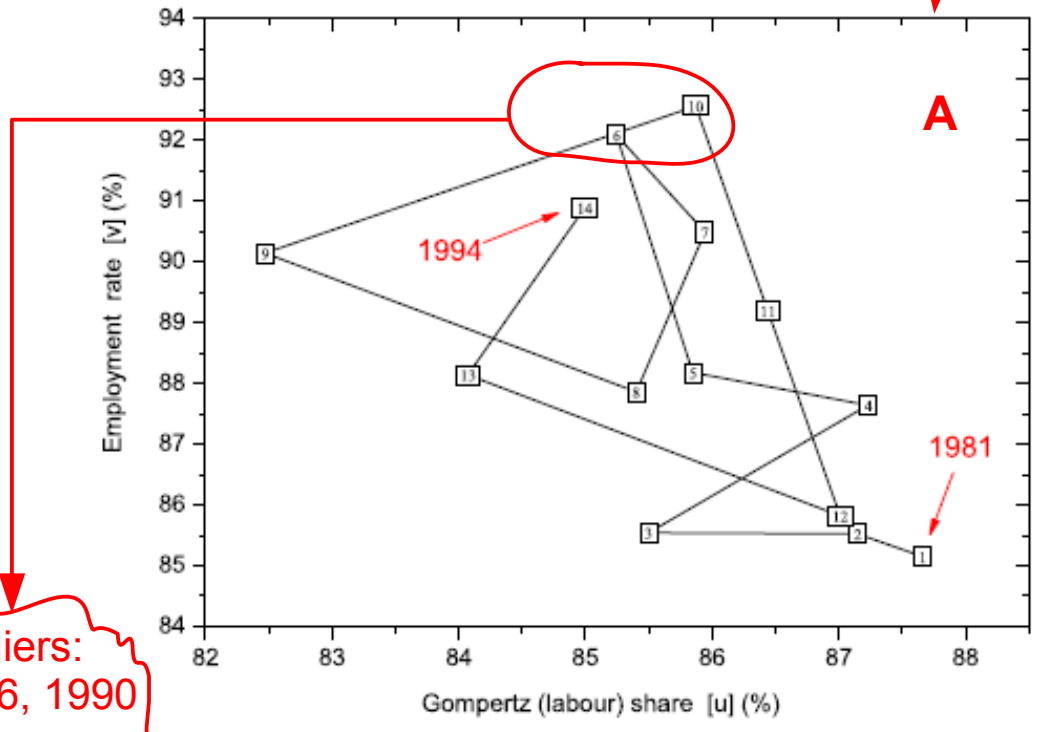
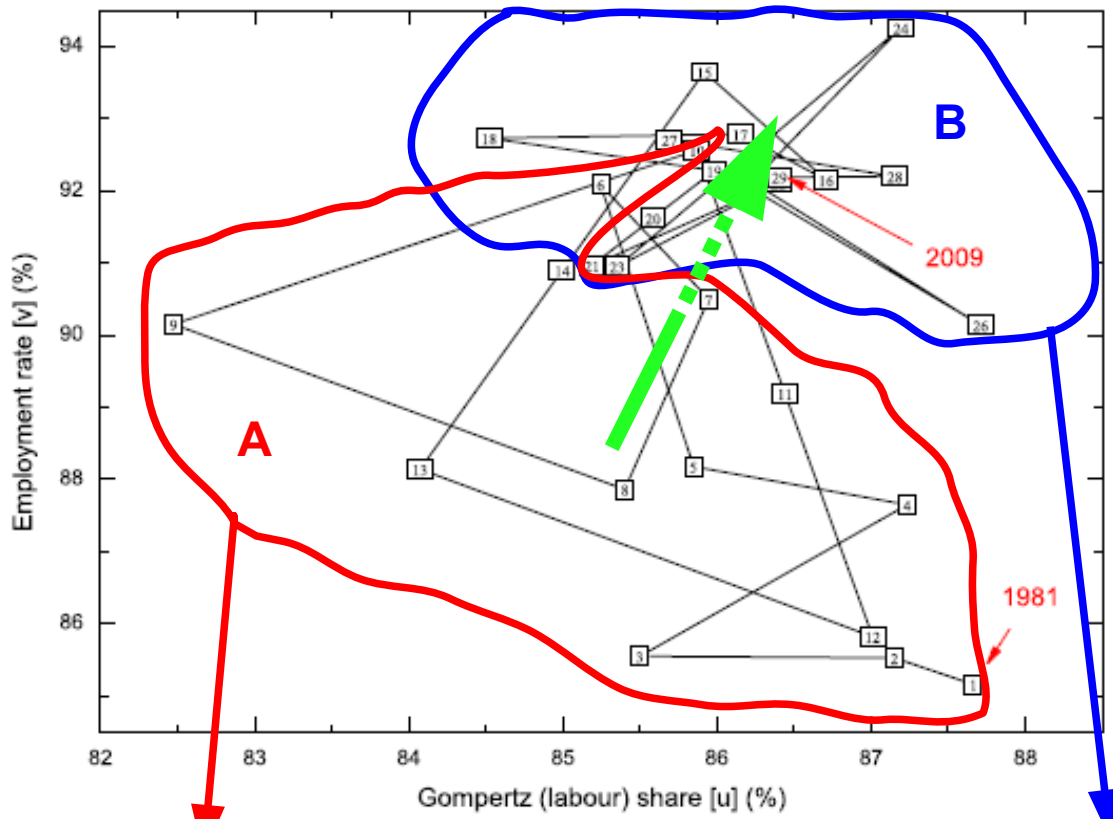


Growing numerical sequence: each number represents one year

- Clockwise cycles, but no single center;
- Two cycling regions, “center” appears to move to the upper region of the plane;
- Qualitative agreement (clockwise cycles), but quantitative disagreement (no single center) with the original model.

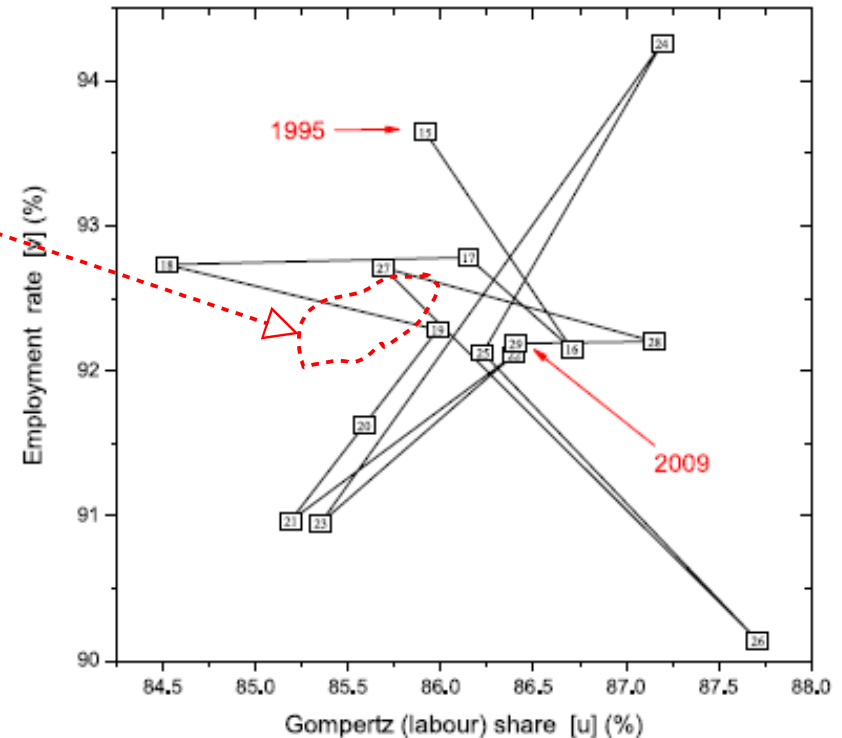
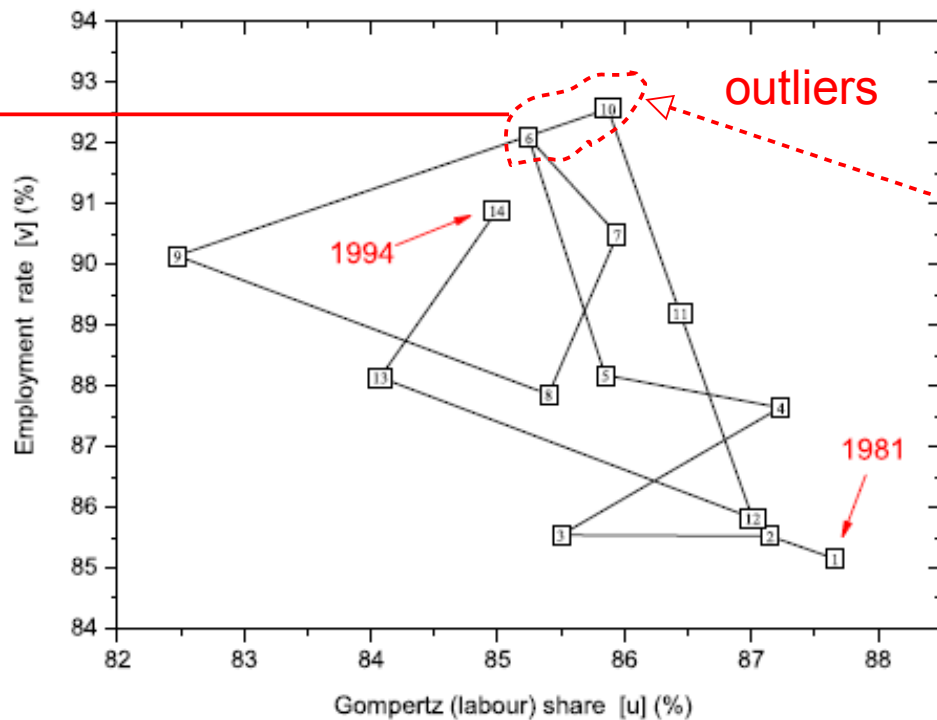
Phase plane evolution (\uparrow) of the Brazilian macroeconomic system

Closer look at phase plane shows that the data can be divided in two distinct regions:
(A) 1981 to 1994; **(B)** 1995 to 2009



Outliers:
1986, 1990

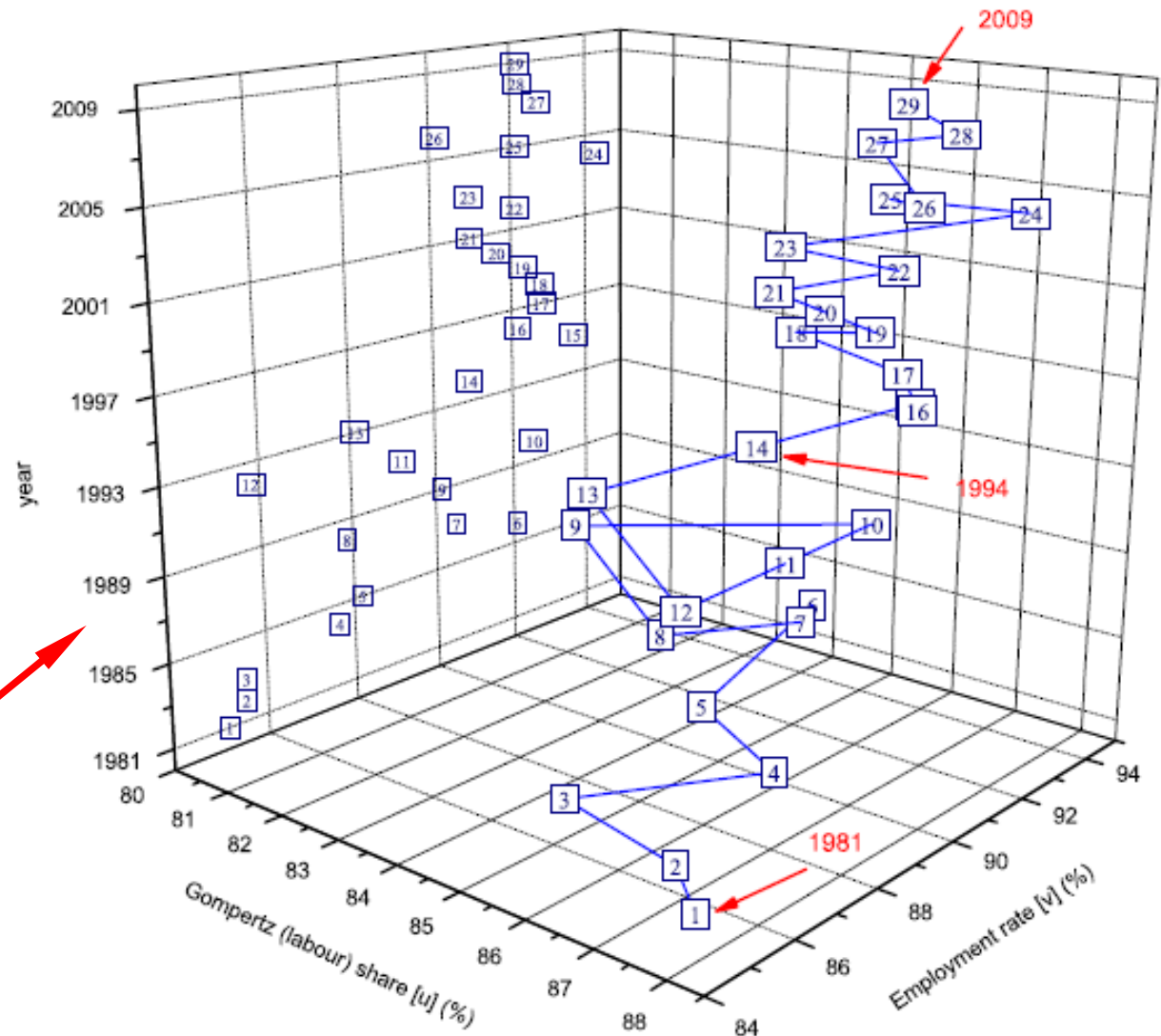
Tentative interpretation of the divided Brazilian u - v phase plane: 1981-1994 (left) and 1995-2009 (right)



- 1994 event: abrupt end of Brazilian hyperinflation;
- Inflation can be considered as an additional tax on labor;
- This may have triggered the system to move into a new position in the phase plane, where employment rate, *i.e.*, Gompertzian (labor) component share, is higher;
- Earlier, failed, attempts to control hyperinflation: 1986 and 1990 (outlier points 6 and 10);
- Runaway inflation started in 1982, so the system may have been in another region in the phase plane before that.

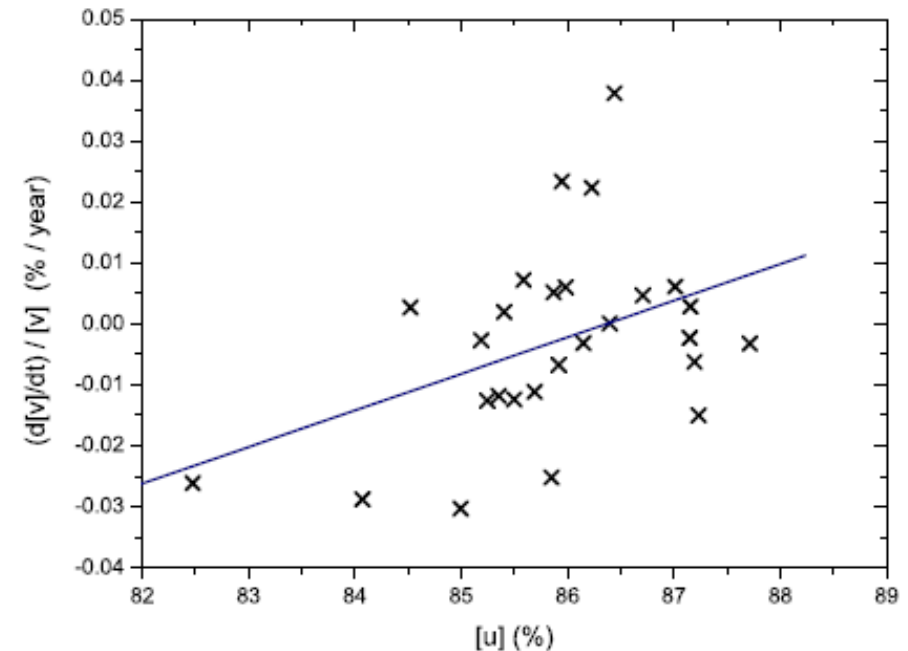
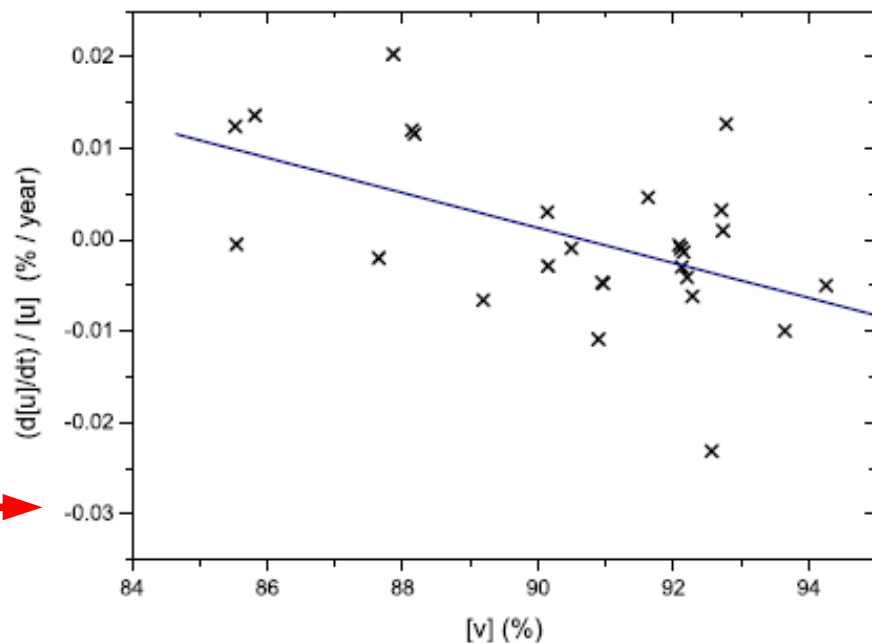
Time evolution of the u - v phase plane

- Previous results in a tri-dimensional plot;
- Points 1 and 2 seem to be the transition from an unspecified earlier region;
- Points 3 to 14 correspond to the hyperinflationary period in the Brazilian economy, abruptly finished in 1995;
- Projection (left vertical plane) also seems to indicate that different regions correspond to different inflationary (income) regimes in Brazil.



Temporal variation of employment rate and workers' share

- Numerical evaluation of the variables' derivatives;
- Straight line fitting to determine the parameters of the model observationally;
- Points show important dispersion;



Observational results imply slopes opposite to what the model is supposed to obey, according to the economic hypotheses.

$$A_1 = 0.17 \pm 0.06, B_1 = -0.0019 \pm 0.0006$$

$$A_2 = -0.52 \pm 0.22, B_2 = 0.006 \pm 0.003$$

Conclusions

- The Goodwin model agrees only qualitatively with the data: it fails on the quantitative front, though this is not a new conclusion;
- It may, however, be useful as a starting point for an improved model showing better data agreement;
- Much theoretical work done on this model by economists have been focused on its economic hypotheses;
- Empirical studies indicate that these hypotheses may, perhaps, not be valid, or need to be modified;
- Perhaps, it is more fruitful to focus on its empirical validation, rather than speculate about hypotheses which may not be substantiated by real data;
- Macroeconomics can be approached at the macro level without any need of the so-called “microeconomic foundations of individual's rationality” and, so, no need to talk about “agent's behavior”, representative or not;
- As the focus is on average individual income evolution, this work can be viewed under an *economical distributive dynamics* perspective;
- The GPD and its exponential approximation provide a useful tool for proposing dynamical models of economic systems;
- It would be very interesting to see similar works carried out using income data of other countries.
- Reference of this work: *Physica A 392 (2013) 2088 - arXiv:1301.1090*

Conclusions

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