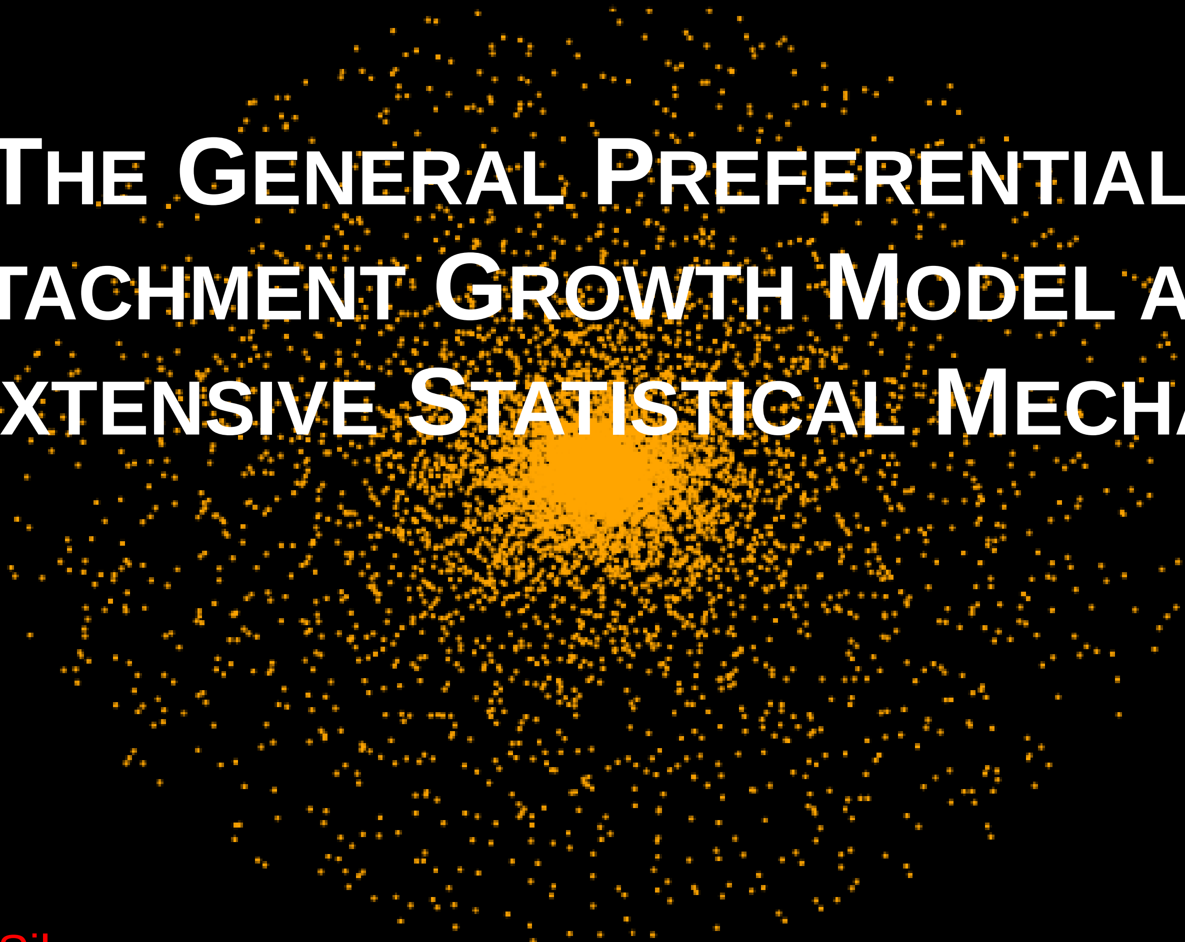
THE GENERAL PREFERENTIAL ATTACHMENT GROWTH MODEL AND NONEXTENSIVE STATISTICAL MECHANICS



L. R. da Silva

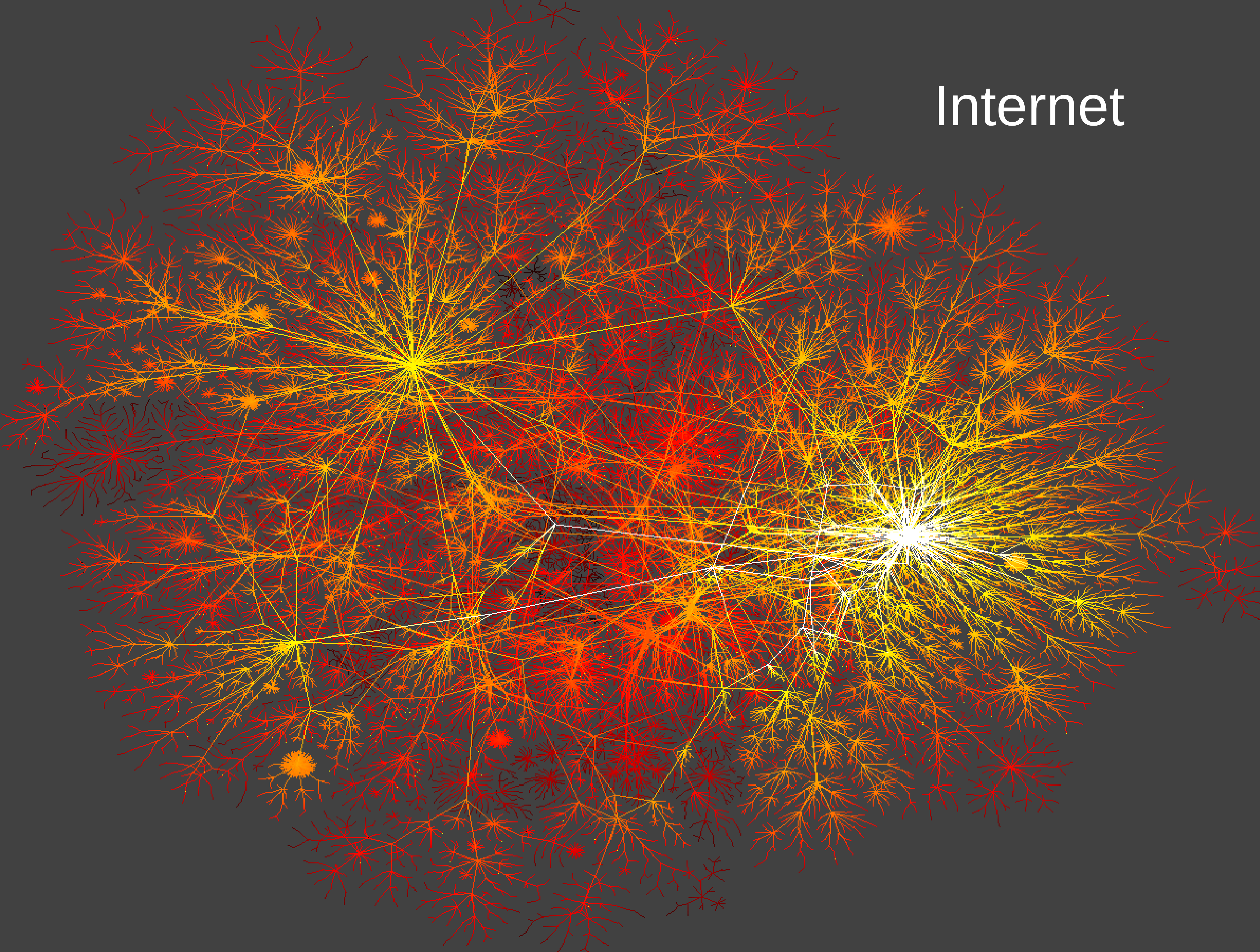
UFRN – DFTE – Natal – Brazil

10/29/13

OUR GOALS

- Growth of an asymptotically scale-free network including metrics.
- Growth of a geographically localized network (around its baricenter).
- To exhibit effects of competition between metrical neighborhood, connectivity and fitness.
- Last but not least, to exhibit the connection between scale-free networks and nonextensive statistics.

Internet

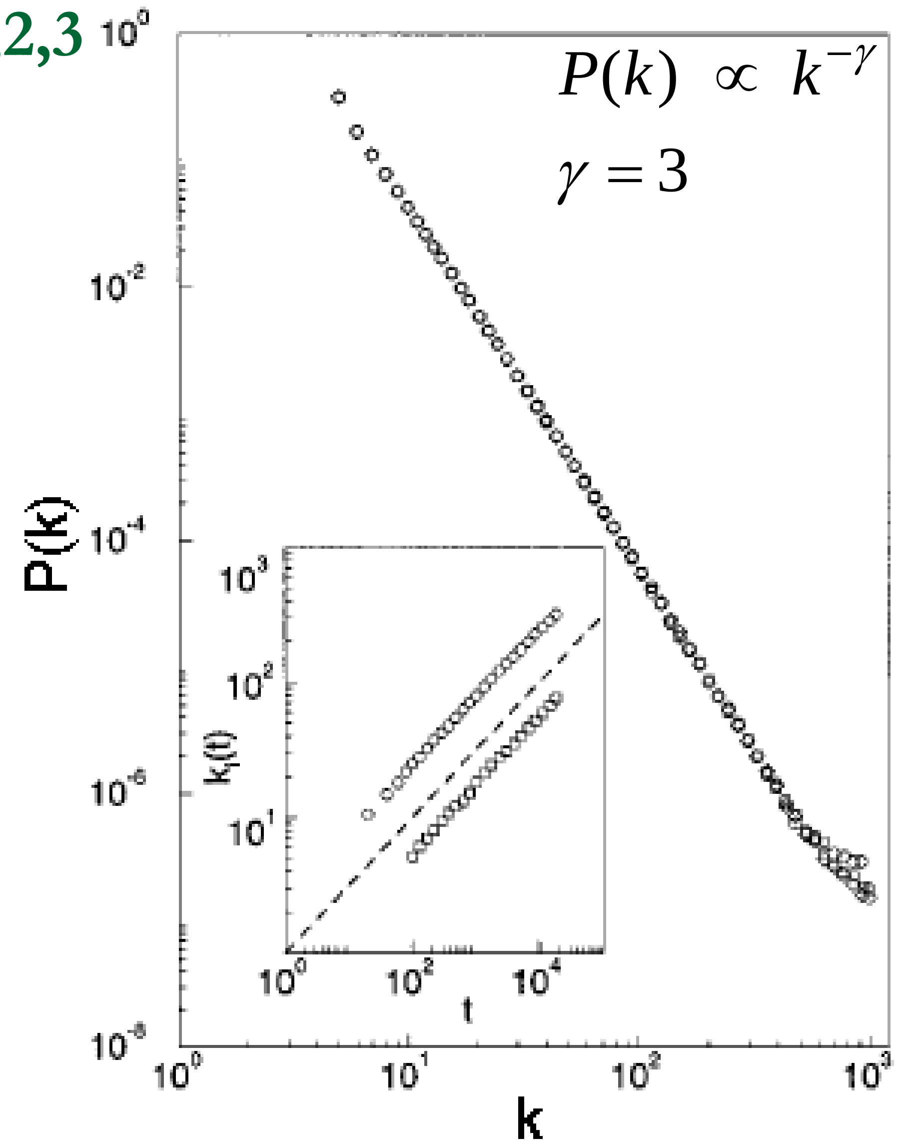


Scale-free Networks[1,2,3]

- Barabási and Albert[1];

$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N'} k_j} \qquad (01)$$

$$< k_i >= \left(\frac{t}{i}\right)^{\beta} \qquad (02)$$



[1]Science **286**, 509 (1999) ; Rev. Mod Phys. **74**, 47 (2002)

[2]M. Boguñá and R. Pastor-Satorras, Physical Review E **68**, 036112 (2003)

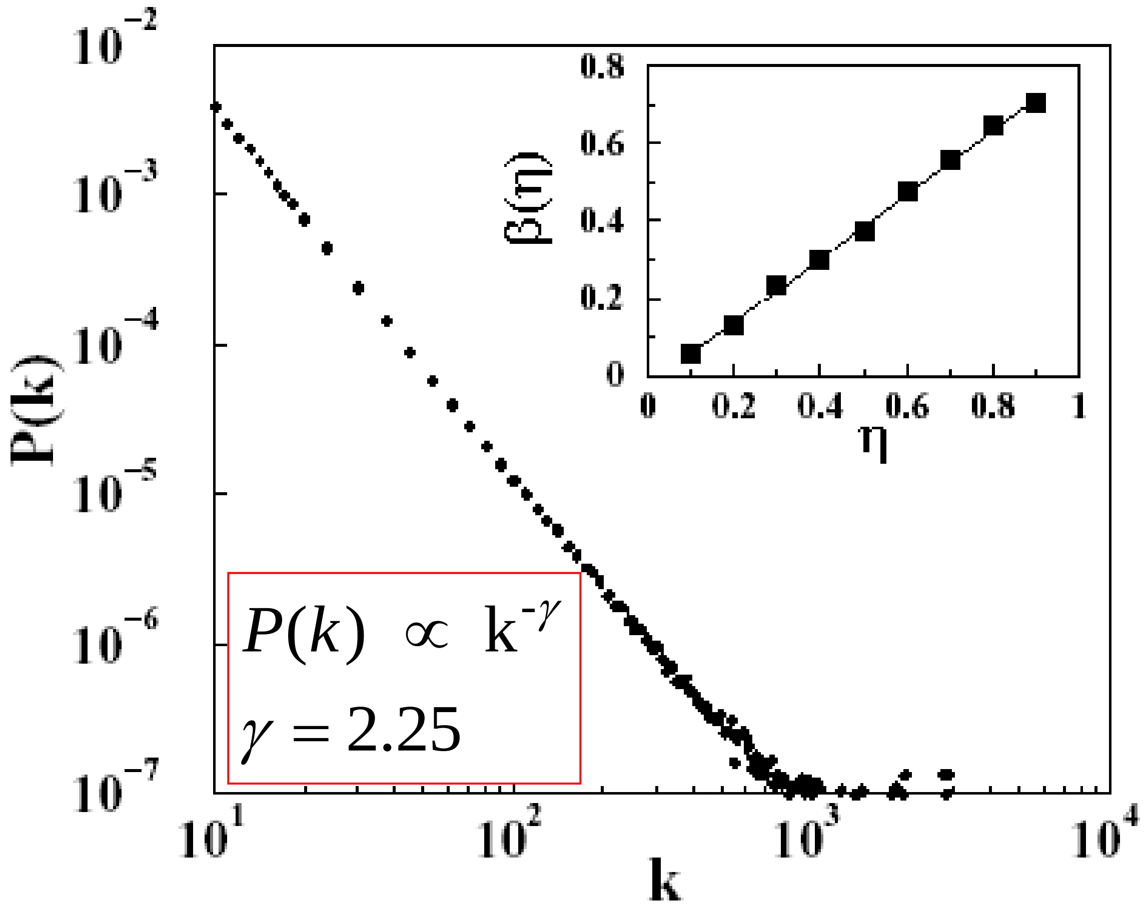
[3]S. Thurner and C. Tsallis, Europhycs Letters **72**, 197 (2005)

Fitness Model

- Bianconi and Barabási[4];
- Albert and Barabási[5];

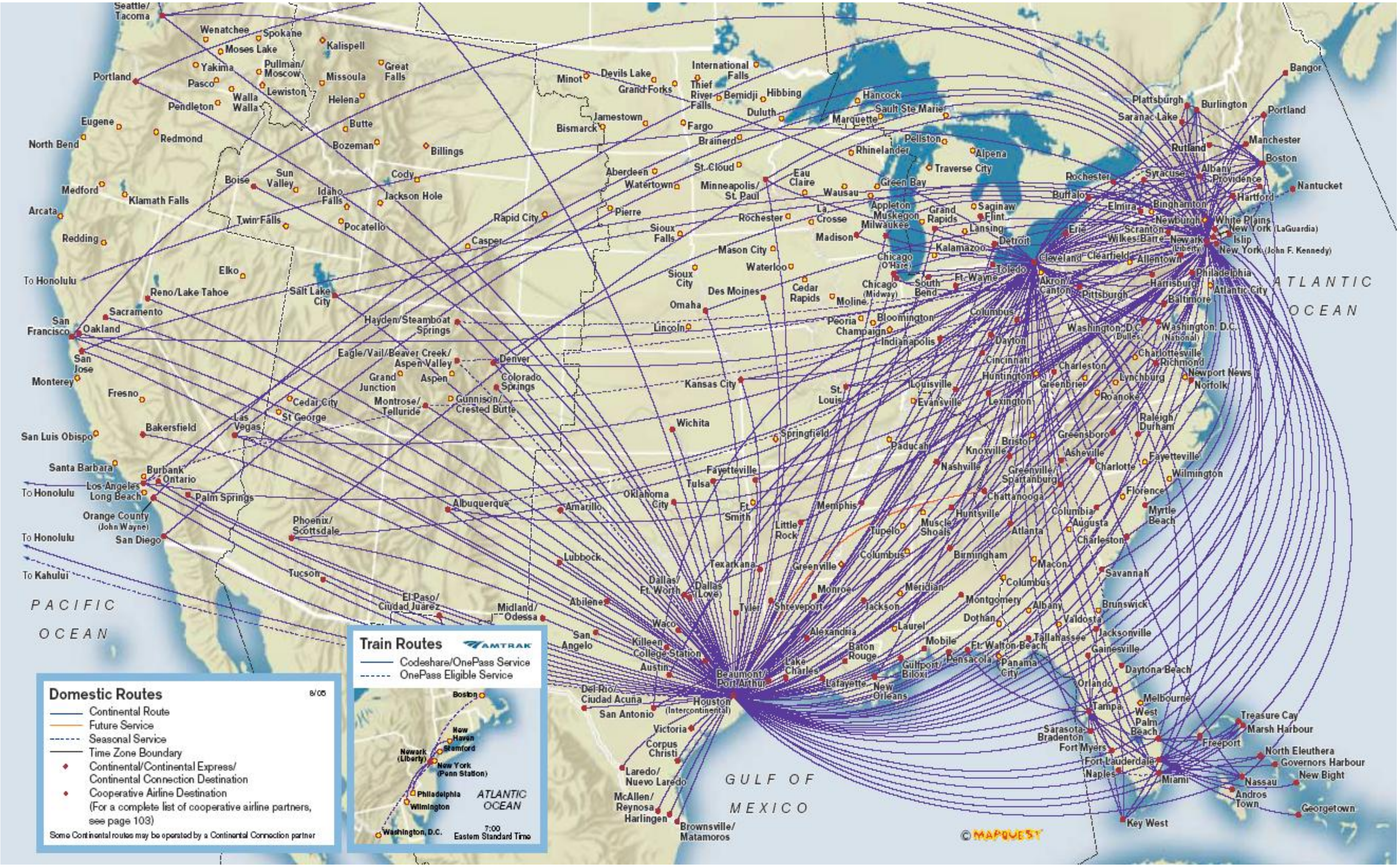
$$\Pi(k_i) = \frac{k_i \eta_i}{\sum_{j=1}^{N'} k_j \eta_j} \quad (04)$$

$$< k_i >= \left(\frac{t}{i}\right)^{\beta(\eta_i)} \quad (05)$$



[4]Europhys. Lett. **54**,436 (2001) ; [5]Rev. Mod Phys. **74**, 47 (2002)

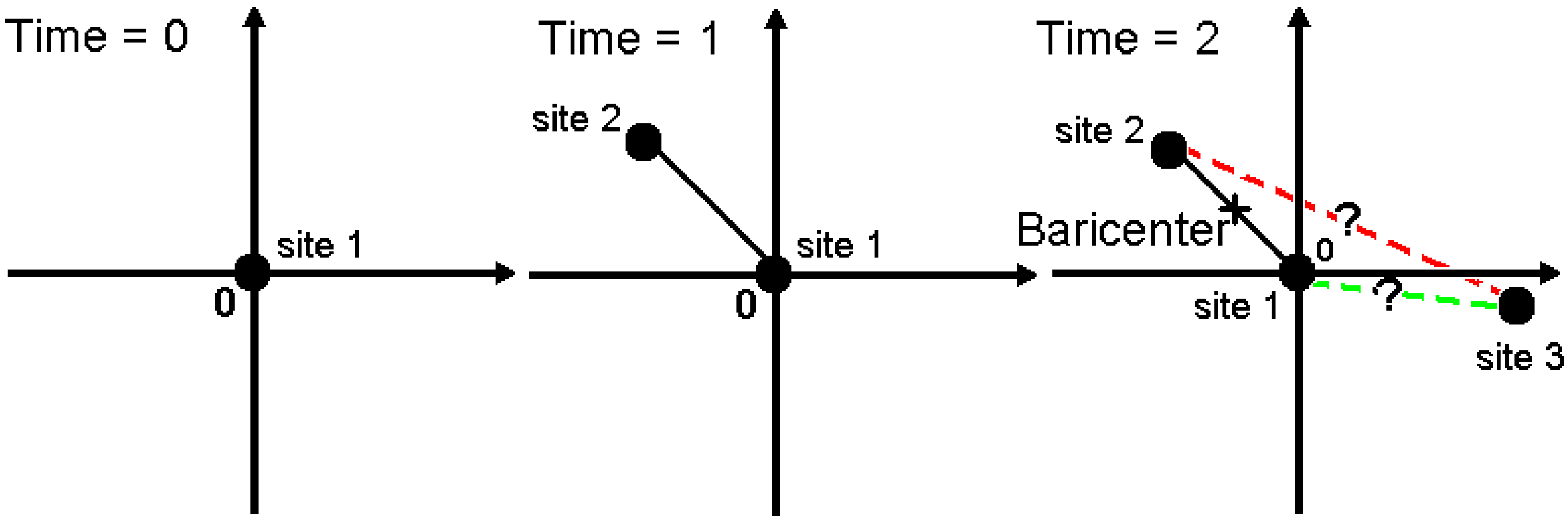
Geographic Model



Continental Airlines

Barabási-Albert Model with Euclidean Distance Power-law Distributed

Network Construction:



$$r = (1-\xi')^{-(2+\alpha_G)}$$

$$\theta = 2\pi\xi''$$

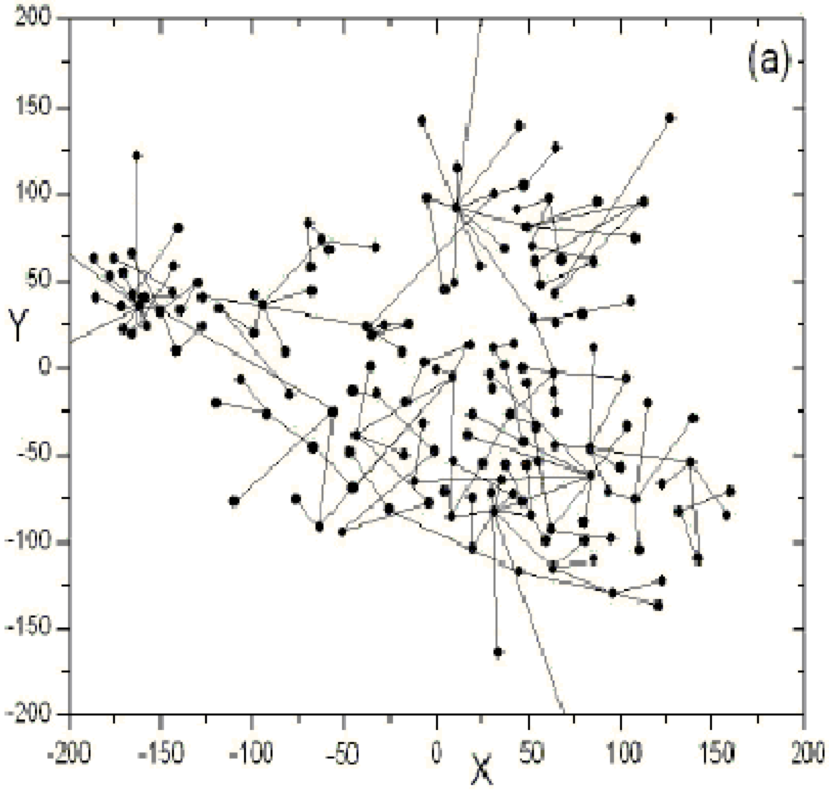
$$\alpha_G > 0$$

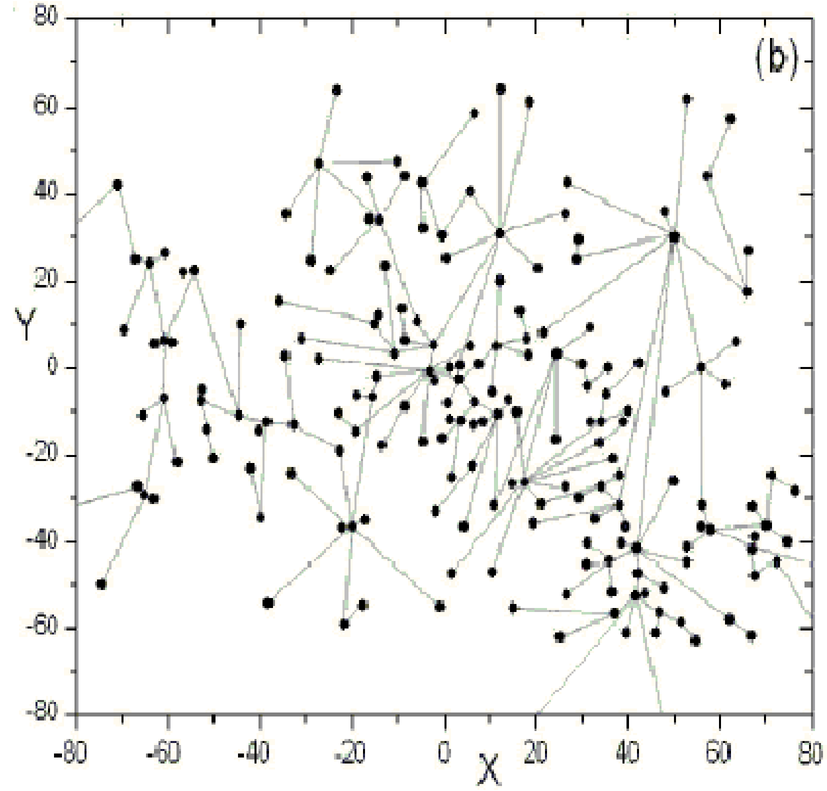
$$P(r) \propto r^{-\gamma_G}$$

$$\gamma_G > 1$$

$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}}$$

Examples

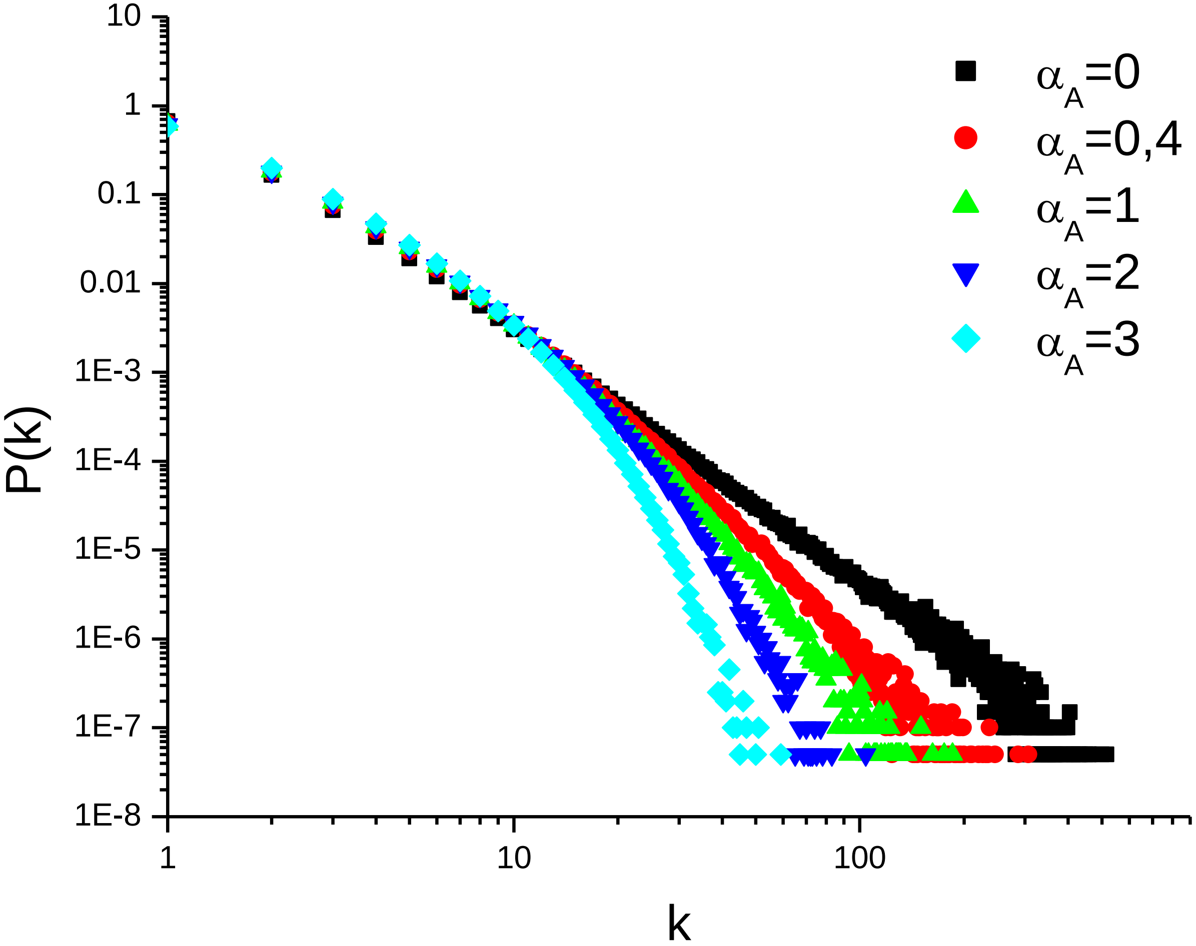




N = 250 nodes (a) $(\alpha_G, \alpha_A) = (1, 0)$ and (b) $(\alpha_G, \alpha_A) = (1, 4)$.
The starting site is at $(X, Y) = (0, 0)$. Notice the spontaneous emergence of hubs.

Barabási-Albert Model with Euclidean Distance Power-law Distributed

$$\Pi(k_i) = \frac{k_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j / r_j^{\alpha_A}} \quad (03)$$

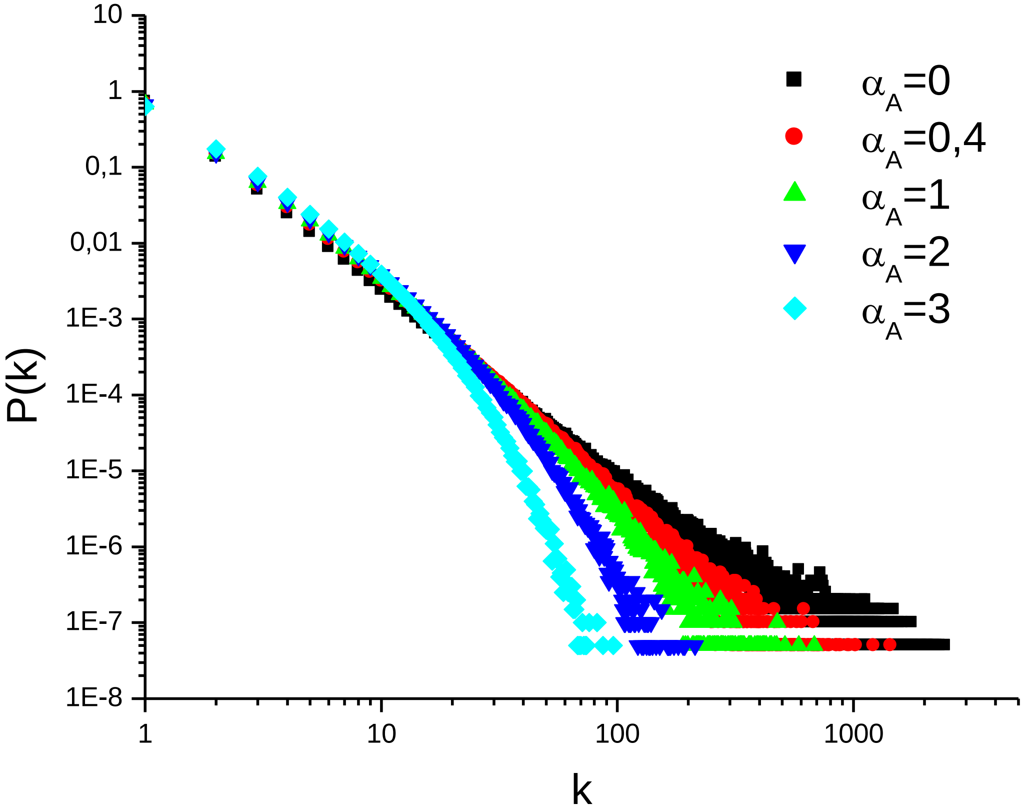


Soares, Tsallis, Mariz and da Silva, Europhys. Lett. **70,** 70 (2005)

Fitness Model with Euclidean Distance Power-law Distributed

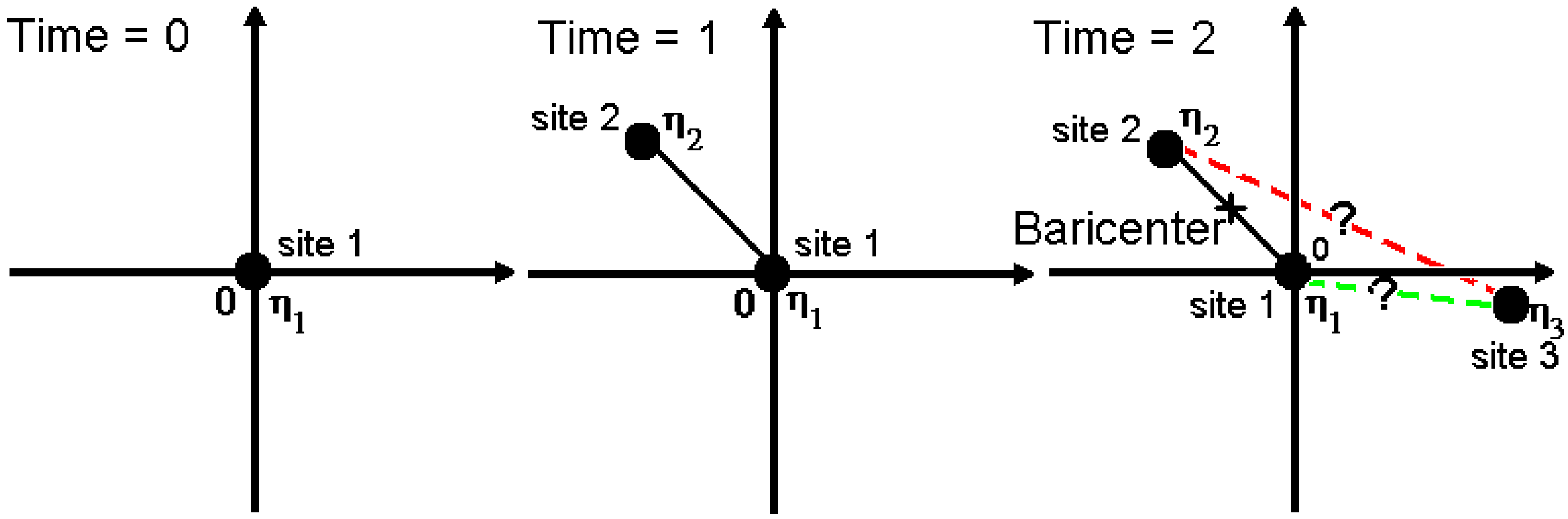
- Inspired in the works of Soares, Tsallis, Mariz and da Silva[3],
  and Bianconi and Barabasi[4].

$$\Pi(k_i) = \frac{k_i \eta_i / r_i^{\alpha_A}}{\sum_{j=1}^{N'} k_j \eta_j / r_j^{\alpha_A}} \qquad (06)$$



Meneses, Cunha, Soares, and da Silva, Progress of Theoretical Physics Supplement **162,** 131 (2006)

Network Construction



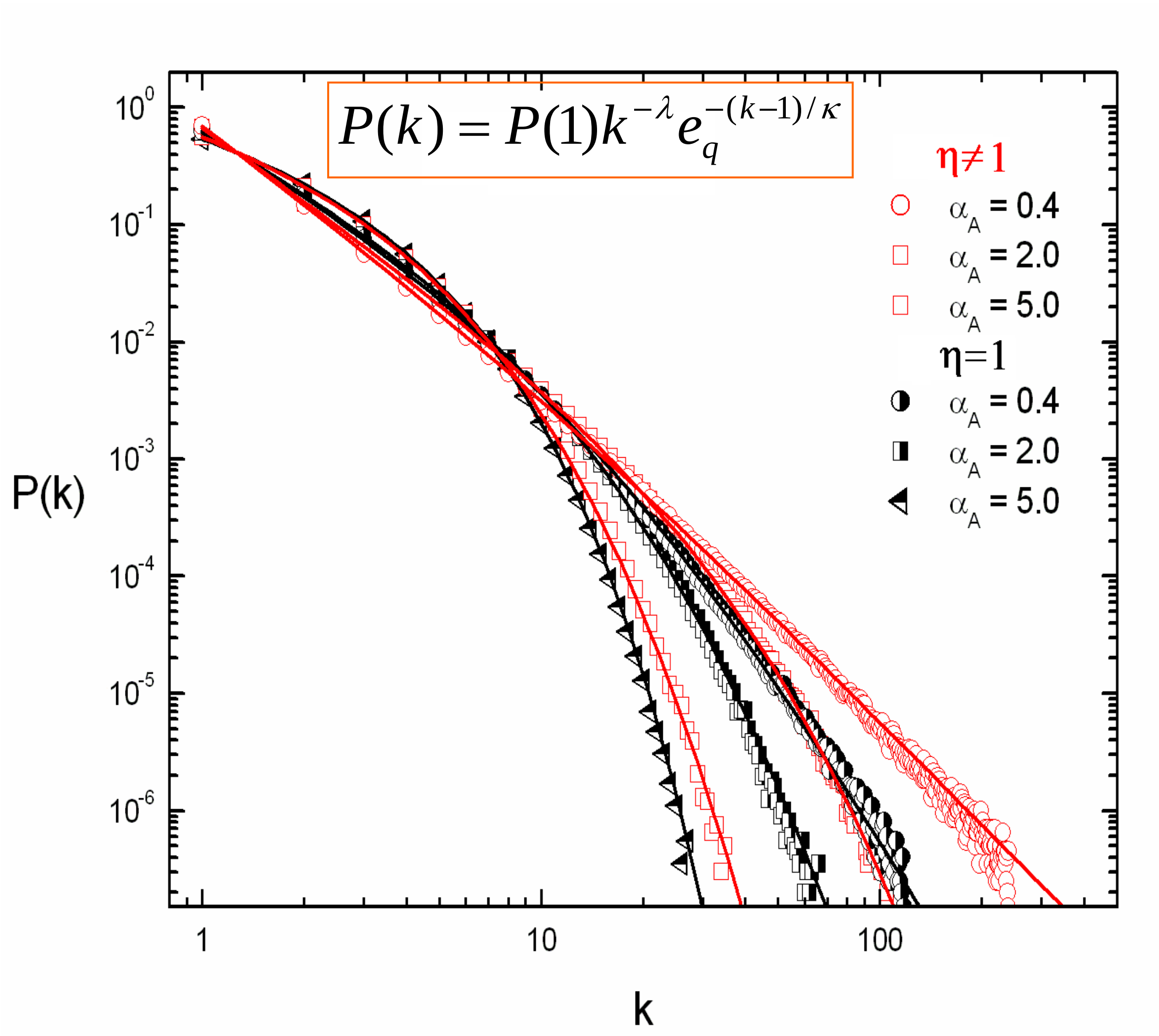
Tsallis Nonextensive Statistical Mechanics

$$s_q = \frac{1 - \int dk [P(k)]^q}{q-1} \qquad (q \in \Re; S_1 = S_{BG})$$

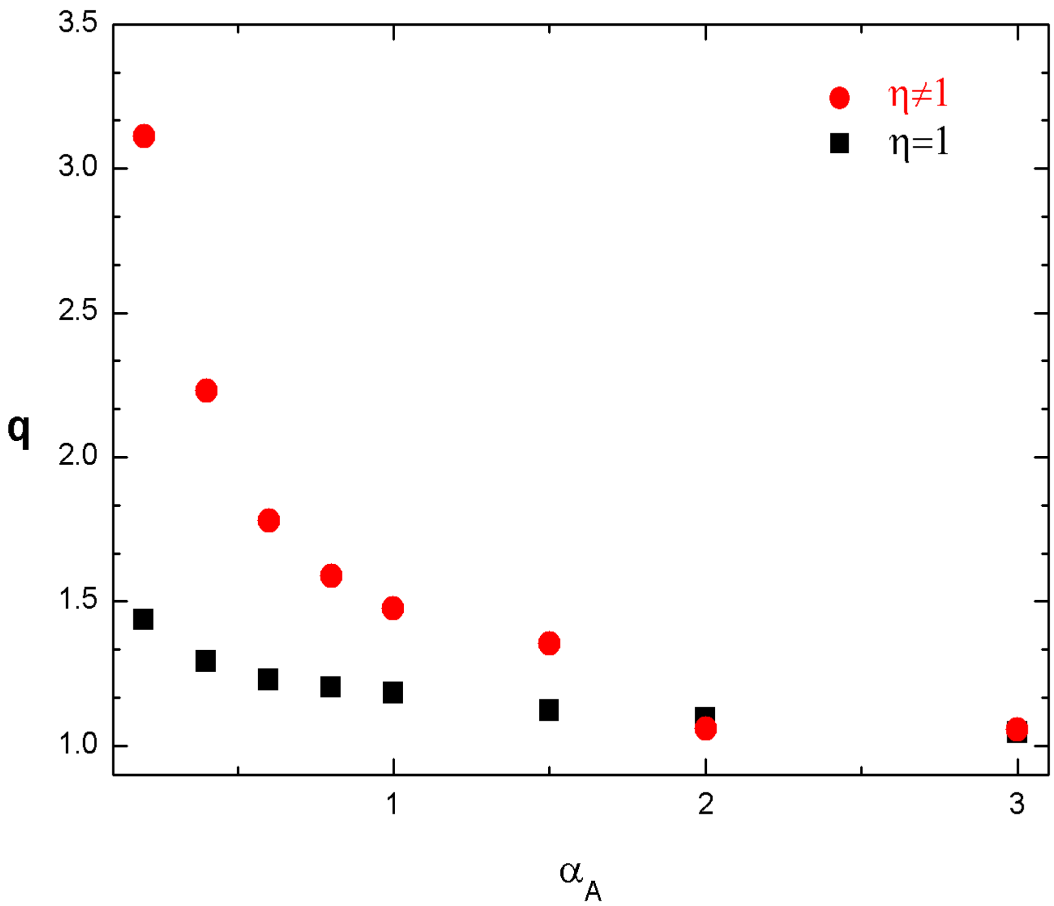
$$\sum_i p_i = 1 \quad \text{and} \quad \frac{\sum_i p_i{}^q \varepsilon_i}{\sum_i p_i{}^q} = U_q.$$

$$P(k) = P(1) k^{-\lambda} e_q^{-(k-1)/\kappa}$$

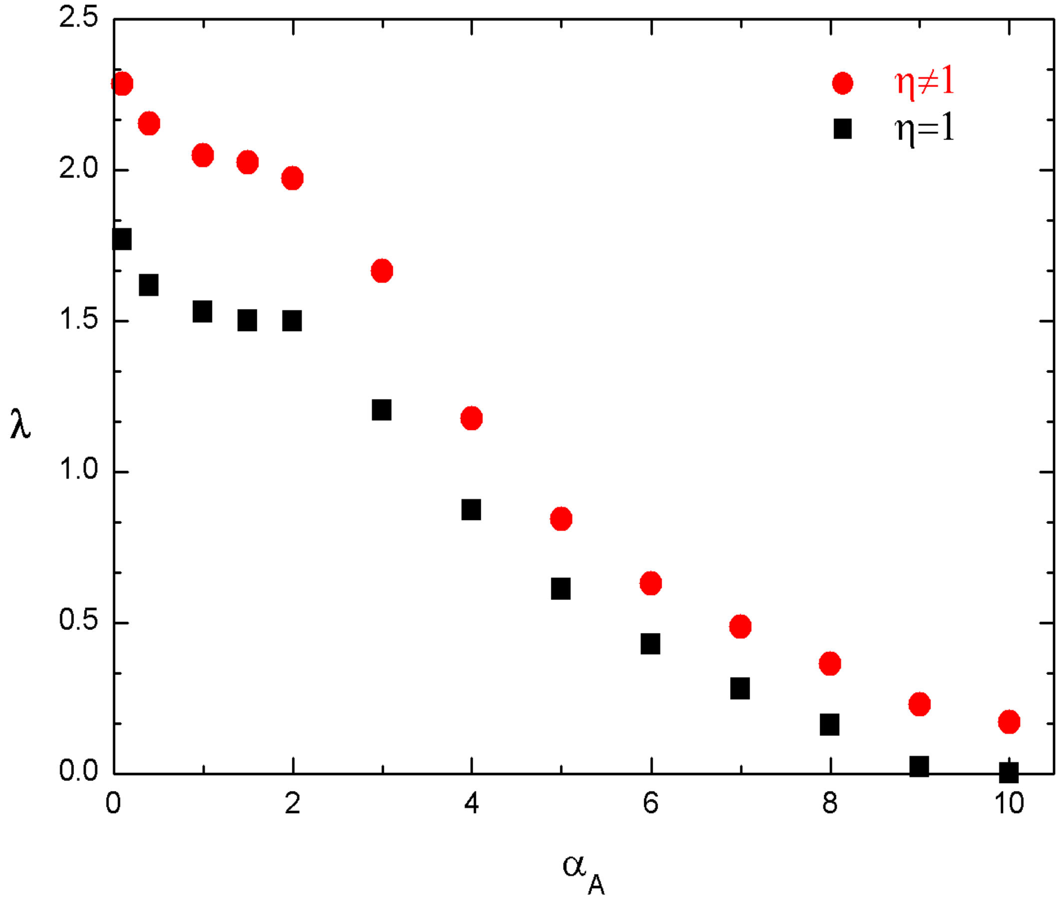
$$e_q^x \equiv [1 + (1-q)x]^{1/(1-q)}$$



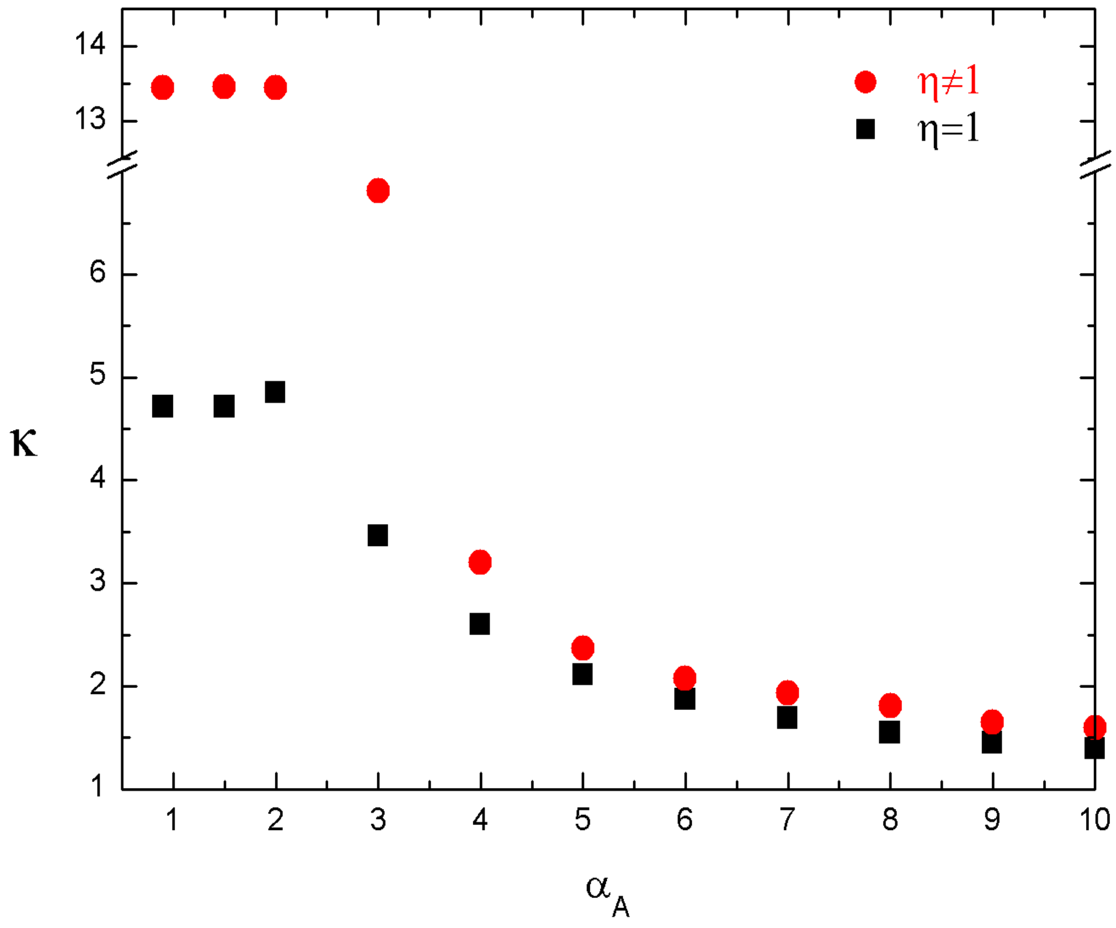
Connectivity distribution *P*(*k*) for typical values α_A for η≠1 and η=1 models. The symbols are numerical results and continuous lines are the best fits according to P(k).



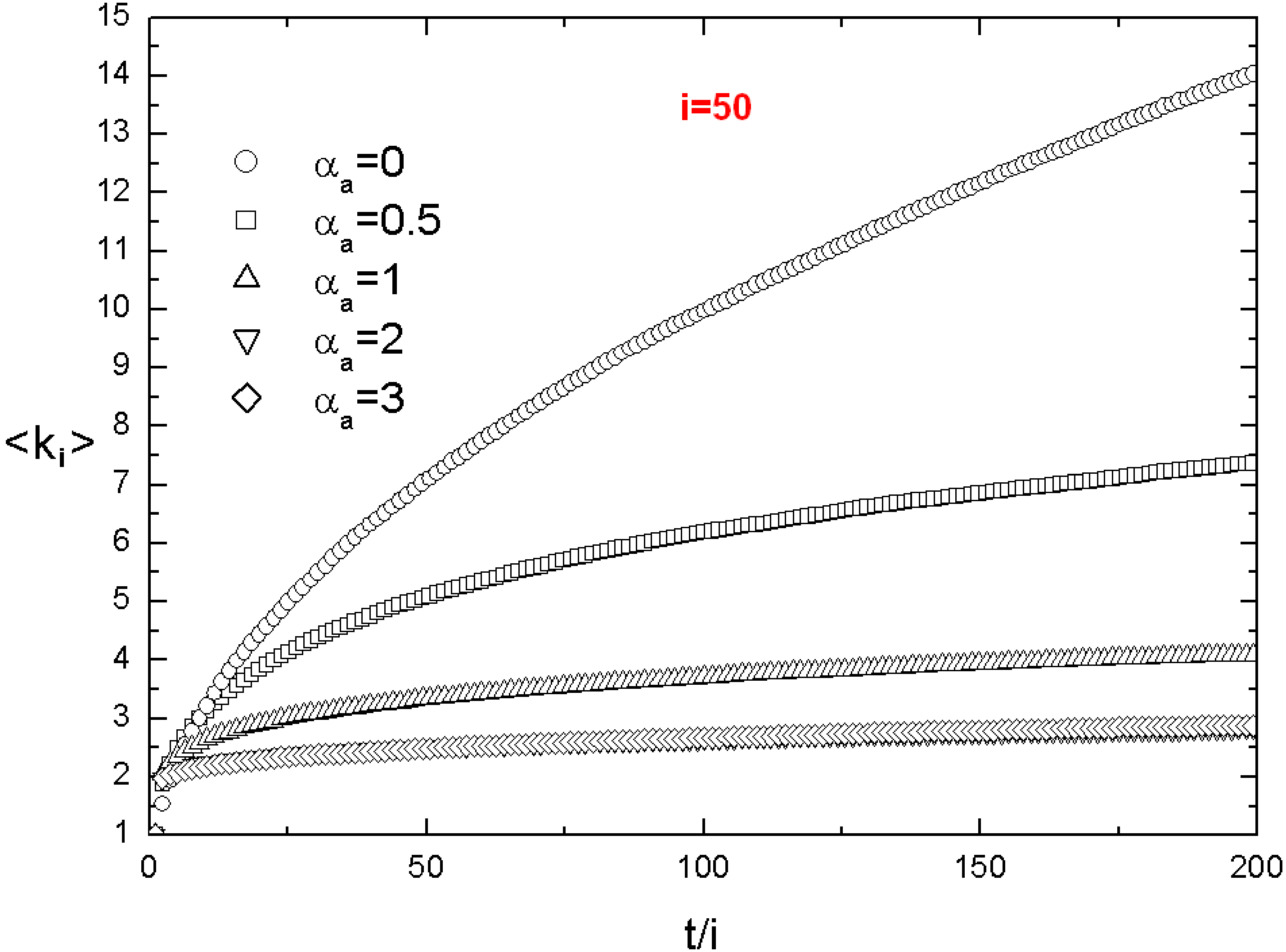
α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



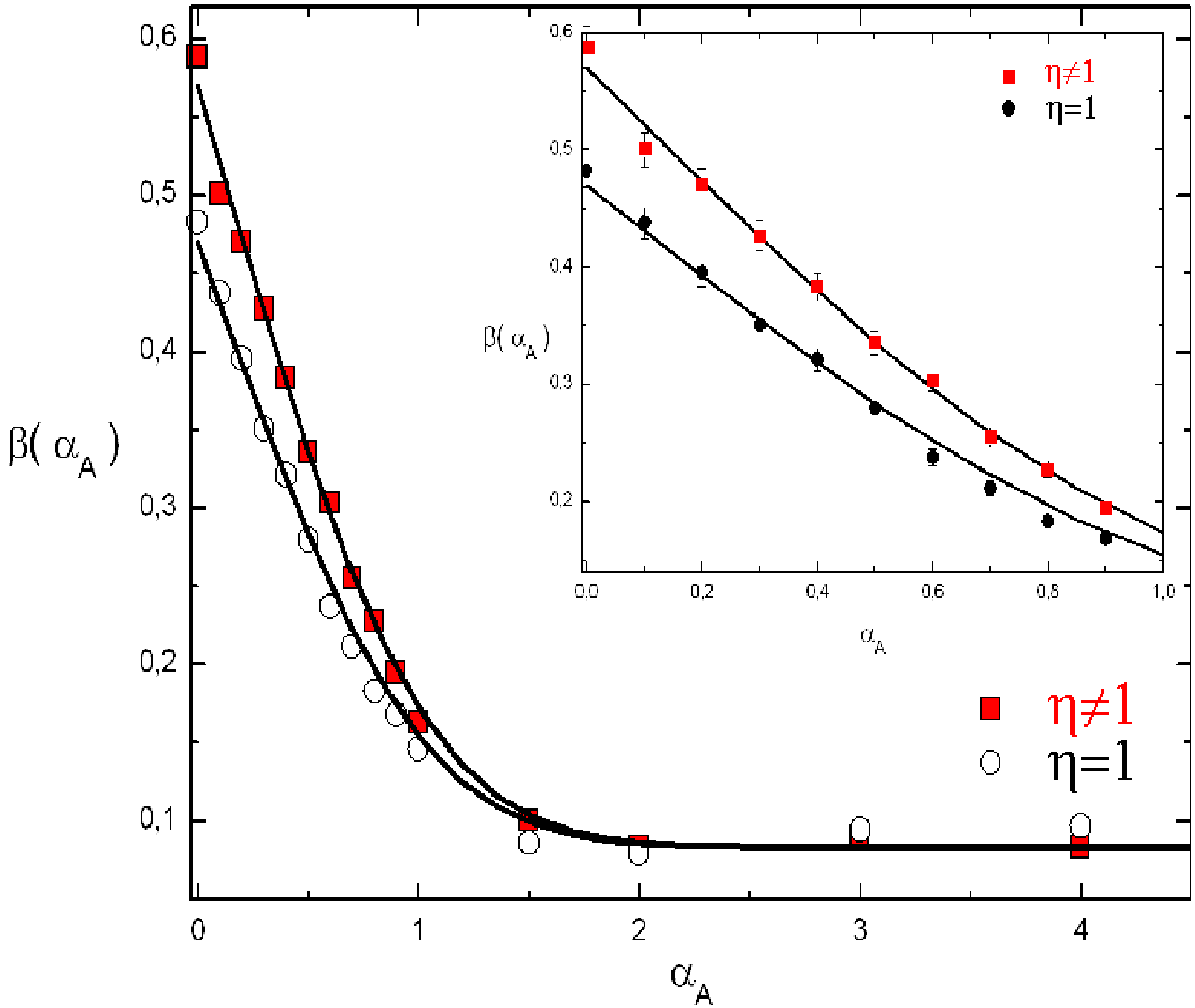
α_A-dependence of λ for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



α_A-dependence of q for both η≠1 and η=1 models. In this graph we observe some kinds of changements of regimes at α_A = 2 (which coincides with the space dimensionality).



Temporal dependence of the average connectivity for $\eta \neq 1$, in 2000 samples.



Average connectivity exponent for α_A values relative to measures on node i = 50.

- The generalized model contains the five previous models:

Model	CONECTIVITY	FITNESS	METRIC
Barabási-Albert	YES	NO	NO
Bianconi et al	YES	UNIFORM	NO
Soares et al	YES	NO	POWER-LAW
Meneses et al	YES	UNIFORM	POWER-LAW
Mendes et al	YES	POWER-LAW	NO
Generalized	YES	POWER-LAW	POWER-LAW

Affinity Model

- Inspired in the works of Bianconi and Barabási; G.A. Mendes and L.R. da Silva.
- The links between similar sites are favored.

$$\Pi_{i\to j} \sim k_j \left(1 - \left|\eta_i - \eta_j\right|\right)$$

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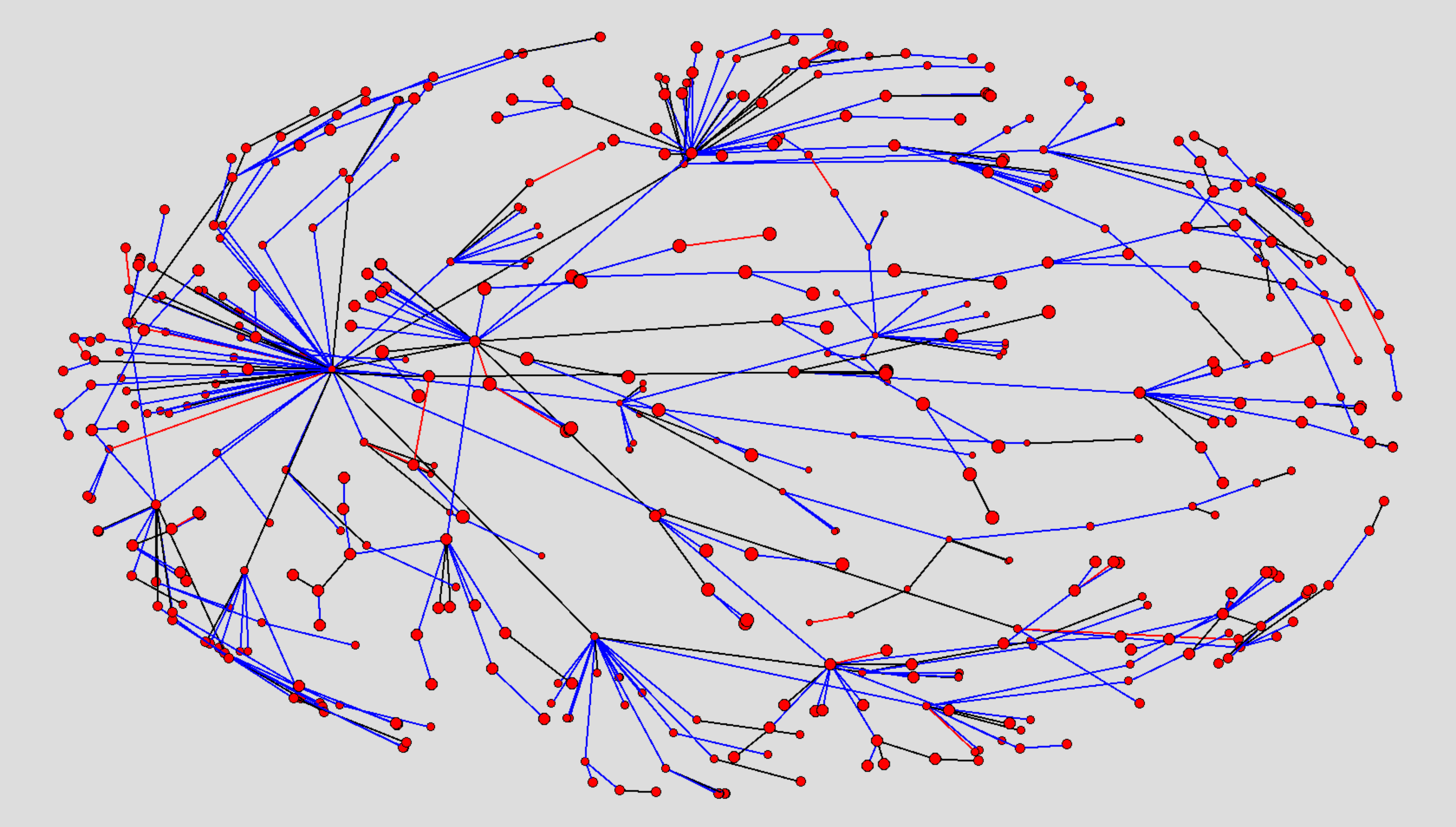
blue: big affinity; black: medium affinity; red: small affinity

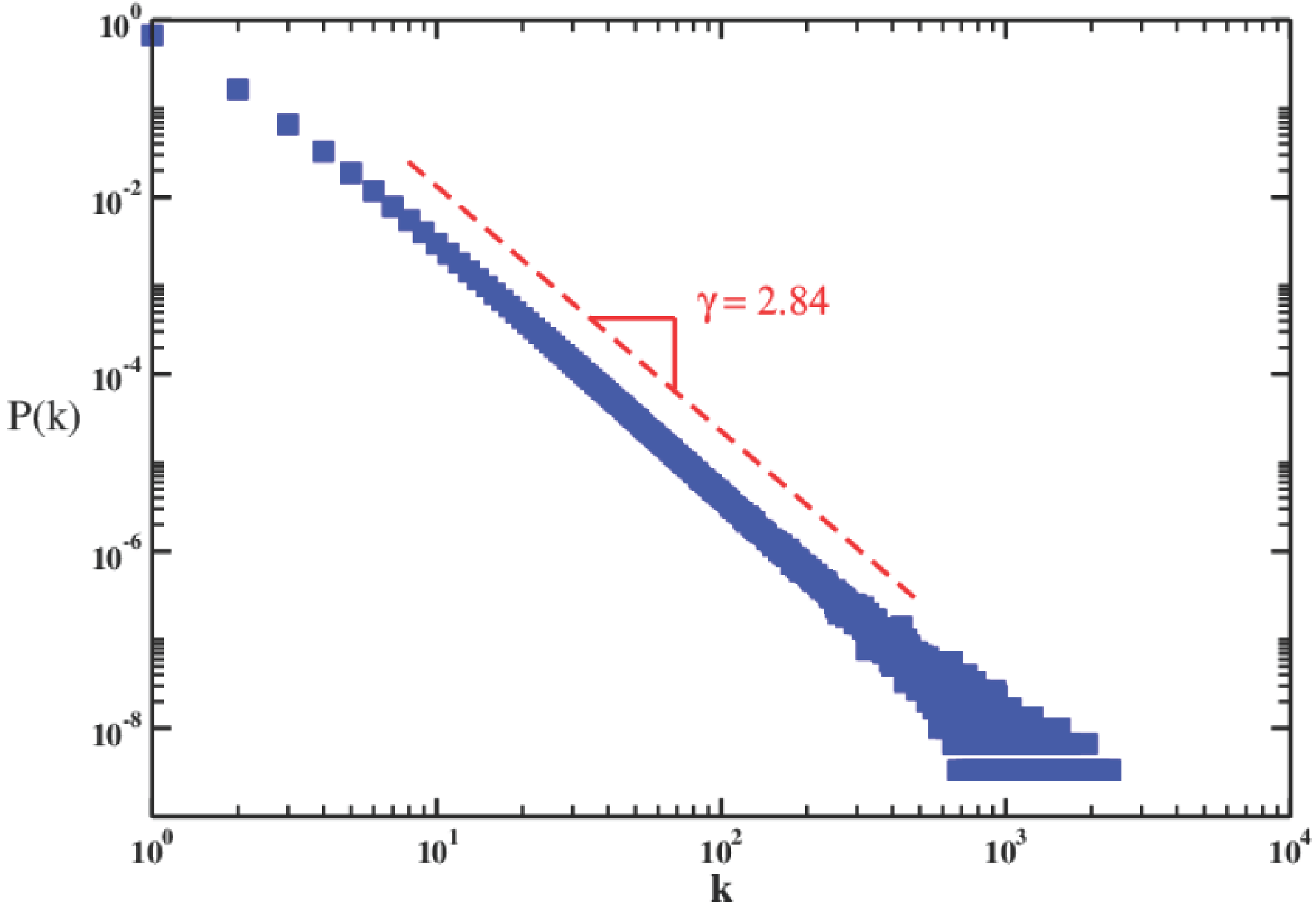


$$\Pi_{i \to j} \sim k_j \left(1 - \left|\eta_i - \eta_j\right|\right)$$

M.L. Almeida, G.A. Mendes, G.M. Viswanathan and L.R. da Silva , EPJ B 86 36 (2013)

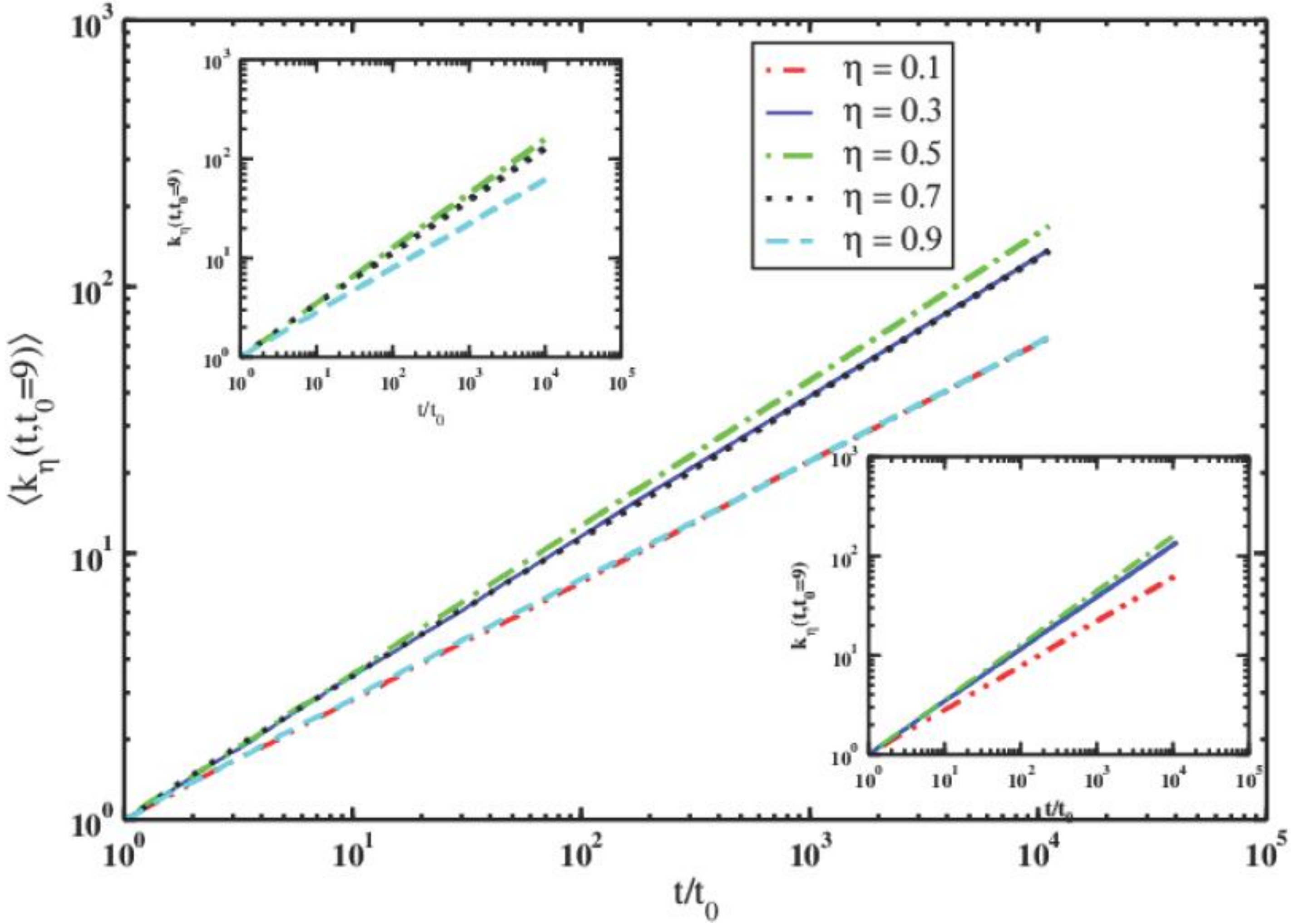
Affinity Model

$$\Pi_{i \to j} \sim k_j \left(1 - \left|\eta_i - \eta_j\right|\right)$$

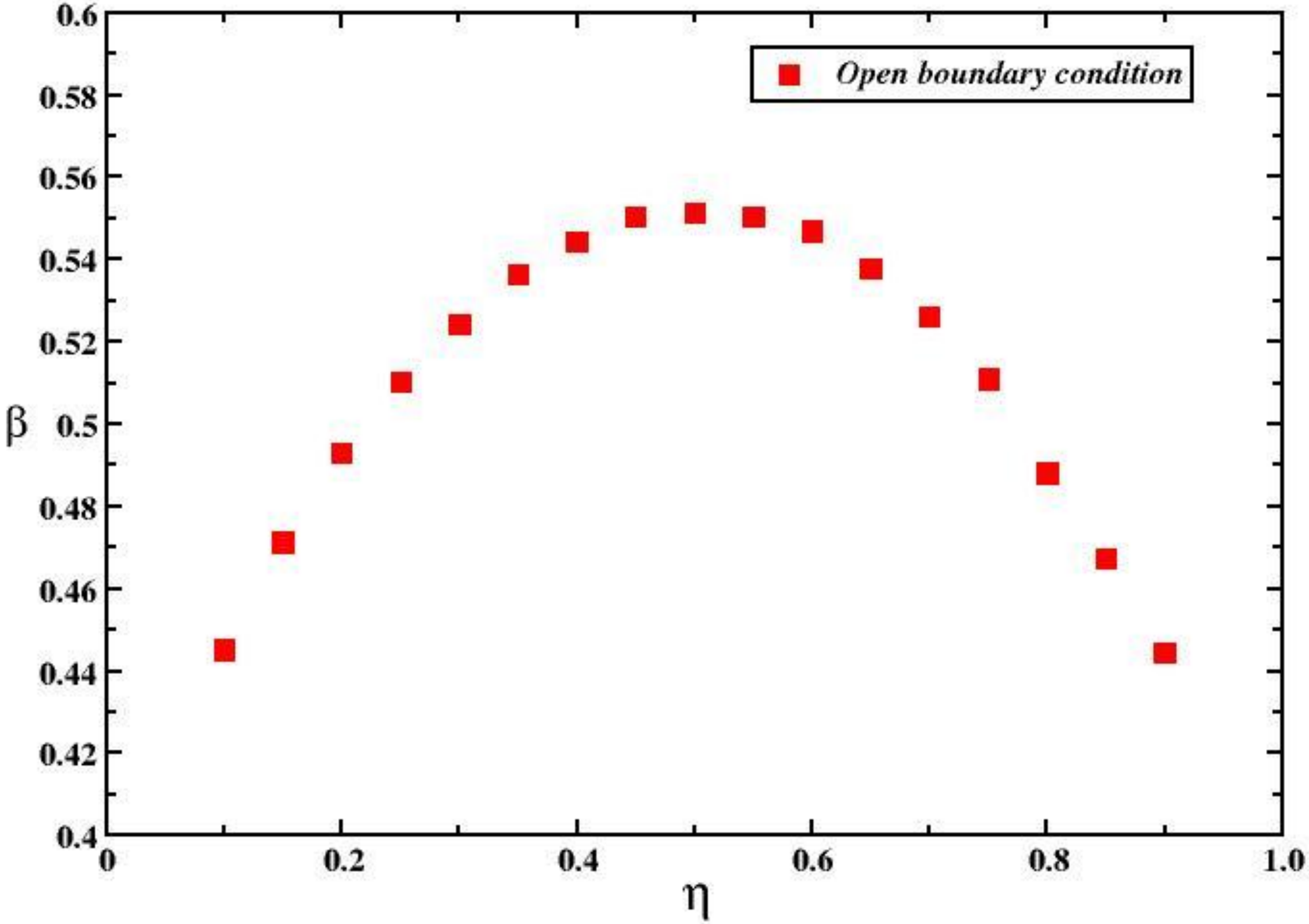


M.L. Almeida, G.A. Mendes, G.M. Viswanathan and L.R. da Silva, EPJ B 86 36 (2013)

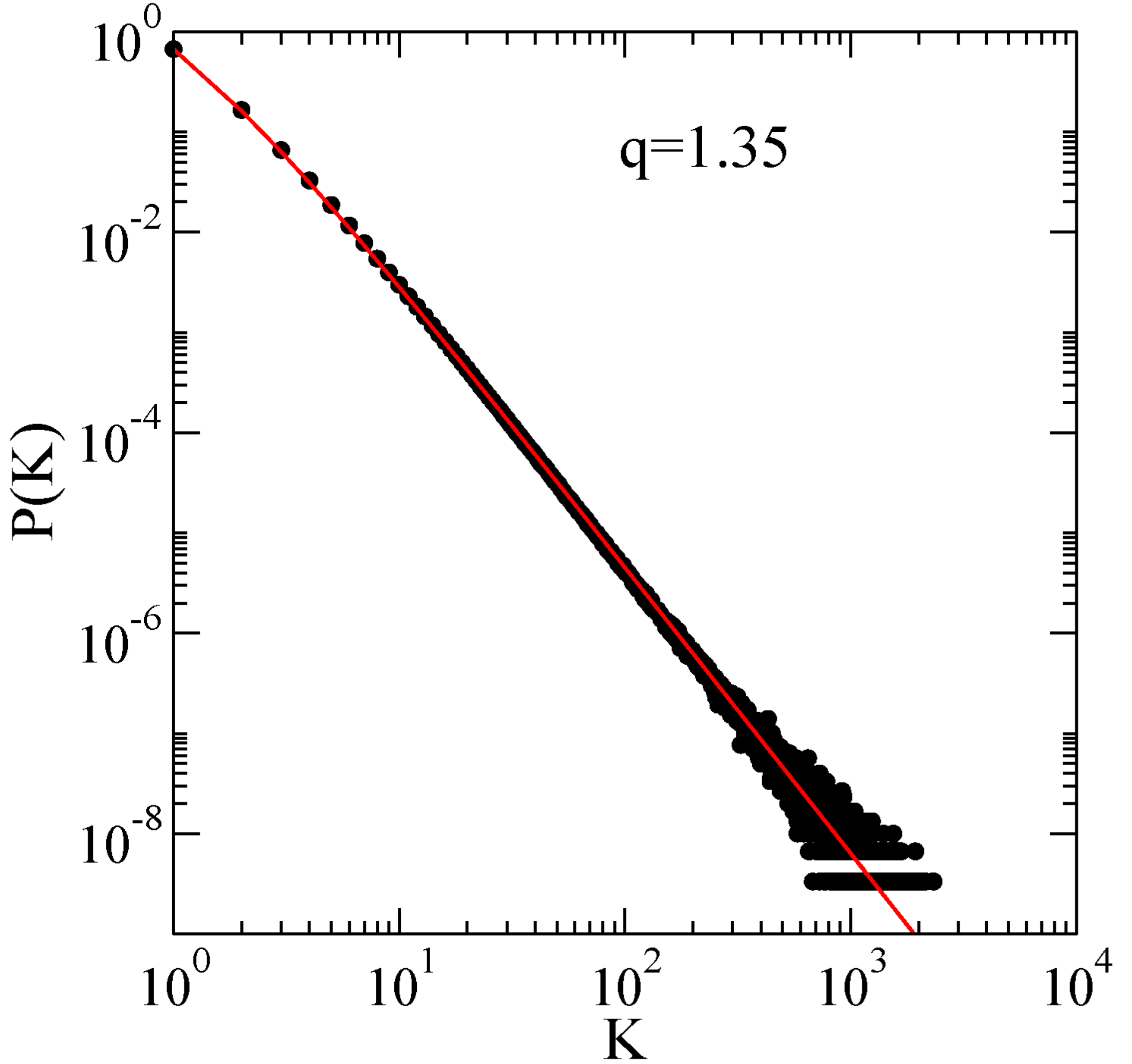
Affinity Model: Connectivity Time Evolution



Affinity Model



Affinity Model: Tsallis Statistics



NATAL MODEL: 1D*, 2D, 3D*

Brito, da Silva and Tsallis

* Work in progress

Natal Model 1D*, 2D, 3D*

Preferential attachment

$$\Pi_i = \frac{k_i r_{ij}^{-\alpha_A}}{\sum_j k_j r_{ij}^{-\alpha_A}}$$

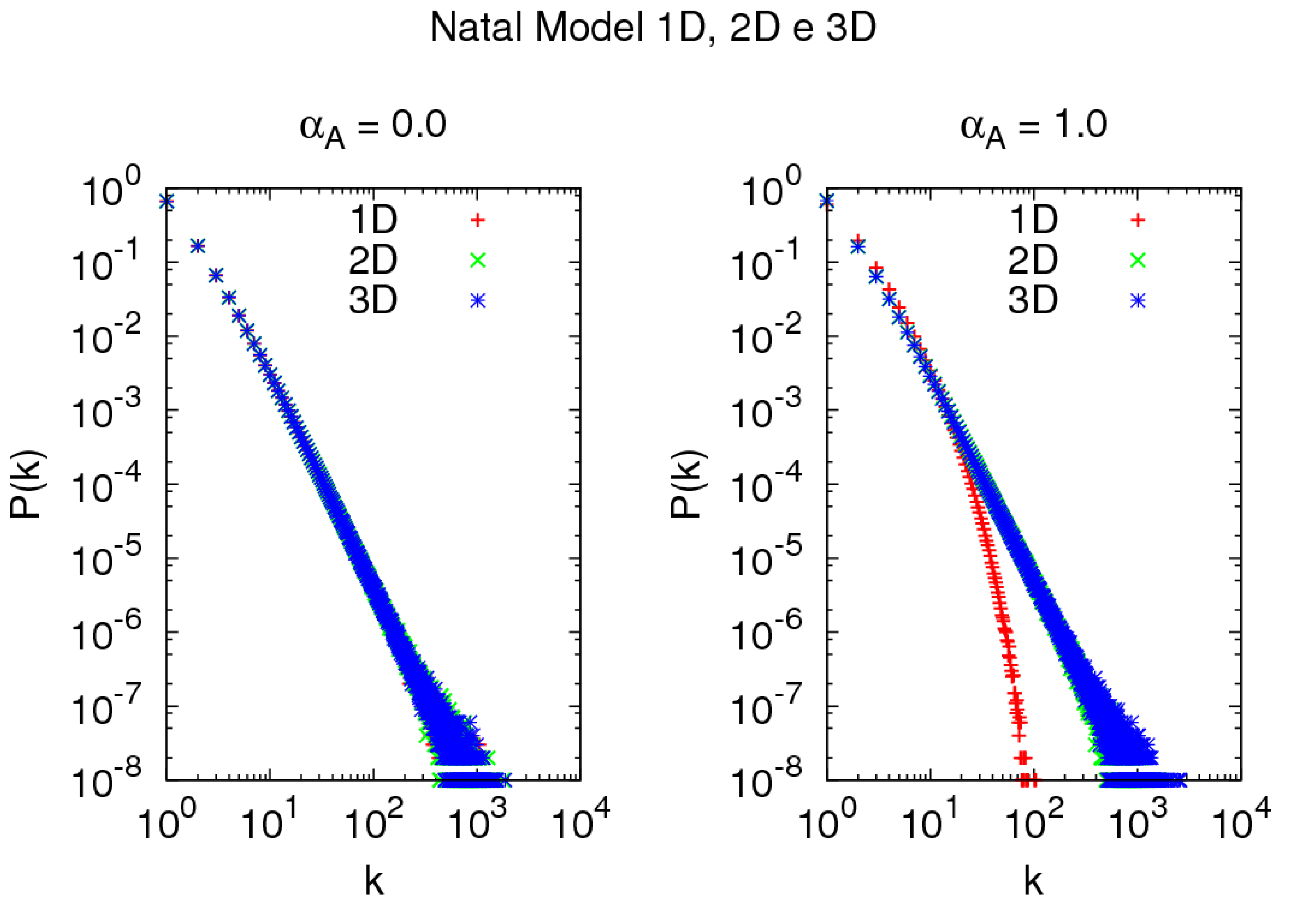
Distance distribution

$$P(r) \propto \frac{1}{r^{2+\alpha_G}}$$

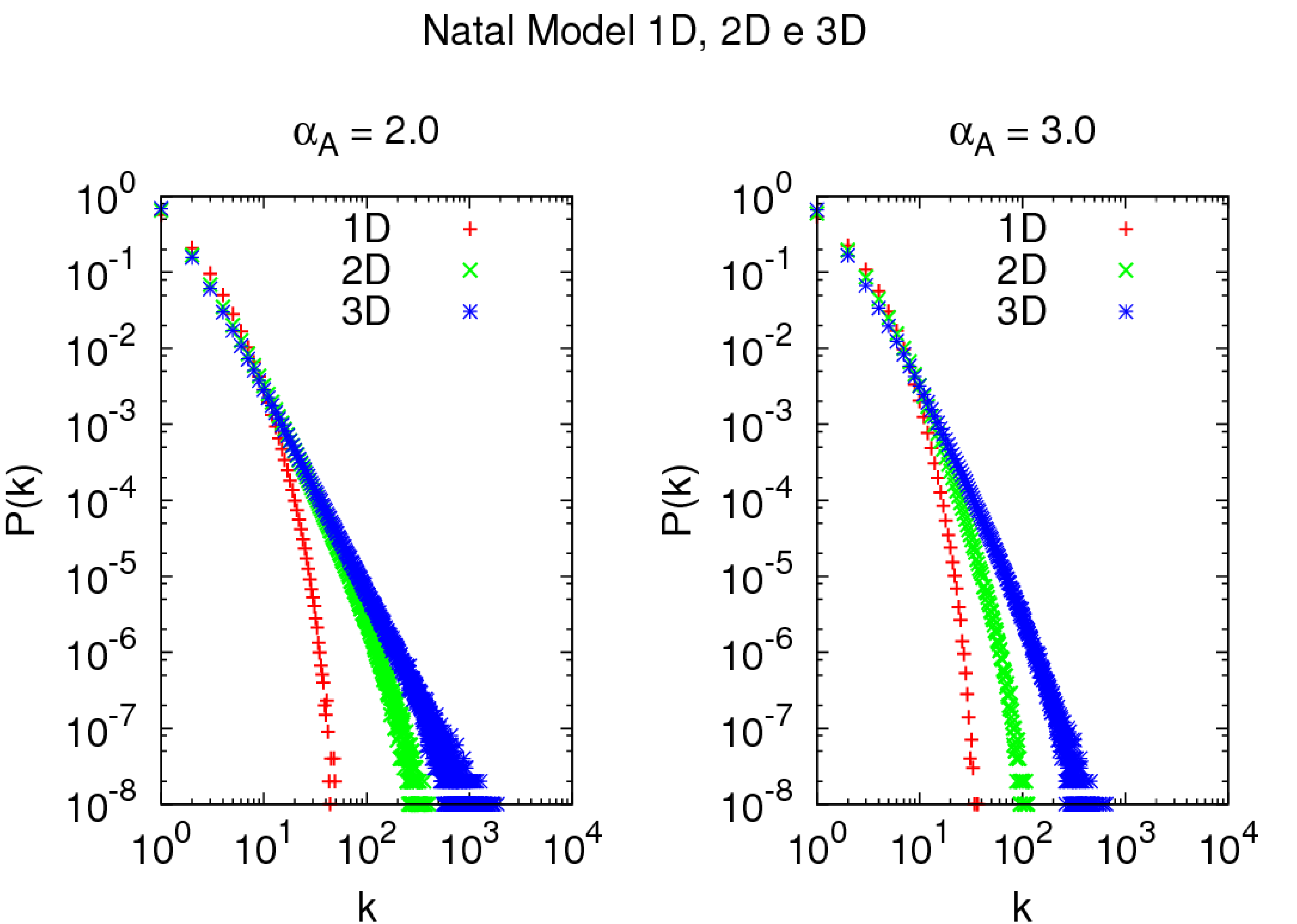
$$\alpha_G = 2$$

*Brito, da Silva, Tsallis, Work in progress

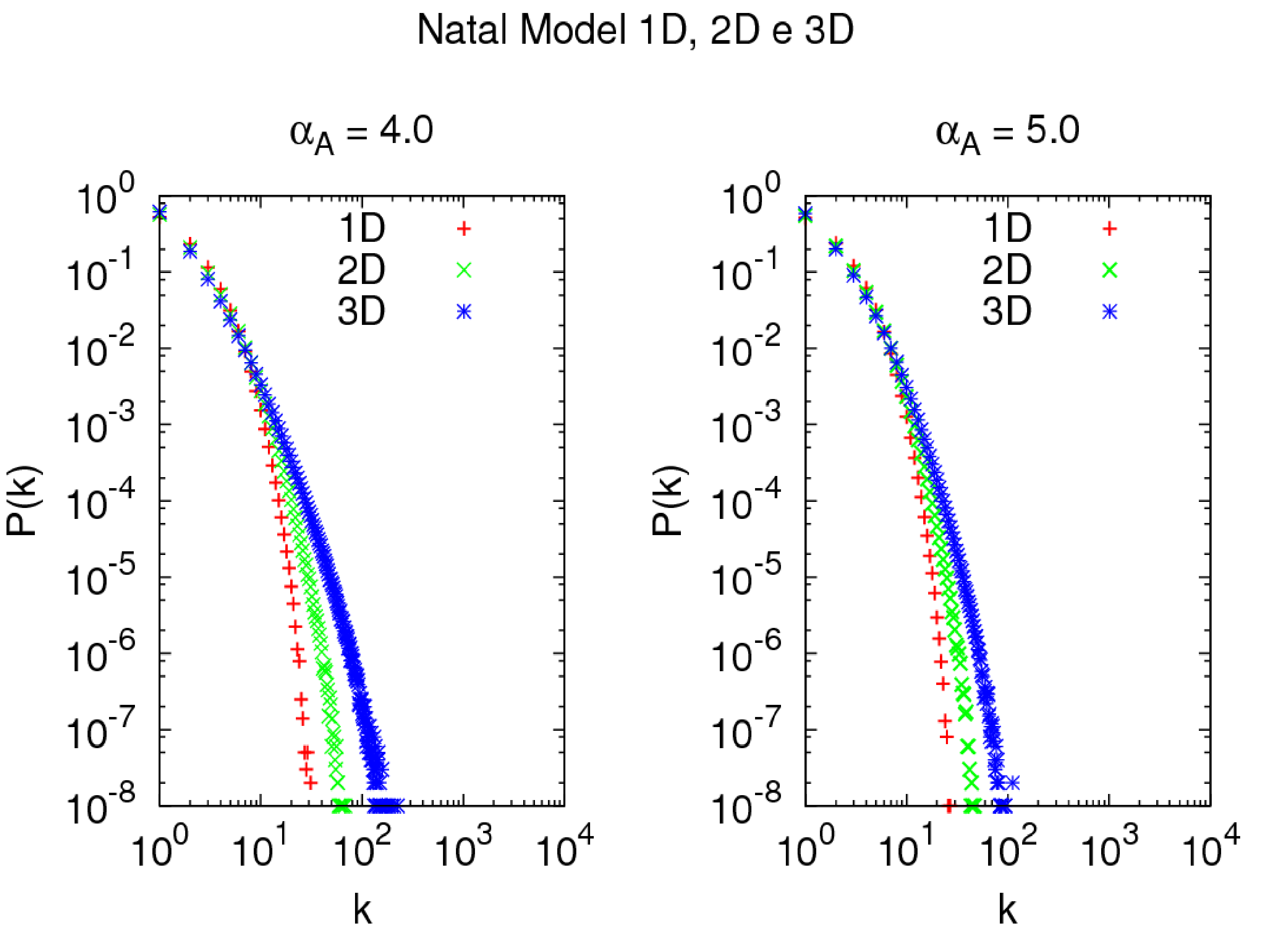
Degree distribution



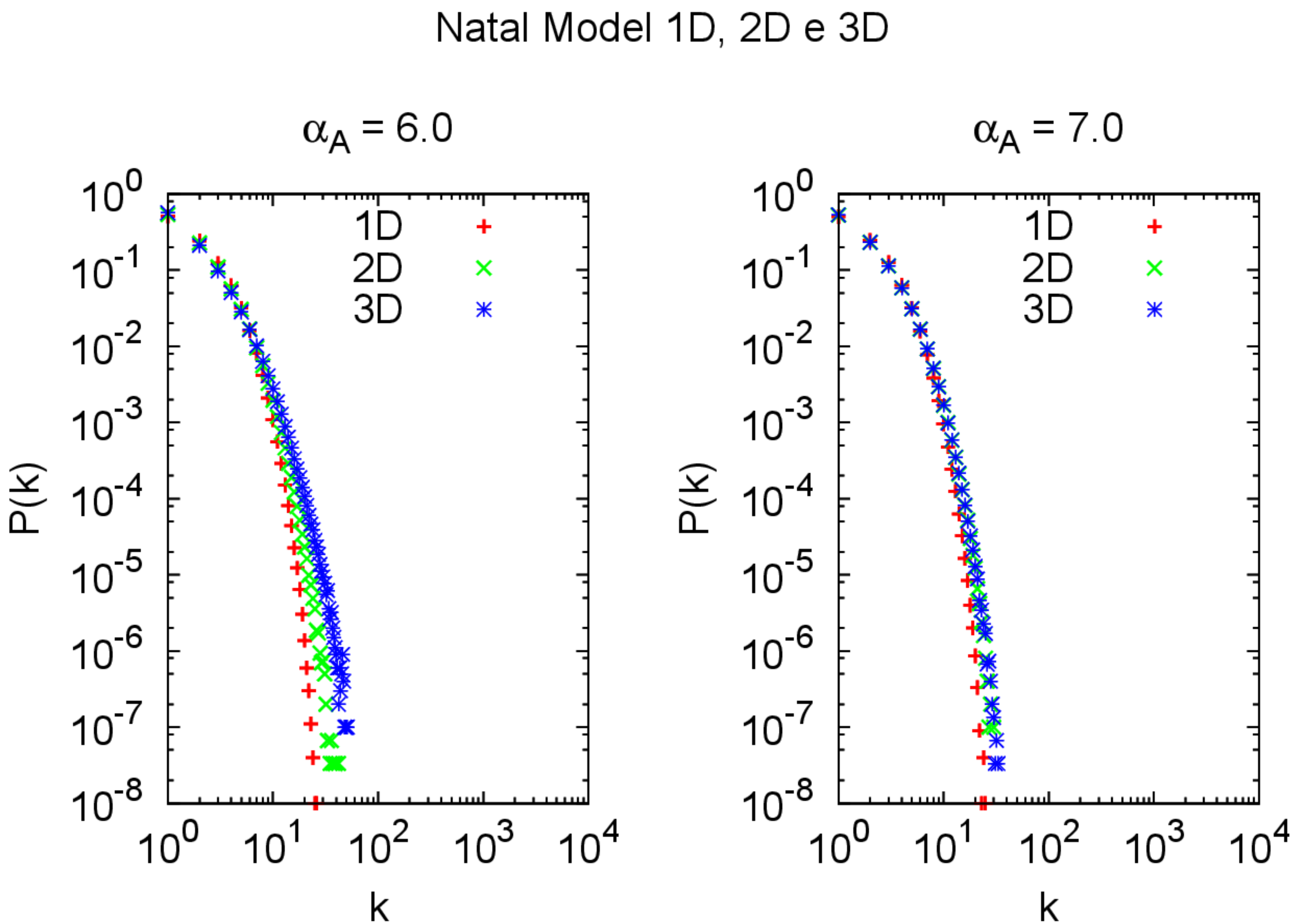
Degree distribution



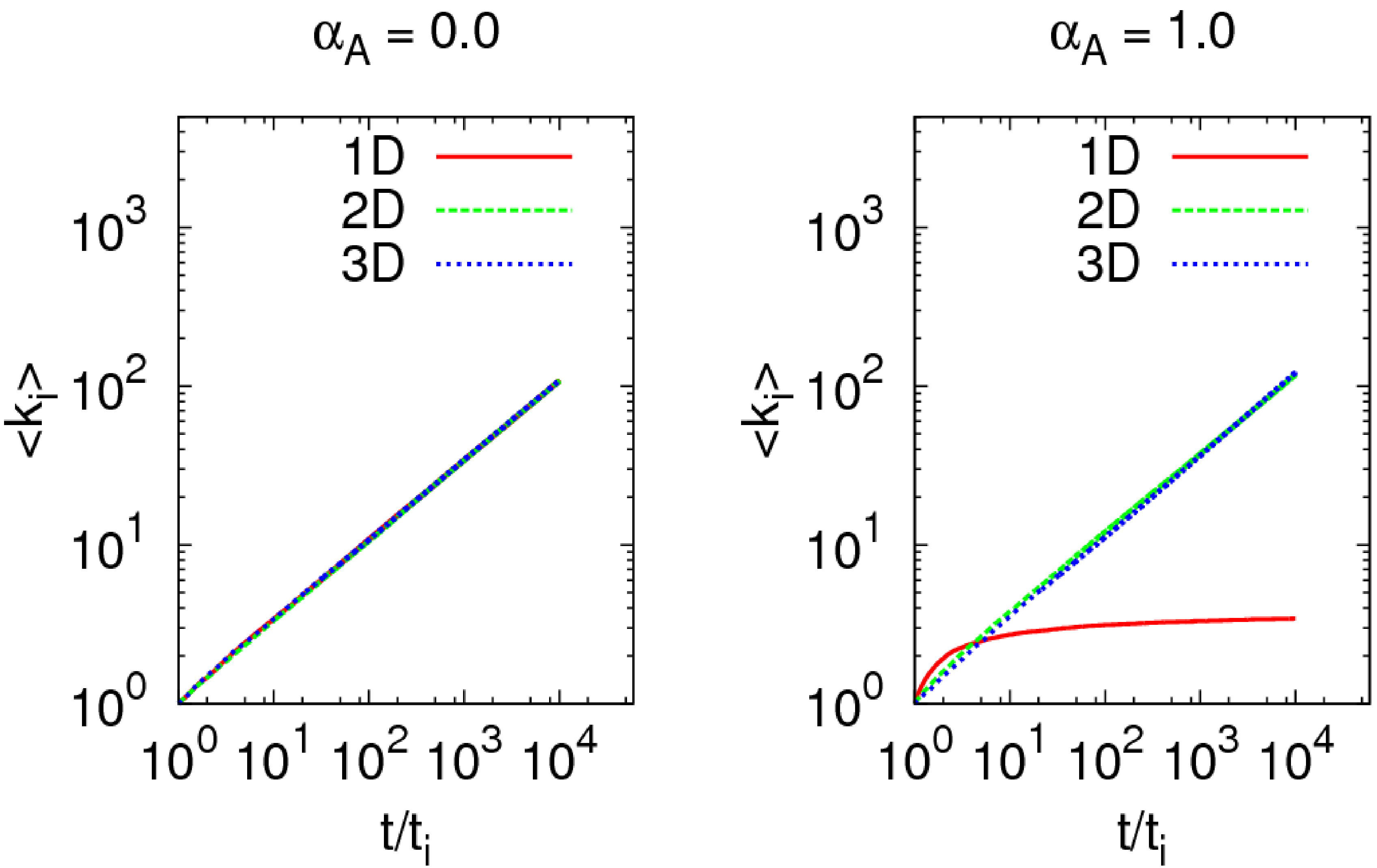
Degree distribution



Degree distribution

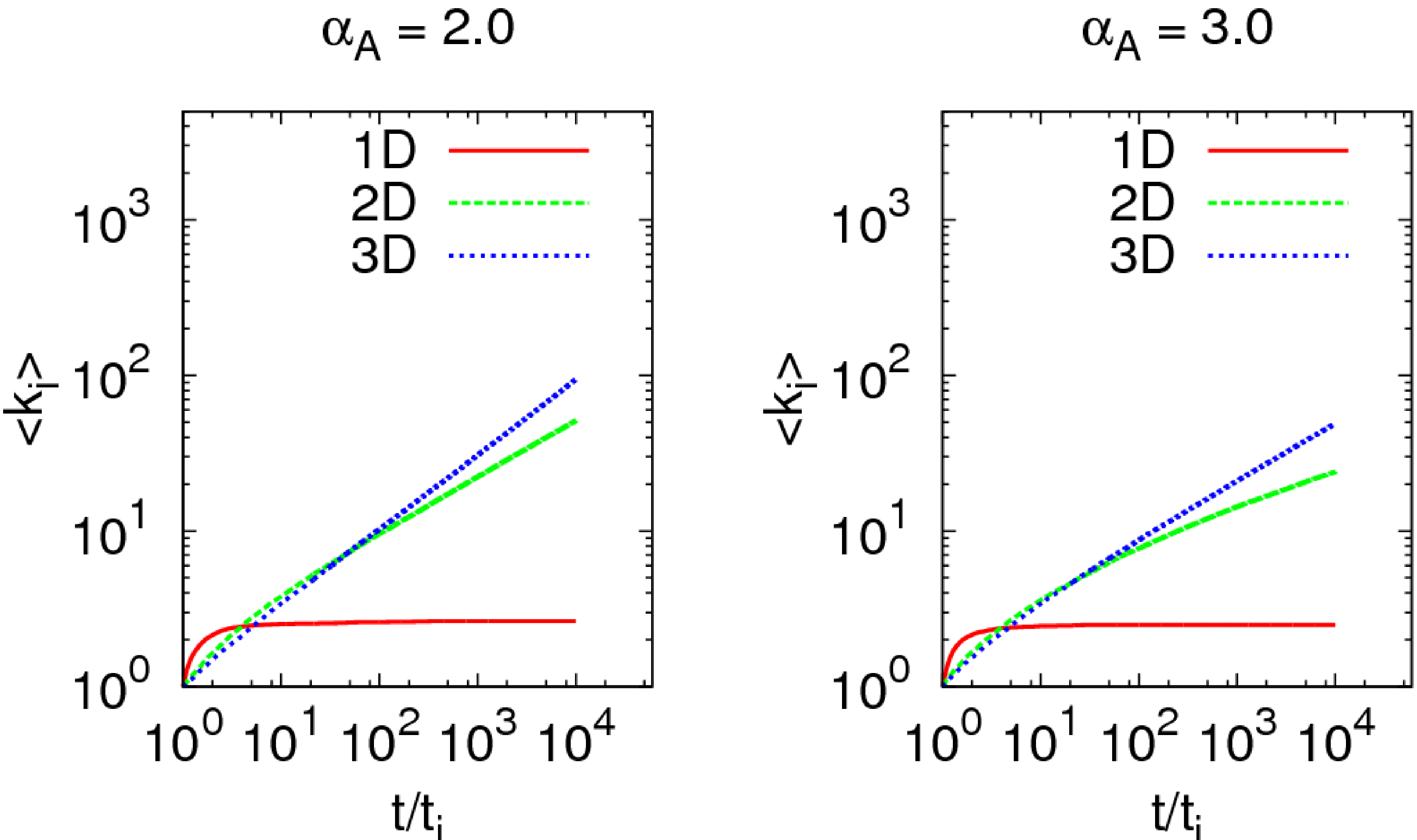


Connectivity Time Evolution – Natal Model 1D, 2D, 3D



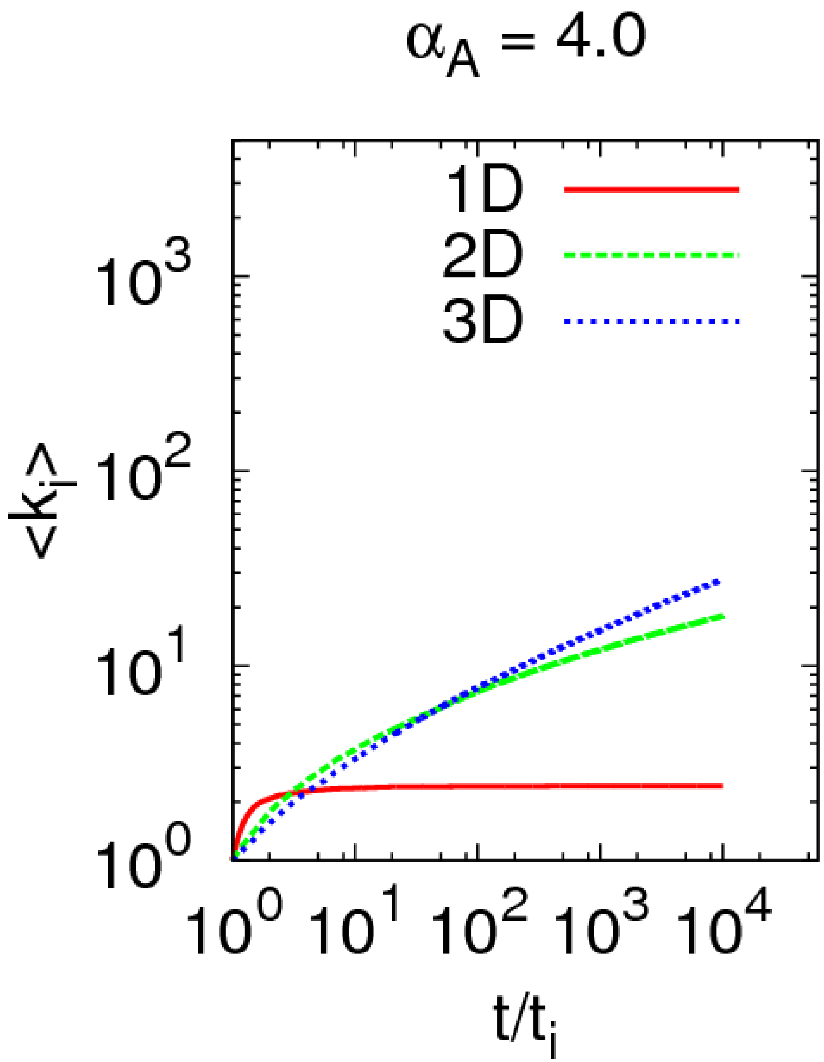
i = 10

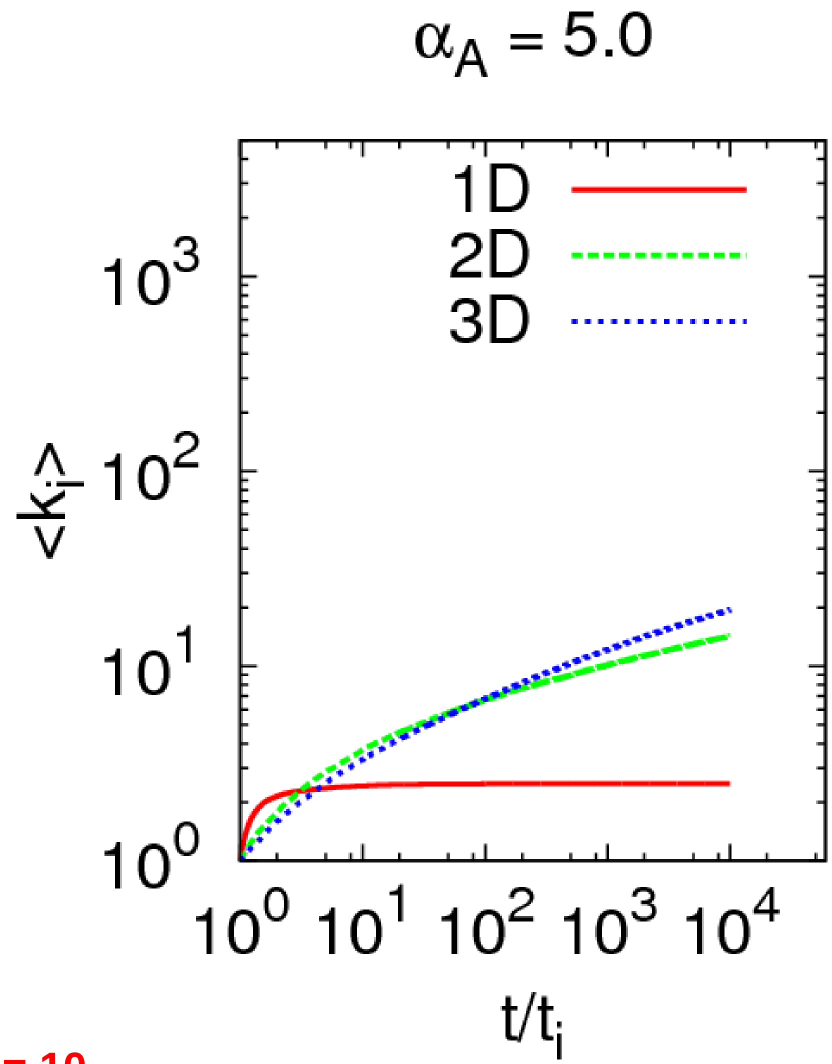
Connectivity Time Evolution – Natal Model 1D, 2D, 3D



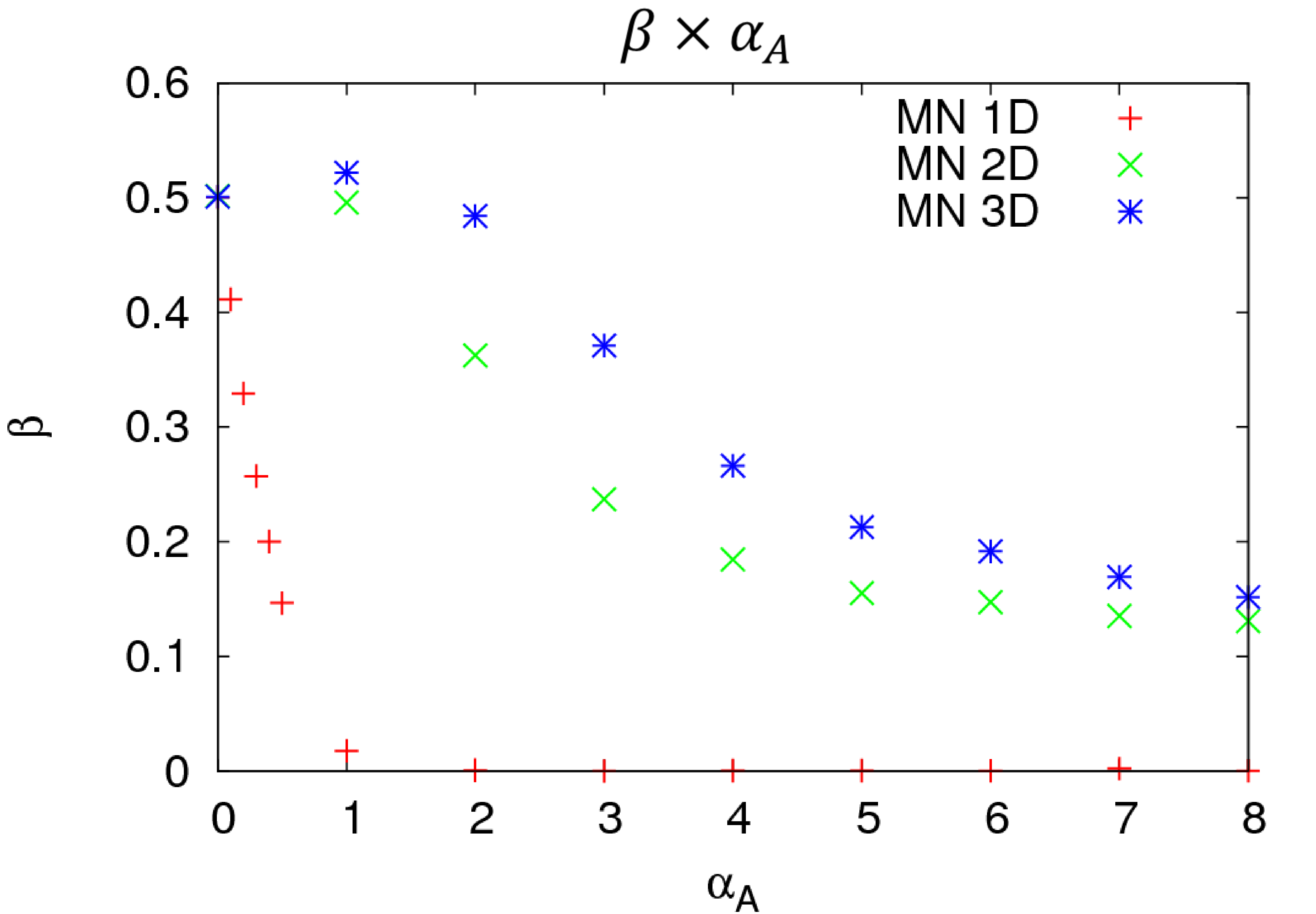
i = 10

Connectivity Time Evolution – Natal Model 1D, 2D, 3D





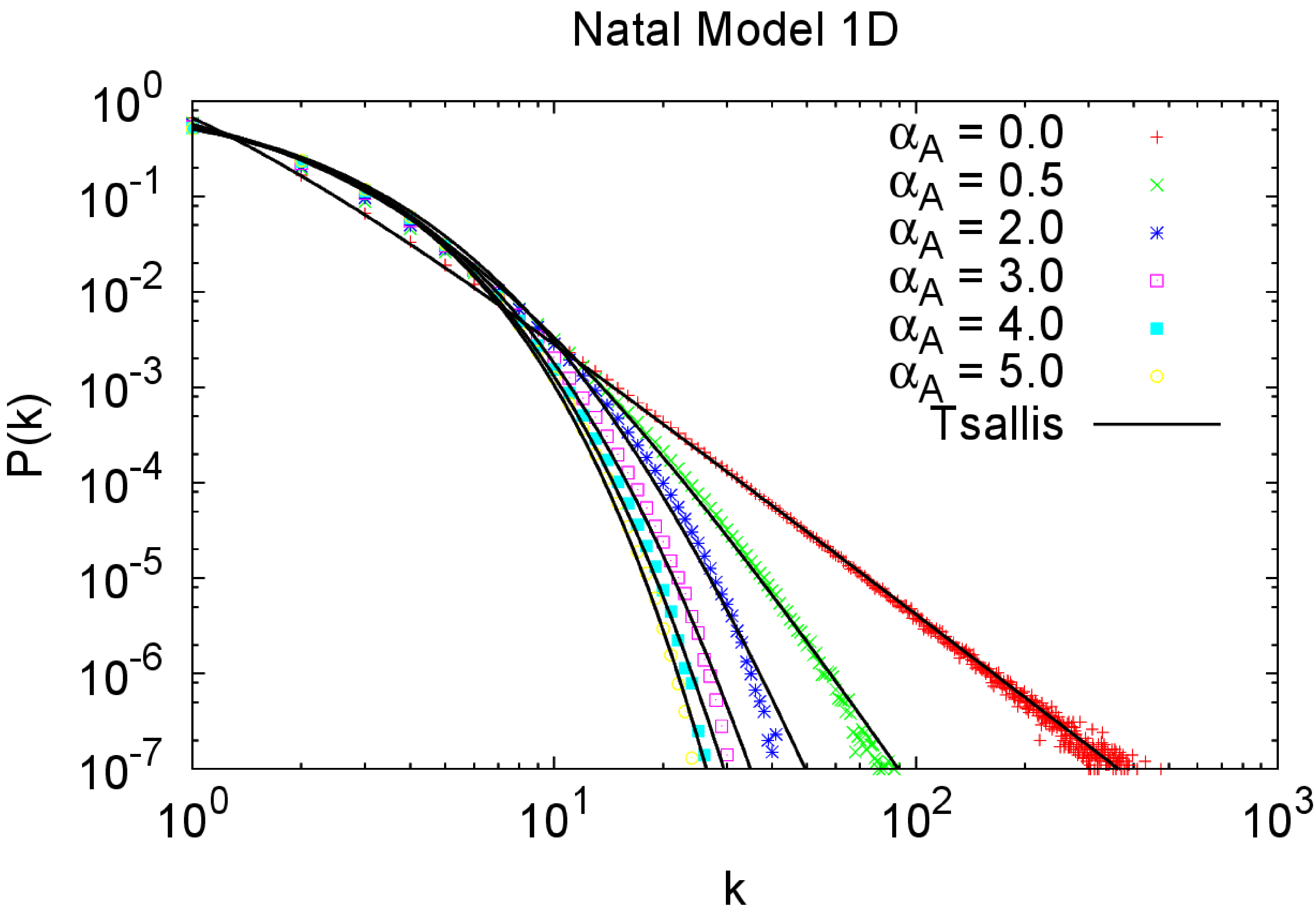
i = 10

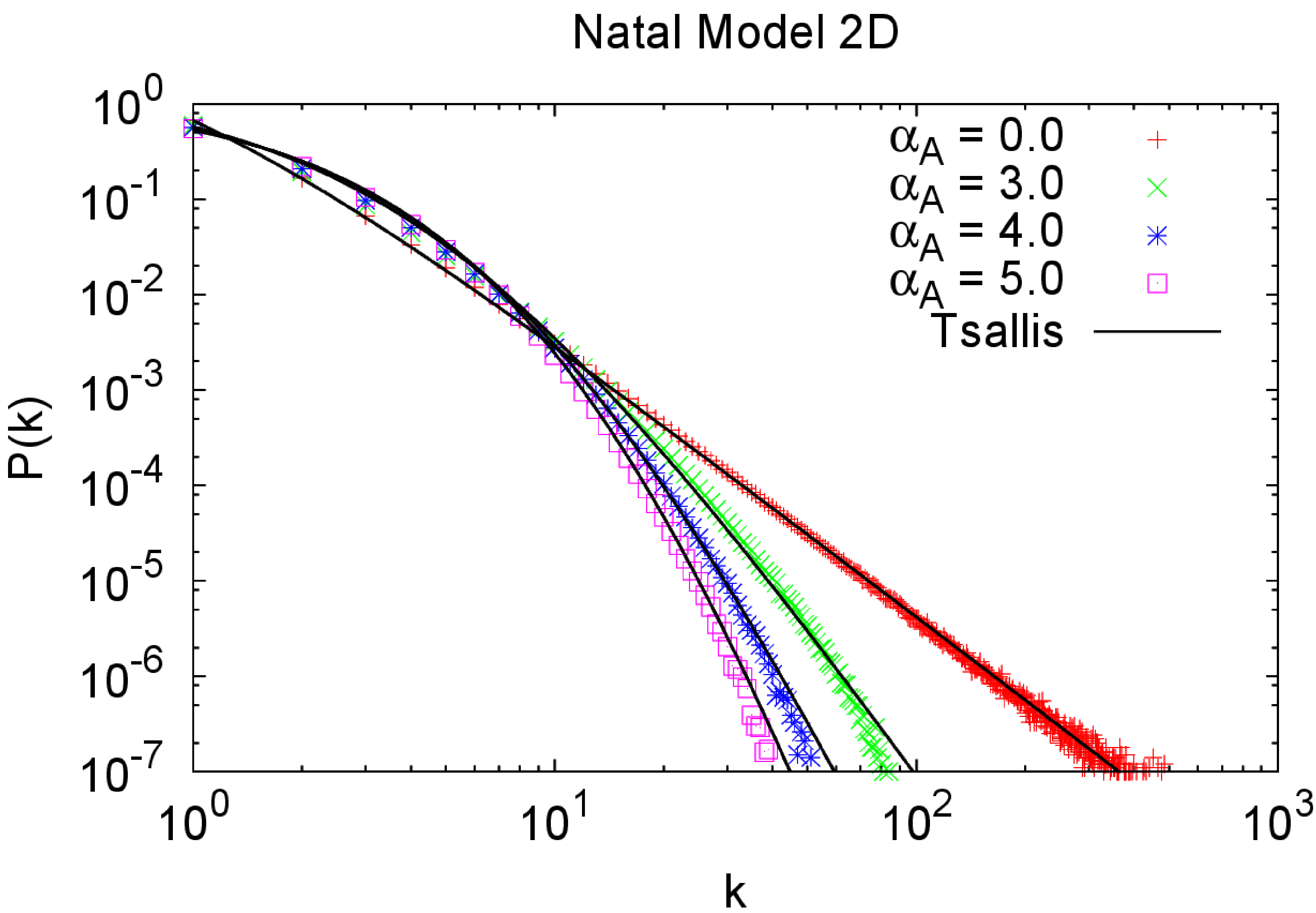

β × α_A
0.6
0.5
0.4
0.3
0.2
0.1
0
β
0
1
2
3
4
5
6
7
8
α_A
MN 1D
MN 2D
MN 3D

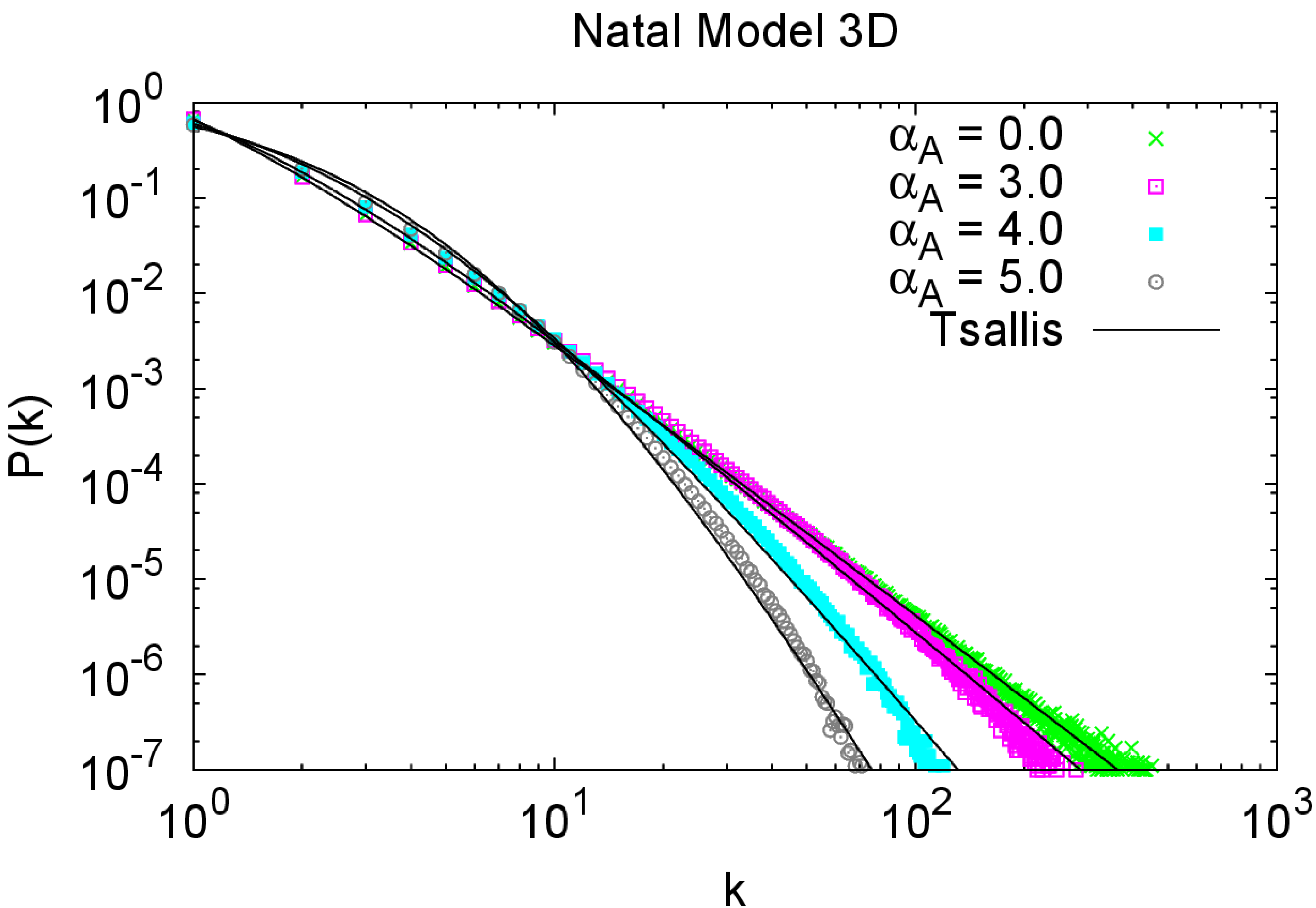
Tsallis Proposal

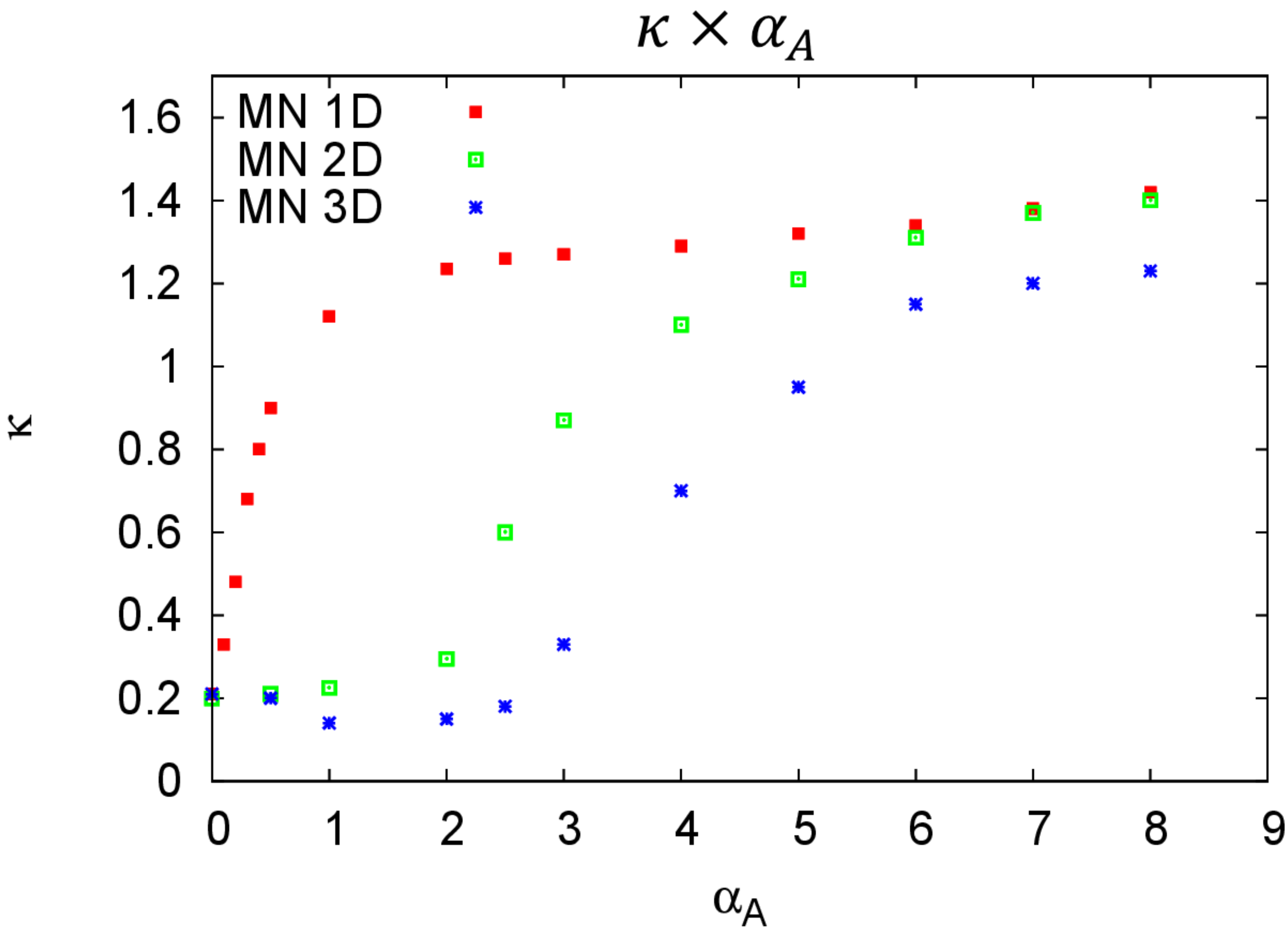
$$P(k) = P_0 e_q^{-k/\kappa}$$

$$e_q^x \equiv [1 + (1-q)x]^{1/(1-q)}$$

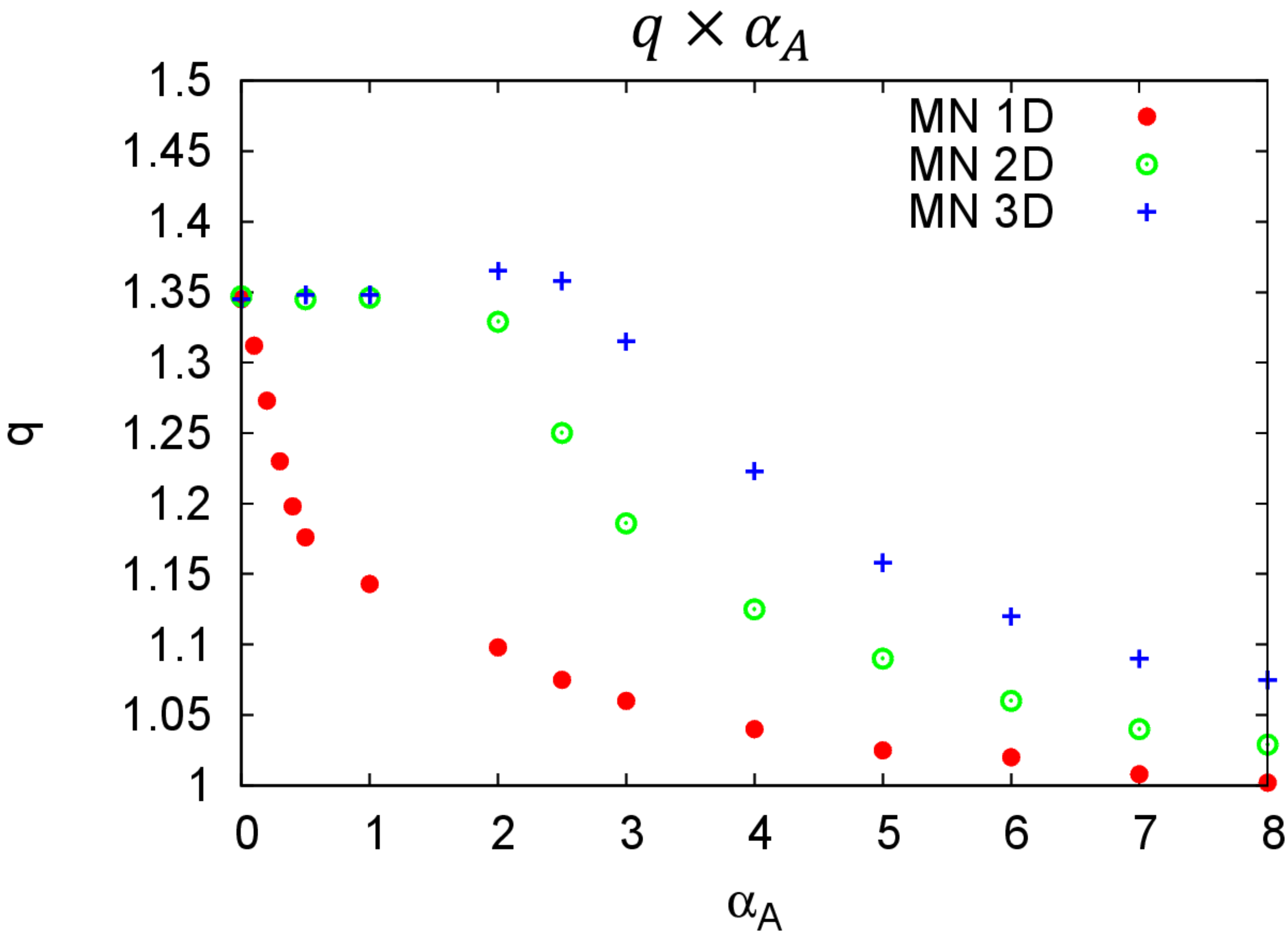

Natal Model 1D
P(k)
k
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^0
10^1
10^2
10^3
α_A = 0.0
α_A = 0.5
α_A = 2.0
α_A = 3.0
α_A = 4.0
α_A = 5.0
Tsallis

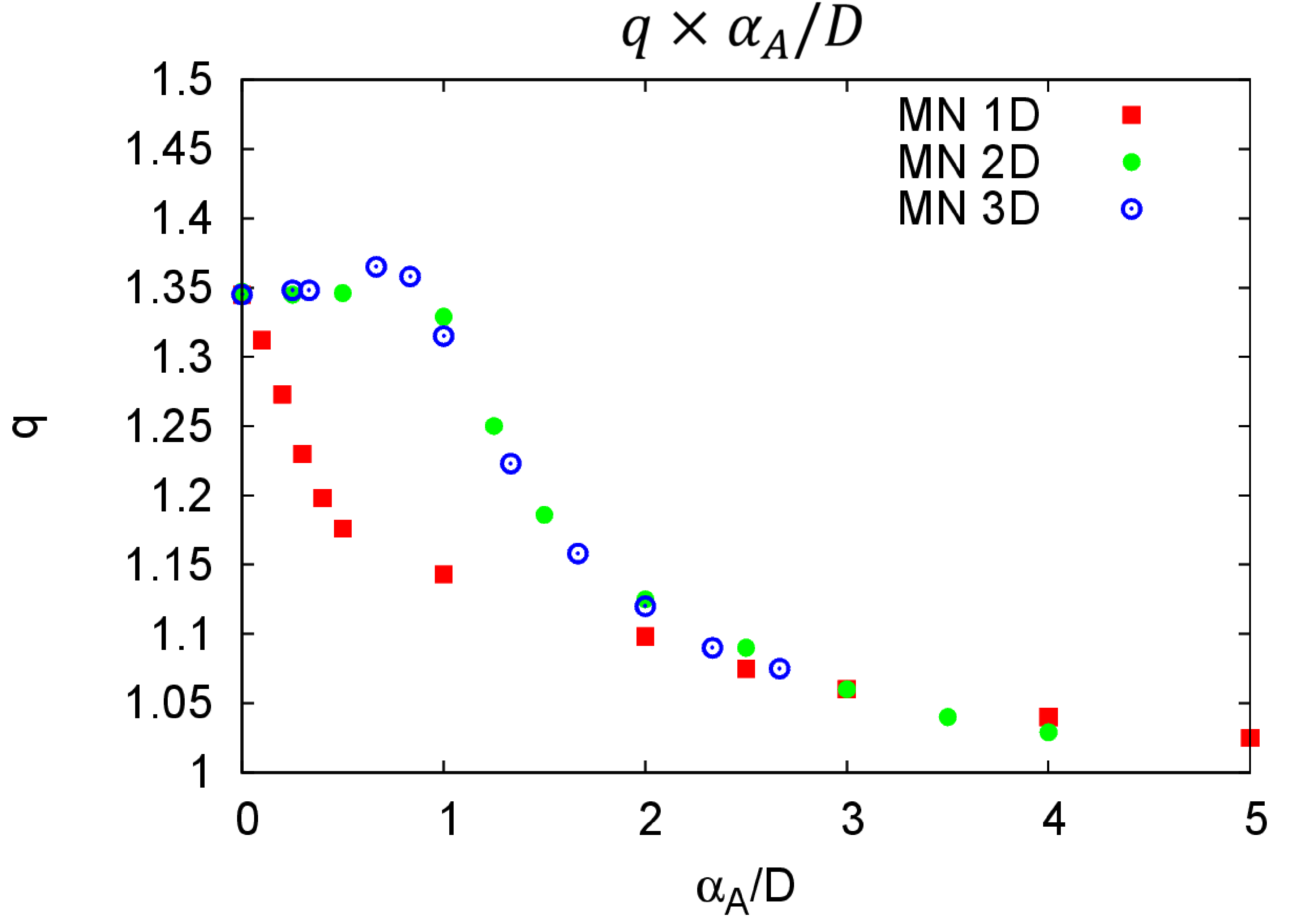

Natal Model 2D
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
P(k)
10^0
10^1
10^2
10^3
k
$\alpha_A = 0.0$
$\alpha_A = 3.0$
$\alpha_A = 4.0$
$\alpha_A = 5.0$
Tsallis


Natal Model 3D
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
P(k)
10^0
10^1
10^2
10^3
k
α_A = 0.0
α_A = 3.0
α_A = 4.0
α_A = 5.0
Tsallis



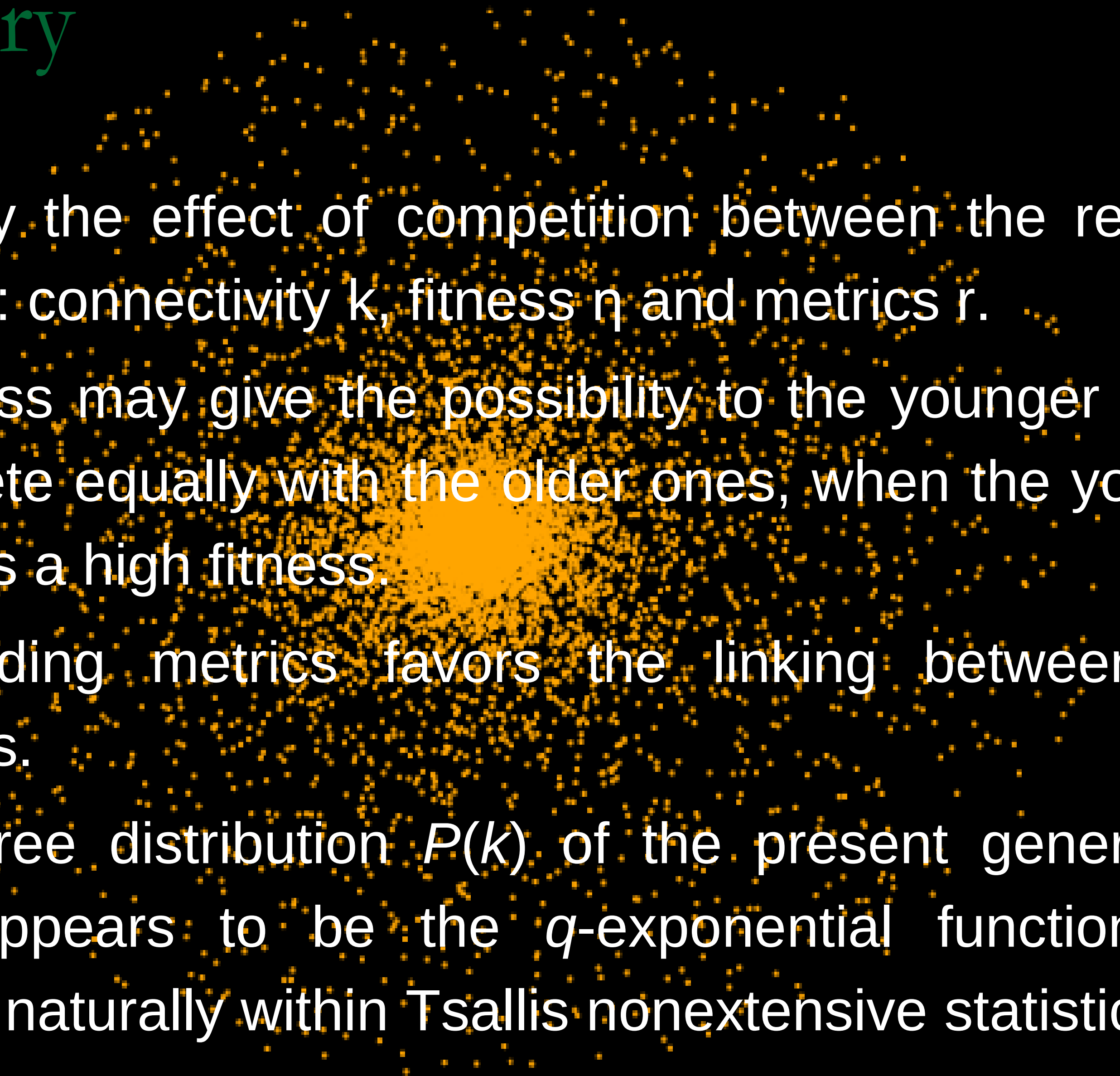
$\kappa \times \alpha_A$
MN 1D
MN 2D
MN 3D
κ
α_A
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
1
2
3
4
5
6
7
8
9


$q \times \alpha_A$
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
q
0
1
2
3
4
5
6
7
8
α_A
MN 1D
MN 2D
MN 3D


$q \times \alpha_A/D$
MN 1D
MN 2D
MN 3D
q
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
1
2
3
4
5
α_A/D

Summary

(a)



- We study the effect of competition between the relevant variables: connectivity k, fitness η and metrics r.
- The fitness may give the possibility to the younger nodes to compete equally with the older ones, when the younger node gets a high fitness.
- By including metrics favors the linking between first neighbors.
- The degree distribution $P(k)$ of the present generalized model appears to be the q-exponential function that emerges naturally within Tsallis nonextensive statistics.

Summary

(b)

- We modify the rule of the preferential attachment of the Bianconi-Barabasi model including a factor which represents similarity of the sites.
- The term that corresponds to this similarity is called the affinity and is obtained by the modulus of the difference between the fitness (or quality) of the sites.
- This variation in the preferential attachment generate very unusual and interesting results.
- We extend the Natal Model (**Geographic Model)** for d=1 e d=3. We calculated P(k) and the respective exponents. We verify P(k) as a Tsallis Statistics approach and we observed a scale low for q versus α/d

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11. A. M. Filho, D. A. Moreira, R. Silva and L. R. da Silva

"Preferential Attachment Growth Model and non-Gaussian Statistics"

Physics Letters A **377** 842 (2013)

70th birthday of Prof. Constantino Tsallis

- It was very fortunate for me to have met Prof. Constantino in 1977 when I arrived in Rio de Janeiro.
- While I came from Natal, he came from Brasília. Since that time my professional and personal life never was the same because one thing is correlated to the other.
- I have no doubt that my life is divided in two parts: B. C. and A. C.
- Congratulations Mr. q and have a very, very, very good time in the future!!! You deserve it !!!

THANK YOU VERY MUCH!