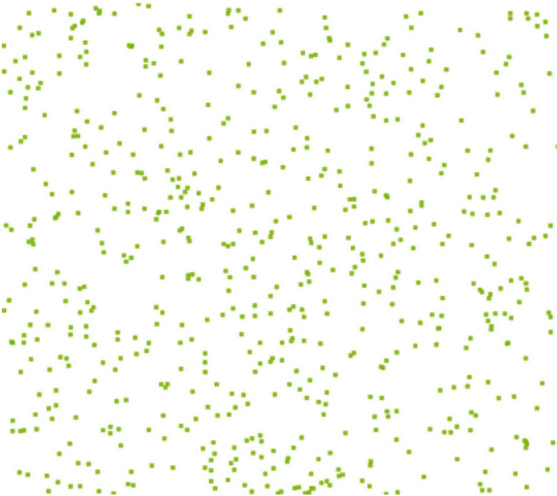
Ergodic Crossover in Partially Self-Avoiding Stochastic Walks

Alexandre Souto Martinez[1], Juliana M. Berbert[1] and Rodrigo Silva González[2]

[1]Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP)
Universidade de São Paulo (USP)
Instituto Nacional de Ciência e Tecnologia em Sistemas Complexos (INCT-SC)
[2]Universidade Federal de Viçosa
campus Rio Paranaiba

Rio de Janeiro, October 29th, 2013

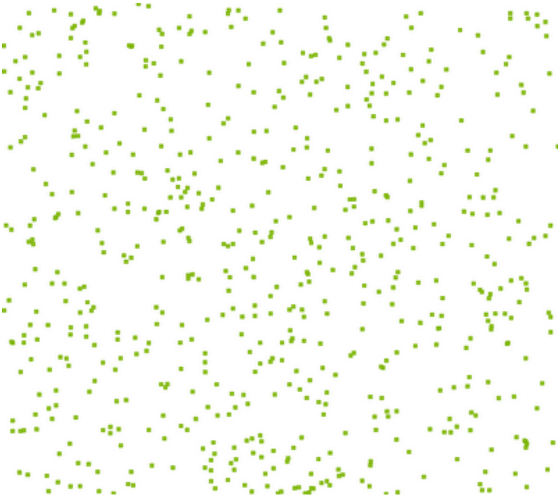
Objective



Random Points

Visit all the N sites randomly distributed in a d-dimensional space

Objective



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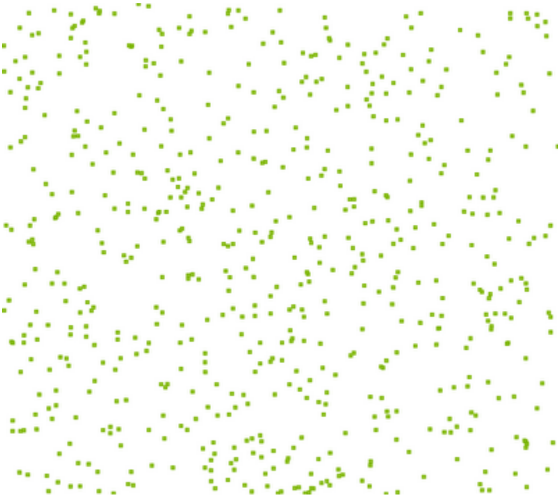

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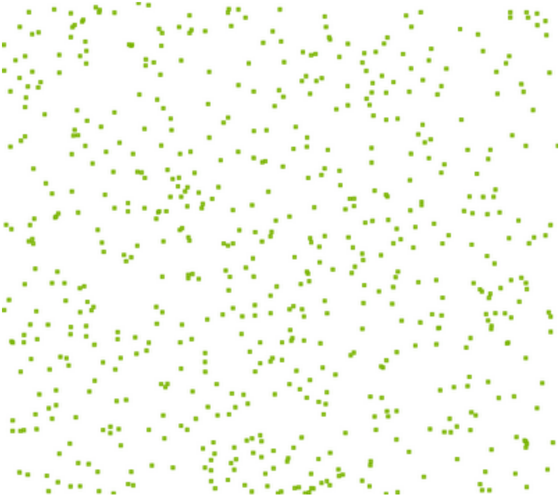
- travellingnot so much!!!!!

Objective

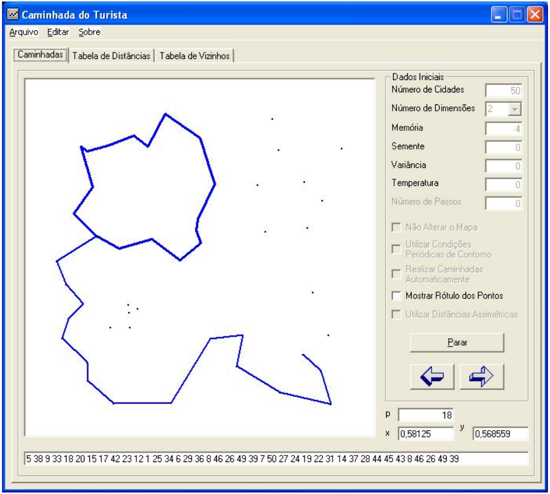


Random Points

Visit all the N sites randomly distributed in a d-dimensional space

- using local information and
- travellingnot so much!!!!!
- not the traveling salesman problem.

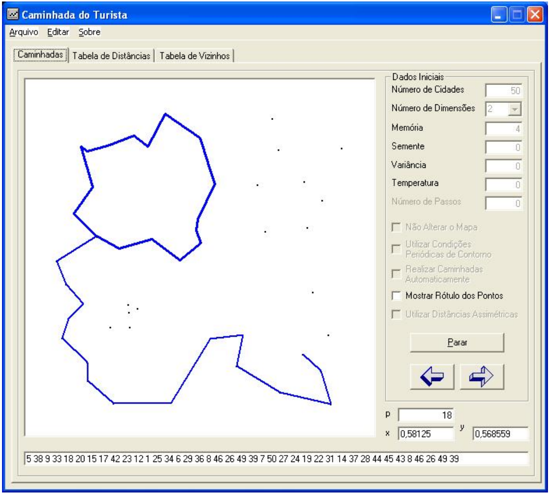
Deterministic walk



Traveler with memory: transient and attractor

go to the nearest site that has not been visited in the μ–th last steps.

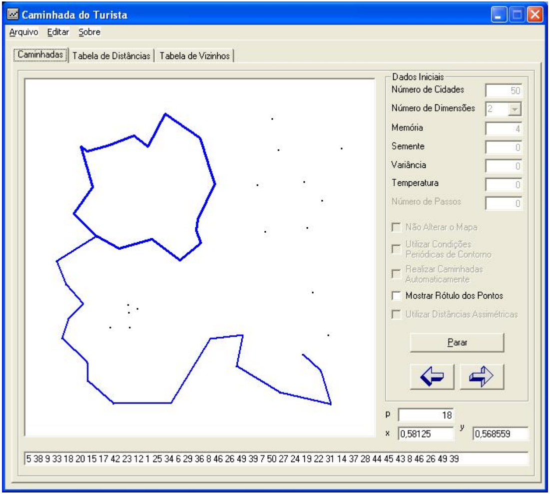
Deterministic walk



Traveler with memory: transient and attractor

- Greedy search strategy: $\mu = N - 1$;

Deterministic walk



Traveler with memory: transient and attractor

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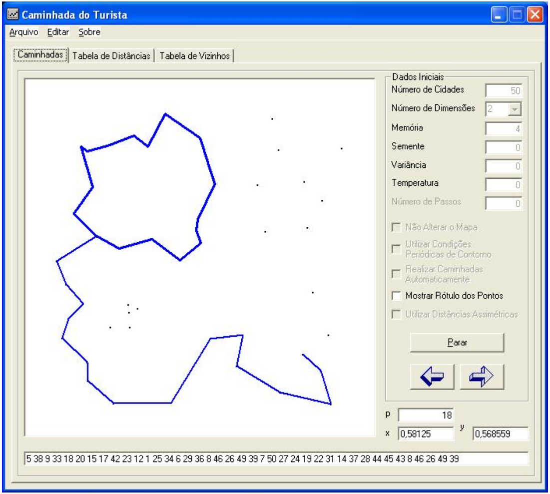
Deterministic walk



Traveler with memory: transient and attractor

- Greedy search strategy: $\mu = N - 1$;
- Extremamenly fast and
- Not too bad, 20% longer than salesman traveler problem.

Partially self-avoiding walk: tourist walk

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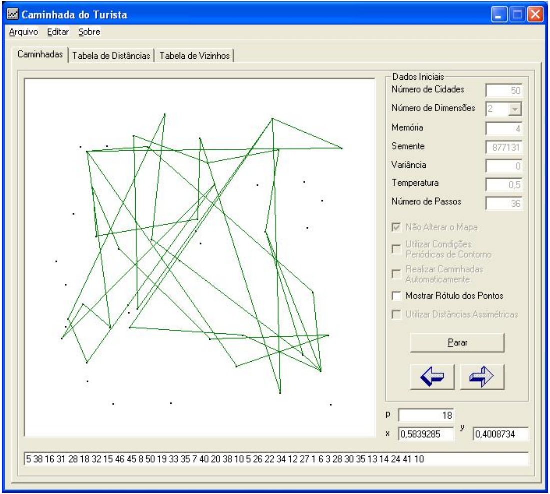
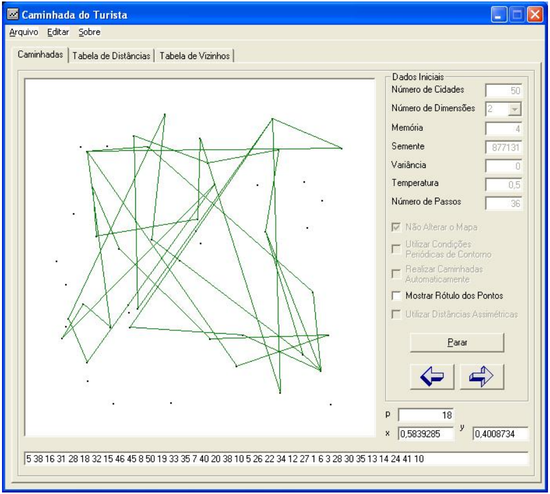
G.F. Lima, A.S. Martinez e O. Kinouchi, Phys. Rev. Lett. **87**, 010603 (2001).



$$W_{j \leftarrow i} = \frac{e^{-\beta E(D_{i,j})}}{Z_i^{(\beta,\mu)}}$$

$$Z_i^{(\beta,\mu)} = \sum_{k=1}^{N}{}' e^{-\beta E(D_{i,k})}$$

Stochastic walk



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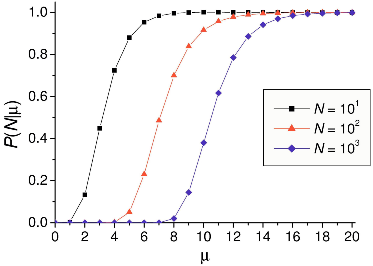
For $\mu = 0$ there is a sharp transition at $T_d = \Gamma(d/2 + 1)/\pi^{d/2}$

A.S. Martinez, O. Kinouchi and S. Risau-Gusman Phys. Rev. E **69**, 017101 (2004).

$$P(N|\mu) = \left(1 - 2^{-\mu}\right)^{N-\mu-1}$$



$$\mu_1 = \log_2 N$$

C. A. S. Terçariol, R. S. González and A. S. Martinez, PRE, 75, (2007)

$d = 1$ exploratory behavior transition

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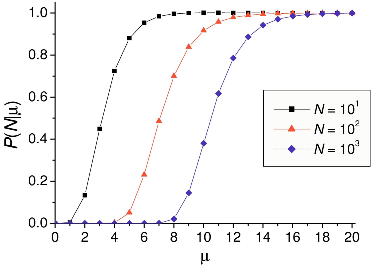

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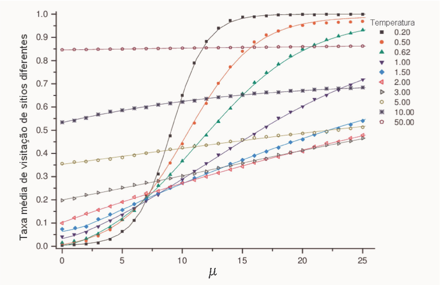
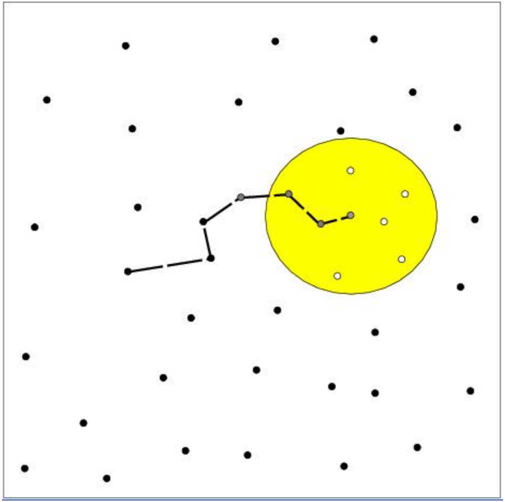
C. A. S. Terçariol, R. S. González and A. S. Martinez, PRE, 75, (2007)

$$\frac{T}{10\,T_0}\left(\frac{N_p}{N}\right)^{5/18} = 1 - \frac{3\mu_1}{2\mu_0}\left(\frac{N_p}{N}\right)^{-1/18}$$



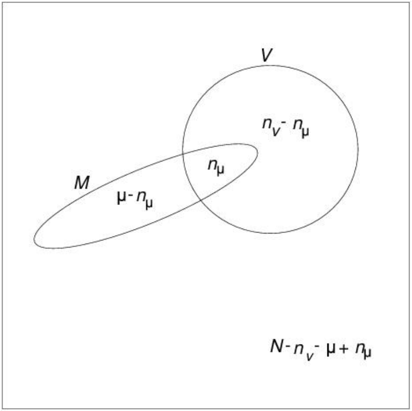
J. M. Berbert e A. S. Martinez, PRE, 81, (2010)

Modified stochastics tourist walk



$V = \frac{\pi^{d/2} T^d}{\Gamma(1+d/2)}$
$W_{j \leftarrow i} = \frac{1}{n_v - n_\mu}$
n_v: sites in the range
n_μ: prohibited sites.

Survival chances



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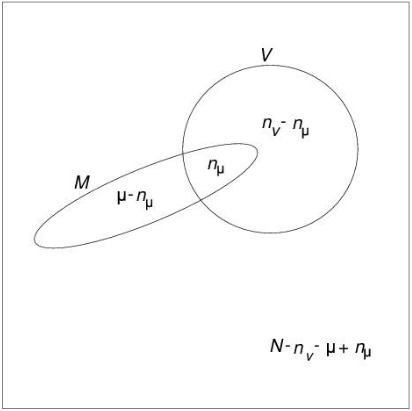
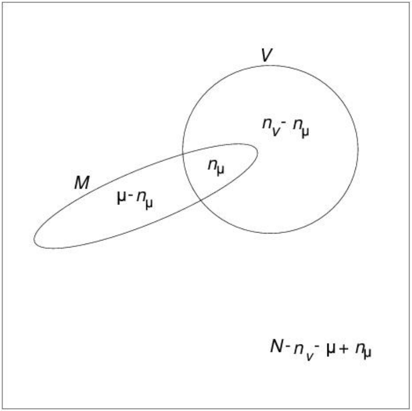


$$A : n_v = k \Rightarrow p(A) = \binom{N}{k} V^k (1 - V)^{N-k}$$

Survival chances



$$A : n_v = k \Rightarrow p(A) = \binom{N}{k} V^k (1-V)^{N-k} \qquad B : n_\mu = k \Rightarrow p(B) = \frac{\binom{n_v}{k}\binom{N-n_v}{\mu-k}}{\binom{N}{\mu}}$$

One-step death probability

$$P(A \cap B) = P(A)P(B) = \binom{\mu}{k} V^k (1 - V)^{N-k}$$

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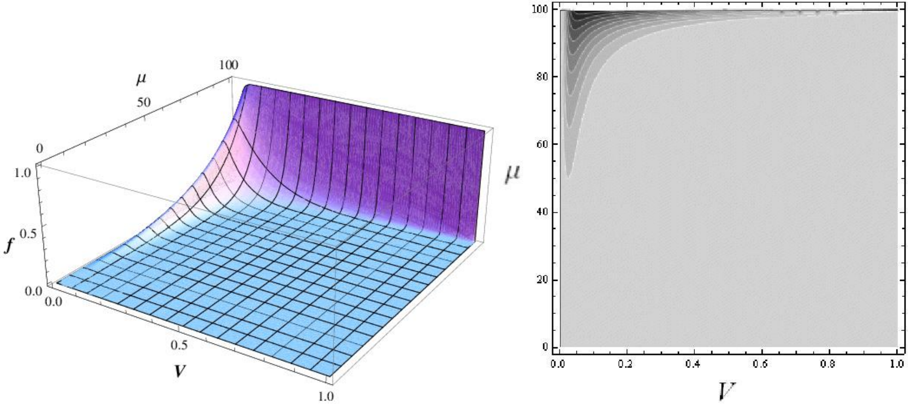
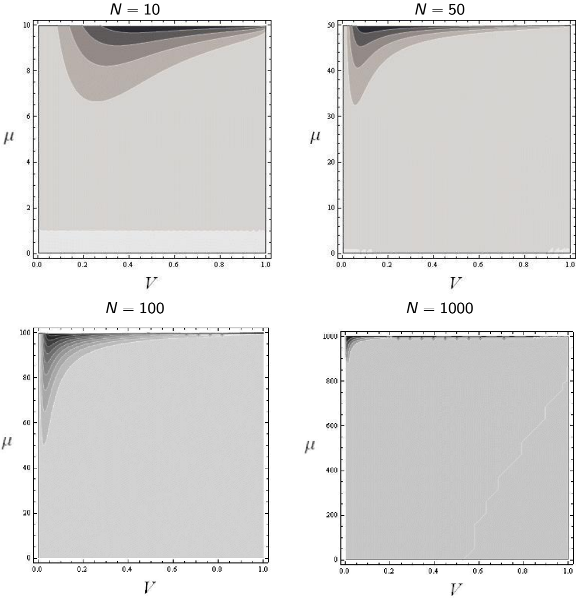
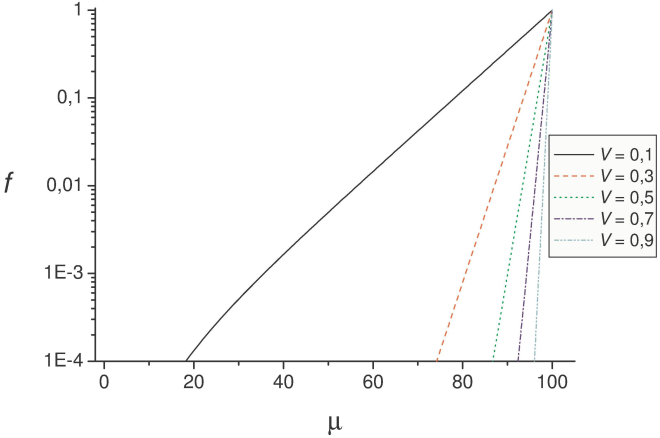


Finite size effect



Survival curves at constant V



Optimal exploration: transition curves

$$(1 - V)^N[-1 + (1 - V)^{-\mu_v} - \mu_v V(1 - V)^{-1}] - f = 0$$

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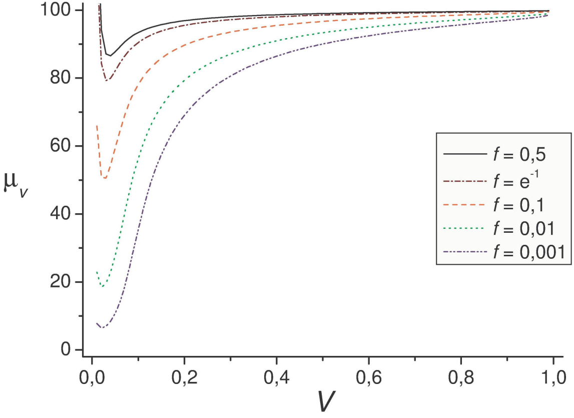

Aging: death probability

$$g = 1 - (1-f)^{N_p} = 1 - \left\{1 - (1-V)^N \left[-1 + (1-V)^{-\mu} - \frac{\mu V}{1-V}\right]\right\}^{N_p}$$

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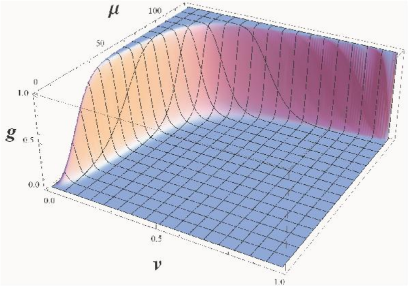
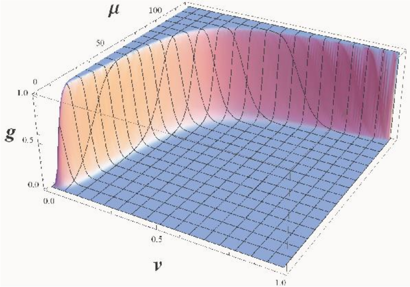
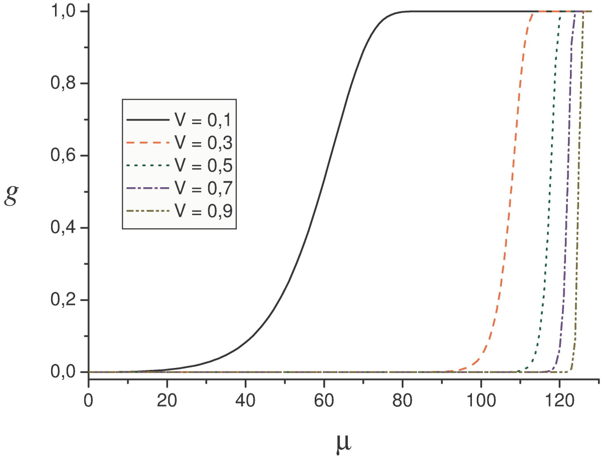
$N_p = N$



$N_p = 100N$

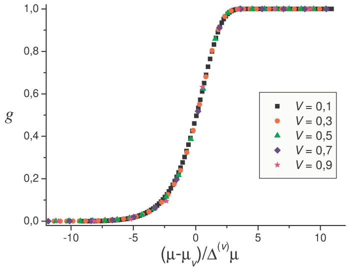


Constant V data collapse

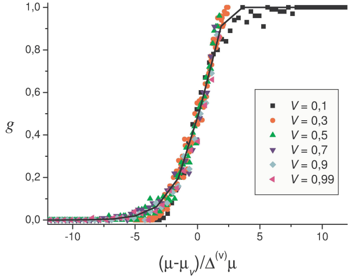


Constant V data collapse

Analitic

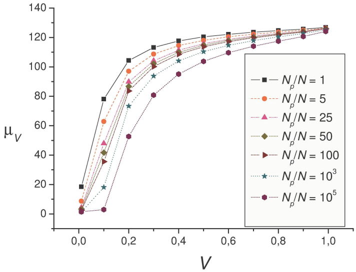


Numeric

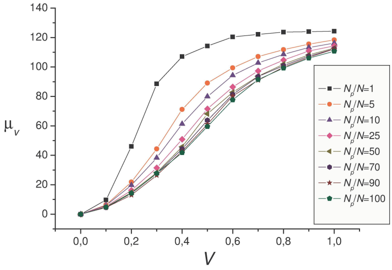


Transition curves displacement

Analítico

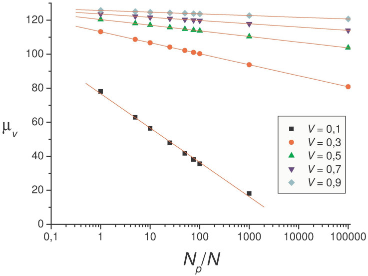


Numérico



Critical memory

Analítico



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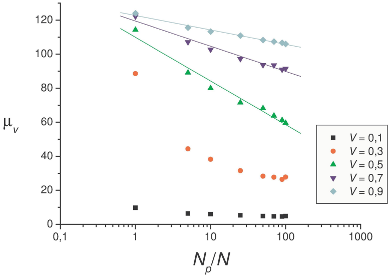

Conclusion

- non-trivial solvable analytical model
- control of finite size effects
- control of aging effect
- may eventually give hints of good parametrization in optimization algorithms considering that optimal exploration occurs along the death/survival (ergodic) transition.