# International Workshop on Foundations of Complexity

**Nonadditive Entropies and Nonextensive Statistical Mechanics**

**Rio de Janeiro – 19 to 23 October 2015**

# Jose Soares Andrade Jr.

## Universidade Federal do Ceara – Fortaleza, Brazil


**[TBA]**


# Roberto F. S. Andrade

## Universidade Federal da Bahia – Salvador, Brazil


### Title

**A thermodynamical formalism describing mechanical interactions**


### Abstract

The dynamical behavior of an overdamped mechanical model devoid from any usual thermal effects is analyzed by a formalism that is similar to usual thermodynamics, and completely independent of any ad-hoc assumption of a probability distribution of states in phase space of the mechanical model. It leads to the definition of a new entropy function, which does not coincide with the usual thermodynamical entropy. The new step making the difference to previous studies of this system is the identification of two non-equivalent mechanical interaction mechanisms, which are defined and identified as work and pseudo-heat. Together with the introduced effective temperature $\Theta$ they make it possible to characterize the equivalent to isothermal, adiabatic, isobaric, and isochoric processes. Three statements, formally analogous to the zeroth, first, and second law of thermodynamics, are issued. The statement of the second law results from the asymmetry in the way energy can be exchanged along the two processes. A Carnot cycle is defined, for which the efficiency is expressed in terms of $\Theta$ in the operating pseudo-heat reservoirs. The analogous Clausius theorem for the system operating an arbitrary reversible cycle is proved, leading to the new entropy function. Consequences of the extension of thermodynamic formalism to mechanical models with different processes of transferring energy are discussed.

# Christian Beck

## Queen Mary College – London, United Kingdom

### Title

**Transitions from one superstatistics to another one**

### Abstract

In this talk, after a brief outline of the superstatistics concept, I will discuss examples where a complex system exhibits a transition from one class of superstatistics to another one. One such example is the statistics of Lagrangian turbulence, which is well described by chi-square superstatistics in the quantum case and by lognormal superstatistics in the classical case [1]. Another example is an observed transition from chi-square to lognormal superstatistics for some share price dynamics as a function of the time scale of returns. A mixed model is discussed which can interpolate between lognormal and chi-square superstatistics [2].

Reference
[1] S. Miah, C. Beck, EPL 108, 40004 (2014)
[2] D. Xu, C. Beck, arXiv:1506.01660

# Tamas S. Biro

## Academy of Sciences – Budapest, HungaryTitle

**Non-extensive Quantum Statistics**

### Abstract

Based on Tsallis entropy (1988) and the corresponding deformed exponential function, generalized distribution functions for bosons and fermions have been used since a while Teweldeberhan et al. (2003) and Silva et al. (2010). However, aiming at a non-extensive quantum statistics further requirements arise from the

symmetric handling of particles and holes (excitations above and below the Fermi level). Naive replacements of the exponential function or ''cut and paste'' solutions fail to satisfy this symmetry and to be smooth at the Fermi level at the same time. We solve this problem by a general ansatz dividing the deformed exponential to odd and even terms and demonstrate that how earlier suggestions, using the κ- and q-exponential, behave in this respect.

Reference

T.S.Biro, K.M.Shen, B.W.Zhang, Physica A 428 (2015) 410

# Ernesto P. Borges

## Universidade Federal da Bahia – Salvador, Bahia

### Title

**Generalized phase-space in quantum mechanics for a position-dependent mass**

### Abstract

We consider a generalized translation operator which implies in the definition of a Hermitian deformed linear momentum operator and its canonically conjugate deformed position operator. These deformed operators are based on a non additive q-algebra. A canonical transformation leads the Hamiltonian of a position-dependent mass particle into another Hamiltonian of a particle with constant mass in a conservative force field of a deformed phase space, that permits the Schroedinger equation in the quantum picture, and the Newton's second law in the classical picture, be rewritten in a simple form with the deformed q-derivatives. Instances of an infinite square potential well and a potential barrier are given. Uncertainty and correspondence principles are analyzed.

# Tassos Bountis

## University of Patras – Greece

### Title

**From mechanical to biological oscillator networks: The role of long range interactions**

### Abstract

In studying one-dimensional particle networks of Classical Mechanics, through Hamiltonian models, we have learned a lot about oscillations under nearest (short range) interactions. Recently, however, through the introduction of long range interactions (LRI), several widely accepted notions concerning chaos and the approach to thermal equilibrium have been challenged based on the discovery of very interesting long lasting metastable states. On the other hand, when LRI (in the form of non-local or all-to-all coupling ) were introduced in systems of biological oscillators, Kuramoto's theory of synchronization in phase oscillators was developed and soon thereafter researchers studied amplitude and phase oscillations in networks of FitzHugh Nagumo and Hindmarsh Rose (HR) neuron models. In these models certain fascinating phenomena of coexistence of synchronous and asynchronous oscillations were discovered, called chimera states. Currently, their synchronization and metastability properties are being widely investigated in HR mathematical models as well as realistic neural networks, similar to what one finds in simple living organisms like the C.elegans worm.

# Evaldo M. F. Curado

## CBPF – Rio de Janeiro, Brazil

### Title

**Correlations and extensivity of the entropy**

### Abstract

We present a generalization of the binomial distribution associated with a sequence of positive numbers. It involves asymmetric and symmetric expressions of probabilities for win-loss sequences of trials. Our approach is based on generating functions and presents constraints of non-negativeness. Our generalizations are illustrated by various analytical and numerical examples. Special attention is dedicated to the analytical asymptotic behavior of several forms of entropy when the number of trials n tends to infinity. We can show analytically one example of combinatorial trials presenting long-term memory in which the entropy linear with n is not the Boltzmann-Gibbs one.

# Airton Deppman

## Universidade de Sao Paulo – Brazil

### Title

**Tsallis statistics and fractal aspects of high energy physics**

### Abstract

One of the most striking features of high energy collisions is the power-law behaviour of the transverse momentum of the generated particles. Perturbative QCD calculations can explain part of those distribution in the high momentum region, however only thermodynamic approaches have shown to be able to give a description of the peak of the distribution, where most of the particle are produced. The Non Extensive Self-Consistent Thermodynamics (NESCT) is so far the only theory that gives a comprehensive description of the whole transverse momentum distribution.
In this talk we review the basic aspects of NESCT and present comparisons between calculations and experimental data. Some recent advances in the theory will be addressed. In addition we explore the possible consequences of the non extensive behaviour of the hot and dense system created in high energy collisions to get some insights on the nature of QCD in non perturbative regime. For this aim we present the thermofractal, a system presenting fractal structure in its thermodynamical functions, and show that Tsallis statistics is the proper framework to deal with such systems. Then we explore the consequences of this structure in hadronic systems.

# Jean-Pierre Gazeau

## Paris Diderot University – Paris, France & CBPF – Rio de Janeiro, Brazil

### Title

**From classical to quantum descriptions of simple physical systems**

### Abstract

What is really Quantization? » That is a recurrent question since the origin of Quantum Physics, with partial answers given by the founding fathers of the theory, like Heisenberg, Born, Jordan, Shrödinger, Dirac, Weyl, Wigner, and many others since then. Starting from three basic examples of classical mechanics, namely the motions of a particle on the line, the half-line, and the circle, we show how to give acceptable quantum descriptions of these toy models. Symmetry and related covariance are essential in that construction. Then we present a general formalism for giving a measure space paired with a separable Hilbert space a quantum version based on a normalized positive operator-valued measure. The latter are built from families of density operators labeled by points of the measure space. We especially focus on various probabilistic aspects of these constructions. Links with Positive-Operator Valued Measure (POVM) and quantum measurement are sketched. Interesting applications to quantum cosmology (« smooth bouncing ») will be briefly presented.

# Maria Eunice Q. Gonzalez

## UNESP – Marilia, Brazil

### Title

**An epistemological enquiry into the issue of inter- and transdisciplinary research: a view from the Complex Systems perspective**

### Abstract

The epistemological approach to reconstructing and/or developing knowledge of the world, based on 'hard data', has been the subject of deep debate amongst philosophers and scientists for centuries. To start with, there are fundamental disagreements about: (a) the types of questions that should be formulated in specific contexts; and (b) the methods of enquiry suitable for dealing with these questions. In this workshop, I am going to consider the nature of these disagreements, arguing that perhaps they could be minimized by adopting the Complex Systems perspective. In this approach, scientific and philosophical research starts with a set of common problems that are of contemporary interest to researchers from different areas. One of the central (ideal) goals of the Complex Systems paradigm is to unite intellectual effort in order to investigate a common problem by taking into consideration data obtained using various perspectives, scales, and dimensions. I will argue that far from introducing undesirable relativism and vagueness in the research, the

objective of investigations developed in the Complex Systems paradigm is to find a second (third, and so on) order pattern of information that comprises order parameters emerging from data obtained at different levels of analysis of the same problem.

# Hans J. Herrmann


## ETH – Zurich, Switzerland & UFC – Fortaleza, Brazil


### Title

**Non-local conditions induce abrupt connectivity**


### Abstract

Abrupt changes in Nature and society can often be traced to the sudden appearance of a finite spanning cluster. While classical percolation exhibits a continuous phase transition, under special conditions the transition can also become of first order. I will illustrate the mechanisms behind the sudden appearance of a macroscopic percolating cluster. Very instructive is the history of the product rule also called "explosive percolation". The scaling of bridge bonds allows to exactly solve the spanning cluster model having a first order transition at $p_c = 1$. Suppressing the largest cluster by forcing a Gaussian size distribution renders an abrupt transition at $p_c < 1$. An interesting relation to the watersheds of random landscapes and SLE can then be established. One can also find exactly the transition from a second to a first order phase percolation transition by establishing a relation to the q-state Potts model using Gliozzi's algorithm. This model appears as a limiting case in the pollution of dielectric plates by metallic dust. Finally, imposing that every new connection must include at least one agent that is weakly connected, i.e. in a small cluster, one obtains a hybrid transition, which is a mixture of first and second order. First appears a macroscopic spanning cluster and then a power-law distribution of cluster sizes with a continuously varying exponent, which can be obtained analytically in the mean-field approximation.


# Henrik J. Jensen


## Imperial College – London, United Kingdom

### Title

**On the relevance of q-distribution functions: The return time distribution of restricted random Walker**

### Abstract

It is a remarkable fact that various types of q-functions often are found to fit data with high accuracy. But exactly why this is possible is not very clear. We'll describe a very simple model consisting of a one-dimensional random walker with position x in the interval [1,L]. The walker moves with probability |X/L|^a.

We'll report on how we find that for a=1 the first passage time distribution is exactly a q-exponential but not for a different from 1. And this despite that the exact analytic solution doesn't appear singular at a=1.

Reference

U. Tirnakli, H.J. Jensen and C. Tsallis, *Restricted random walk model as a new testing ground for the applicability of q-statistics*, EPL **96**, 40008 (2011). arXiv:1105.6184.

Jaleh Zand, Ugur Tirnakli and Henrik J Jensen, *On the relevance of q-distribution functions: The return time distribution of restricted random walker.* To appear in J Phys. A. arXiv:1506.02011

# George Livadiotis

## Southwest Research Institute – San Antonio, USA

### Title

**Kappa distributions: Connection with non-extensive statistical mechanics**

### Abstract

Classical particle systems reside at thermal equilibrium with their velocity distribution function stabilized into a Maxwell distribution. On the contrary, collisionless and correlated particle systems, such as the space and astrophysical plasmas, are characterized by a non-Maxwellian behavior, typically described by the so-called kappa distributions. Empirical kappa distributions have become increasingly widespread across space and plasma physics. However, a breakthrough in the field came with the connection of kappa distributions to the solid statistical framework of Tsallis non-extensive statistical mechanics. Understanding the statistical origin of kappa distributions was the cornerstone of further theoretical developments and applications, some of which will be presented in this talk: (i) The physical meaning of thermal parameters, e.g., temperature and kappa index; (ii) the *multi*-particle description of

kappa distributions; (iii) the phase-space kappa distribution of a Hamiltonian with non-zero potential; and (iv) the *Sackur-Tetrode* entropy for kappa distributions, and the second quantization constant, $\hbar^{*}\sim10^{-22}$ Js.

Reference:
Livadiotis, G., (2015), "Application of the theory of Large-Scale Quantization to the inner heliosheath", *J Phys Conf Ser*, 577, 012018, (7pp).
Livadiotis, G., (2015), "Kappa distribution in the presence of a potential energy", *J Geophys Res*, 120, 880-903, (24pp).
Livadiotis, G., (2015), "Statistical background and properties of kappa distributions in space plasmas", *J Geophys Res*, 120, 1607–1619, (13pp).
Livadiotis, G., (2015), "Kappa and $q$ indices: Dependence on the degrees of freedom", *Entropy*, 17, 2062-2081, (20pp).

# Renio S. Mendes

## Universidade Estadual de Maringa – Brazil

### Title

**Nonlinear Kramers equation and applications**

### Abstract

Stationary and time-dependent solutions of a nonlinear Kramers equation, as well as its associate nonlinear Fokker-Planck equations, are investigated within the context of Tsallis non extensive statistical mechanics. Since no general analytical time-dependent solutions are found for such a nonlinear Kramers equation, an ansatz is considered and the corresponding asymptotic behavior are studied an compared with those known for the standard linear Kramers equation. Applications are discussed, namely the center of pressure of velocity distribution of a person during quiet standing and the motion of Hydra cells in two-dimensional cellular aggregates.

# Fernando D. Nobre

## CBPF – Rio de Janeiro, Brazil

### Title

**Thermodynamic approach for a physical system described by nonextensive statistical mechanics**

### Abstract

A system of interacting vortices is currently used in the literature to model flux lines, and their associated vortices, in disordered type-II superconductors. Recent studies have shown that this system is related to nonextensive statistical mechanics. Lately, the concept of an effective temperature Θ was introduced for a system of interacting particles, and more particularly, for vórtices in type-II superconductors. The quantity Θ was shown to represent an appropriate definition of effective temperatura for this system, exhibiting properties very similar to those of the usual thermodynamic temperature T, being:
(a) A positive quantity by definition; (b) Thermodynamically conjugated to a generalized entropy, characteristic of nonextensive statistical mechanics; (c) Proportional to the density of vortices, yielding the desirable possibility for varying Θ since recent experimental researches in type-II superconductors led to considerable advances in the ability of controlling many properties of these vortices, including their density; (d) Characterized by values that are much higher than typical room temperatures, so that the thermal noise can be neglected as a good approximation; (e) Physically interpreted in terms of the variance of the vortex positions. In this presentation we will show a consistent thermodynamic formalism for this system, based on the effective temperature Θ. In order to achieve this, we propose a form for the first law of thermodynamics, showing its consistency through the construction of a Carnot cycle. Moreover, we consolidate the first-law proposal by following the usual procedure for obtaining different potentials, i.e., applying Legendre transformations for distinct pairs of independent variables.

# Angelo Plastino

## Universidad Nacional de La Plata – Argentina

**Title**

**Hypergeometric connotatios of quantum equations**

**Abstract**

We show that the Schrödinger and Klein-Gordon equations can both be derived from a Hypergeometric differential equation. The same applies to non linear generalizations of these equations.

# Angel R. Plastino

## Universidad del Noroeste de Buenos Aires – Argentina

**Title**

**Twenty years of the S_q-NLFP connection.**

**Abstract**

In 1995 [Plastino-Plastino, Physica A 222 (1995) 347] it was shown that there is a close relationship between Tsallis non-additive entropy and a non-linear Fokker-Planck equation (NLFP) endowed with a power-law diffusion term. It was shown that in the case of quadratic potentials the NLFP equation admits exact, time dependent *q*-Gaussian solutions that can be regarded as maxent densities optimizing Tsallis entropy under simple constraints. We review various extensions of that result, including *q*-maxent schemes generating semi-analytical, approximate solutions of the NLFP equation. We also discuss other related nonlinear evolution equations admitting exact *q*-Gaussian solutions, such as reaction-diffusion equations with power-law diffusion terms.

# Alessandro Pluchino

## University of Catania – Italy

### Title

**Exploring and modelling financial markets by self-organized criticality**

### Abstract

Building on similarities between earthquakes and extreme financial events, we use a self-organized criticality (SOC) generating model to study herding and avalanche dynamics in financial markets. In the first part of the talk we consider a community of interacting investors, distributed in a small-world network, who bet on the bullish (increasing) or bearish (decreasing) behavior of an *exogenous* market specified according to the S&P 500 historical time series. We show that the size of herding-related avalanches in the community can be strongly reduced by the presence of a relatively small percentage of traders, randomly distributed inside the network, who adopt a random investment strategy. This suggests a promising strategy to limit the size of financial bubbles and crashes. In the second part of the talk we show that, considering heterogeneous traders, composed by chartists and fundamentalists, and focus on the role of informative pressure on market participants, our SOC model is also able to generate *endogenously* a realistic price dynamics and to reproduce well-known stylized facts of financial markets, even in comparison with real price time series. Among them, fat-tailed distributions of price returns are shown to be well fitted by q-gaussian curves. Also in this case we introduce a variable number of random traders in order to study their possible beneficial role in stabilizing the market.

# Silvio M. D. Queiros

## CBPF – Rio de Janeiro, Brazil

### Title

**Beyond the q-Gaussian: generating distributions within the non-extensive statistical mechanics framework**

### Abstract

Probability distributions that stem from the standard (equilibrium) statistical mechanics probability are mainly described by scale dependent exponential function(al)s, e.g., the Boltzmann weight and Maxwell distribution. However, for systems in different conditions, namely nonequilibrium - where we can still define a local temperature - and out of equilibrium - for which we cannot define a temperature - when systematically measures data which are compatible with asymptotic power-law distributions, particularly distributions that emerge from optimising Tsallis entropy like the q-exponential distribution and the q-gaussian.
In my talk, I will review other processes - either in variable or probability space - that yield distributions,

other than those aforementioned, which also fit the non-extensive statistical mechanics scenario, namely the q-Gamma and the q-log-Normal. Extensions of such models to other problems will be introduced as well.


# Andrea Rapisarda

## University of Catania – Italy


### Title

**Selective altruism in collective games**


**Coauthors:** Dario Zappalà and Alessandro Pluchino


### Abstract


We study the emergence of altruistic behaviour in collective games. In particular, we take into account Toral's version of collective Parrondo's paradoxical games, in which the redistribution of capital between agents, who can play different strategies, creates a positive trend of increasing capital. In this framework, we insert two categories of players, altruistic and selfish ones, and see how they interact and how their capital evolves. More in detail, we analyse the positive effects of altruistic behaviour, but we also point out how selfish players take advantage of that situation. The general result is that altruistic behaviour is discouraged, because selfish players get richer while altruistic ones get poorer. We also consider a smarter way of being altruistic, based on reputation, called "selective altruism", which prevents selfish players from taking advantage of altruistic ones. In this new situation it is altruism, and not selfishness, to be encouraged and stabilized. Finally, we introduce a mechanism of imitation between players and study how it influences the composition of the population of both altruistic and selfish players as a function of time for different initial conditions and network topologies adopted.



# Alberto Robledo

## UNAM – Mexico City, Mexico

### Title

**Statistical-mechanical structures in the dynamics at or near the onset of chaos**

### Abstract

The dynamics towards attractors in low-dimensional iterated nonlinear maps (habitually) involves a contraction of phase space. We study this circumstance along the three routes to chaos in low-dimensional nonlinear maps where the attractors at the transitions, between regular and chaotic behavior, drive phase-space contraction for ensembles of trajectories. For chaotic attractors we find two kinds of statistical–mechanical structures associated with the dynamics separated by a crossover episode. The structures correspond, respectively, to the dynamics towards and the dynamics within the attractor, and the crossover reflects the arrival at the attractor. In the first regime the associated partition function measures the rate of approach of trajectories to the attractor and the statistical weights are deformed exponentials. In the second regime the partition function is made of position distances within the attractor and the statistical weights acquire exponential features. The time duration of the first regime increases as the extent of the attractor diminishes. When this contraction is extreme and the dimension of the attractor becomes fractal, as it is the case for the onset of chaos, we observe only the first regime. We illustrate this circumstance for properties of systems that find descriptions in terms of nonlinear maps. These are size-rank functions, urbanization and similar processes, and settings where frequency locking takes place.

## Misako Takayasu

## Tokyo Institute of Technology – Japan

### Title

**Various power laws in business firms and evolution dynamics of their trading network**

### Abstract

It is known that statistics of business firms are characterized by power law distributions and power law scalings among various quantities such as annual sales, number of employees and number of business partners [1]. I will review these results based on high quality business firms data provided by Teikoku Databank in Japan. Complicated trading relation among firms is described by evolution model based on creation-annihilation-aggregation processes [2], and basin structure of money flow can be estimated by a generalized gravity interaction model [3]. Percolation analysis is applied to this trading network, and precise value of percolation threshold and critical exponents are evaluated [4].

Reference:

[1] Hayafumi Watanabe, Hideki Takayasu, Misako Takayasu,

"Relations between allometric scalings and fluctuations in complex systems: The case of Japanese firms", **Physica A**, 392, 741-756, 2013.

[2] Wataru Miura, Hideki Takayasu, Misako Takayasu
"Effect of Coagulation of Nodes in an Evolving Complex Network"
**Phys. Rev. Lett.** 108, 168701 (2012), 5 pages.
[3] Koutarou Tamura, Hideki Takayasu, Misako Takayasu,

"Extraction of conjugate main-steam structures from a complex network flow"

**Phys. Rev. E**91, 042815, 7 pages, 2015.

[4] Hirokazu Kawamoto, Hideki Takayasu, Henrik J Jensen, Misako Takayasu

"Precise Calculation of a Bond Percolation Transition and Survival Rates of Nodes in a Complex Network", **PLoSONE**, 10(4): e0119979, 16 pages, 2015.

# Hideki Takayasu

## Sony Computer Science Laboratories – Tokyo Insitute of Technology – Japan

### Title

**Microscopic mechanism of financial market price motion and an origin of power law distribution**

### Abstract

By analyzing high quality order-book data of Foreign Exchange Market, detail mechanism of random walk of market price has been clarified [1]. It is shown that both buy-side and sell-side of order-book have layered structures; the inner-layer which directly drive the motion of market price in short time scale, and the outer-layer which works oppositely as resistivity in long time scale. Langevin equation is naturally derived from the data for description of market price motion, demonstrating that the market price has effective inertial mass and surrounding orders show viscous resistivity, quite similar to colloidal particle's random walk in water molecules.

Contrary to the case of colloial random walks in which all parameters are regarded to be constant, parameters of financial random walks seem to change time by time, both exogenously by news or endogenously by market's mood. In order to characterize the whole property for a long period a grand-superposition should be introduced based on time-dependent Langevin equation, or equivalently by PUCK model [2]. The resulting statistics are characterized by power law distributions.

Reference

[1] Yoshihiro Yura, Hideki Takayasu, Didier Sornette, Misako Takayasu

"Financial Brownian Particle in the Layered Order-Book Fluid and Fluctuation -Dissipation Relations" , **Phys. Rev. Lett.** 112, 098703, 5 pages (2014, OPEN ACCESS)
[2] Misako Takayasu, Hideki Takayasu

"Continuum Limit and Renormalization of Market Price Dynamics Based on PUCK Model",

**Progress of Theoretical Physics Supplement** No. 179 (2009), 1-7.

# Piergiulio Tempesta

## Universidad Complutense de Madrid – Spain

### Title

**Formal Groups, Generalized Entropies and L-Series**

### Abstract

We will show that there exists an intrinsic group-theoretical structure behind the notion of entropy [2]-[4]. It comes from the requirement of composability of an entropy with respect to the union of two statistically independent systems. We propose a formulation of the celebrated Shannon-Khinchin axioms, in which a new composability axiom replaces the traditional additivity axiom. The theory of formal groups [1] offers a natural language for our group-theoretical approach to generalized entropies.

At the same time, we will propose a simple universal class of trace-form entropies, satisfying the first three SK axioms. This class contains many well known examples of entropies and infinitely many new ones, a priori multi-parametric. Due to its specific relation with the Lazard universal formal group of algebraic topology, this new family of entropies is called the universal-group entropy [2].

We shall also prove that the celebrated Renyi entropy is the first example of a new class of non trace-form entropies, that we call the Z-entropies [3]. Each of them generalizes both the celebrated Boltzmann and Rényi entropies, which are recovered in suitable limits.

All these entropies are strictly composable, i.e. possess a composition law for any possible probability distribution. This property defines again a group-theoretical structure, which determines crucially the statistical and thermodynamical properties of the underlying entropies. A new class of divergences and applications to information theory are proposed.

The theory of group entropies will also be related with the theory of L-series and generalized Bernoulli polynomials [5].

Reference:

[1] M. Hazewinkel, Formal Groups and Applications, Academic Press, New York, 1978.

[2] P. Tempesta, Beyond the Shannon-Khinchin formulation: The composability axiom and the universal group entropy, preprint arxiv: 1407.3807, Annals of Physics, in press (2015).

[3] P. Tempesta, A new class of composable generalized entropies from formal groups: the Z-entropies, preprint arxiv: 1507.07436 (2015).

[4] P. Tempesta, A theorem on the existence of generalized entropies, Proc. Royal Society A, to appear (2015).

[5] P. Tempesta, The Lazard formal group, universal congruences and special values of zeta functions, Transactions of the American Mathematical Society 367, 7015-7028 (2015).

# Stefan Thurner

## University of Vienna – Austria

### Title

**Entropy for complex systems and its use in path dependent non-ergodic processes**

### Abstract

[TBA]

# Ugur Tirnakli

## Ege University – Izmir, Turkey

### Title

**A case study of ergodicity and non-ergodicity in Hamiltonian systems: The Standard Map**

### Abstract

As it is evident from its indisputable success, Boltzmann-Gibbs statistics is the correct way of thermostatistically approaching ergodic systems. On the other hand, nontrivial ergodicity breakdown and strong correlations typically drag the system into out-of-equilibrium states where Boltzmann-Gibbs statistics fails. For a wide class of such complex systems, it has been shown in recent years that the correct approach seems to use Tsallis statistics instead. Here we discuss how the dynamics of the paradigmatic conservative (area-preserving) standard map exhibits, in an exceptionally clear manner, the crossing from one statistics to the other. Our results unambiguously illustrate the domains of validity of both Boltzmann-Gibbs and Tsallis statistics.

# Constantino Tsallis

CBPF – Rio de Janeiro, Brazil & Santa Fe Institute - USA

### Title

**Universality in Complex Systems**

### Abstract

Similarly to standard critical phenomena, complex systems exhibit ubiquitously a variety of robust universal laws. We will focus on some of them in connection to anomalous diffusion, inter-occurrence times in financial, geophysical and biological phenomena, role played by dimensionality in classical many-body Hamiltonian systems, asymptotically scale-free networks, and others. Nonadditive entropies and their associated distributions (q-exponentials, q-Gaussians) emerge naturally in such systems. In addition to that, we shall mention some subtleties that must be taken into account in the task of determining from first principles the nonadditive entropy which is adequate for a given complex system.

# Oral Communications

## Zochil Gonzalez Arenas

Universidade do Estado do Rio de Janeiro - Brazil


### Title

**Generalized Fokker-Planck equation: normal and fractional derivatives**

Zochil Gonzalez Arenas & Constantino Tsallis

Different diffusive scenarios are described by the general diffusion equation

$$\frac{\partial}{\partial t}p(x,t) = D\frac{\partial^{\alpha}}{\partial x^{\alpha}}p^{2-q}(x,t)\ , \tag{1}$$

for the probability density function $p(x,t)$ of finding a particle at site $x$ at time $t$ and the diffusion constant $D > 0$, depending on the chosen values for the parameters $\alpha$ and $q$. In the linear case ($q = 1$), normal diffusion is described for $\alpha = 2$ and Gaussian distributions are obtained as its solutions for the initial condition $p(x,0) \equiv \lim_{t\to 0+} p(x,t) = \delta(x)$, while anomalous diffusion can be studied with $1 < \alpha < 2$ and Lévy stable distributions are obtained. Otherwise, the nonlinear case ($q \neq 1$, $2 - q > 0$) has also been studied and some of its solutions are associated with nonextensive statistical mechanics.

In this work, we briefly review a nonlinear and inhomogeneous Fokker-Planck equation with "normal" derivatives within a general stochastic prescription[a]. We show that its stationary state, for a given class of constitutive relations between drift and diffusion, has a $q$-Gaussian form[b]. Then, we focus in nonlinear Fokker-Planck equation with fractional derivatives. In view of better understanding of ($q$,$\alpha$)– stable distributions[c], we are interested in carefully characterizing the solutions, for long times, of the previously defined diffusion equation with spatial fractional derivatives in the Riemann-Liouville sense and nonlinear terms, for different values of the parameters $\alpha$ and $q$. We used the singular initial condition $p(x,0) = \delta(x)$ and numerically solved the equation (1). Preliminaries results are presented.


## Allbens P. F. Atman

Institute for Science and Technology for Complex Systems & CEFET-MG - Minas Gerais, Brazil

### Title

**Experimental validation of nonextensive scaling law in confined granular media**

### Abstract

We address the relationship between the statistical fluctuations of grain displacements for a full quasistatic plane shear experiment, and the corresponding anomalous diffusion exponent, alpha. We experimentally validate a particular case of the so-called Tsallis-Bukman scaling law, alpha= 2/(3 - q), where q is obtained by fitting the probability density function (PDF) of the measured fluctuations with a q-Gaussian distribution, and the diffusion exponent is measured independently during the experiment. Applying an original technique, we are able to evince a transition from an anomalous diffusion regime to a Brownian behavior as a function of the length of the strain-window used to calculate the displacements of grains in experiments. The outstanding conformity of fitting curves to a massive amount of experimental data shows a clear broadening of the fluctuation PDFs as the length of the strain-window decreases, and an increment in the value of the diffusion exponent of the anomalous diffusion. Regardless of the size of the strain-window considered in the measurements, we show that the Tsallis-Bukman scaling law remains valid, which is the first experimental verification of this relationship for a classical system at different diffusion regimes. We also note that the spatial correlations show marked similarities to the turbulence in fluids, a promising indication that this type of analysis can be used to explore the origins of the macroscopic friction in confined granular materials.

## Ogunsua Babalola

Federal University of Technology – Akure, Nigeria

### Title

**On the efficacy of the use of non-extensive statistical mechanics in the study of the upper atmospheric dynamics**

### Abstract

The dynamical behavior of the ionosphere has been examined using the non-extensive statistical mechanics approach. The Tsallis entropy has been applied to the study of the ionospheric processes and dynamics to establish its response to ionospheric behavior. The global positioning system (GPS) total electron content (TEC) data was engaged in these tests employing the use of Tsallis entropy which is being used comparatively with other nonlinear dynamical analysis techniques to carry out various tests on the data. The results obtained from the computation of Tsallis entropy were compared with that of the computed Lyapunov exponents. The results show the usefulness of Tsallis entropy the in study of natural systems like the ionosphere, with the response of both quantifiers exhibiting self organized criticality (SOC) phenomenon at different regions due to changes in energy input.

## Debarshee Bagchi

CBPF – Rio de Janeiro, Brazil

### Title

**Sensitivity to initial conditions of d-dimensional long-range-interacting Fermi-Pasta-Ulam model: Universal scaling**

Debarshee Bagchi & Constantino Tsallis

### Abstract

We introduce a d-dimensional generalization of the celebrated β-Fermi-Pasta-Ulam model which allows for long-range nonlinear interaction between the oscillators whose coupling constant decays with distance algebraically. We have focused on the sensitivity to initial conditions, more precisely on the first-principle (based on Newton's law) calculation of the maximal Lyapunov exponent as a function of the number of particles. We obtain a universal scaling law which is consistent with Boltzmann-Gibbs statistics and the q-statistics.

## Philippe Claudin

Laboratoire de Physique et Mécanique des Milieux Hétérogènes – Paris, France

### Title

**Rheology and jamming of granular systems and suspensions**

### Abstract

The rheology of dense dry granular flows can be described in terms of the inertial number, which compares the shear rate to the microscopic rearrangement time. By means of numerical simulations, we show how the constitutive relation can be generalized to the case of athermal and Brownian suspensions. We also discuss the limitations of this approach when non-local effects are present, and propose an extended framework to account for non-locality.

## Max Jauregui

CBPF – Rio de Janeiro, Brazil


### Title

**Paradoxical asymptotic independence in a system with strong correlations.**


### Abstract

Using a simple probabilistic model which presents q-Gaussian distribuitions as limiting distributions, we show that a part of finite size of the strongly correlated system becomes stochastically independent as the system grows.



## Ervin K. Lenzi

Universidade Estadual de Ponta Grossa - Brazil


### Title

**Fractional Diffusion Equation, Electrical Response, and Equivalent Circuits**


### Abstract

We present an extension of the Poisson-Nernst-Planck diffusional model in the context of the fractional diffusion equations. By using the impedance spectroscopy response obtained from this extension, for a typical electrolytic cell, a connection with equivalent circuits containing constant phase elements (CPE) is established. We also show that, depending on the choice of the equivalent circuit, the action of these elements can be the same as the one obtained by using integro-differential boundary conditions to describe anomalous diffusive processes in the framework of generalized models. The predictions are also compared with experimental data.



## Kenric Nelson

Raytheon and Boston University – Watertown, USA

### Title

**On the average uncertainty of systems with coupled states**

Kenric P. Nelson, Sabir Umarov, and Mark A. Kon

### Abstract

The increased uncertainty and complexity of nonlinear systems have motivated investigators to consider generalized approaches to defining an entropy function. In this paper, we show that important insights are achieved by determining average uncertainty in the probability domain and then propose an improved normalization of the Tsallis entropy. The Shannon entropy transforms to the weighted geometric mean of the probabilities. The weighted geometric mean of the exponential and Gaussian distributions is equal to the density at the location plus the scale (i.e. at the width of the distribution). The Rényi, Tsallis, and normalized Tsallis entropies each transform to the identical weighted generalized mean of the probabilities. The generalized Pareto and student-t distributions have the maximum entropy for these generalized entropies given constraints on the scale. In analog with the geometric mean, the appropriate weighted generalized mean also equals the density of these distributions at the location plus scale. We define the source of nonlinearity, which is a function of the fluctuations in the variance, such that it is equal to the shape parameter of the generalized Pareto and the inverse of the degree of freedom of the student-t. Coupling of states is modeled with an additive coupling and coupling of probabilities is modeled with a multiplicative coupling. Together {nonlinear source, additive coupling, multiplicative coupling} define the coupled entropy. For scale parameter equal to one, the coupled entropy is equal to one for the generalized Pareto and is close to a linear function of the nonlinear source for the student-t.

## Peter Rapcan

CBPF – Rio de Janeiro, Brazil

### Title

**In search of generalized Gibbs states for subsystems of critical quantum spin chains.**

Peter Rapcan & Constantino Tsallis * In collaboration with Yuriel Núñez Fernández, Karen Hallberg and Andrej Gendiar

### Abstract

It is well known that, for a critical quantum spin chain in its fundamental state, the block density matrix is not a Gibbs state featuring the system Hamiltonian restricted to the block. We search* for the appropriate expression for the state of the block in terms of the block Hamiltonian. In particular the quantum Ising chain is considered.

## Guiomar Ruiz Lopez

Universidad Politecnica de Madrid - Spain


### Title

**Towards a q-generalized large deviation theory for complex systems**


### Abstract

The theory of large deviations constitutes a mathematical cornerstone in the foundations of Boltzmann-Gibbs statistical mechanics, based on the additive entropy S_{BG}. An elementary large-deviation connection is provided by N independent binary variables, which, in the N>>1 limit yields a Gaussian distribution. The probability of having n ne N/2 out of N throws is governed by the exponential decay exp{-Nr}, where the rate function r is directly related to the relative BG entropy. To deal with a wide class of complex systems, nonextensive statistical mechanics has been proposed, based on the non-additive entropy S_q (q real; S_1=S_{BG}). Its optimization yields a q-exponential generalized weight q-exp{-beta_q E_i}. We numerically study large deviations for a strongly correlated model that provides, in the N >>1 limit, Q-Gaussian distributions, ubiquitously observed in nature (Q = 1 recovers the independent binary model). We show that its corresponding large deviations are governed by q-exp{-N r_q}. This q-generalized illustration opens wide the door towards a desirable large-deviation foundation of nonextensive statistical mechanics.



## Gabriele Sicuro

CBPF – Rio de Janeiro, Brazil


### Title

**Robustness of the q-Gaussian family**

Gabriele Sicuro, Piergiulio Tempesta, Antonio Rodriguez & Constantino Tsallis


### Abstract

We introduce three deformations, called $\alpha$-, $\beta$- and $\gamma$-deformation respectively, of a N-body probabilistic model, first proposed by Rodríguez et al. (2008), having q-Gaussians as $N\to\infty$ limiting probability distributions. The proposed $\alpha$- and $\beta$-deformations are asymptotically scale--invariant, whereas the $\gamma$-

deformation is not. We prove that, for both $\alpha$- and $\beta$-deformations, the resulting deformed triangles still have q-Gaussians as limiting distributions, with a value of q independent (dependent) on the deformation parameter in the $\alpha$-case ($\beta$-case). In contrast, the $\gamma$-case, where we have used the celebrated Q-numbers and the Gauss binomial coefficients, yields other limiting probability distribution functions, outside the q-Gaussian family. These results suggest that scale--invariance might play an important role regarding the robustness of the q-Gaussian family.

Reference
arXiv1506.02136 to appear on Annals of Physics.

## Luciano da Silva

Universidade Federal do Rio Grande do Norte – Natal, Brazil

### Title

**Role of dimensionality of complex networks with metrics: Connection with nonextensive mechanics**

S. G. A. Brito, L. R. Da Silva & C. Tsallis

### Abstract

Networks are observed in many fields of science, from social sciences to physics, from biology to economics, to cite but a few. In 2005 it was proposed (Soares, Tsallis, Mariz and Silva) a two-dimensional geographic network model with preferential attachment given by $\prod_{ij} \propto k_i / {r_{ij}}^{\alpha_A}$ [1]. This model exhibits competition between metric neighborhood and connectivity. The results showed that the connectivity distribution is given by $P(k) = P(0)e_q^{-k/\kappa}$, where ${e_q}^x \equiv [1 + (1-q)x]^{1/(1-q)}$. In this way the connection was exhibited between scale-free networks and nonextensive statistics. We now look for possible universal scaling laws in this model. How would the value of the index $q$ of the connectivity distribution depend on the dimension $d$ and the index $\alpha_A$ characterizing the range? In this work we numerically analyze the geographic growth model network at various dimensions (d=1,2,3,4) and verify the existence of a law of corresponding states, more precisely that the index $q(\alpha_A$,d) can be written as a function of the single variable $\alpha_A/d$.

Reference
D.J.B. Soares, C. Tsallis, A.M. Mariz and L.R. da Silva. “Preferential Attachment Growth Model and Nonextensive Statistical Mechanics”. Europhysics Letters **70**, 70 (2005).

# Roseli Wedemann

Universidade Estadual do Rio de Janeiro – Brazil


## Title

**Nonlinear Fokker-Planck Equation, q-Maxent Distributions, and Non-Potential Force Fields**

R. Wedemann & A. R. Plastino

## Abstract


We investigate a multi-dimensional nonlinear Fokker-Planck equation endowed with a power-law diffusion term and a drift term not arising from a potential function (that is, describing "curl forces"). The drift term consists of two parts: a gradient term coming from a pontential, and a curl term. We determine the conditions for having stationary solutions of the Tsallis maximum entropy form. In the case of quadratic potentials and linear non-potential drifts, we obtain time dependent solutions exhibiting a multidimensional q-Gaussian form. A two-dimensional example is provided.


Reference


R. S. Wedemann, R. Donangelo, and L. A. V. Carvalho, Chaos 19, 015116, 2009; A.R. Plastino and A. Plastino, Physica A 222, 347, (1995); Evaldo Curado, Fernando Nobre, Entropy, 13, 1928-1944, (2011); C. Tsallis and D. A. Stariolo, Physica A, 233, 395—406, (1996).