Latin-American School on Foundations of Complexity

Nonadditive Entropies and Nonextensive Statistical Mechanics

Rio de Janeiro – 4 to 30 October 2015

Christian Beck

University of London – United Kingdom

Title

Superstatistical methods for complex systems

Abstract

In my lectures I will provide an easy-going introduction to the superstatistics concept. Many complex driven nonequilibrium systems are effectively described by a superposition of several statistics on different time scales, in short a 'superstatistics'. A simple example is a Brownian particle moving in a spatially inhomogeneous medium with temperature fluctuations on a large scale, but the concept is much more general. Superstatistical systems typically have marginal probability distributions that exhibit fat tails, for example power law tails or stretched exponentials. Tsallis q-statistics is obtained for special cases of temperature fluctuations. In most applications one finds three relevant universaliy classes: Lognormal superstatistics, chi-square superstatistics (equivalent to q-statistics) and inverse chi-square superstatistics. In the past 12 years superstatistical techniques have been applied to a variety of complex systems, for example turbulence (Lagrangian, Eulerian, environmental, quantum and classical), hydroclimatic fluctuations, pattern formation in Raleigh Benard convection, share price dynamics, train delay statistics, rainfall statistics, complex networks, quantum mechanical systems described by path integrals, scattering processes in high energy physics, as well as medical and biological applications. In my lectures I will first outline the most important theoretical concepts related to superstatistics, making contact with generalized versions of statistical mechanics for complex systems, and then discuss several of the above applications.

Reference

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S. Miah, C. Beck, Statistics of Lagrangian quantum turbulence, Phys. Rev. E 031002(R) (2013)
P. Rabassa, C. Beck, Extreme value laws for superstatistics, Entropy 16, 5523 (2014)

Tamas Biro

Academy of Sciences – Budapest, Hungary

Title – First lecture

Finite reservoir models and the Tsallis distribution

Abstract – Lecture 1

Finite heat reservoir capacity and temperature variance lead to modifications of the well known canonical exponential weight factor. In certain cases, however, even small systems may show a Boltzmannian exponential. We analyze different simple models of particle production covering a microcanonical phase space evenly and leading to different ("deformed exponential") factors in the single particle energy spectra. We obtain the Tsallis distribution in some models exactly, and approximately in a general thermodynamical approach.

Reference

T.S.Biro, G.G.Barnafoldi, P.Van, Physica A 417 (2015) 215

Title – Lecture 2

Generalized entropy formulas and their construction

Abstract – Lecture 2

Based on the previous lecture we demonstrate that a deformed entropy can be constructed and used for fulfilling the zeroth law of thermodynamics by demanding a special form of generalized additivity: K(S12)=K(S1)+K(S2). This requirement leads to a second order differential equation for the entropy deformation, K(S). The generalized q-entropy formula arises as a p-weighted sum of K(− ln p). It contains the Tsallis, Renyi and Boltzmann–Gibbs–Shannon expressions as particular cases. For diverging variance of the temperature we obtain a novel entropy formula leading to tails like the Gompertz distribution.

Reference

T.S.Biro, P.Van, G.G.Barnafoldi, K.Urmossy, Entropy 16 (2015) 6497

Title – Lecture 3

Non-extensive entropic distances in a kinetic model view

Abstract – Lecture 3

We connect particle production in hadronization with network models in a combinatoric approach. A Markovian master equation governs both descriptions and arrives at a detailed balance state including Poissonian and negative binomial distributions. We suggest an entropic distance formula generalizing the Kullback-Leibler divergence with deformed logarithmic function.

Title – Lecture 4

q-parameter for particle spectra from black hole Renyi entropy

Abstract – Lecture 4

By regarding the Hawking–Bekenstein entropy of Schwarzschild black hole horizons as a non-extensive Tsallis entropy, its formal logarithm, the Renyi entropy, is considered. The resulting temperature – horizon radius relation has the same form as the one obtained from a (3+1)-dimensional black hole in anti-de Sitter space using the original Boltzmannian entropy formula. In both cases the temperature has a minimum. A semi-classical estimate of the horizon radius at this minimum leads to a Bekenstein bound for the q-parameter in the Renyi entropy of micro black holes, which is surprisingly close to fitted q-parameters of cosmic ray spectra and power-law distribution of quarks coalescing to hadrons in high energy accelerator experiments.

Reference

T.S.Biro, V.G.Czinner, Phys.Lett. B 726 (2013) 861

Tassos Bountis

University of Patras – Greece

Title – Lecture 1

Order and chaos in N- degree-of-freedom Hamiltonian systems

Abstract – Lecture 1

These lectures start with a brief introduction in the foundations of Hamiltonian Systems by discussing in a pedagogical way the simple cases of N=1 and N=2 degrees of freedom. In particular, I will present the basic notions of simple periodic orbits and their local stability properties and wonder whether the dynamics in their vicinity have anything to do with more global phenomena that we will later call "strong" or "weak" chaos. I will then turn to examine these ideas in the framework of specific examples of N–Degree– of–Freedom Hamiltonians with N arbitrary. I shall introduce the method of Lyapunov exponents (LEs) and review more recent chaos detection indicators which have important advantages over the LEs. We will focus on the case of 1-dimensional lattices and discuss the concept of dynamical complexity as exemplified in the important examples of localization phenomena and hierarchies of order and chaos in configuration space as well as Fourier space and their relationship with "weak" and "strong chaos" as N becomes arbitrarily large.

Title – Lecture 2

Complex aspects in Hamiltonian dynamics and statistics

Abstract – Lecture 2

In the first part of this longer lecture I will enter into a more detailed discussion of the **chaos detection methods** mentioned in the previous lecture; in particular, I will compare LEs with the spectrum of **the GALI$_k$(t) indicators** that distinguish more efficiently order from chaos in multidimensions. I will derive analytical estimates for the asymptotic behavior of the GALI$_k$(t) indices, describe several instances of their numerical verification and explain how they are used in specific applications to identify **low - dimensional tori** and weakly diffusive dynamics. However, such indicators are not able by themselves to distinguish "'weak" from "strong" chaos. This can only be done by the use of **non-extensive (better yet, non-additive) thermodynamics** to which we turn in the second part of the lecture. More specifically, we will explain how one computes sums of chaotic variables **in the sense of the Central Limit Theorem**, and use Tsallis' entropy approach to obtain probability density functions (pdfs), which are very well approximated by **q-Gaussian functions**. These functions and, in particular, the value of their entropic parameter $1 \leq q < 3$ help us **define**

weak chaos and obtain: (a) pdfs of what we call quasi-stationary states (QSS) in area–preserving maps (b) QSS in FPU 1-dimensional lattice models that shed light on questions of energy equipartition relevant to solid state physics, (c) identify the occurrence of dynamical transitions in microplasma models and (d) distinguish weakly from strongly chaotic orbits in a barred galaxy model of galactic dynamics.

Title – Lecture 3

Chaotic diffusion in disordered lattices and the role of long range interactions

Abstract – Lecture 3

In the first part of the lecture I will apply what we learned in the previous lectures to study diffusive motion in a Klein Gordon (KG) one–dimensional lattice governed by quartic on–site potential and harmonic interactions with nearest neighbors **in the presence of disorder**. As wave packets **spread subdiffusively** one might expect that the dynamics will ultimately relax on a high – dimensional KAM torus where the motion would be quasiperiodic. Instead, our statistical analysis shows that the particles execute at the beginning **weakly chaotic motion with q(>1) – Gaussian pdfs,** but the global dynamics ultimately **converges to strong chaos,** where q=1 and Boltzmann-Gibbs (BG) statistics reigns! Last, but not least, we will discuss the importance of introducing to our models **Long Range Interactions (LRI) of the form $1/r^{\alpha}$**, r being the inter particle distance and $0 \leq \alpha < \infty$ is a positive parameter. Using again our favorite FPU 1-dimensional lattice, I will show that the dynamics for **LRI with $0 < \alpha < 1$ is weakly chaotic** and becomes **strongly chaotic for $\alpha > 1$**, while **a "Phase Diagram"** can be drawn separating BG from Tsallis thermostatistics.

Reference:

1. "Complex Hamiltonian Dynamics", in English (Synergetics series of Springer Verlag, April 2012).
2. Ch. Skokos, T. Bountis and C. Antonopoulos, "Geometrical Properties of Local Dynamics in Hamiltonian Systems: The Generalized Alignment (GALI) Method", Physica D **231**, 30 (2007).
3. C. Antonopoulos and T. Bountis, "Detecting Order and Chaos by the Linear Dependence Index (LDI) Method", ROMAI **2** (2) Journal, 1 – 13 (2007).
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8. T. Bountis, G. Chechin and V. Sakhnenko, "Discrete Symmetries and Stability in Hamiltonian Dynamics", International J. of Bifurc. Chaos, **6** , 1539-1582 (June, 2011). http://arxiv.org/abs/1005.4308

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Evaldo M. F. Curado

CBPF – Rio de Janeiro, Brazil

Title

Entropy, correlations and extensivity

Abstract

1. Historical introduction and physical motivation

2. Uncorrelated system - binomial distribution

3. Correlated system - deformed binomial distribution

4. Mathematical formalism to construct a deformed binomial distribution

5. Extensivity - analytical studies of Boltzmann-Gibbs, Tsallis

and Rényi entropies for some deformed binomial distributions

6. Extensivity and correlations - discussions

Maria Eunice Q. Gonzalez

UNESP – Marilia, Brazil

Title

Information and Complexity

Abstract

In my lectures, I will introduce the main topics of Information Theory and its possible applications to problems related to the foundations of complex systems. Emphasis is going to be given to the semiotic Theory of Information (mainly developed by Charles S. Peirce), which enables qualitative and quantitative analysis of aspects of natural, formal, and social systems in their multiple constitutive layers. From the Peircean semiotic perspective, information is characterized in terms of its triadic relation to signs, understood as vehicles of forms. I will argue that one advantage of the semiotic approach to information is that it is not restricted to the domain of symbols or any specific conventional languages, hence allowing inter/transdisciplinary studies of the dynamics of complex systems. I will also discuss the four general principles of information processing in complex decentralized systems, proposed by Melaine Mitchell in the paper "Complex Systems: Network thinking":

(1) "Global information is encoded as statistics and dynamics of patterns over the system's components."
(2) "Randomness and probabilities are essential."
(3) "The system carries out a fine-grained, parallel search of possibilities."
(4) "The system exhibits a continual interplay of bottom-up and top-down processes." Mitchell (2006, pp. 1208-1210)

Reference:

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Henrik J. Jensen

Imperial College – London, United Kingdom

Title

Intermittent dynamics in complex system

Abstract

The systemic level dynamics of may complex systems exhibit a non-steady pace. Periods of little activity are interrupted by brief bursts of change. Examples are earthquakes, finance, macroevolution, brain activity etc. We'll discuss a number of examples (spin glasses, superconductors, biological evolution, hungry ants) with emphasis on cases where the dynamics is non-stationary and where the abrupt events, for simplicity called quakes, leads to a change in the properties of the systems. The time at which the quakes occur are often found to follow so-called record statistics and we'll discuss the properties of this kind of statistics and its consequences.

At the end of the series of lectures we'll turn to attempts to predict the occurrence of the quakes and explain how a combination of ordinary linear stability analysis (though of very high dimensional systems) with simulations (or observation) may be able to forecast with a significant the approaching dramatic transformative bursts of activity.

Reference

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Renio S. Mendes

Universidade Estadual de Maringa - Brazil

Title

Diversity index, Tsallis entropy, mean values, and anomalous diffusion

Abstract

We discuss some introductory aspects of nonextensive statistics. We start focusing on an ecological context: diversity indexes. We show that these indexes can be used as a motivation to introduce entropic forms. In this context, we explore the use of Tsallis entropy. The following point to be investigated is the use of generalized mean values in connection with generalized exponentials. A special attention is given to q-exponentials and q-Gaussians. After the discussion of these aspects, we direct our discussion to anomalous diffusion, mainly related to nonlinear diffusion equations that can be connected with q-Gaussians. Applications are inserted along of our studies.

Fernando D. Nobre

CBPF – Rio de Janeiro, Brazil

Title

Nonlinear Fokker-Planck equations and generalized entropies for complex systems

Abstract

In these talks we will explore important topics of nonequilibrium statistical mechanics, associated with time-evolution equations of the Fokker-Planck type. First, we will review briefly the linear Fokker-Planck equation, its time-dependent and stationary-state solutions, as well as its relation to the Boltzmann-Gibbs entropy.

These well-known results will be generalized for nonlinear Fokker-Planck equations, and particularly, their relations with generalized entropic forms will be emphasized. Physical consequences and possible applications will be discussed. A system of interacting vortices, which is currently used in the literature to model flux lines in disordered type-II superconductors, will be analyzed in its stationary state. It will be shown that this system is closely related to nonextensive statistical mechanics, and particularly, a consistent thermodynamic framework will be presented.

Angel R. Plastino

Universidad del Noroeste de Buenos Aires - Argentina

Title – Lecture 1

Quantum entanglement in atomic models

Abstract – Lecture 1

We review the roles played by the Tsallis and the Renyi entropies as the basis of quantitative entanglement measures or indicators. We apply these tools to the study of quantum entanglement in various atomic models. We consider both exactly soluble models, such as the Moshinsky and the Krandall ones, and also more realistic systems like the Helium isoelectronic series.

Title – Lecture 2

Aspects of the NRT non-linear Schroedinger equation

Abstract – Lecture 2

We discuss various aspects of the NRT nonlinear Schroedinger equation. This equation is closely related to the nonextensive thermostatitical formalism and admits soliton-like, power-law traveling solutions of the q-plane wave form. We consider some symmetry properties exhibited by the NRT equation, related to Galilean transformation, and to transformations involving uniformly accelerated reference frames. We explore a family of exact, analytical q-Gaussian wave-packet solutions of the NRT equation that incorporates the aforementioned q-plane wave solutions as particular instances.

Title – Lecture 3

Entropic measures and the conservation of information

Abstract – Lecture 3

The conservation of information associated with the evolution of closed physical systems is nowadays regarded as a fundamental principle of Nature. We discuss various aspects of this feature of fundamental physical laws, focusing on generalizations of Landauer's principle, and on classical analogues of basic properties of quantum information, such as the one given by the celebrated no-cloning theorem.

Andrea Rapisarda

University of Catania – Italy

Title

Chaotic dynamics and statistical mechanics in systems with-long range interactions

Abstract

Long-range interacting systems, both of conservative and dissipative type, have been extensively studied in this last decade. I will address the dynamical features of these systems both in the equilibrium and out-of-equilibrium regime, illustrating a possible interpretation in terms of Tsallis generalized statistical mechanics. The importance of long-range interactions and the connections to Tsallis statistics will be discussed also in systems showing self-organized criticality and in coupled maps at the edge-of-chaos.

Alberto Robledo

UNAM – Mexico City, Mexico

Title

Generalized statistical mechanics at the onset of chaos

Abstract

Transitions to chaos in archetypal low-dimensional nonlinear maps offer real and precise model systems in which to assess proposed generalizations of statistical mechanics. The known association of chaotic dynamics with the structure of Boltzmann–Gibbs (BG) statistical mechanics has suggested the potential verification of these generalizations at the onset of chaos, when the only Lyapunov exponent vanishes and ergodic and mixing properties cease to hold. There are three well-known routes to chaos in these deterministic dissipative systems, period- doubling, quasi-periodicity and intermittency, which provide the setting in which to explore the limit of validity of the standard BG structure. It has been shown that there is a rich and intricate behavior for both the dynamics within and towards the attractors at the onset of chaos and that these two kinds of properties are linked via generalized statistical-mechanical expressions. Amongst the topics presented are: (i) permanently growing sensitivity fluctuations and their infinite family of generalized Pesin identities; (ii) the emergence of statistical-mechanical structures in the dynamics along the routes to chaos; (iii) dynamical hierarchies with modular organization; and (iv) limit distributions of sums of deterministic variables. The occurrence of generalized entropy properties in condensed-matter physical systems is illustrated by considering critical fluctuations, localization transition and glass formation. We complete our presentation with the description of the manifestations of the dynamics at the transitions to chaos in various kinds of complex systems, such as, size-rank functions and complex network images of time series.

Piergiulio Tempesta

Universidad Complutense de Madrid – Spain

Title

Groups, Entropies and Number Theory

Abstract

Entropy is a fundamental notion, at the heart of modern science. In the second half of the twentieth century, its range of applicability has been extended from the traditional context of classical thermodynamics to new areas such social sciences, economics, biology, quantum information theory, linguistics, etc. More recently, the role of entropy in the theory of complex systems has been actively investigated. From one side, several studies were devoted to axiomatic formulations, aiming at clarifying the foundational aspects of the notion of entropy. From the other side, many researchers pursued the idea of generalizing the classical Boltzmann-Gibbs statistical mechanics.

Consequently, a plethora of new entropic forms, designed for extending the applicability of BG entropy to new contexts, was introduced.

The purpose of these Lecture Notes is to present a theory of generalized entropies, based on a simple, unifying principle: the existence of a group-theoretical structure underlying the notion of entropy. This structure is responsible of essentially all the main mathematical and thermodynamical properties of generalized entropies.

Lecture I: On the mathematical foundations of the theory of generalized entropies

First, we shall discuss several foundational aspects at the heart of the notion of entropy. Then we shall introduce a new formulation of the classical Shannon-Khinchin axioms, where the fourth one is replaced by the Composability Axiom. It amounts to require that the composition of statistically independent subsystems can be performed according to a formal group law. This property arises as a crucial one in order for an entropy to be considered as thermodynamically admissible.
Consequently, the theory of formal group laws, originated in the context of algebraic topology by the work of Bochner, Novikov, Quillen, Serre, etc., will be outlined, and its relation with thermodynamics elucidated.

Lecture II: From groups to entropies: the composability axiom and the universal group entropy

We shall use the group-theoretical approach to classify the known trace-form entropies in relation with a general entropic function, called the universal group entropy, due to its connection with Lazard's universal formal group.

In particular, several new cases of multi-parametric trace-form entropies will be discussed as a realization of the proposed universal construction.

Lecture III: Information theory of group entropies: Rényi's entropy and the Z-class of entropies

The important class of entropies which are not of trace-form type will be analyized. It will be shown that the group approach allows to define a tower of infinitely many new entropies, called Z-entropies, containing the Rényi entropy as the first nontrivial case. All of them are composable and thermodynamically admissible.The information-theoretical content of these entropies will be discussed.

Lecture IV: Formal groups and zeta functions: A possible root from Number Theory to Statistical Mechanics.

In the last Lecture, the emerging multiple connections among formal groups, entropies and analytic number theory will be sketched. In particular, a large class of Dirichlet series and Bernoulli polynomials will be constructed. They are directly related with the same algebraic structures used to define the universal group entropy and the Z-entropies.

Stefan Thurner

University of Vienna – Austria

Title – Lecture 1

What are complex systems?

Abstract – Lecture 1

[TBA]

Title – Lecture 2

Information theory for complex systems

Abstract – Lecture 2

[TBA]

Title – Lecture 3

The maximum entropy principle for complex systems

Abstract – Lecture 3

[TBA]

Title – Lecture 4

Non-ergodic Markov processes and power laws

Abstract – Lecture 4

[TBA]

Ugur Tirnakli

Ege University – Izmir, Turkey

Title – Lecture 1

Low-dimensional dynamical systems in the vicinity of chaos threshold: Part I

Abstract – Lecture 1

It is now known from the literature that the analysis of the sensitivity to initial conditions of low-dimensional dissipative dynamical systems allows one to deduce the index qsen (sen stands for sensitivity) which can also be obtained from the analysis of the entropy increase rates. These two methods are discussed here in details. On the other hand, investigating the phase space contraction of the same dynamical systems, one can also introduce another index qrel (rel stands for relaxation) which is the second foot of the famous q-triplet. We also give here the details of this method.

Title – Lecture 2

Low-dimensional dynamical systems in the vicinity of chaos threshold: Part II

Abstract – Lecture 2

It is evident that one of the important ingredients of statistical mechanics and of probability theory is the central limit theorem. For deterministic dynamical systems, a central limit theorem is valid only if the system is sufficiently mixing. Recently the leading-order correction to the central limit theorem for the fully developed logistic map and cubic map has been analytically deduced and found to be in perfect agreement with numerical results. On the other hand, due to strong temporal correlations, a standard central limit theorem is not valid when the system is at the edge of chaos. It is also shown that the probability distribution

of sums of iterates of the logistic map at the edge of chaos can be well approximated with q-Gaussian. In order to better analyse the entire distribution (both the central part and the tails) to see under what conditions there is convergence to a q-Gaussian, we performed a closer look at the problem. A theoretical argument has been provided on determining the optimum value of N (the number of iterates) in order to achieve the

best convergence to a q-Gaussian. These results are consistent with a large number of already available analytical and numerical evidences that the edge of chaos is well described in terms of Tsallis entropy and its associated concepts.

Title – Lecture 3

Emergence of q-Gaussians in complex systems: Case study of SOC and non-SOC models

Abstract – Lecture 3

The probability density functions of avalanche size differences (or return distributions) of several models of complex systems are shown to converge to the shape of q-Gaussians, which are the standard distributions obtained in nonextensive statistical mechanics.

An exact relation connecting the exponent tau of avalanche size distribution and the q value of appropriate q-Gaussian has been derived. Making use of this relation, for each case, one can easily determine q parameter values of the appropriate q-Gaussians a priori from one of the well known exponents of the model system. This clearly rescues the q parameter to be a fitting parameter anymore, which is a rather rare achievement in the literature.

Constantino Tsallis

CBPF – Rio de Janeiro, Brazil & Santa Fe Institute, USA

Title

Statistical mechanics for complex systems: Foundations and applications

Abstract

Boltzmann-Gibbs entropy and statistical mechanics is one of the pillars of contemporary physics. It applies extremely successfully to the so called simple systems, essentially ergodic. When we wish to study complex systems, particularly nonergodic ones, a more powerful theory is needed. For a wide class of such complex systems, nonadditive entropies and the associated statistical mechanics are being currently used and studied.

Recent aspects related to its foundations and applications are now available. An overview will be presented. Foundations concern nonlinear dynamics, central limit theorems, large deviation theory, probabilistic correlations, calculation of the index q from first principles, among others. Applications concern long-range-interacting many-body classical Hamiltonian systems (e.g., XY rotator and Fermi-Pasta-Ulam oscillator models), over- damped motion of repulsively interacting vortices in type-II superconductors, high energy physics (e.g., distributions of momenta in high energy collisions at CERN/LHC, Brookhaven/RHIC), granular matter (position fluctuations in two-dimensional shear motion), plasma physics (e.g., distributions of velocities), financial laws (e.g., distributions of price returns and of interoccurrence times), geophysics (e.g., seismic analyses of diverse geographical areas), biology (e.g., chemical distances between classes of nucleotides in DNA sequences of modern and archaic bacteria and Homo Sapiens), cold atoms, image and signal processing, among others.

Reference

A regularly updated bibliography is available at http://tsallis.cat.cbpf.br/biblio.htm