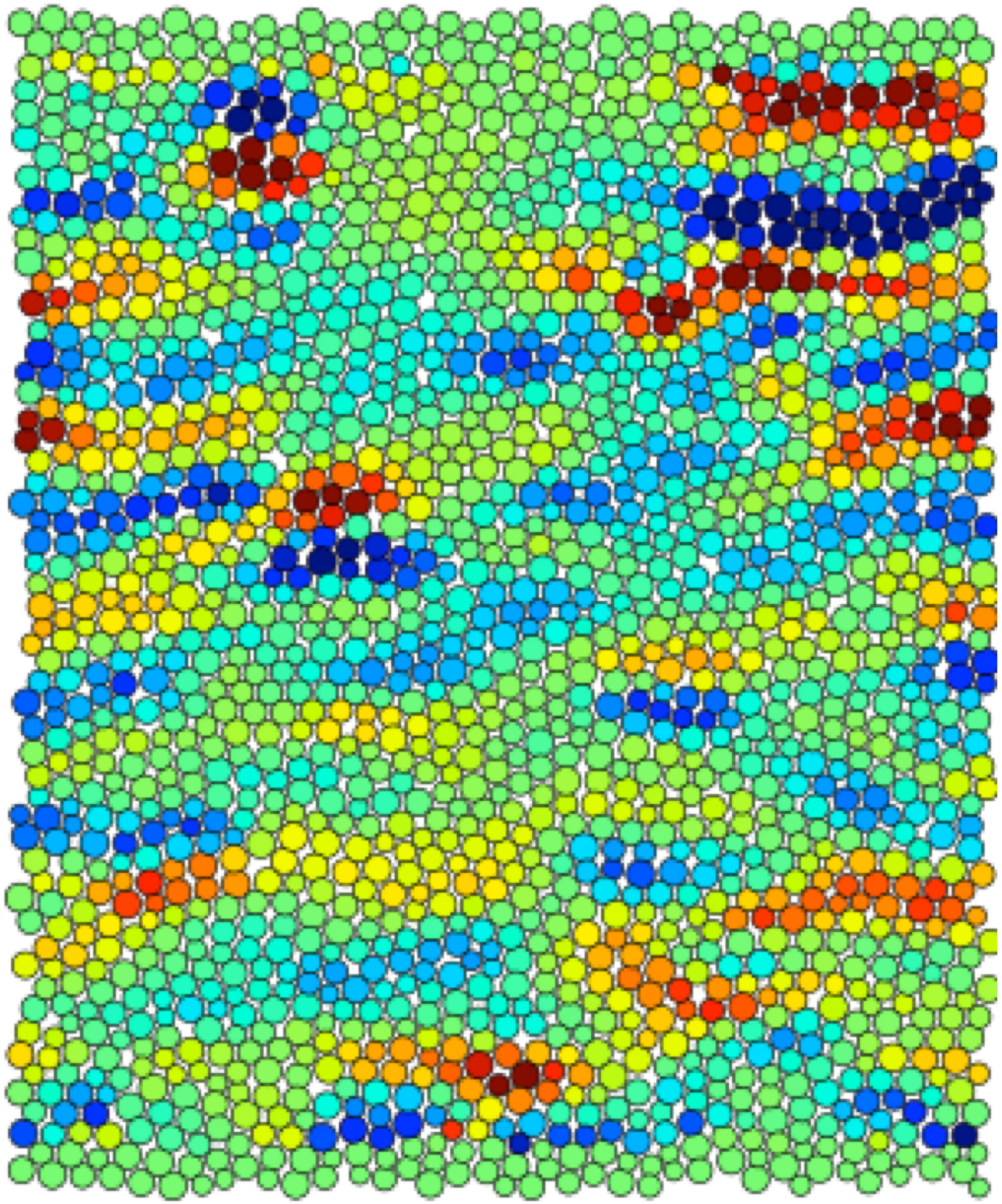
# Rheology and jamming of granular systems and suspensions

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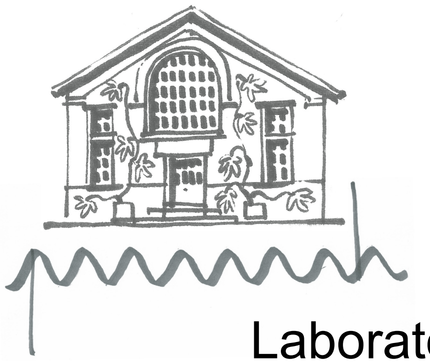


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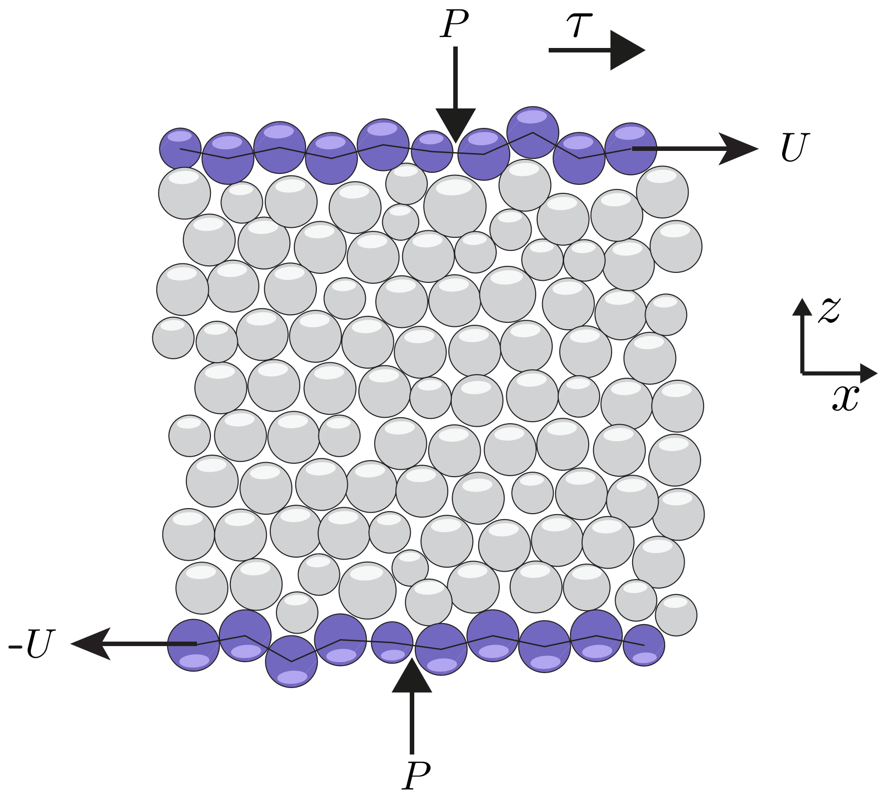
# Homogeneous shear flow of rigid particles




Inertial number

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}$$

effective friction $\mu = \frac{\tau}{P}$

volumic fraction $\phi$

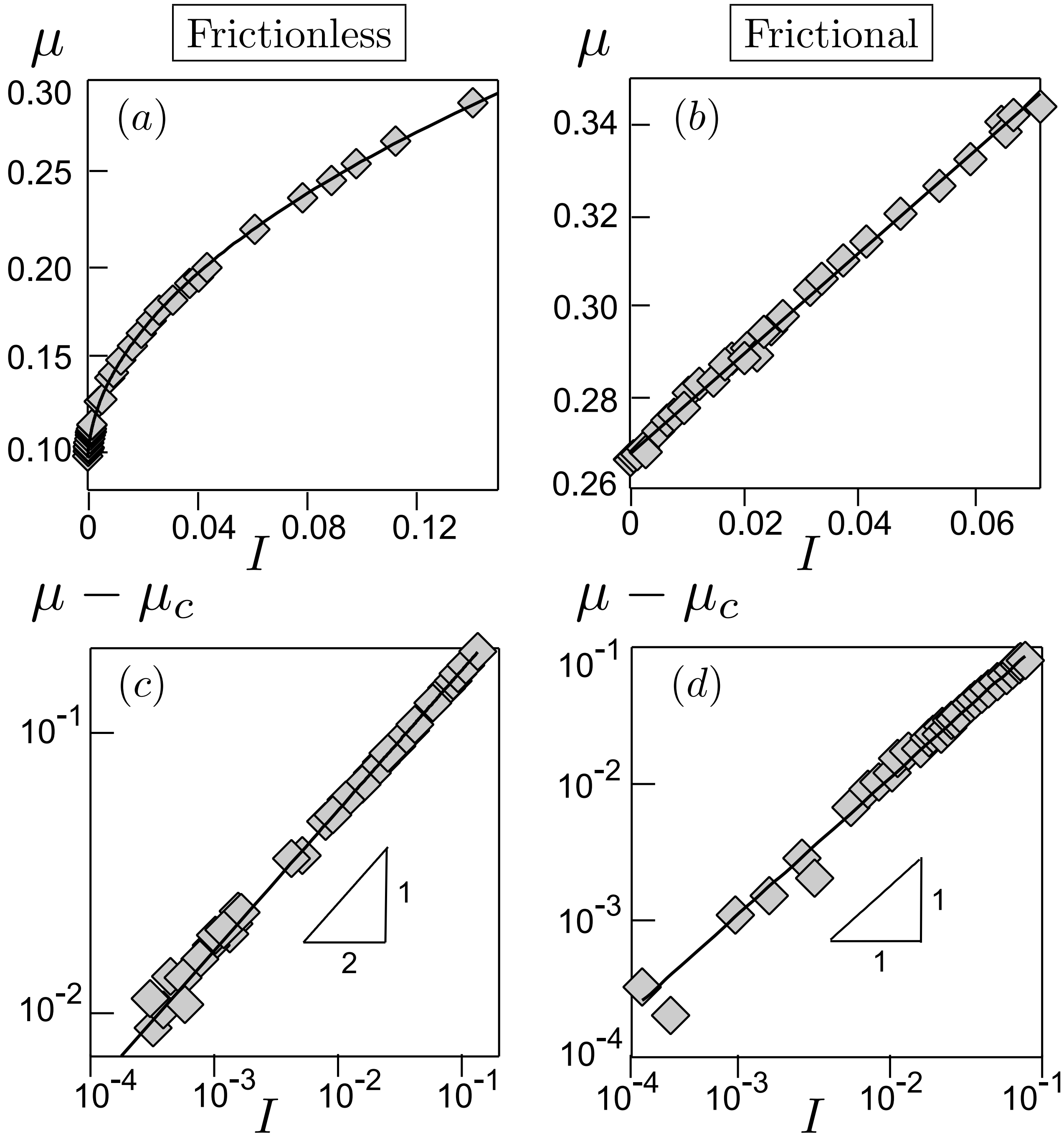
coordination number $Z$


GdR MiDi, EPJE **14**, 341 (2004).

da Cruz et al., PRE **72**, 021309 (2005).

Jop et al., Nature **441**, 727 (2006).

# Local rheology

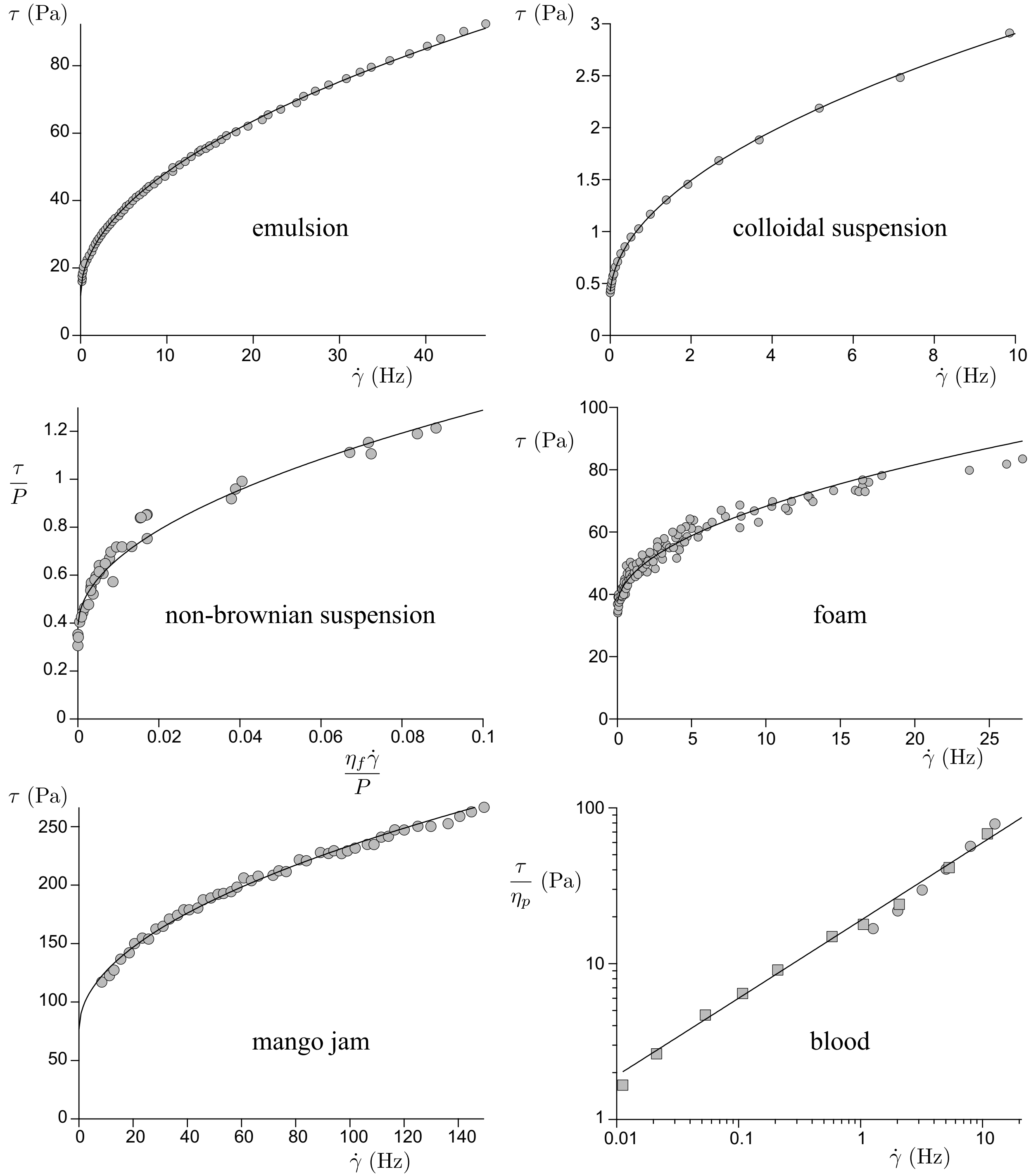



$$\mu = \mu_c + aI^\alpha$$

$$\phi = \phi_c - bI^\alpha$$

$$\mathcal{Y} = \frac{\mu}{\mu_c}$$

# Rheologies of amorphous materials




Goyon et al., Nature **454**, 84 (2008).

Willenbacher et al., Soft Matter **7**, 5777 (2011).

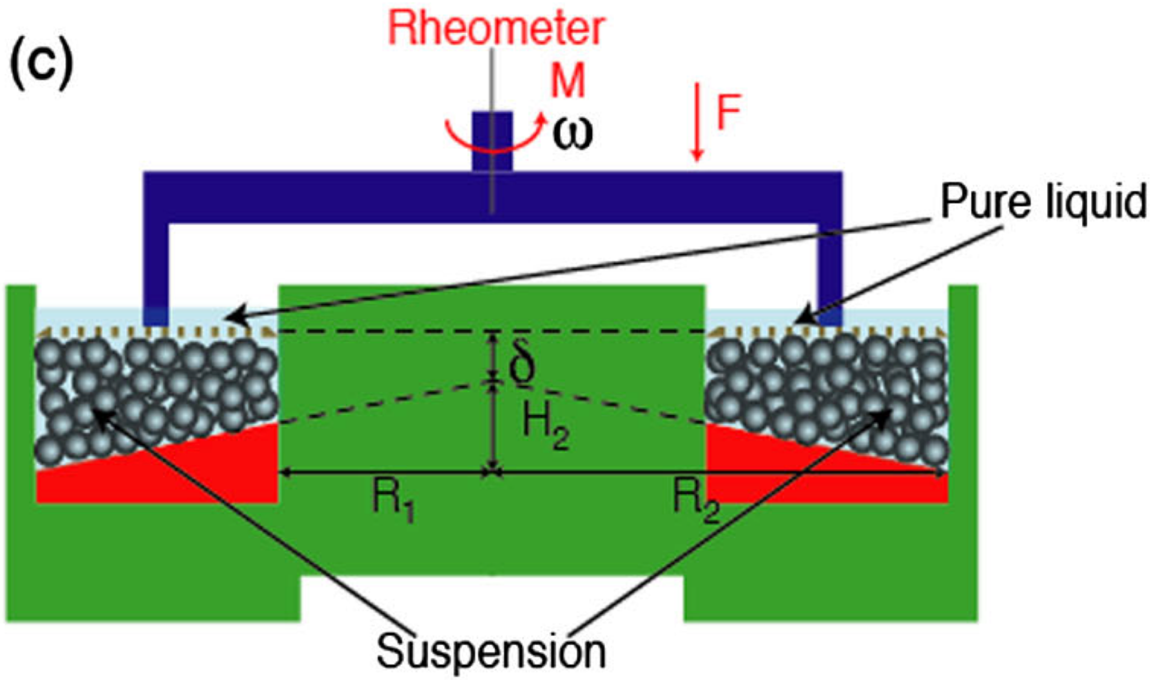
Boyer et al., PRL **107**, 188301 (2011).

Ovarlez et al., EPL **91**, 68005 (2010).

Basu & Shivhare, J. Food. Eng. **100**, 357 (2010).

Chien Science **168**, 977(1970).

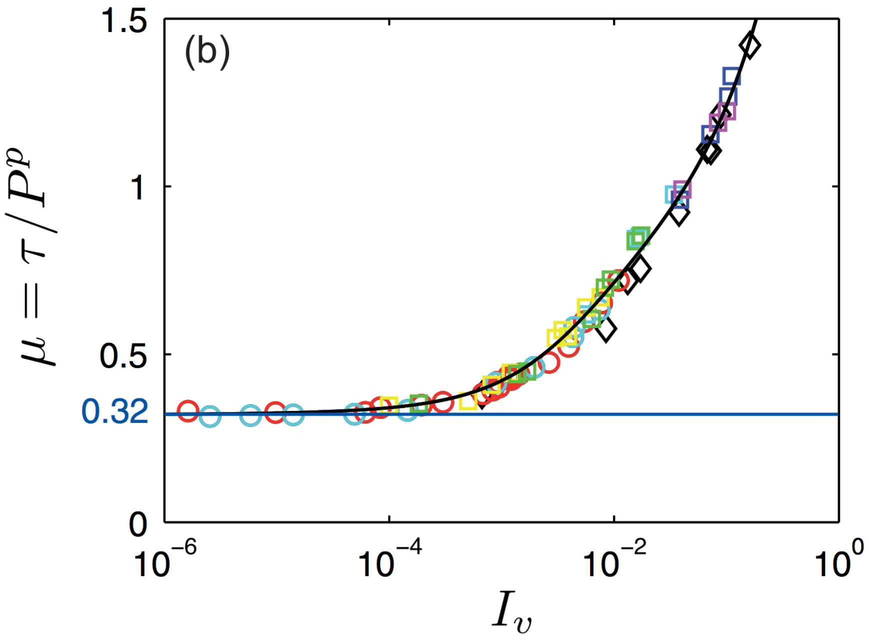
# Rheology of dense granular suspensions

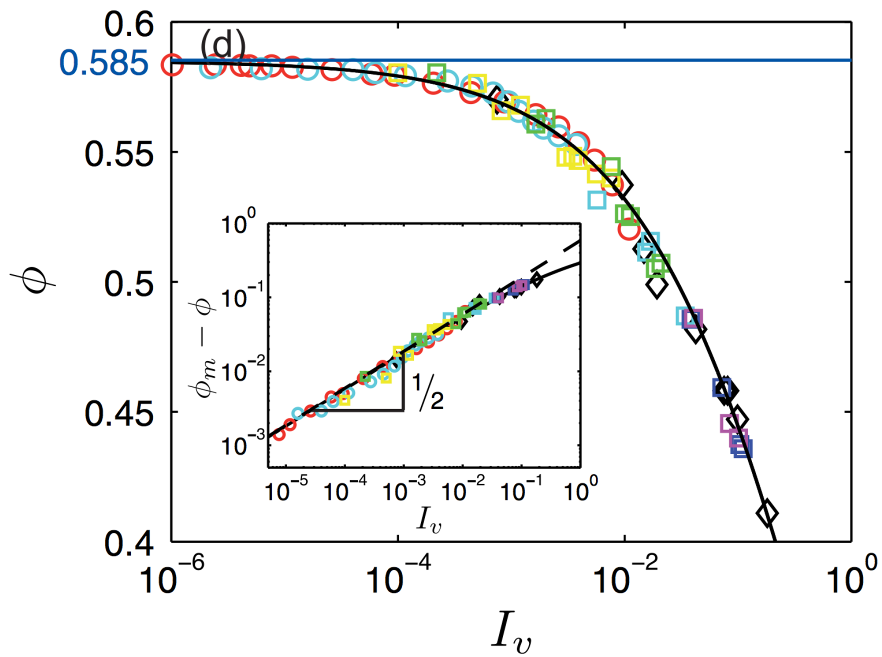



Boyer et al., PRL **107**, 188301 (2011).

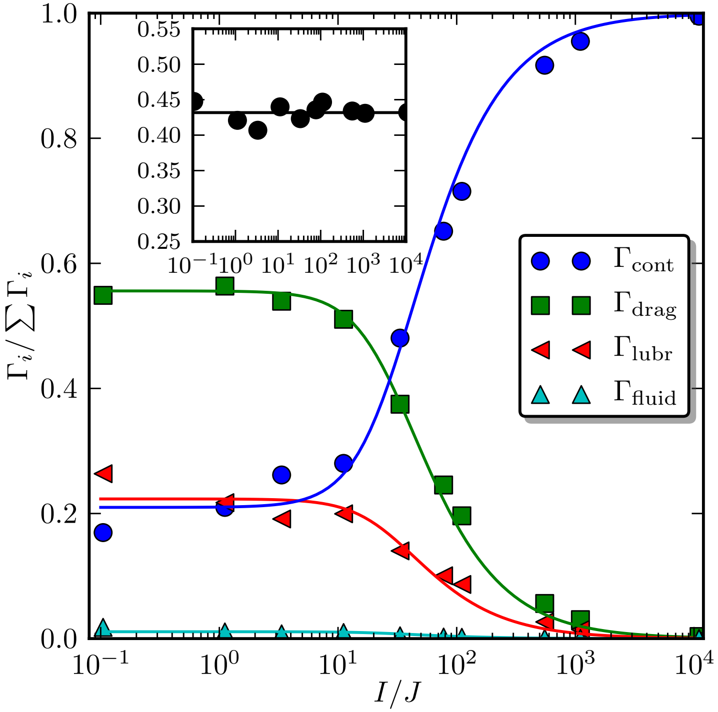
$$J = \frac{\eta_f \dot{\gamma}}{P}$$

$(I_v \equiv J)$



# Transition from viscous to inertial regime




Adding dissipation

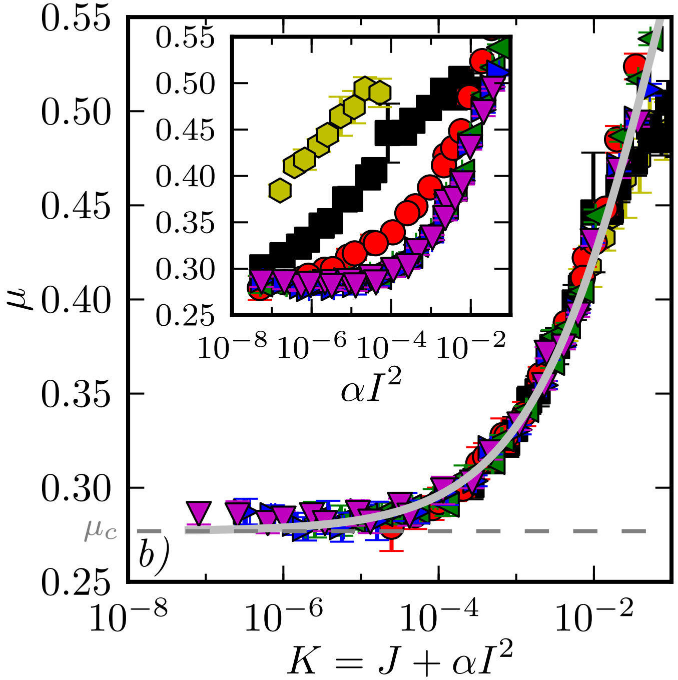
$$\tau = f_\tau(\phi)\left(\eta_f \dot\gamma + \alpha \rho d^2 \dot\gamma^2\right)$$

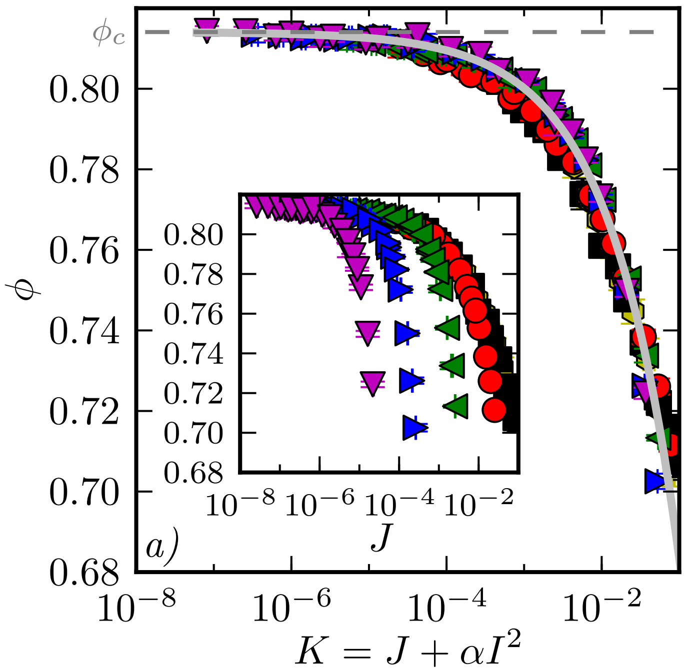
$$P = f_P(\phi)\left(\eta_f \dot\gamma + \alpha \rho d^2 \dot\gamma^2\right)$$

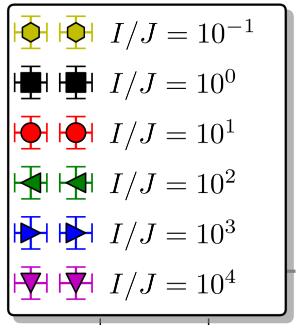
Stokes number

$$I^2/J = \dot\gamma d^2 \rho / \eta_f \simeq 1/\alpha$$

$$\boxed{K = J + \alpha I^2}$$



# Brownian particles

Péclet number

$$\mathrm{Pe} = \frac{\eta_f \dot{\gamma}}{k_B T / d^3}$$

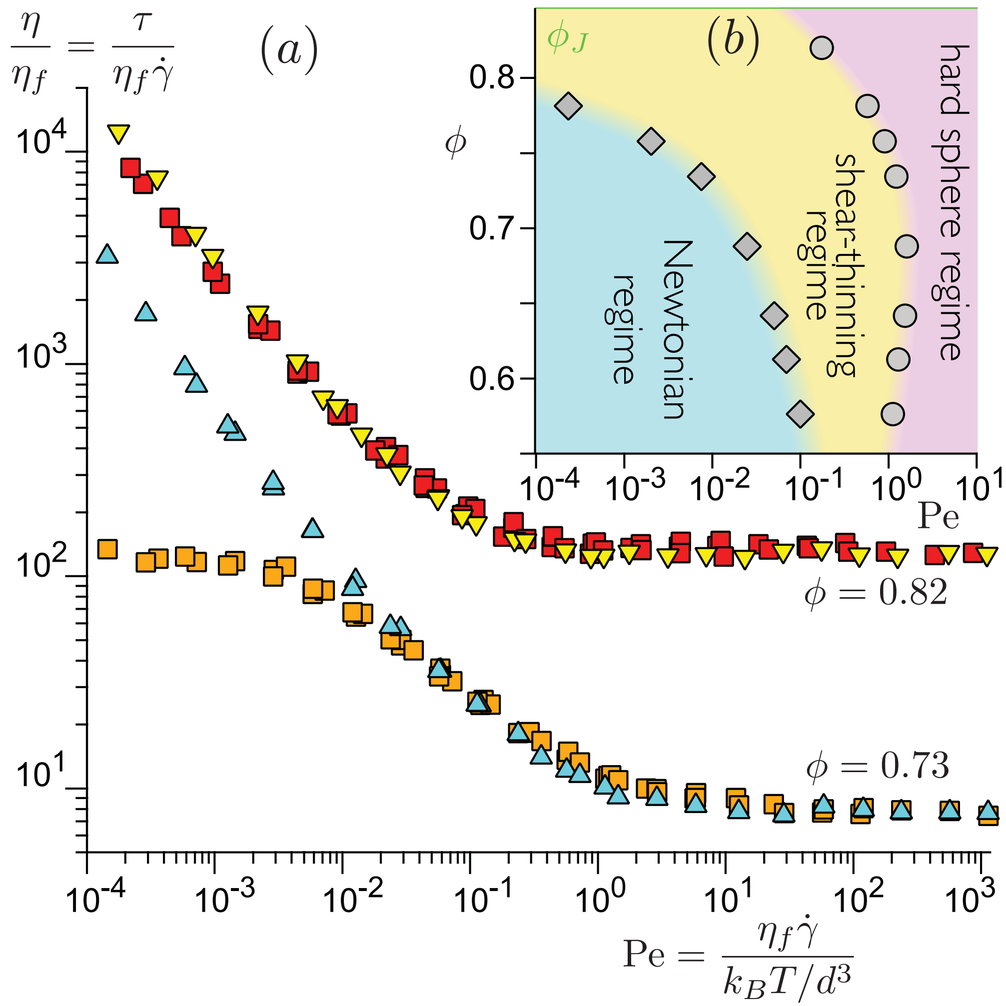
Overdamped Langevin equation

$$-\frac{3\pi\eta_f}{(1-\phi)}\boldsymbol{\delta u}_i + \sum_{j \neq i} \boldsymbol{f}_{ij} + \boldsymbol{\xi}_i = 0$$

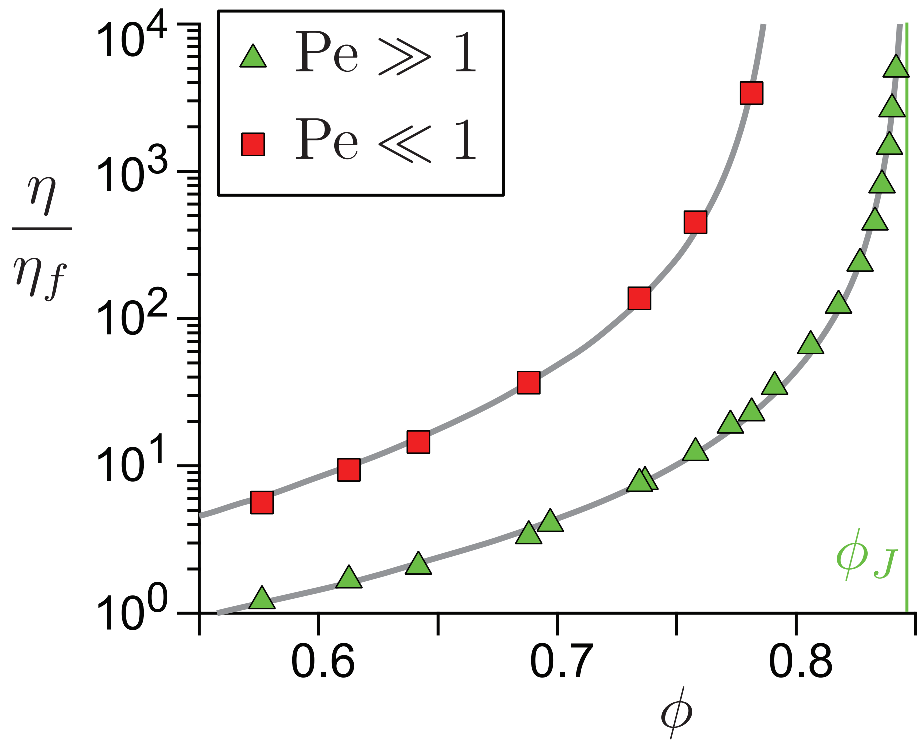
$$\boldsymbol{\delta u}_i \equiv \boldsymbol{u}_i - \dot{\gamma} z \mathbf{e}_x$$

$$\langle \boldsymbol{\xi}_i(t) \boldsymbol{\xi}_j(t') \rangle = 6\pi\eta_f k_B T/(1-\phi)\delta_{ij}\delta(t-t')$$

# Rheology



$\frac{\eta}{\eta_f} = \frac{\tau}{\eta_f \dot{\gamma}}$
(a)
(b)
$\phi_J$
Newtonian regime
shear-thinning regime
hard sphere regime
Pe
$\phi = 0.82$
$\phi = 0.73$
$\mathrm{Pe} = \frac{\eta_f \dot{\gamma}}{k_B T / d^3}$

$\mathrm{Pe} \gg 1$
$\mathrm{Pe} \ll 1$
$\frac{\eta}{\eta_f}$
$\phi_J$
$\phi$

# Athermal analogue

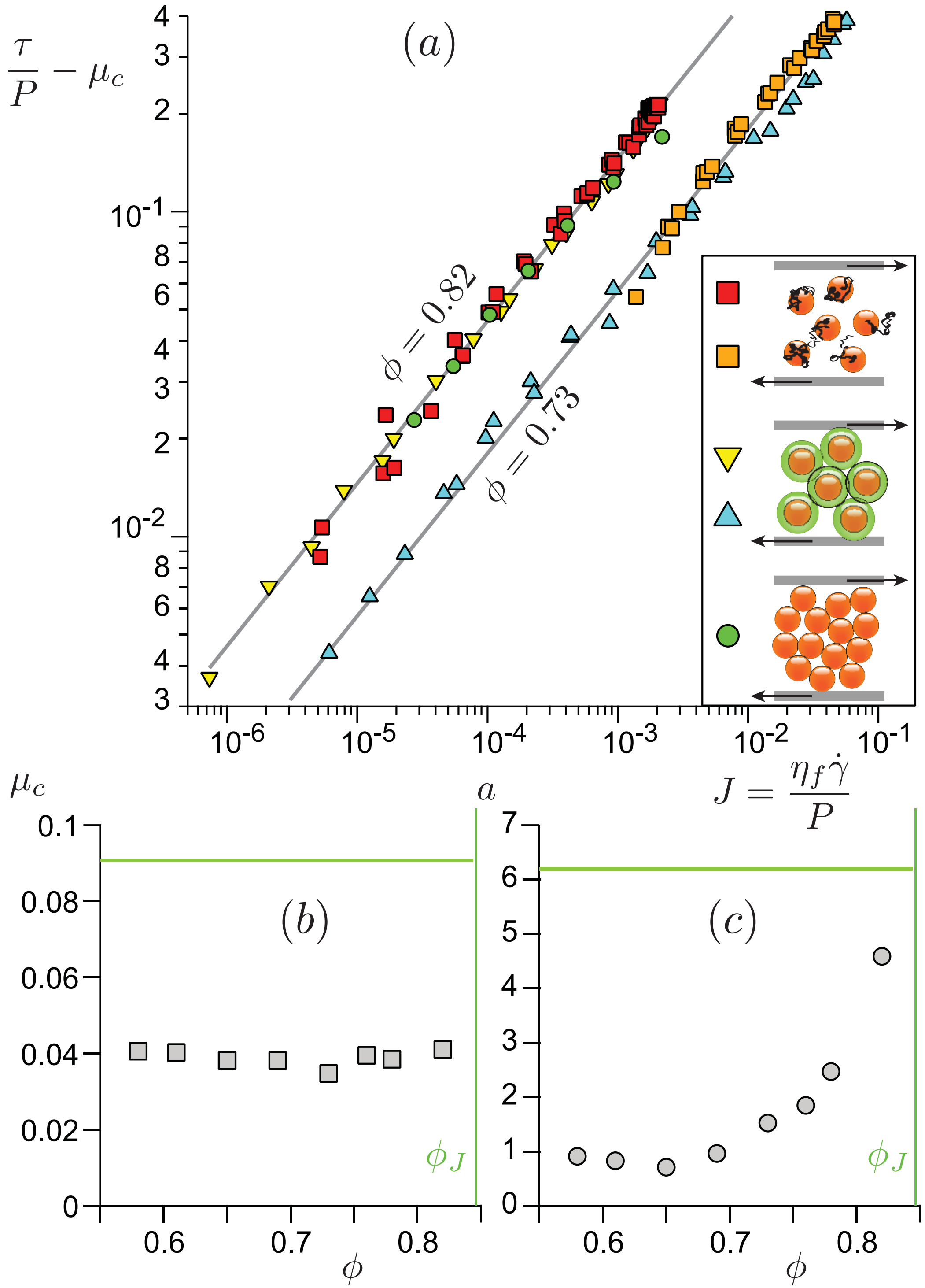
Effective potential

$$\varphi(r) = -k_B T \ln \frac{r-d}{d}$$

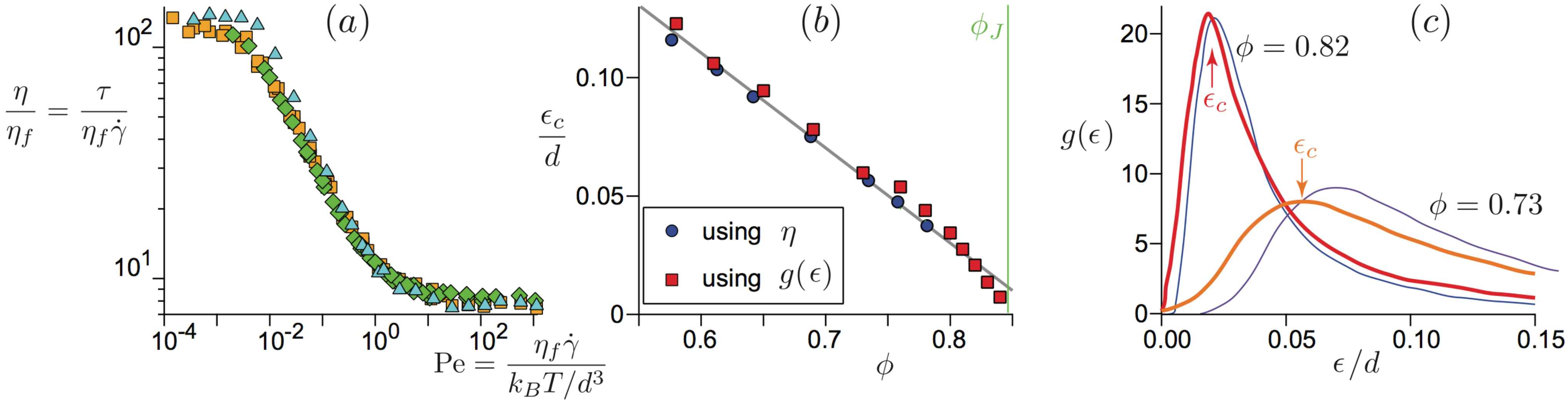
Brito & Wyart EPL **76**, 149 (2006).

Herschel-Bulkley rheology

$$\frac{\tau}{P} = \mu_c + aJ^{0.5}$$

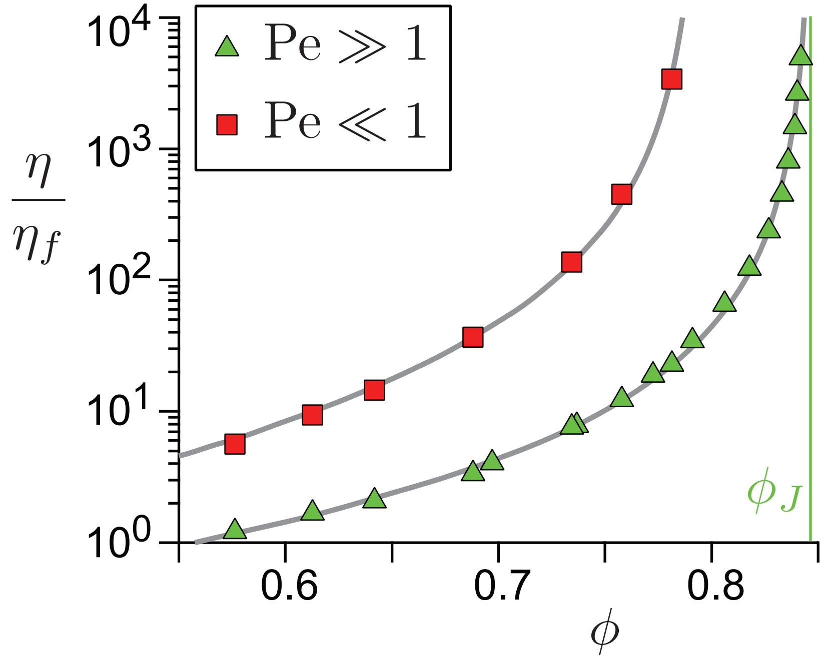


# Phenomenological cut-off length



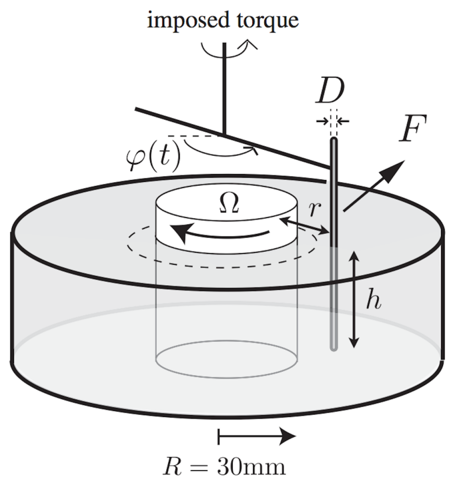

$$\eta_B(\phi) = \eta_H \left[ \left(1 + \frac{\epsilon_c}{d}\right)^2 \phi \right]$$

■ ▲

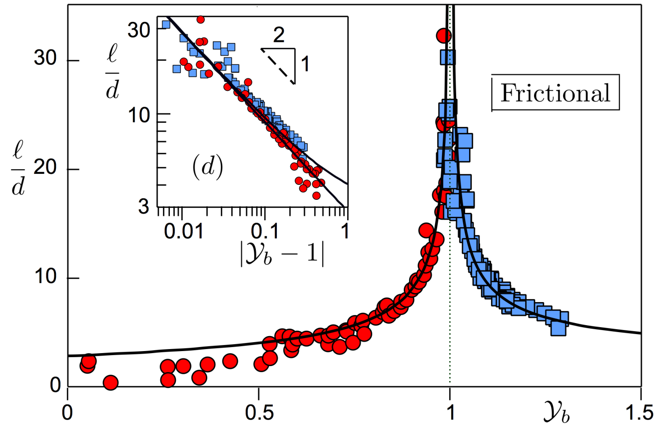


# Beyond the local rheology

Reddy et al., PRL **106**, 108301 (2011).




$$\mathcal{Y}_b = \frac{\mu}{\mu_c} = \frac{\tau}{\mu_c P}$$




- Non-local effects
- Spatial relaxation length
- Non-local constitutive relations

Order parameter: fluidity $f$

$$\frac{\tau}{P} = \mu(I)\,[1 - \chi(\kappa)]$$

$$\chi(\kappa) = \nu\kappa + \mathcal{O}(\kappa^2)$$

$$\kappa = \frac{d^2\nabla^2 f}{f}$$

$$f = I = \frac{|\dot{\gamma}|d}{\sqrt{P/\rho}}$$

# Related publications


[1] *Transition from viscous to inertial regime in dense suspensions*
M. Trulsson, B. Andreotti and P. Claudin
Phys. Rev. Lett. **109**, 118305 (2012).

[2] *A non-local rheology for granular flows across yield conditions*
M. Bouzid, M. Trulsson, P. Claudin, E. Clément and B. Andreotti
Phys. Rev. Lett. **111**, 238301 (2013).

[3] *Dynamic compressibility of dense granular shear flows*
M. Trulsson, M. Bouzid, P. Claudin and B. Andreotti
Europhys. Lett. **103**, 38002 (2013).

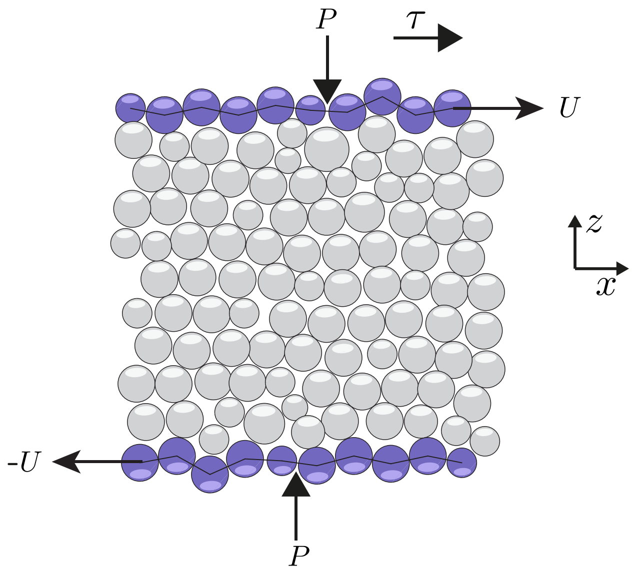
[4] *Microrheology to probe non-local effects in dense granular flows*
M. Bouzid, M. Trulsson, P. Claudin, E. Clément and B. Andreotti
Europhys. Lett. **109**, 24002 (2015).

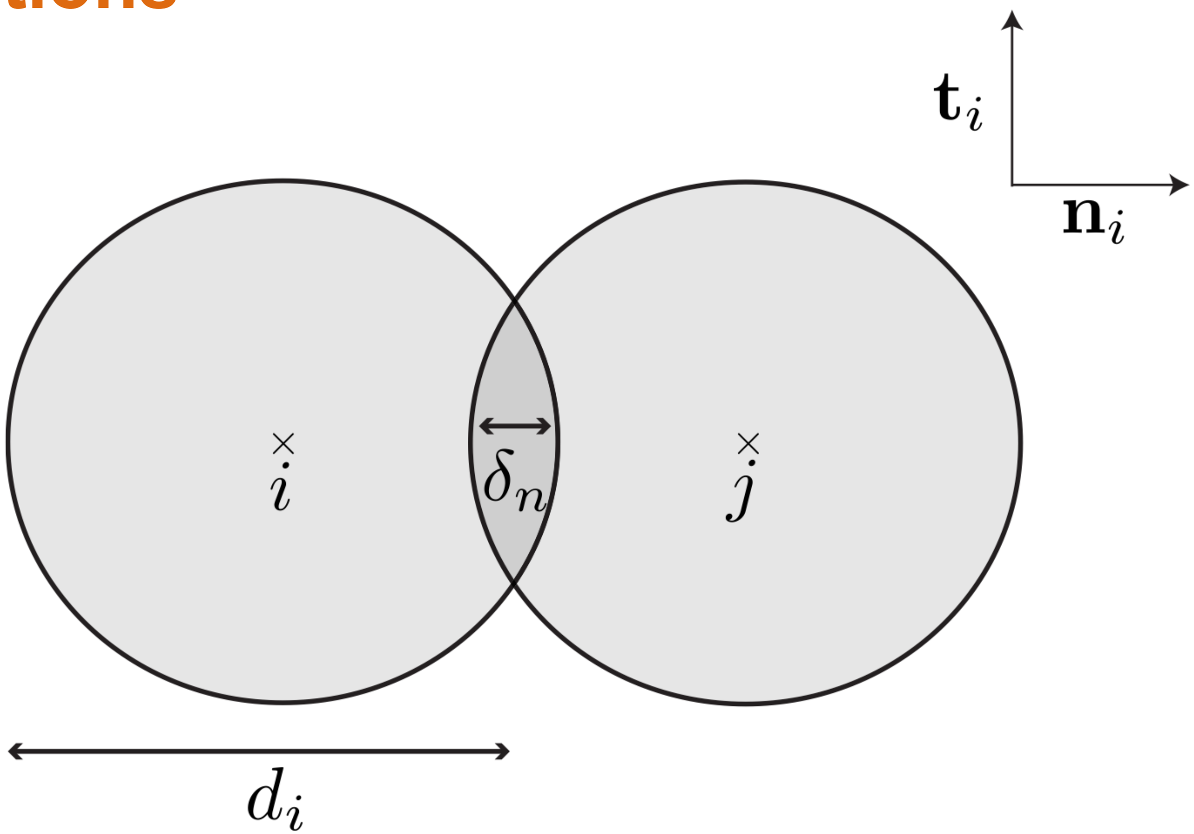
[5] *Athermal analogue of sheared dense Brownian suspensions*
M. Trulsson, M. Bouzid, J. Kurchan, E. Clément, P. Claudin and B. Andreotti
Europhys. Lett. **111**, 18001 (2015).

[6] *Non-local rheology in dense granular flows -- Revisiting the concept of fluidity*
M. Bouzid, A. Izzet, M. Trulsson, E. Clément, P. Claudin and B. Andreotti
Accepted EPJE.


`www.pmmh.espci.fr/~claudin/publications.html`

# DEM numerical simulations




$$m_i \frac{\mathrm{d}^2 \vec{r}_i}{\mathrm{d}t^2} = \sum_{j \neq i} \vec{F}_{ij} + \vec{F}_{\mathrm{ext}}$$

$$I_i \frac{\mathrm{d}\vec{\omega}_i}{\mathrm{d}t} = \sum_{j \neq i} d_i \vec{n}_{ij} \times \vec{F}_{ij}$$

contact laws

$$\boxed{\frac{k_n}{P}}$$

# Change of representation

$$\frac{\tau}{P} = \mu_c \left(1 + aI^n\right)$$

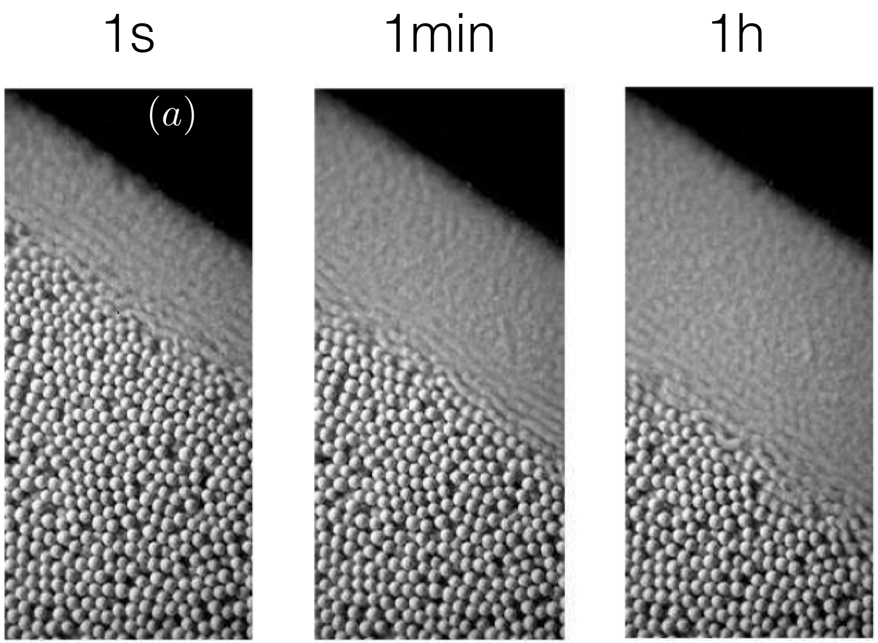
$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}$$

$$\phi = \phi_c - bI^n$$

$$P = b^{2/n} \frac{\rho \dot{\gamma}^2 d^2}{(\phi_c - \phi)^{2/n}}$$

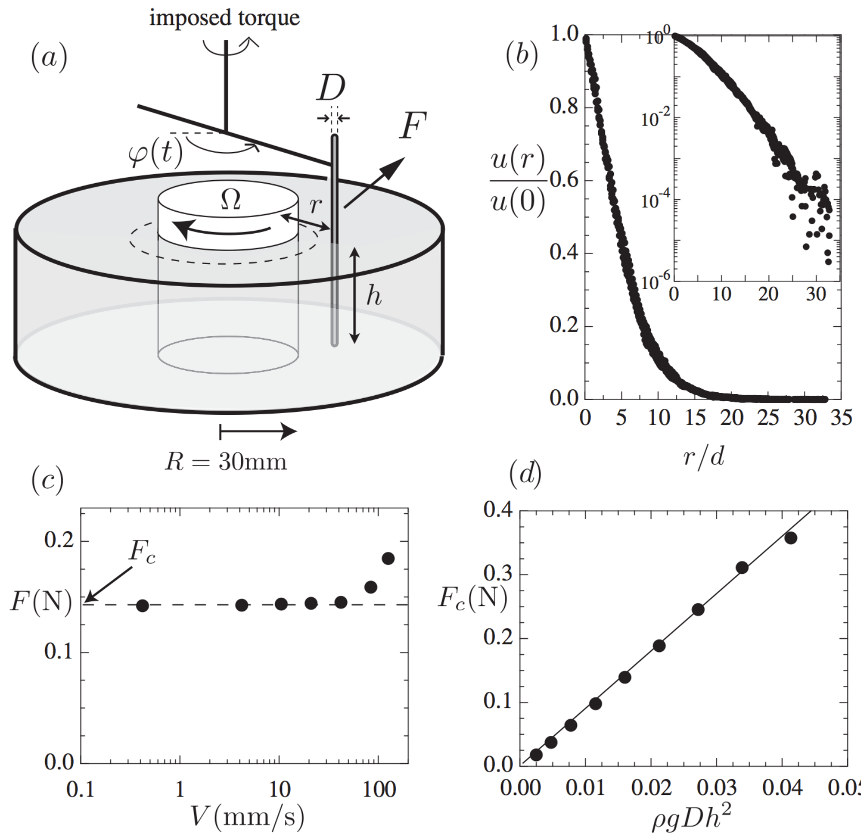
$$\tau = \mu_c b^{2/n} \left(1 + \frac{a}{b}\left(\phi_c - \phi\right)\right) \frac{\rho \dot{\gamma}^2 d^2}{(\phi_c - \phi)^{2/n}}$$

# Non-Local flowing effects



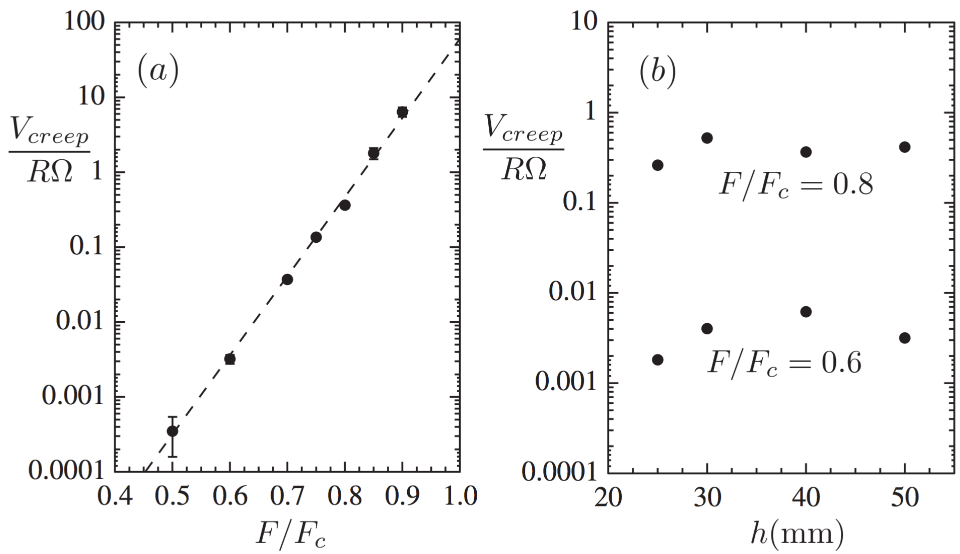

Komatsu et al., PRL **86**, 1757 (2001).

Reddy et al., PRL **106,** 108301 (2011).

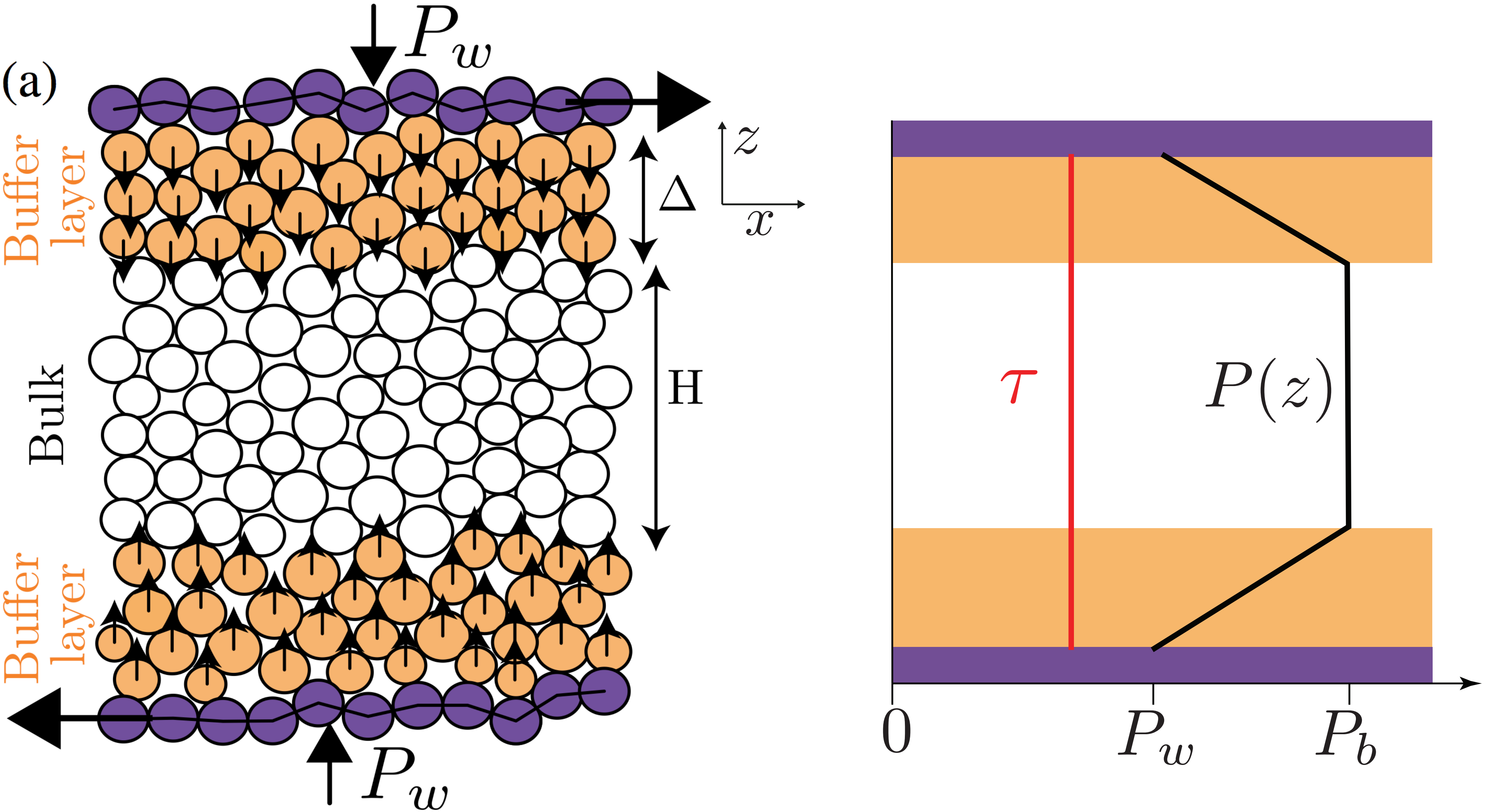



Goyon et al., Nature **454**, 84 (2008).

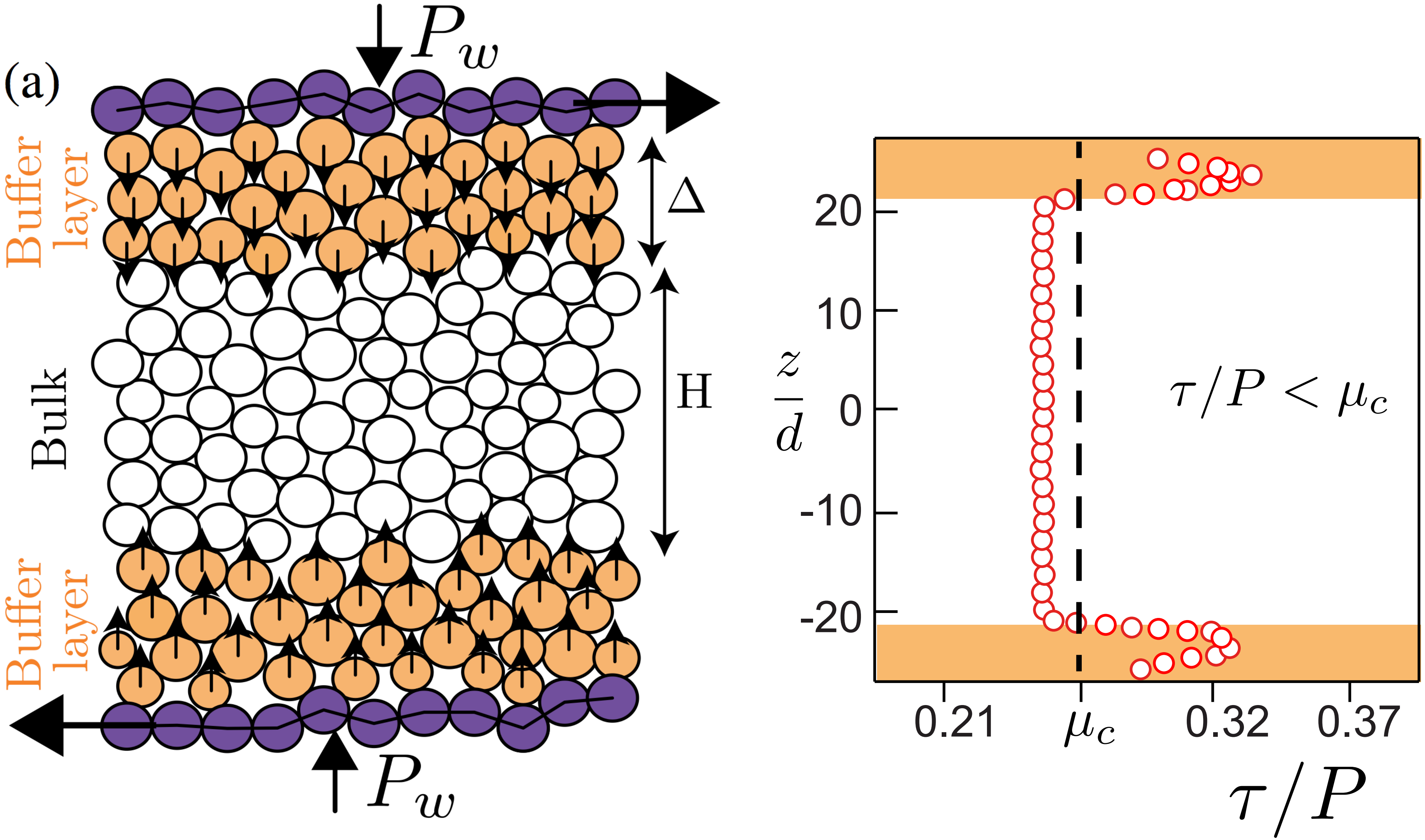
Nichol et al., PRL **104** 078302 (2010).



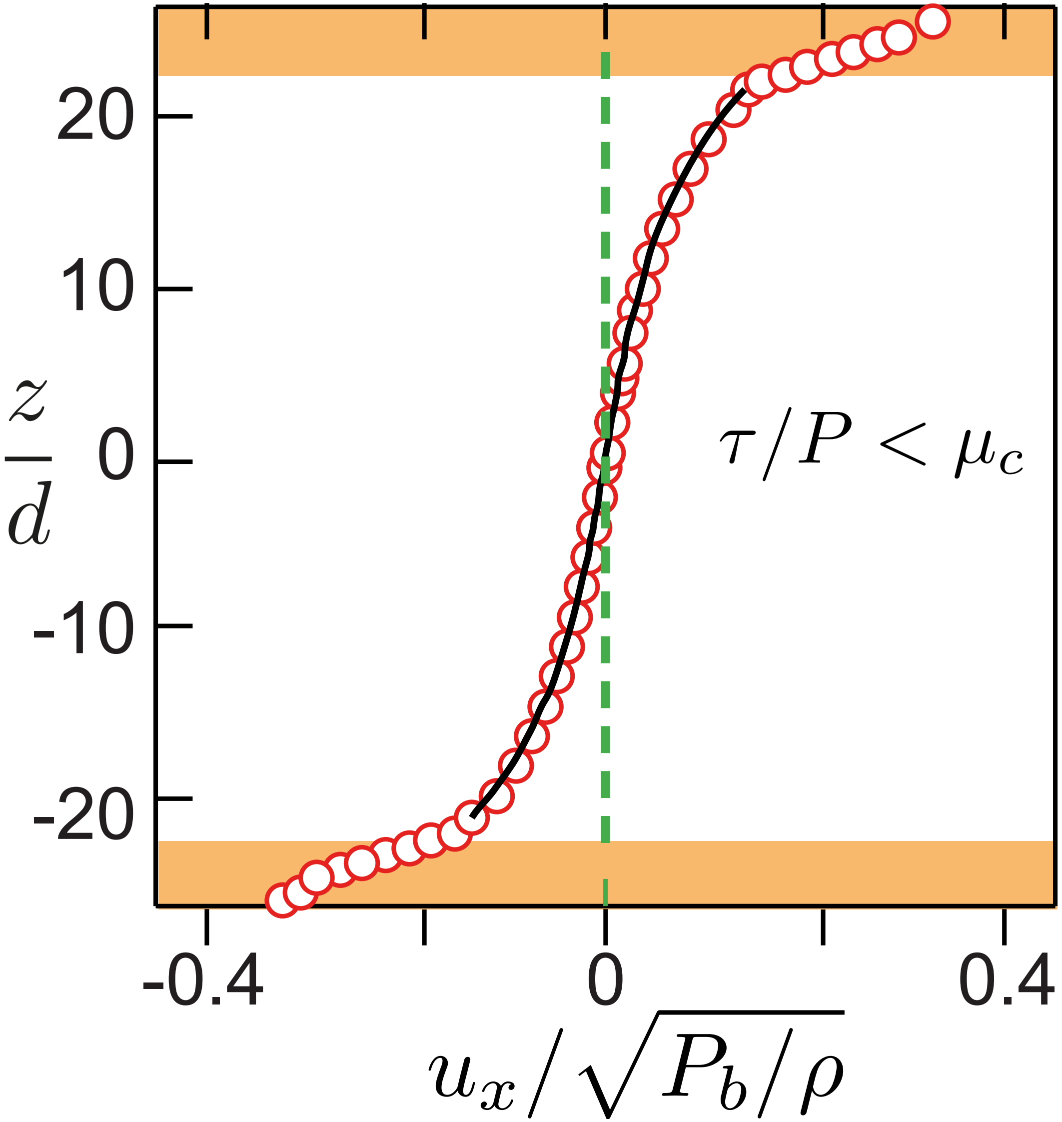
# Numerical set-up



# Can one create a solid/liquid interface?



# E pur si muove!

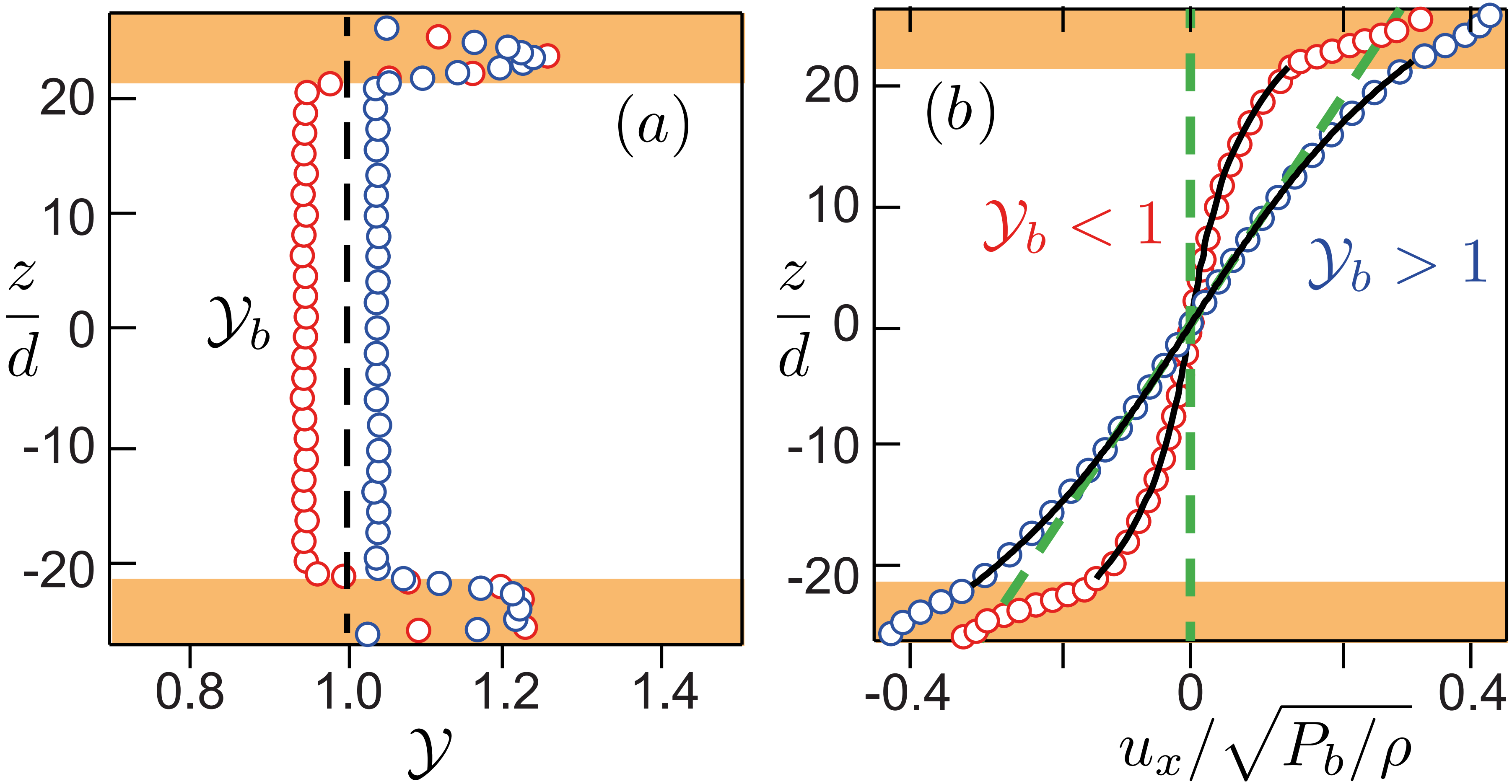



$\tau/P$ (or $\mathcal{Y}$ )
does not control the
solid/liquid transition

spatial relaxation

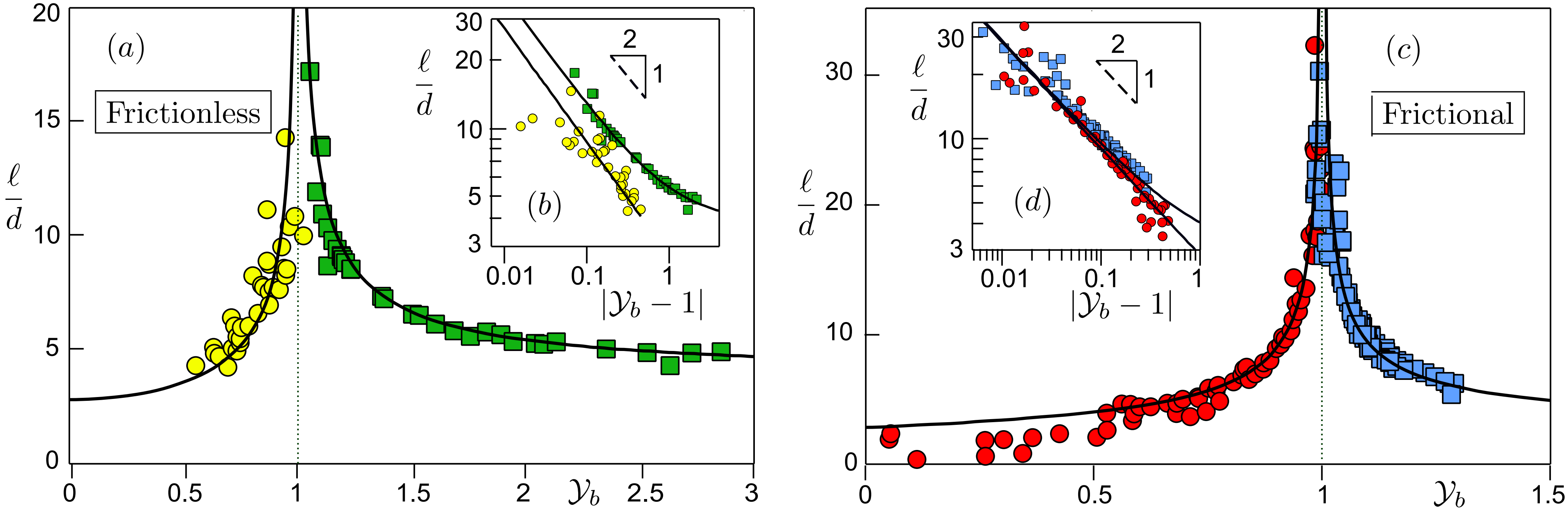
$$e^{\pm z/\ell}$$

# Relaxation length (1)




$$u_x(z) = \dot{\gamma}_\infty z + \frac{u_x(H/2) - \dot{\gamma}_\infty H/2}{\sinh(H/(2\ell))} \sinh(z/\ell)$$

# Relaxation length (2)




A single liquid phase: the divergence is NOT the signature of change of state.

# Fluidity

Order parameter: fluidity

$f = 0$ solid

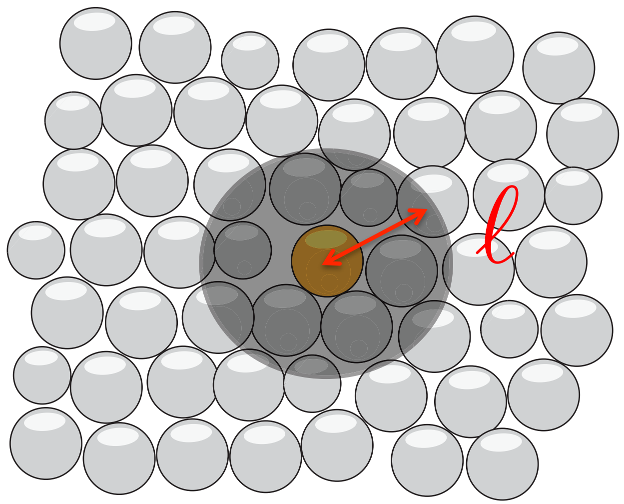
$f \neq 0$ liquid

Fluidity must depend on
state variables only

$I$ $Z$ $\phi$ ~~$\tau$~~

Relative environment fluidity

$$\kappa = \frac{d^2 \nabla^2 f}{f}$$



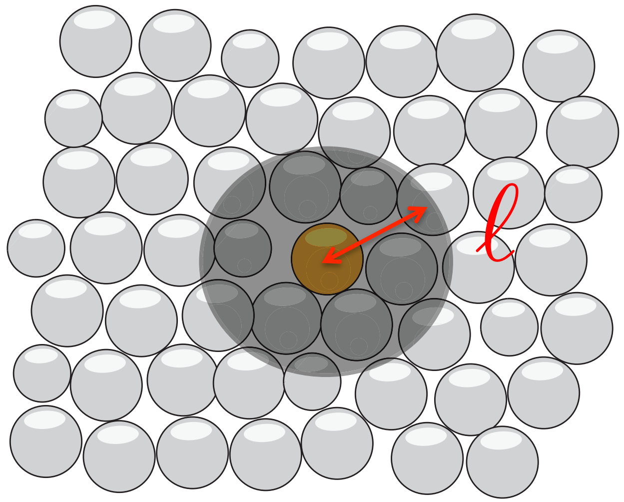
# Non-local rheology

$$\frac{\tau}{P} = \mu(I)\left[1 - \chi(\kappa)\right]$$

$$\kappa = \frac{d^2 \nabla^2 f}{f}$$

$$\chi(\kappa) = \nu\kappa + \mathcal{O}(\kappa^2)$$

$$f = I = \frac{|\dot{\gamma}| d}{\sqrt{P/\rho}}$$



# Linearisation

$$\mu = \mu_c + aI^\alpha$$

$$\mathcal{Y} = \frac{\mu(I)}{\mu_c}\left[1 - \nu\kappa\right]$$

$$\kappa \equiv d^2 \frac{\nabla^2 I}{I}$$

$\mathcal{Y}_b > 1$

$$I = I_b + \delta I$$

$$\ell^2 \frac{\mathrm{d}^2 \delta I}{\mathrm{d}z^2} - \delta I = 0$$

$$\ell_> = d\sqrt{\frac{\mathcal{Y}_b \nu}{\alpha(\mathcal{Y}_b - 1)}}$$

$\mathcal{Y}_b < 1$

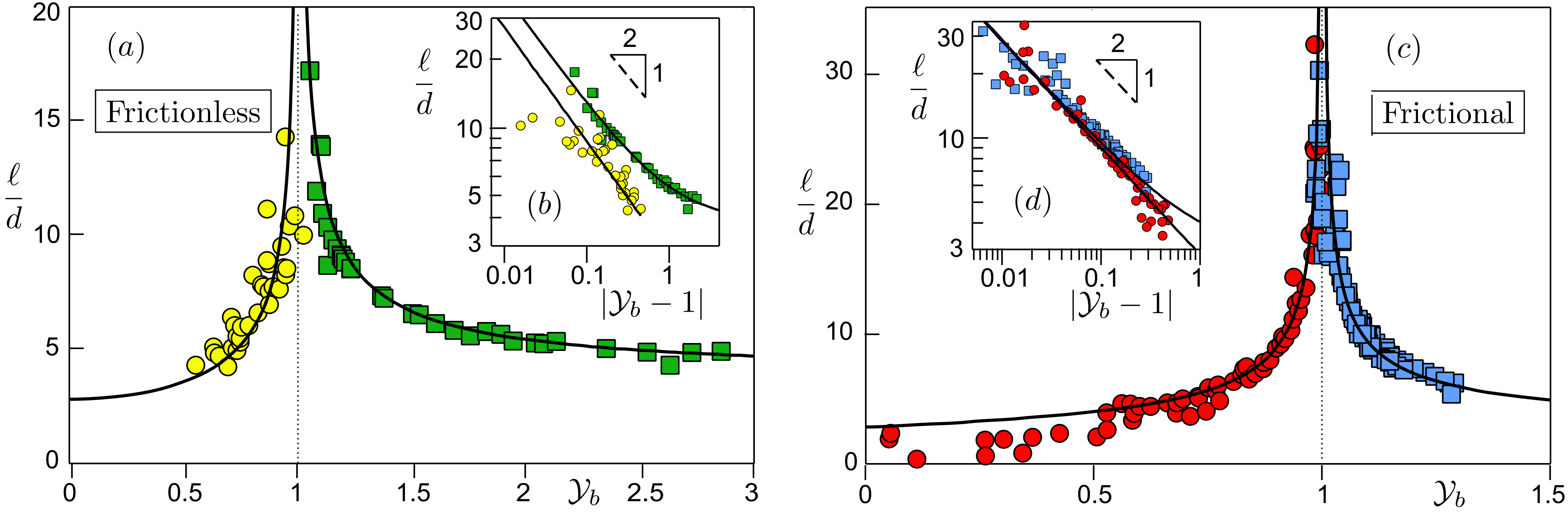
$$I = \delta I \qquad (I_b = 0)$$

$$\kappa = (1 - \mathcal{Y}_b)/\nu$$

$$\ell^2 \frac{\mathrm{d}^2 \delta I}{\mathrm{d}z^2} - \delta I = 0$$

$$\ell_< = d\sqrt{\frac{\nu}{\mathcal{Y}_b - 1}}$$

# Relaxation length (3)




$$\ell_> = d\sqrt{\frac{\mathcal{Y}_b \nu}{\alpha(\mathcal{Y}_b - 1)}}$$

$$\ell_< = d\sqrt{\frac{\nu}{\mathcal{Y}_b - 1}}$$

$$\nu \simeq 8$$

# Some other approaches

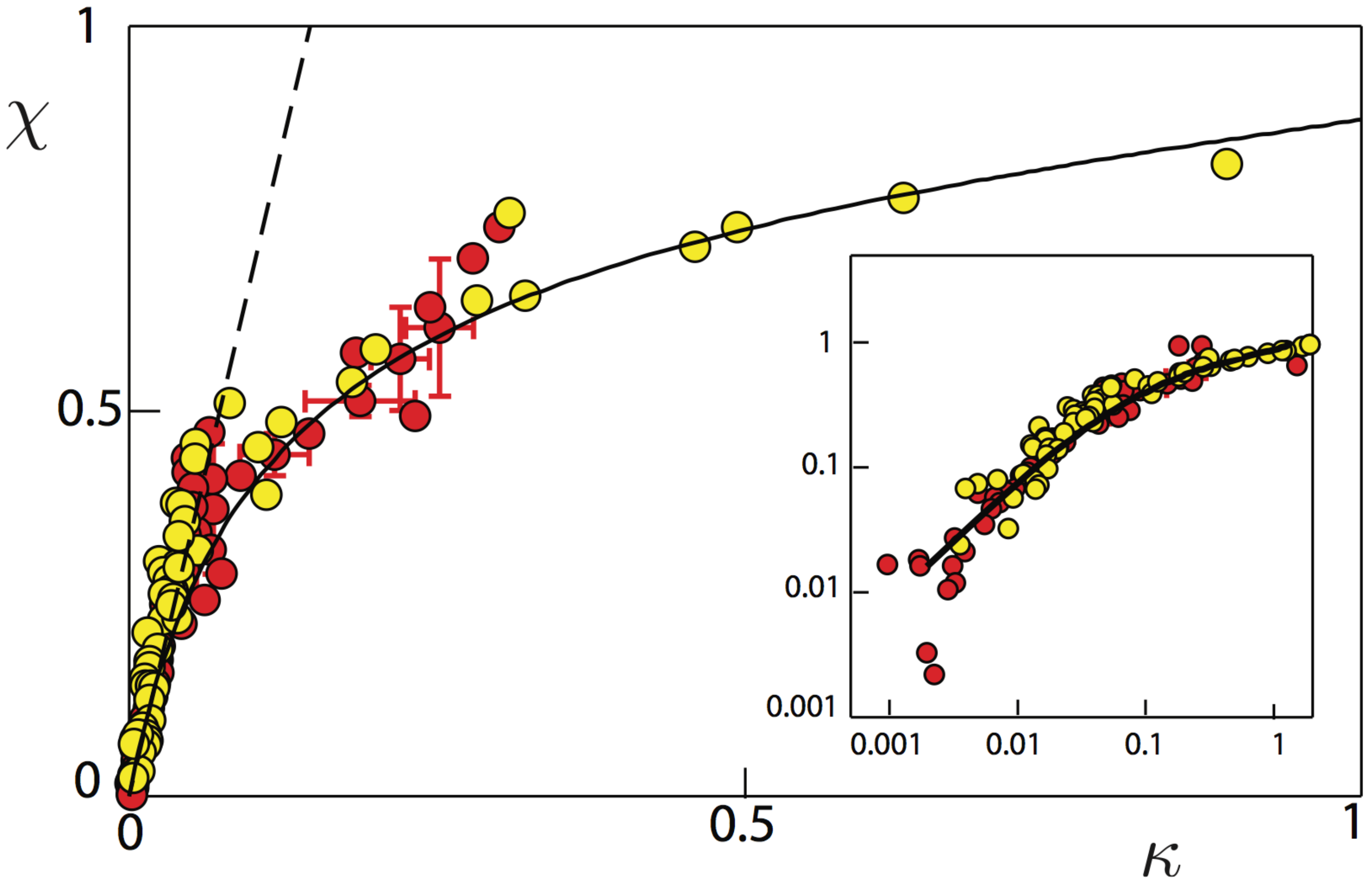
- Phase field — Aranson & Tsimring, PRE **64**, 1757 (2001).
- Mechanically activated plastic events — Pouliquen & Forterre, Phil. Trans. R. Soc. A. **367**, 5091 (2009).
- Kinetic elasto-plastic — Bocquet et al., PRL **103**, 036001 (2009). Kamrin & Koval, PRL **108**, 178301 (2012).

→ Choice for fluidity

Continuity and boundary conditions

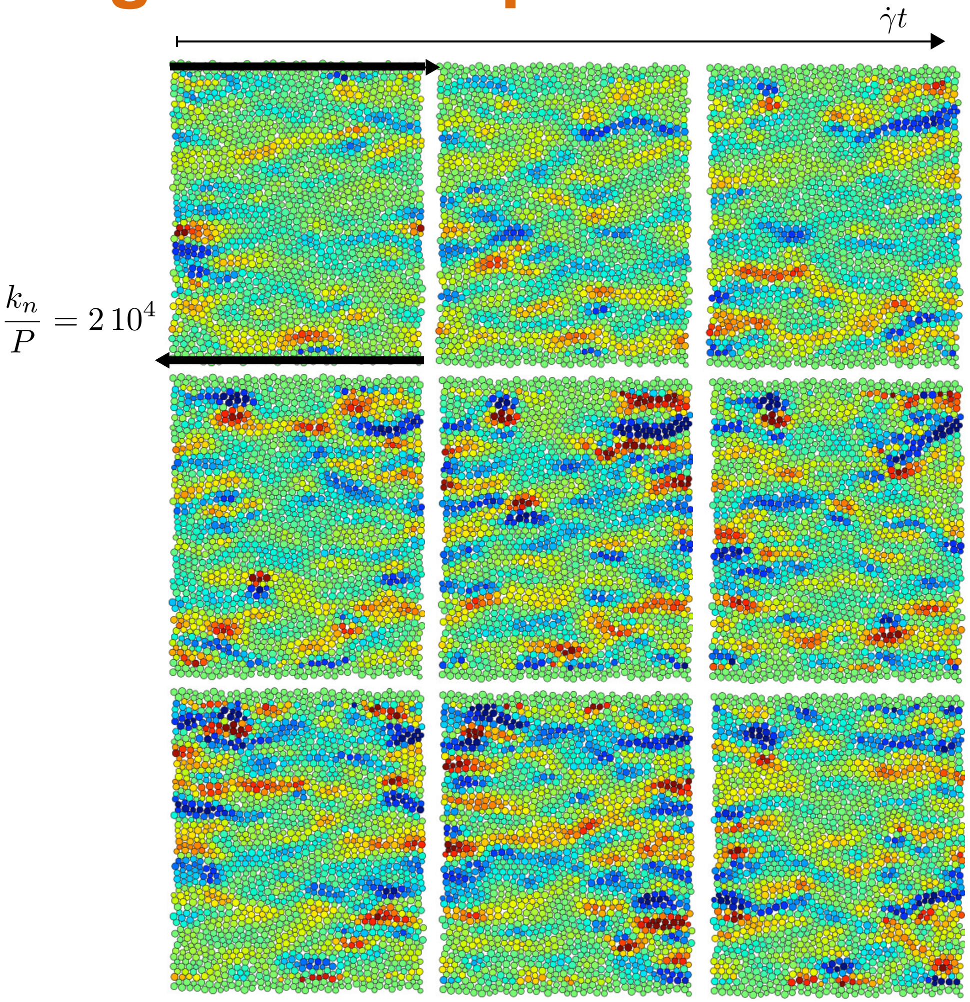
Assumptions and mechanisms

# Non-linear non-local function

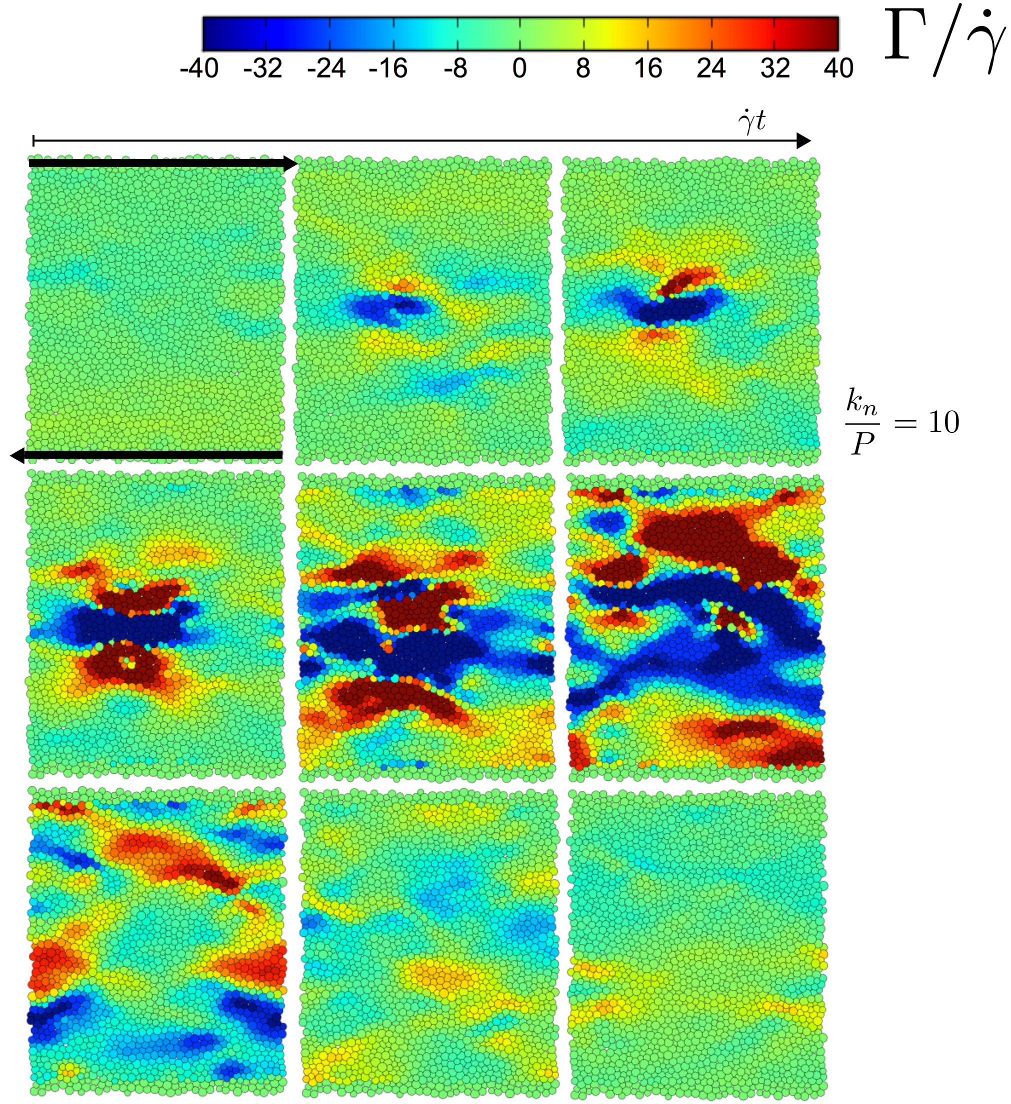



At linear order $\chi(\kappa) = \nu\kappa + \mathcal{O}(\kappa^2)$ with $\nu \simeq 8$

# Rigid *vs* soft particles




No localized plastic events
Permanent spatial heterogeneities




Localized plastic events
Very intermittent dynamics

Local contribution of a region to the mean shear rate $\dot{\gamma}$ :

$$\dot{\Gamma}(\vec{r},t) = \frac{\sum_{j=1}^{N} [u_i(\vec{r},t) - u_j(\vec{r},t)][z_i(t) - z_j(t)] \exp(\frac{-||\Delta\vec{r}||^2}{2\delta^2})}{\sum_{j=1}^{N} [z_i(t) - z_j(t)]^2 \exp(\frac{-||\Delta\vec{r}||^2}{2\delta^2})}$$

# Rigid *vs* soft particles