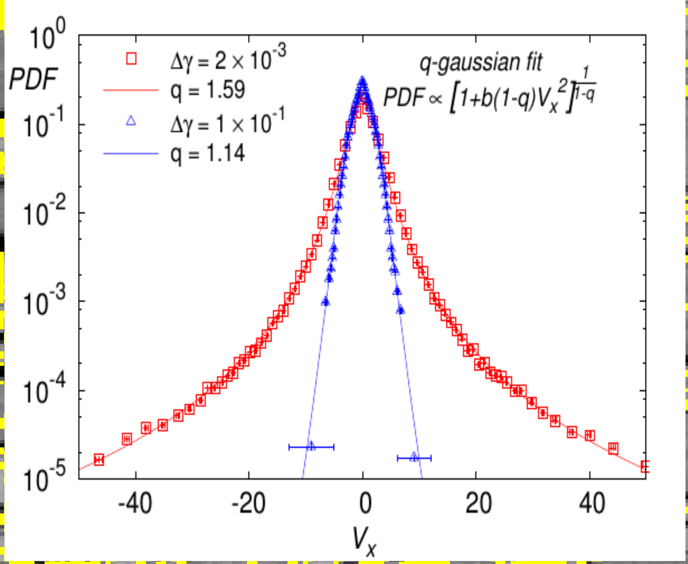
Experimental validation of nonextensive scaling law in confined granular media



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Outline

1 – Granular Materials

Basic concepts and phenomenology

Physics of “granulence”

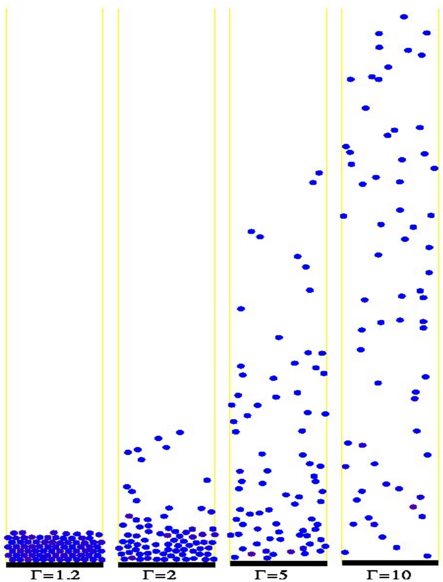
2 – Experimental setup and Results

3 – Tsallis-Bukman scaling law validation

Basic Concepts

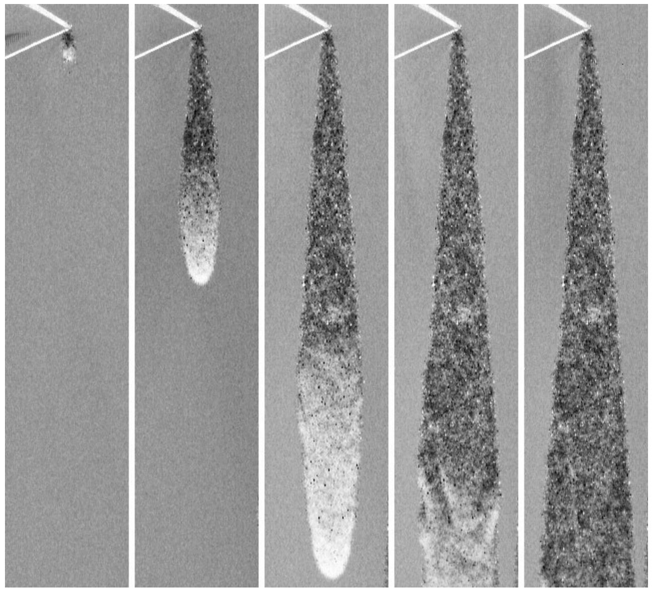
Classical states of the matter

Gas



Granular under vibration:
Granular temperature

Liquid



Avalanches in granular displacements

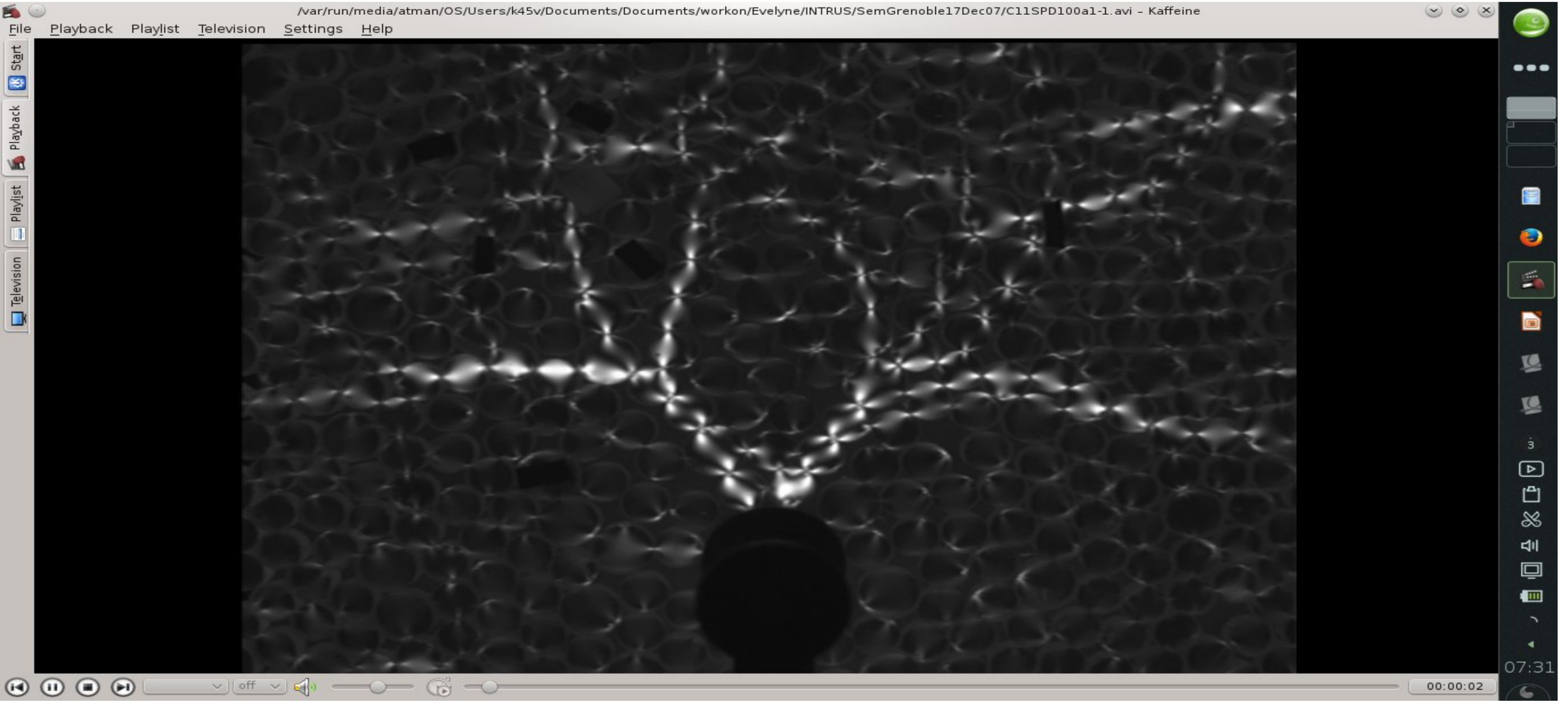
Solid

Sandpiles
(J-B.Métais)

A.Daerr, S.Douady , NATURE **399** 241 (1999)

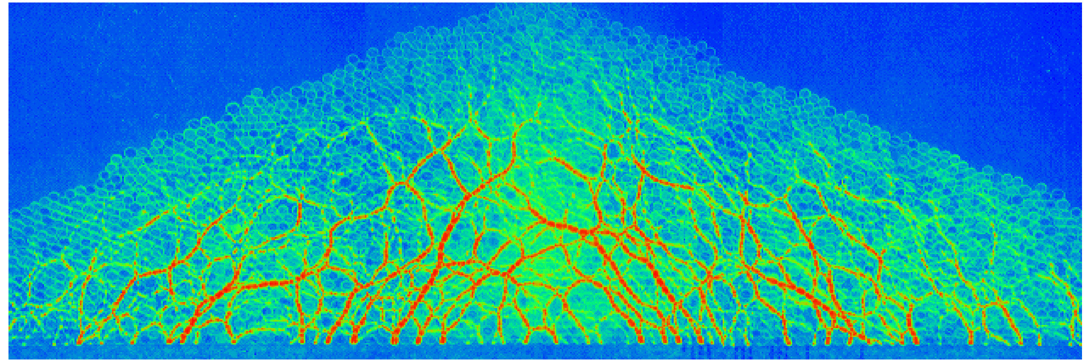
Phenomenology

Force chains network



Flow fields around an intruder immersed in a 2D dense granular layer
E. Kolb, P. Cixous, J. C. Charmet Granular Matter (2014) **16** (2).

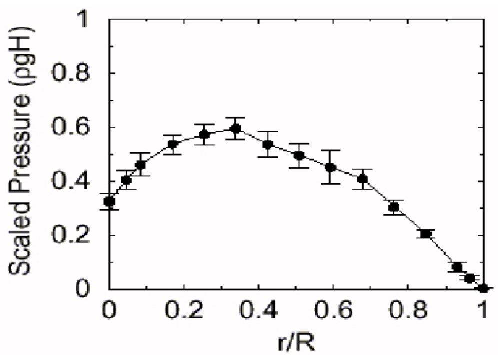
Phenomenology

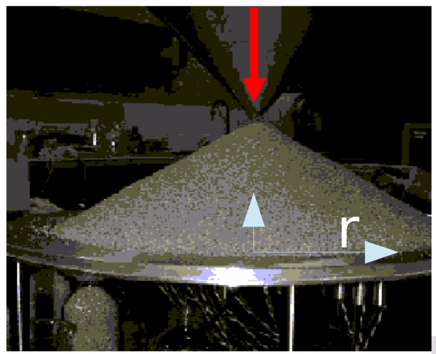


Static of granular materials:

- history dependence;
- textures

Stress distribution depends on preparation history

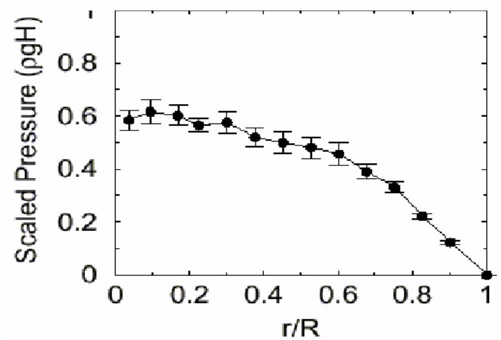


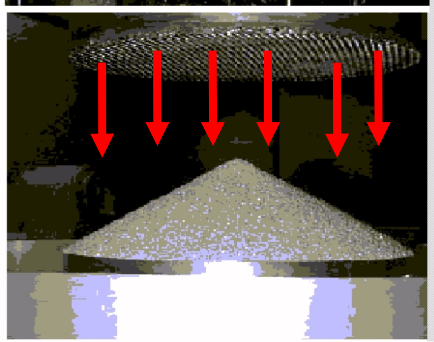


Vanel et al. PRE **60** R5040 (1999)

Sandpile Stress Dip Effect

Point source deposition
Pressure dip





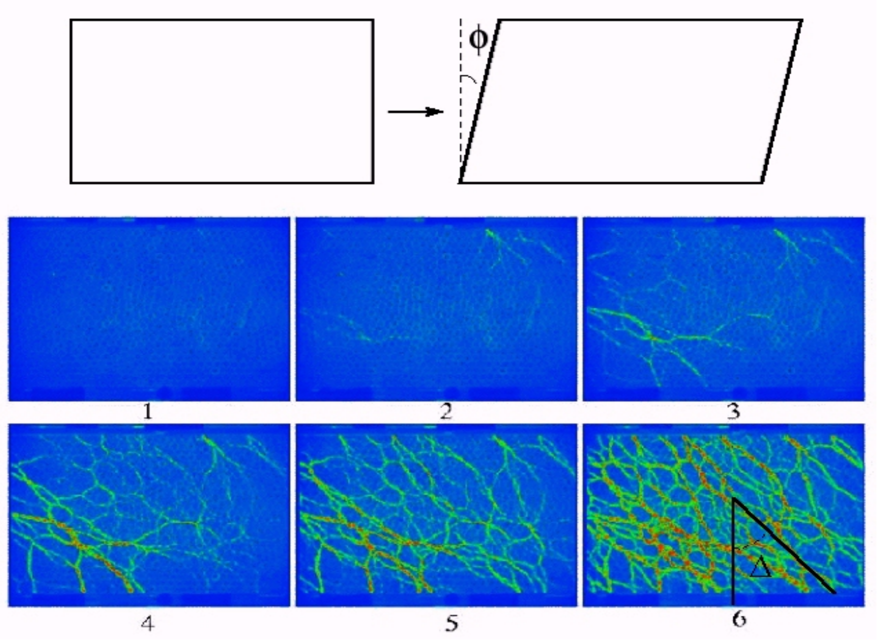
Vertical rain deposition
no pressure dip

Influence of shear strain

Geng et al PhysicaD **182**, 274 (2003)

2 D shearing cell with birefringent material;
piling of pentagonal photo-elastic grains

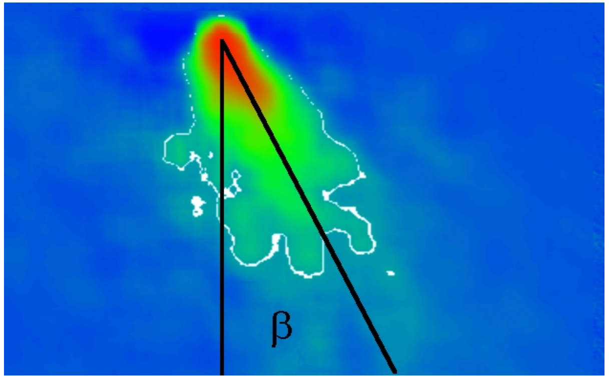
$\phi = 5°$



Force chains orient to oppose shearing: $\Delta = 45^o$

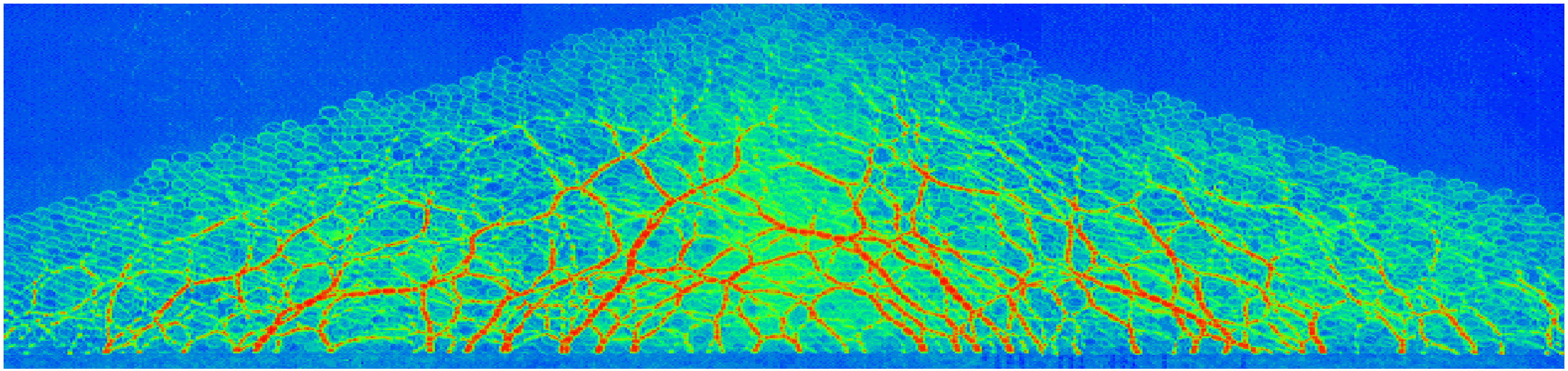
Response function deviation angle : $\beta = 22°$ (2D)

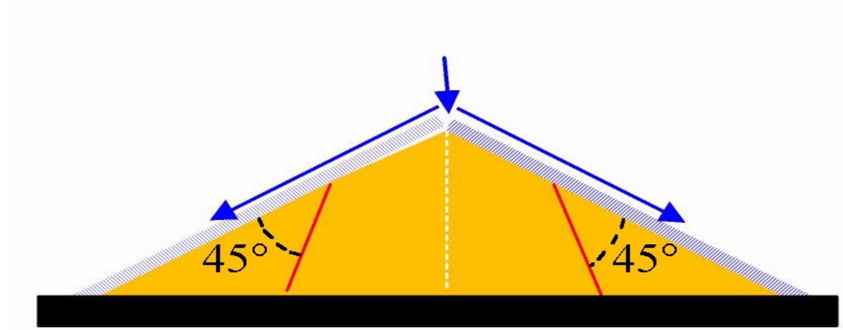
$\beta = 8°$ (3D)



A quantitative explanation for the sandpile stress dip

Texturing effect due to shearing in the avalanching process





Anisotropic elasticity model
Orthotropic axis
$\Delta = 45°$ / avalanche direction
Sand pile slope angle : $\theta = 30°$

From the stress response function (back) to the sandpile pressure 'dip',
A. P. F.Atman, G. Reydellet, P. Brunet, P. Claudin, J. Geng, R. P. Behringer and E. Clèment, *Eur. Phys. J. E*, **17**, 93-100 (2005).

Phenomenology

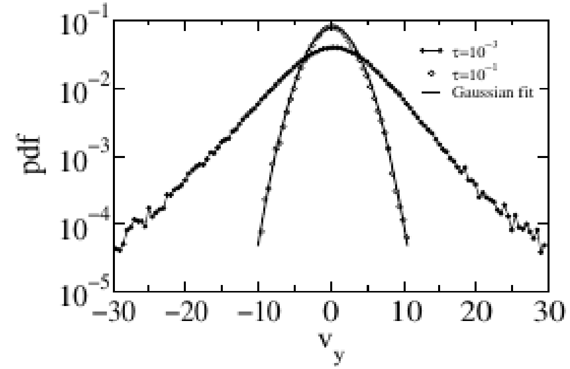
"Granulence"

Velocity fluctuations in sheared confined granular systems which share scaling characteristics of fluid turbulence (in spite of their different physical origins):

1) Scale-dependent probability distribution with non-Gaussian broadening at small time scales;

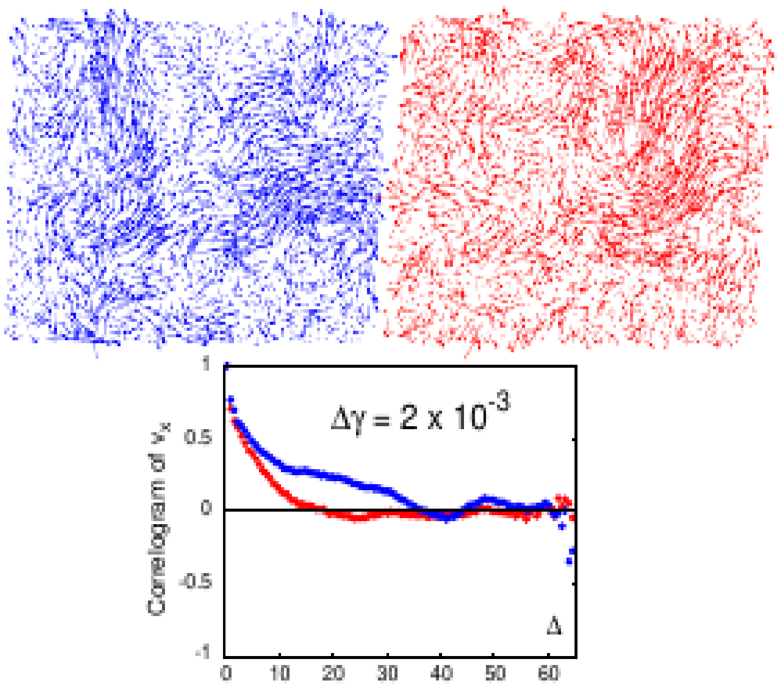
2) Power-law spectrum, reflecting long-range correlations and the self-affine nature of the fluctuations;

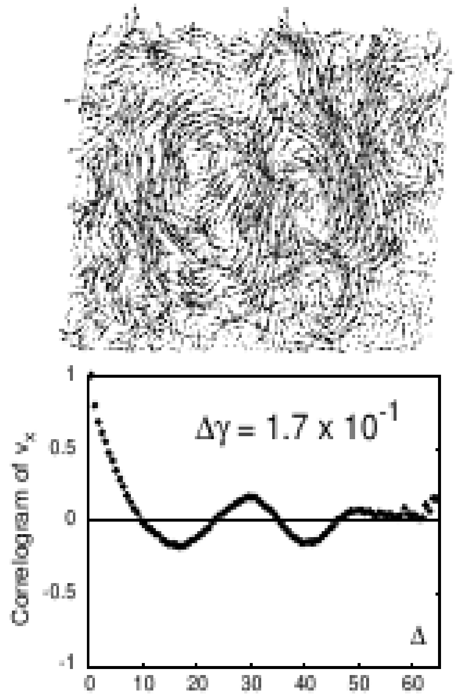
3) Superdiffusion with respect to the mean background flow.

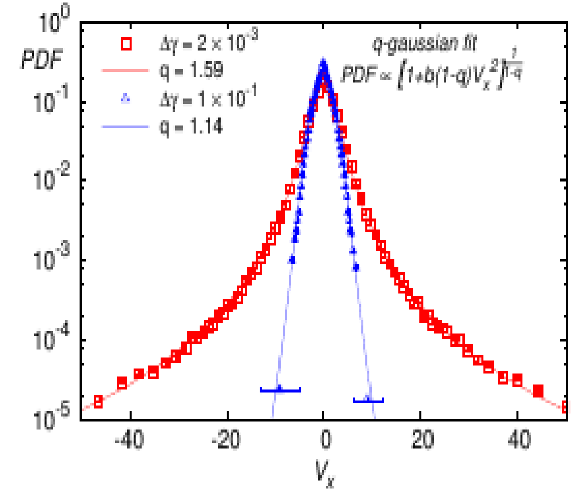


Radjaï & Roux - PRL **89**, 064302 - 2002
"Turbulencelike fluctuations in quasistatic flow of granular media"

Experimental evidence of “granulence”







An experimental assessment of displacement fluctuations in a 2D granular material subjected to shear.
V. Richefeu, G. Combe, and C. Viggiani, Géotechnique Letters 2, 113 (2012).

Experimental evidence of “granulence”. G. Combe, V. Richefeu, G. Viggiani, S. A. Hall, A. Tengattini, and A. P. F. Atman, in POWDERS AND GRAINS 2013: Proceedings of the 7th International Conference on Micromechanics of Granular Media, edited by Aibing Yu , Kejun Dong, Runyu Yang and Stefan Luding (2013), vol. 1542 of AIP Conference Proceedings, pp. 453–456.

Anomalous diffusion

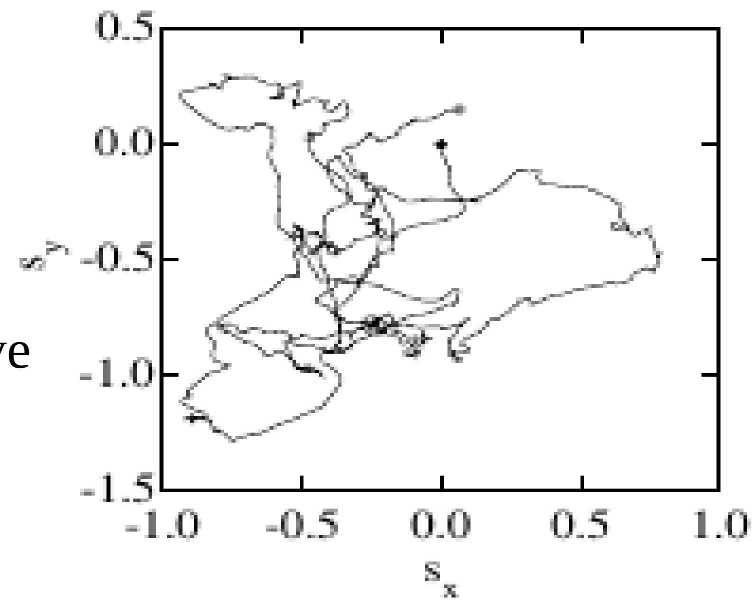
$$\langle \vec{x}^{\,2} \rangle \sim t^{\alpha}$$

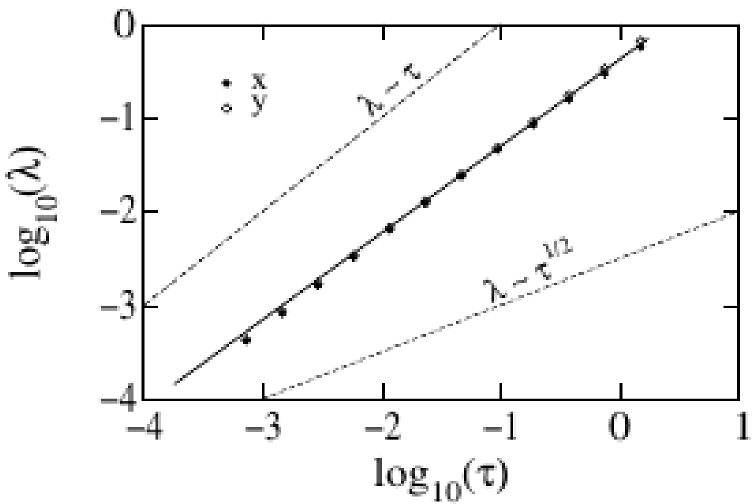
$\alpha = 1$ → normal (Gaussian) diffusion;

$\alpha = 2$ → ballistic diffusion;

$1<\alpha<2$ → “anomalous” diffusion.

$\alpha > 1$ → superdiffusive; $\alpha < 1$ → subdiffusive





Radjaï & Roux - PRL **89**, 064302 - 2002

Anomalous diffusion

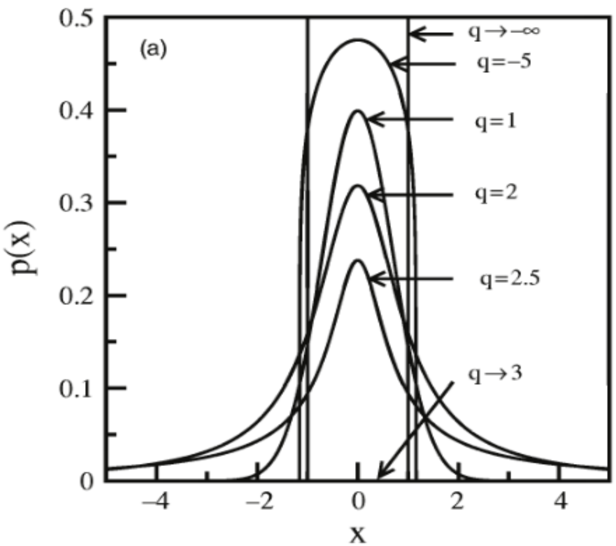
Another possible generalization of the heat equation:
Porous media equation!

$$\frac{\partial p(x,t)}{\partial t} = D_q \frac{\partial^2 [p(x,t)]^{2-q}}{\partial x^2} = D \frac{\partial^2 [p(x,t)]^{\nu}}{\partial x^2} \quad (\nu \in \mathbb{R}).$$

q-Gaussians!

$$p_q(x,t) = p_q(x/[D_q\, t]^{\frac{1}{3-q}}),$$

where p_q is the q-Gaussian:



$$p_q(x) = \frac{1}{\sqrt{\pi\, A_q}}\, e_q^{-x^2/A_q} = \frac{1}{\sqrt{\pi\, A_q}} \frac{1}{\left[1 + (q-1)\frac{x^2}{A_q}\right]^{\frac{1}{q-1}}},$$

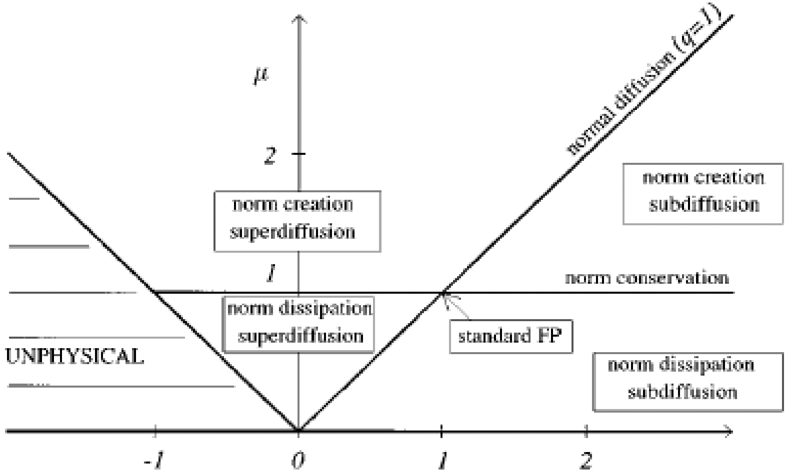
$$A_q = \begin{cases} \dfrac{\sqrt{q-1}\,\Gamma\left(\frac{1}{q-1}\right)}{\Gamma\left(\frac{3-q}{2(q-1)}\right)} & \text{if } 1 < q < 3, \\ 2 & \text{if } q = 1, \\ \dfrac{\sqrt{1-q}\,\Gamma\left(\frac{5-3q}{2(1-q)}\right)}{\Gamma\left(\frac{2-q}{1-q}\right)} & \text{if } q < 1. \end{cases}$$

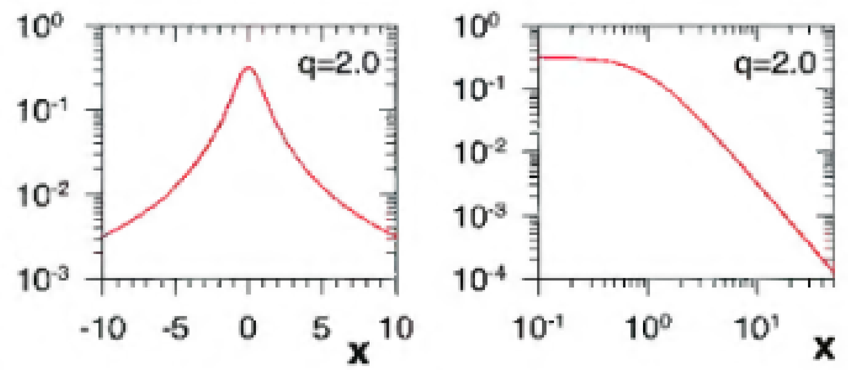
A.P.F.Atman, *Experimental validation of nonextensive scaling law in confined granular media,*

Tsallis-Bukman Scaling Law

$$\langle x^2 \rangle \propto t^{\frac{2}{3-q}} .$$ Thus, we have $$\alpha = \frac{2}{3-q}$$

which is know as the Tsallis-Bukman scaling law.



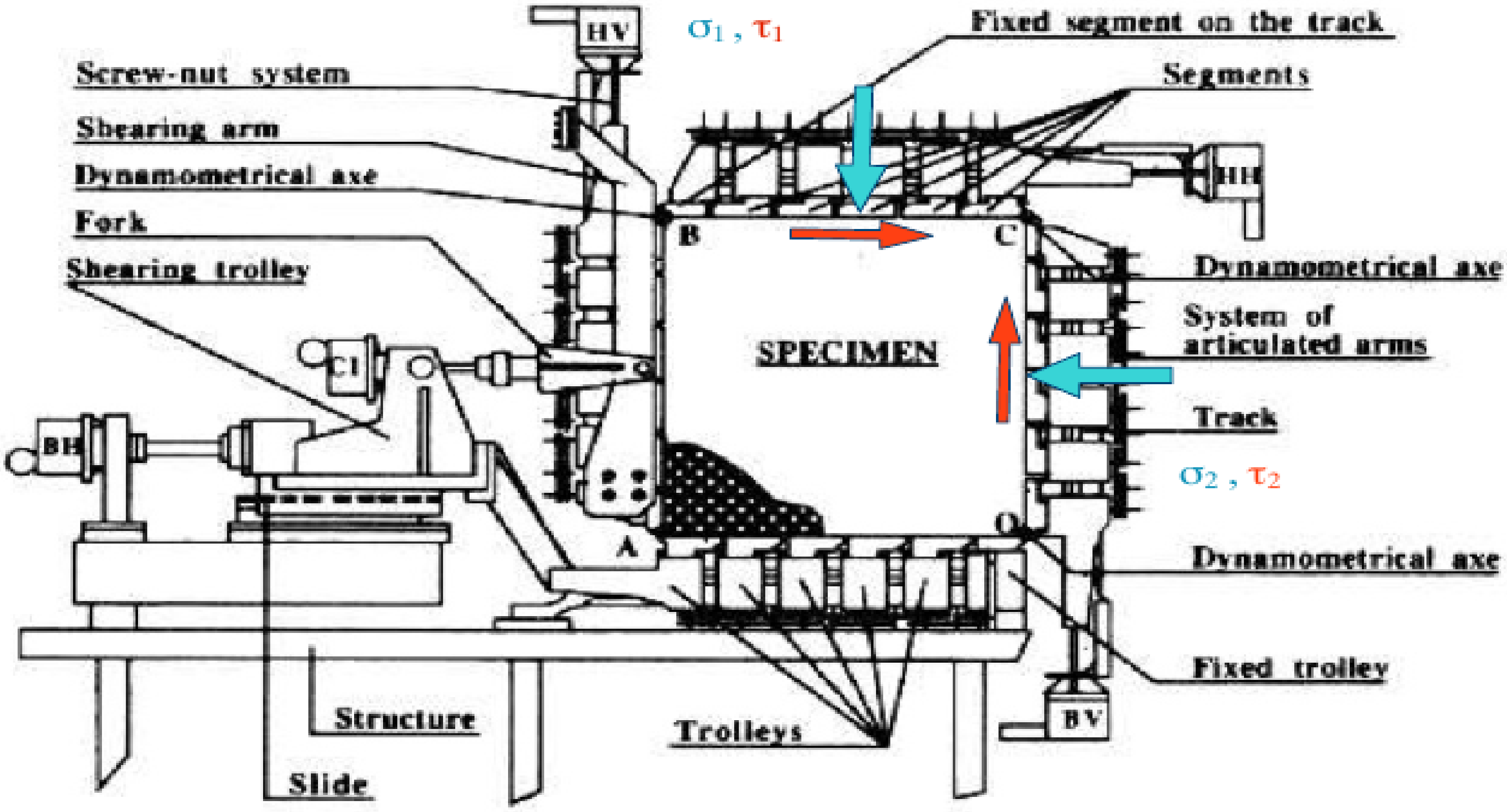


C. Tsallis, *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World* (Springer, 2009), 1st ed., ISBN 0387853588.

C. Tsallis and D. J. Bukman, *Physical Review E* **54**, R2197 (1996).

A. R. Plastino and A. Plastino, *Physica A* **222**, 347

Experimental setup: 1γ2ε aparatus

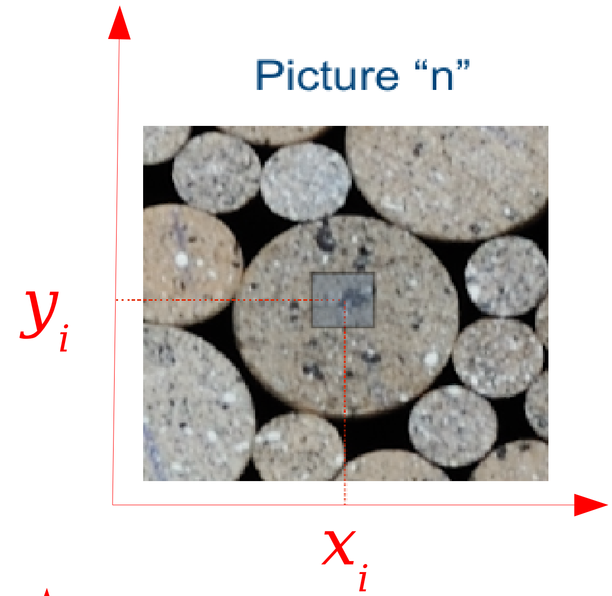


A.P.F.Atman, *Experimental validation of nonextensive scaling law in confined granular media,*
Foundations of Complexity – Nonadditive Entropies and Nonextensive Thermodynamics– CBPF, Rio de Janeiro, 21 OCT 2015

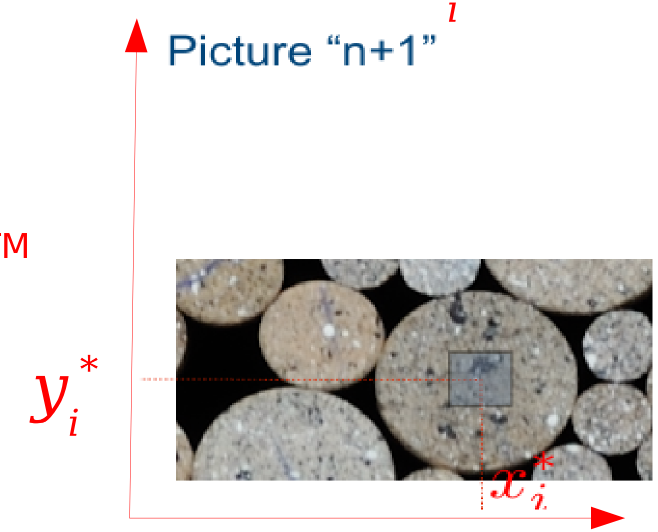
1γ2ε : what do we measure

- Macroscopic level (sample scale):
• Lengths and tilt of the walls ➟ Strain tensor
• Forces at the corners ➟ Stress tensor

- Microscopic level (grain scale):
• Position and rotation of grains
• Contacts list ➟ Fabric tensor



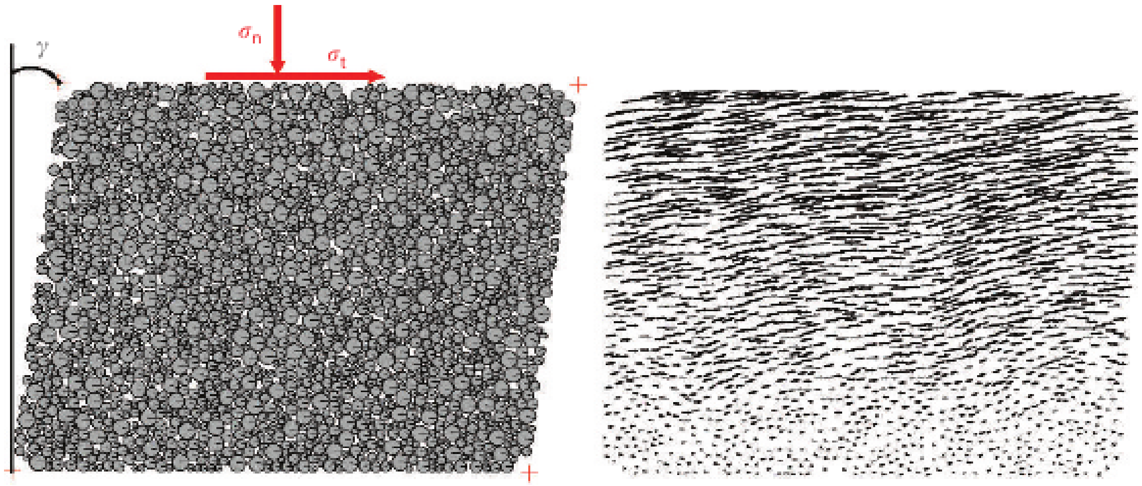
How?



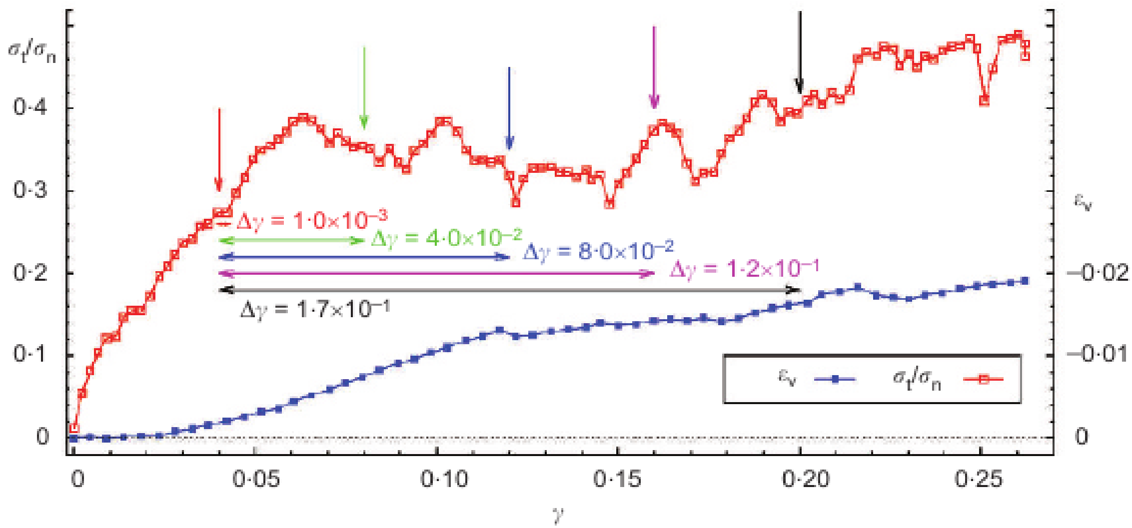
DIC – Digital Image Correlation – TRACKER™
Sub-pixel precision (~0.05 pixel)

Tracker: a Particle Image Tracking (PIT) technique dedicated to nonsmooth motions involved in granular packings
G. Combe and V. Richefeu, in POWDERS AND GRAINS 2013: vol. 1542 of AIP Conference Proceedings, p. 461.

Results



(a) (b)

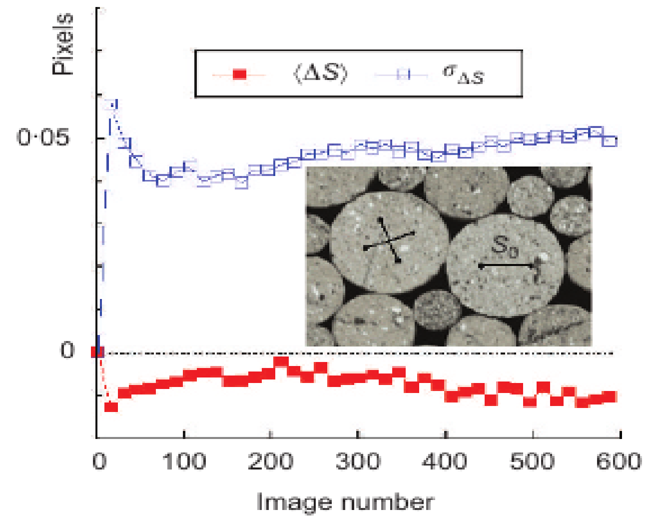


(c)

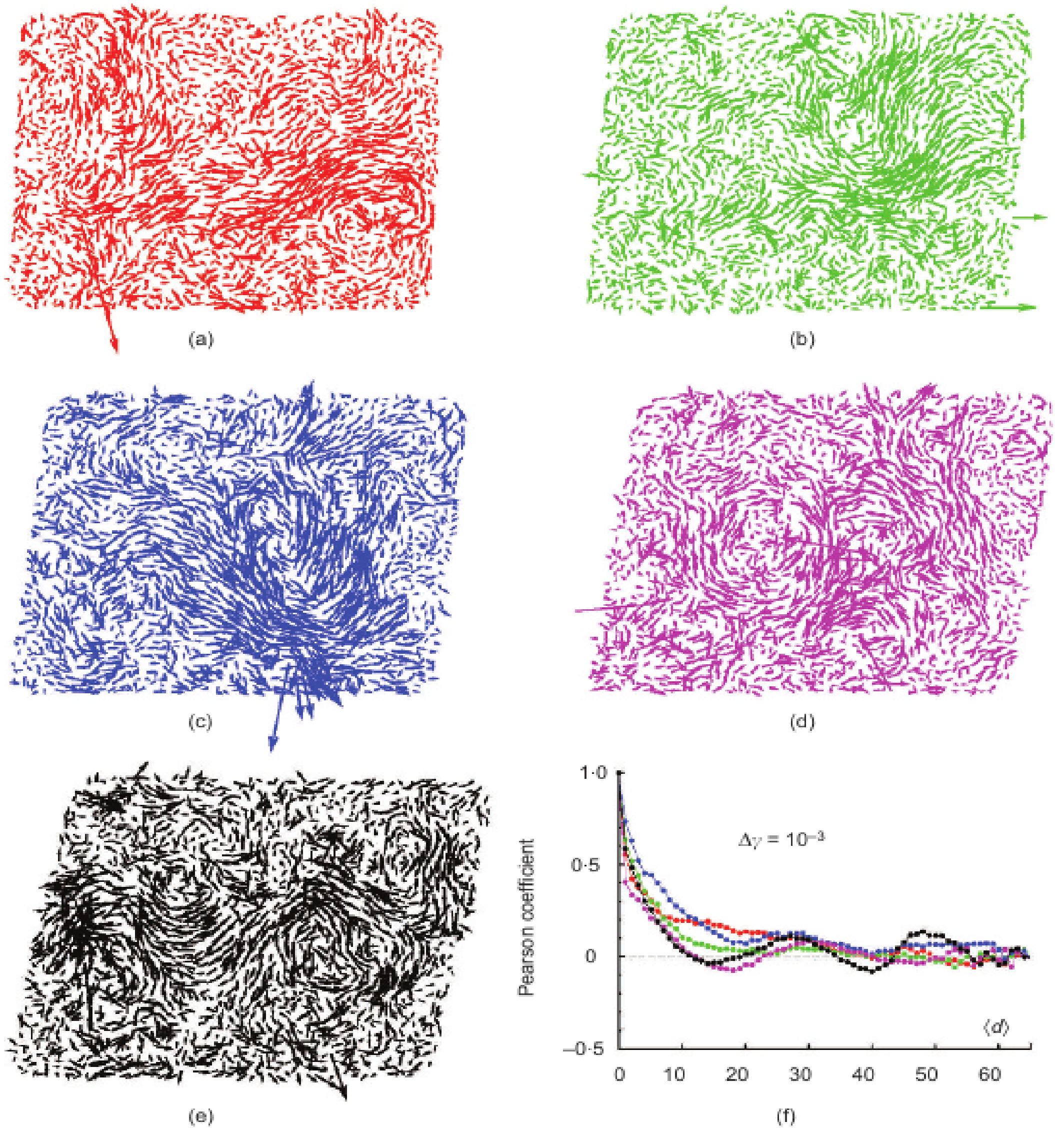
Displacement field;

Stress – strain relation;

Positions and rotations with high degree of precision.



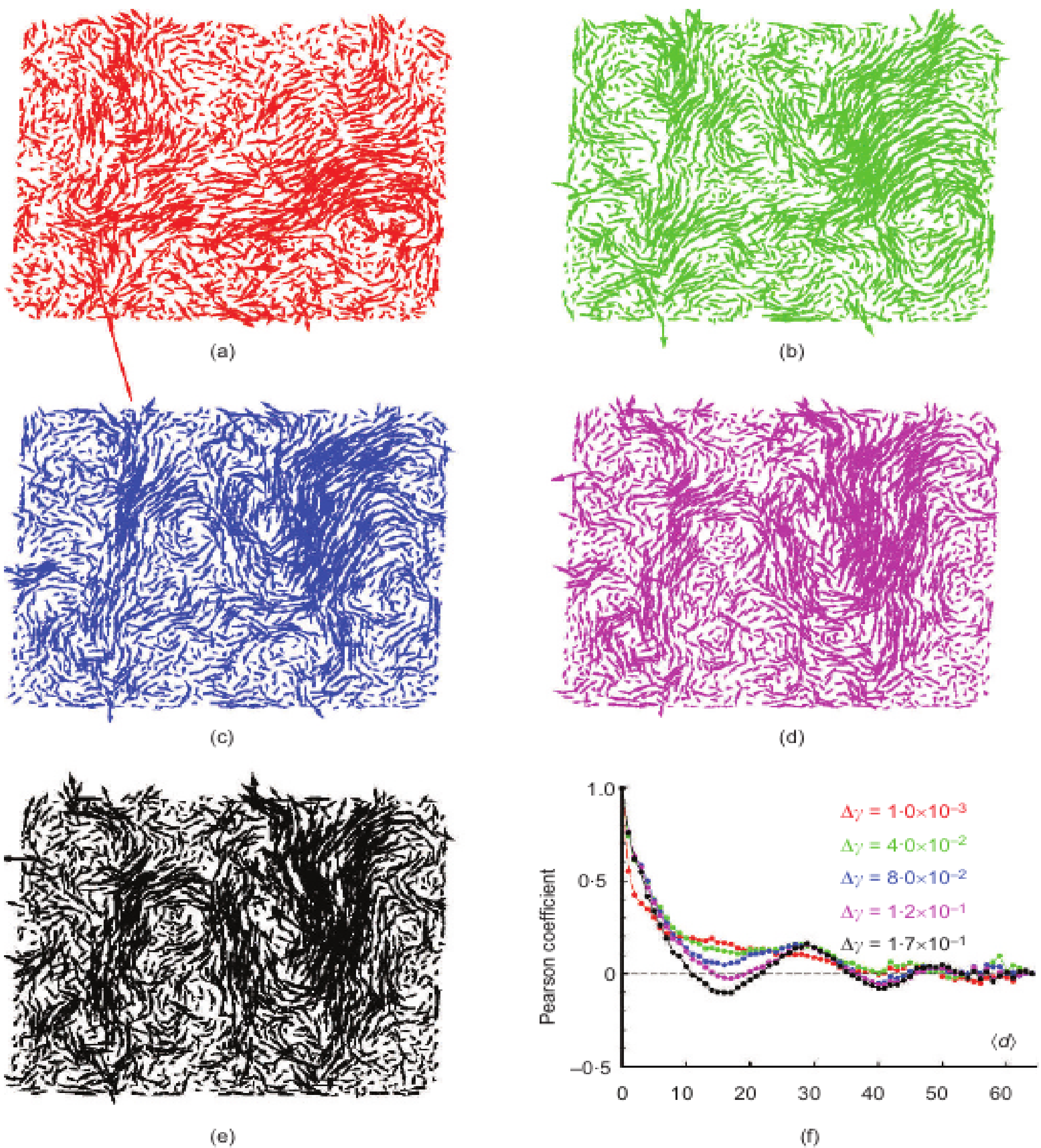
Results



(a)–(e): maps of normalised displacement fluctuations for a constant strain window $\Delta\gamma = 10^{-3}$, at different values of γ: (a) $\gamma = 0.04$; (b) $\gamma = 0.08$; (c) $\gamma = 0.2$; (d) $\gamma = 0.16$; (e) $\gamma = 0.20$; (corresponding to the coloured vertical arrows in the previous figure).

(f): Spatial correlograms computed for each fluctuation map as a function of the mean grain diameter <*d*>.

Results



(a)–(e): maps of normalised displacement fluctuations for different strain window $\Delta\gamma$, from $\gamma = 10^{-4}$ to $\gamma = 10^{-4} + \Delta\gamma$ (corresponding to the coloured horizontal arrows in the stress-strain figure);
(a) $\Delta\gamma = 10^{-3}$; (b) $\Delta\gamma = 4\mathrm{x}10^{-2}$; (c) $\Delta\gamma = 8\mathrm{x}10^{-2}$; (d) $\Delta\gamma = 1.2\mathrm{x}10^{-1}$; (e) $\Delta\gamma = 1.7\mathrm{x}10^{-1}$;

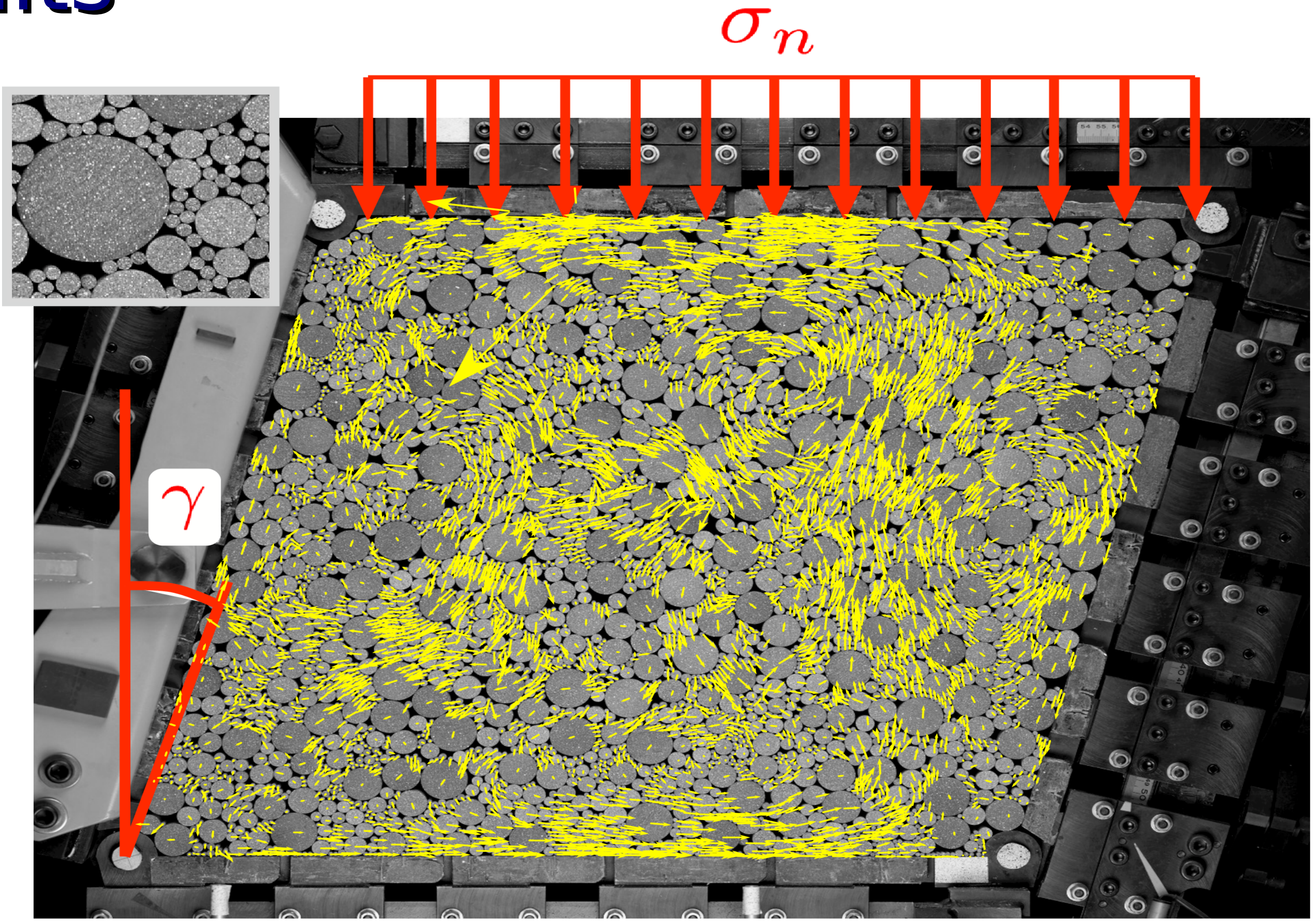
(f): spatial correlograms computed for each fluctuation map as a function of mean grain diameter <d>.

Results

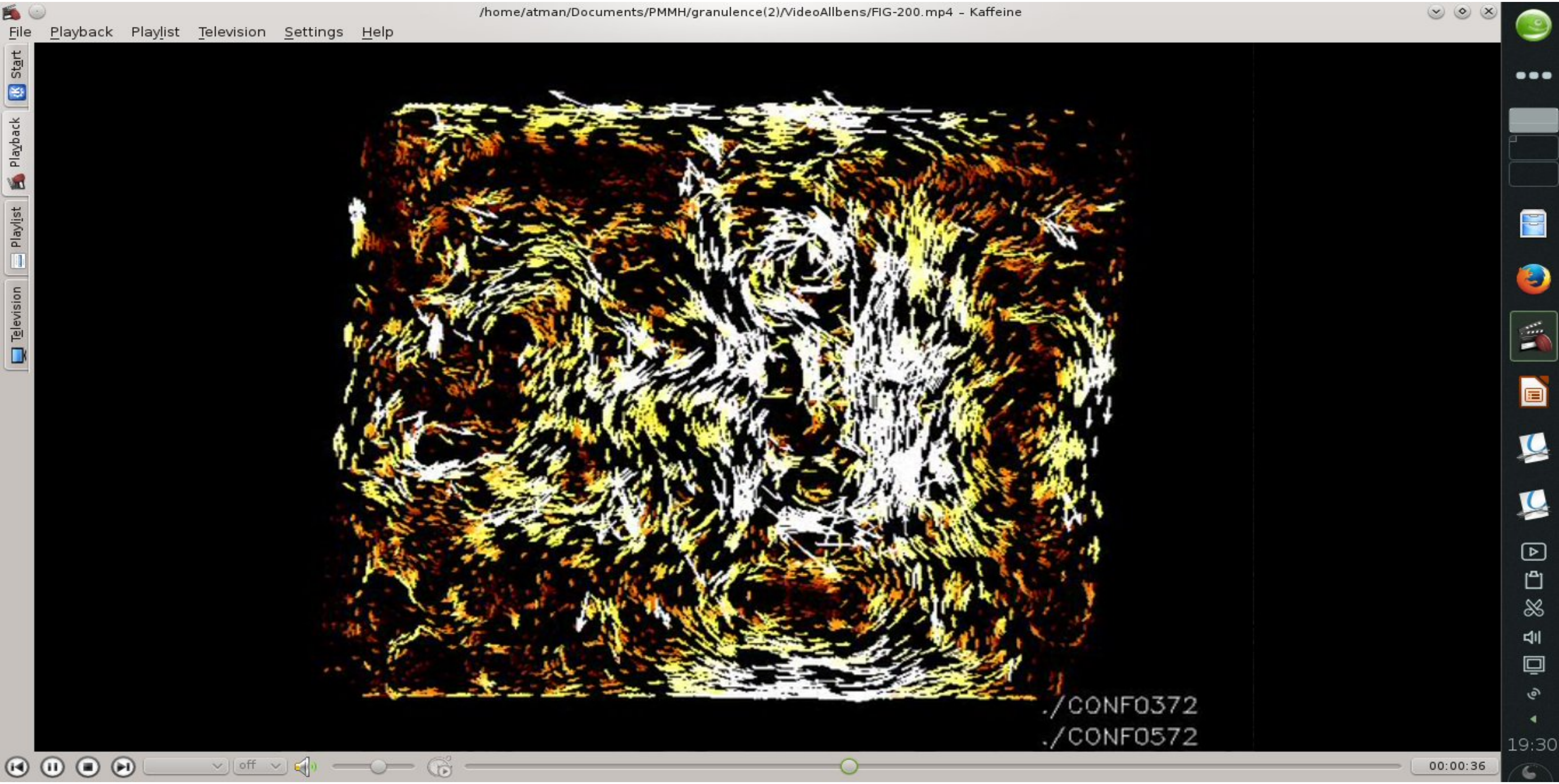
5471 wooden rollers;

10 different diameters , ranging from 3 up to 30 mm;

0<γ<15°



Results



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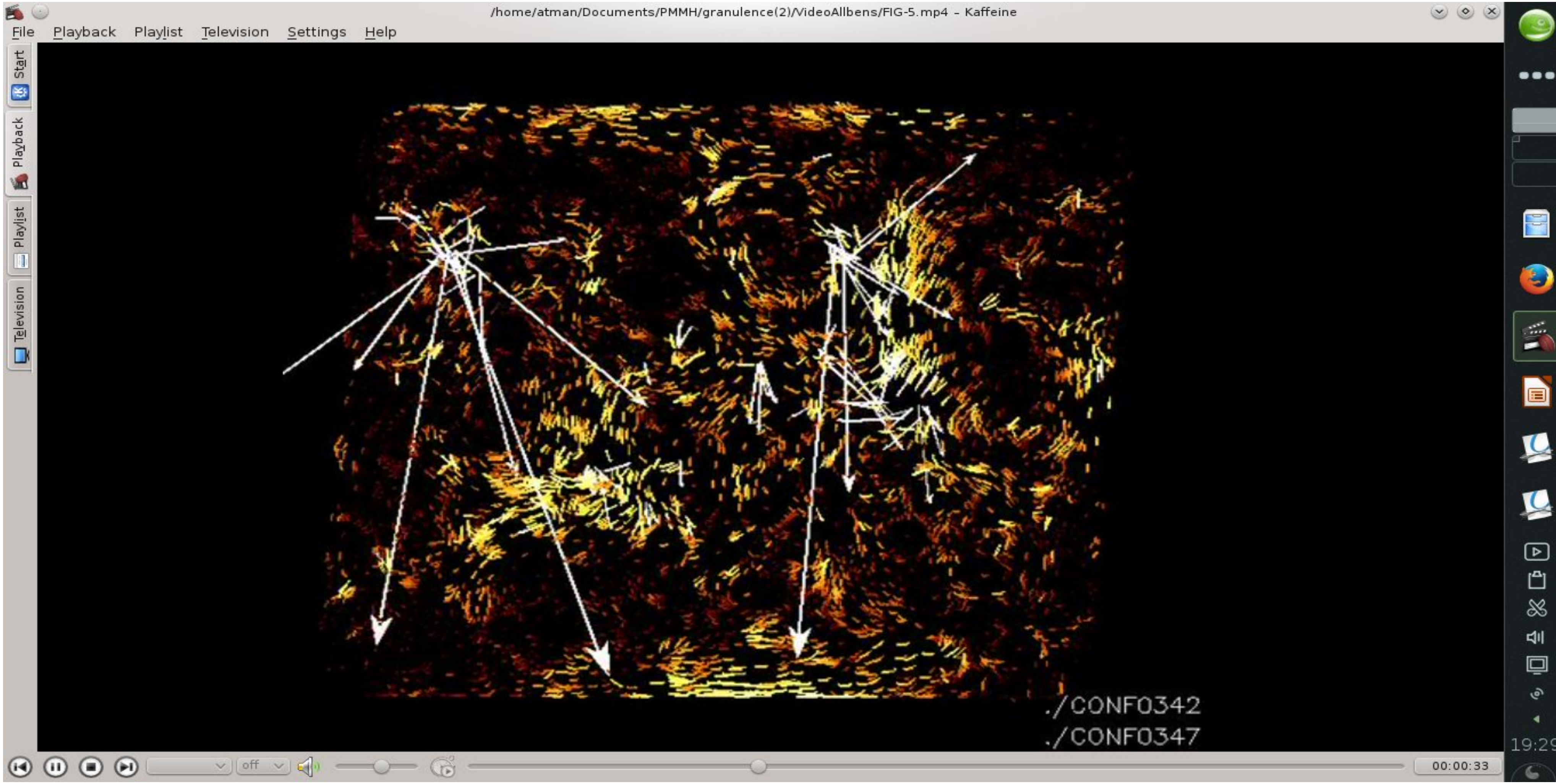
Results


/home/atman/Documents/PMMH/granulence(2)/VideoAllbens/FIG-5.mp4 – Kaffeine
File Playback Playlist Television Settings Help
Start
Playback
Playlist
Television
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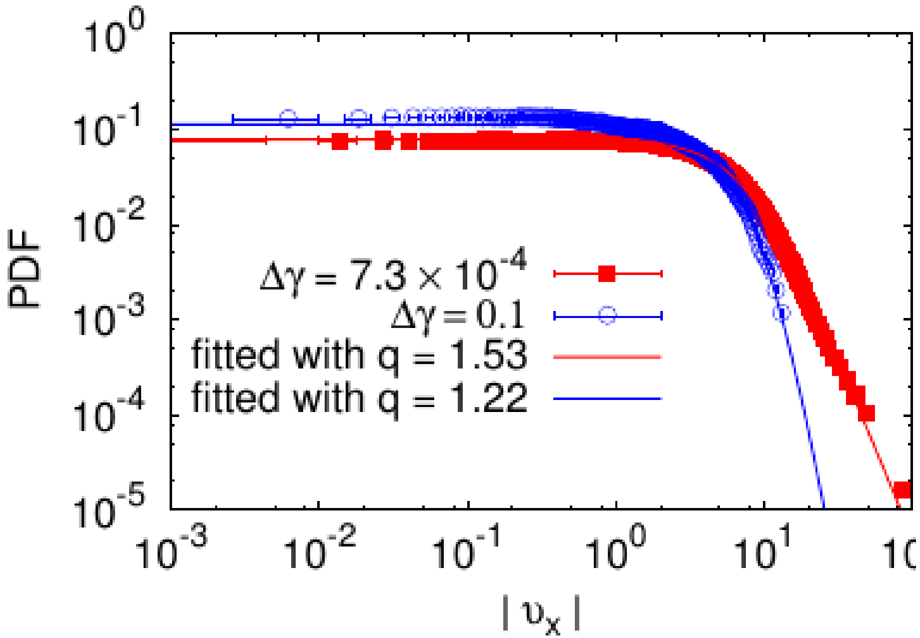
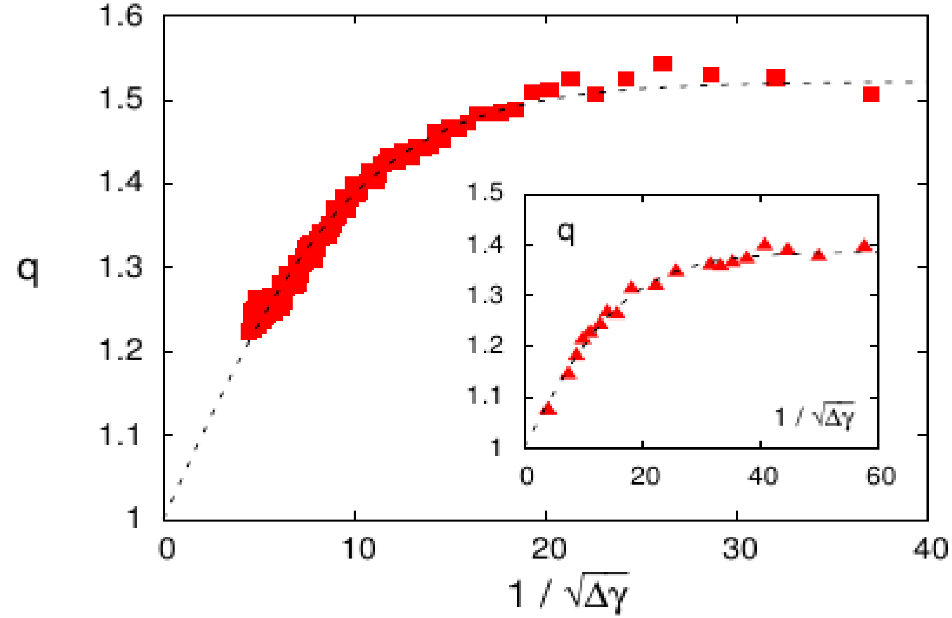
Validation of Tsallis-Bukman scaling law



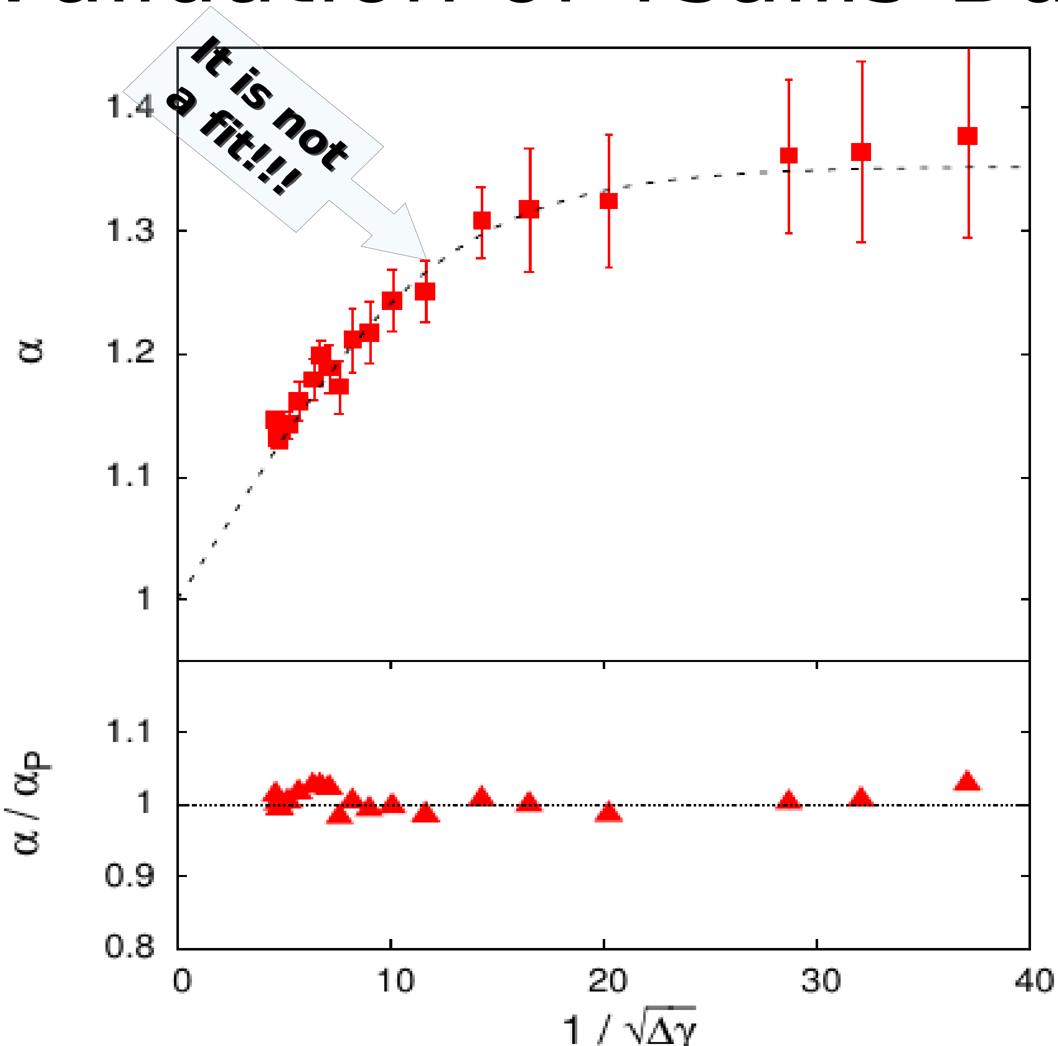
Fit of the PDF with q-Gaussian distribution for each $\Delta\gamma$;

Dependence of the *q* value parameter in function of the control parameter $\Delta\gamma$.



Squares – experiment
Triangles - simulation

Validation of Tsallis-Bukman scaling law



Anomalous diffusion:

$$\langle x^2 \rangle \propto t^\alpha$$

Particular case of Tsallis-Bukman scaling law:
$\mu=1$; $\eta=2-q$

predicted diffusion exponent α_p:

$$\alpha_P = \frac{2}{3-q}$$

arXiv:1507.07268v2
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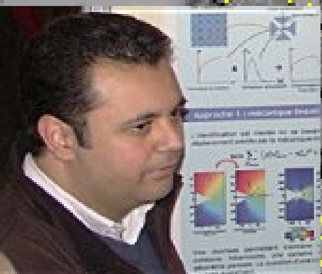
Gaël Combe, Vincent Richefeu, Marta Stasiak, Allbens P.F. Atman

That's all Folks!!!



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Gaël Combe

Thanks for your attention!!!

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