



## Research article

# The emergence of $k\pi$ skyrmions and their spin wave modes in a ferromagnetic disk

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## ABSTRACT

Using micromagnetic simulations, we show that varying the perpendicular anisotropy constant  $K_z$  and the Dzyaloshinskii–Moriya exchange constant  $D_{\text{int}}$  one can stabilize  $k\pi$  skyrmions ( $k = 1, 2, \dots, 5$ ) in a ferromagnetic nanodisk. We have studied their spin wave modes, and obtained the spatial profile of the radial and azimuthal modes, for different values of  $D_{\text{int}}$ . Hysteresis curves and the variation of the skyrmion topological charges were obtained as a function of the value of a planar applied magnetic field. The curves show how the application of the field induces the conversion between different  $k\pi$  skyrmions.

## 1. Introduction

Skyrmionic textures in ferromagnetic nanostructures have gained interest in recent years [1–14].

Unlike the simple skyrmion ( $k = 1$ ), the spatial profile of the  $k\pi$  skyrmions ( $k > 1$ ) has additional out-of-plane domains [1,2].

A  $2\pi$  skyrmion can emerge in thin cylindrical and polygonal geometries [12,14], and recently Yang et al. [3] showed that it can also be formed in hemispherical shells.

A  $k\pi$  skyrmion has topological charge  $|Q| = 1$  for  $k$  odd and  $Q = 0$  for  $k$  even. Due to this property, for example, a  $2\pi$  skyrmion (skyrmionium), with  $Q = 0$ , can move without suffering deflections induced by the skyrmion Hall effect (SkHE), and it can reach higher velocities when compared to a simple skyrmion [15], properties relevant to applications in Spintronics.

The study of the spin wave modes is an important issue, due to their applications in magnonics [16]. Spin wave modes can be used in the process of creating and switching  $2\pi$  skyrmions [3,17–19] and in signal processing and transmission [6,20,21].

Spin wave modes for a simple skyrmion have been studied by several authors [22–26]. Additionally, the stability of  $k\pi$  skyrmions ( $k \geq 2$ ) in a ferromagnetic nanodisk has only been studied as a function of a perpendicular magnetic field [27], Dzyaloshinskii–Moriya interaction [28] and mechanical strains [29]. Thus, the knowledge of the influence of the perpendicular uniaxial anisotropy and the Dzyaloshinskii–Moriya interaction on the stability and spin wave modes of  $k\pi$  skyrmions is still lacking. Thus, the main goal of this work is to

study the influence of the Dzyaloshinskii–Moriya interaction (DMI) on the ground state of a ferromagnetic nanodisk and on the spin wave modes. The other problem that we have investigated is the variation of the magnetization and stability of the  $k\pi$  skyrmions submitted to a cycle of planar magnetic fields.

## 2. Results and discussion

The micromagnetic simulations were performed using the open source code Mumax3 [30].

A cobalt cylindrical ferromagnetic layer with diameter  $D = 150$  nm and thickness  $L = 1$  nm, coupled to a nonmagnetic layer, was simulated. The ferromagnetic layer was discretized in  $1 \times 1 \times L$  nm<sup>3</sup> size cells. Typical parameters of cobalt were used [10–14]: saturation magnetization  $M_s = 5.8 \times 10^5$  A/m, exchange stiffness  $A_{\text{ex}} = 15$  pJ/m and damping constant  $\alpha = 0.3$ . In order to reduce staircase effects due to the circular geometry, we have used a value of edgesmooth parameter equal to 8. We have used values of perpendicular uniaxial anisotropy constant  $K_z$  and  $D_{\text{int}}$  ranging from 0.50 MJ/m<sup>3</sup> to 1.5 MJ/m<sup>3</sup> and from 0 mJ/m<sup>2</sup> to 7 mJ/m<sup>2</sup>, respectively. [10–14]. The range of possible  $D_{\text{int}}$  values is an open question in the literature. Theoretical works, using ab-initio calculations, predict that it is possible to obtain large values of  $D_{\text{int}}$  in the systems Co/NM, NM/Co/NM multilayers, where NM is a nonmagnet [31–33]. For example, in Ref. [32], using ab-initio calculations it is shown that higher values of  $D_{\text{int}}$ , of approximately

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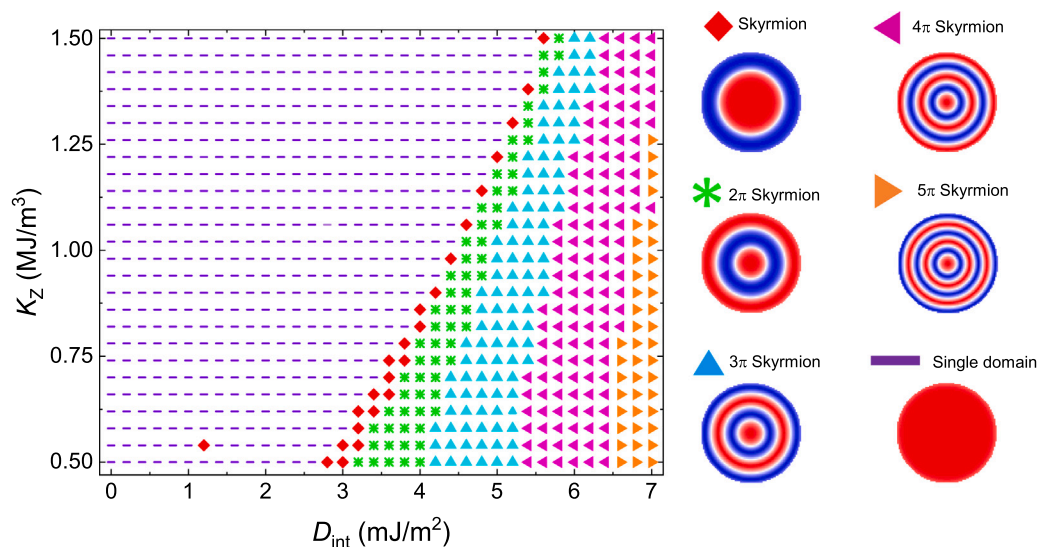


Fig. 1. Phase diagram of the ferromagnetic nanodisk obtained from micromagnetic simulations.

6 mJ/m<sup>2</sup>, can be obtained when cobalt is between a nonmagnetic material and an insulator (e.g., MgO/Co/Pt). Using electric fields, values of just over 7 mJ/m<sup>2</sup> can be obtained. However, experimental results for similar systems show lower values, of approximately 2.5 mJ/m<sup>2</sup> [34]. This difference can be attributed to the disordered interface and grain structure due to the fabrication process of the nanostructure, e.g., in the case of sputtered thin films, as mentioned in Ref. [32].

The first step was to obtain a phase diagram (Fig. 1) showing the ground state of the cylindrical ferromagnetic layer. This phase diagram was obtained for zero magnetic field. For this purpose, in our simulations, we have considered six initial magnetic configurations: perpendicular single domain and  $k\pi$  skyrmion (for  $k = 1, 2, \dots, 5$ ). For each initial magnetic configuration it was obtained its respective final magnetic configuration. The values of the energies of these final configurations were compared in order to obtain the lowest value, which corresponds to the ground state of the nanodisk. This stage was repeated for each combination of parameters ( $K_z$ ,  $D_{int}$ ). The results of this stage are shown in the phase diagram in Fig. 1.

In Fig. 1, it is possible to observe that  $k\pi$  skyrmions emerge for values of  $D_{int}$  ranging from 2.8 mJ/m<sup>2</sup> to 7 mJ/m<sup>2</sup> for  $K_z = 0.50$  MJ/m<sup>3</sup>. However, when  $K_z$  increases up to  $K_z = 1.50$  MJ/m<sup>3</sup>, that range is reduced to values of  $D_{int}$  between 5.6 mJ/m<sup>2</sup> and 7 mJ/m<sup>2</sup>. This behavior is expected, due to the fact that the perpendicular anisotropy aligns the magnetic moments in the  $z$ -direction, favoring the perpendicular single domain configuration. On the other hand, for a fixed value of  $K_z$ , we can see that  $k$  increases with the increase of  $D_{int}$ , due to the fact that the Dzyaloshinskii–Moriya interaction favors the twisting of the magnetic moments, leading to the formation of skyrmions with extra out-of-plane domains. Note that, for  $K_z > 1.22$  MJ/m<sup>3</sup>, it was not possible to obtain a 5 $\pi$  skyrmion.

We have measured the diameters<sup>1</sup> of the cores of the  $k\pi$  skyrmions for all values of  $D_{int}$  used in this work, where it was possible to obtain them (not necessarily as the ground state of the disk). These values are shown in Fig. 2. In all cases, it is possible to observe a smooth increase in the values of the diameters with the increase of the values of  $D_{int}$ . It is possible that due to the Dzyaloshinskii–Moriya interaction, the energy of the domain wall regions is reduced, favoring the increase of the out-of-plane domains. Of course, the diameters of the cores are limited by the diameter of the disk.

<sup>1</sup> In the Supplementary Material are shown the definitions of  $D_1$ ,  $D_2$ ,  $D_3$ ,  $D_4$  and  $D_5$ .

The next step was to obtain the power spectra of the spin wave modes of the  $k\pi$  skyrmions<sup>2</sup> (for  $k = 2, \dots, 5$ ) as a function of  $D_{int}$ . For that, we have chosen a fixed value of  $K_z = 0.8$  MJ/m<sup>3</sup> and a range of values of  $D_{int}$  in which it is possible to obtain the desired  $k\pi$  skyrmion (not necessarily as the ground state of the disk). The simulation starts by considering a  $k\pi$  skyrmion as initial magnetic configuration for the nanodisk and allowing it to relax. After that, in order to excite the spin wave modes, a sinc magnetic field pulse  $B_z = B_0 \text{Sinc}(2\pi f_c(t-t_0))$  (for radial modes) and  $B_x = B_0 \text{Sinc}(2\pi f_c(t-t_0))$  (for azimuthal modes) was applied, where  $B_0 = 5$  mT is the magnetic field intensity,  $f_c$  is the cut-off frequency and  $t_0 = 1$  ns is the time delay. The temporal evolution of the spatial profiles (considering all cells of the nanodisk) of the  $z$ -component and  $x$ -component of the magnetization were stored every 5 ps for a total of 10 ns. For this stage a lower damping constant  $\alpha = 0.01$  was used, in order to obtain a better resolution of the spin wave modes.

Each power spectrum was obtained performing a fast Fourier transform (FFT) from the temporal evolution of the spatial profile (considering all cells of the nanodisk) of the  $z$ -component (for radial modes), and  $x$ -component of the magnetization (for azimuthal modes). The power spectra for radial spin wave modes<sup>3</sup> of the  $k\pi$  skyrmions ( $k = 2, 3, 4$  and 5), as a function of  $D_{int}$  are shown in Fig. 3

In Fig. 3 we can see that in a general way, the values of the frequencies of the radial spin wave modes increase with the increase of  $D_{int}$ . The values of the frequencies for the minimum and maximum value of  $D_{int}$  are shown in the Supplementary Material.

In order to gain more information about the spin wave modes, the spatial distribution profiles of the amplitudes of FFT and phases of the oscillation of the spin wave modes (see Fig. 4) were obtained. The oscillations occur within the walls that bound the Skyrmions. Thus, these regions determine the position of the rings of the oscillations. Also, these regions are narrow, and consequently, the rings are also narrow. The width of these regions is of approximately 5 nm.

#### 2 $\pi$ skyrmion:

For mode 1, the oscillation amplitudes of the radial spin wave modes are distributed in two rings around the center of the disk ( $D_{int} = 3.3$  mJ/m<sup>2</sup>). The outer diameter formed by the maximum oscillation amplitude has a larger intensity than the inner diameter formed by

<sup>2</sup> We have not considered  $k = 1$ , because this case has been well studied. See for example Refs. [3,23–25,35].

<sup>3</sup> The fourth mode for a 4 $\pi$  and 5 $\pi$  skyrmion is not described because it appears in a shorter range of values of  $D_{int}$ .

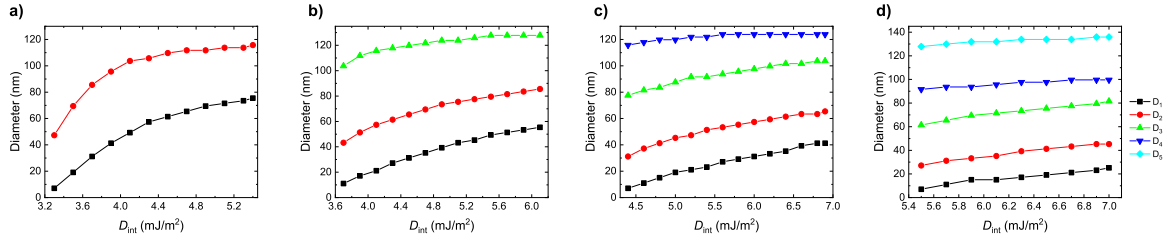


Fig. 2. Diameters of the  $k\pi$  skyrmions as a function of the  $D_{\text{int}}$  constant, (a) for a  $2\pi$  skyrmion, (b) for a  $3\pi$  skyrmion, (c) for a  $4\pi$  skyrmion and (d) for a  $5\pi$  skyrmion.

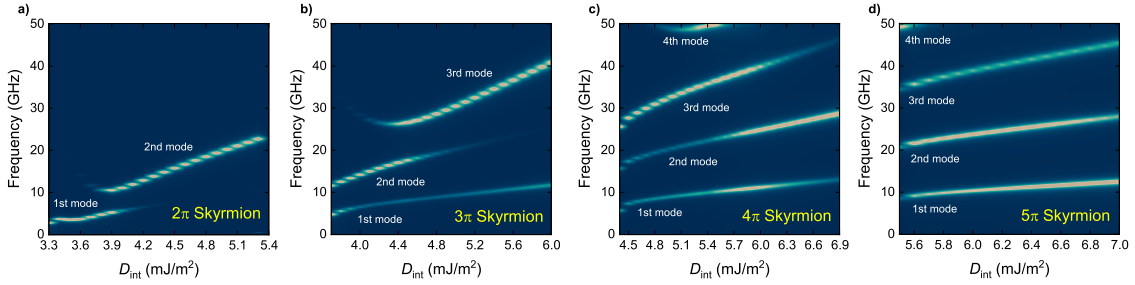


Fig. 3. Power spectra of the  $k\pi$  skyrmions as a function of the Dzyaloshinskii–Moriya exchange constant  $D_{\text{int}}$  for radial modes, (a) for a  $2\pi$  skyrmion, (b) for a  $3\pi$  skyrmion, (c) for a  $4\pi$  skyrmion and (d) for a  $5\pi$  skyrmion.

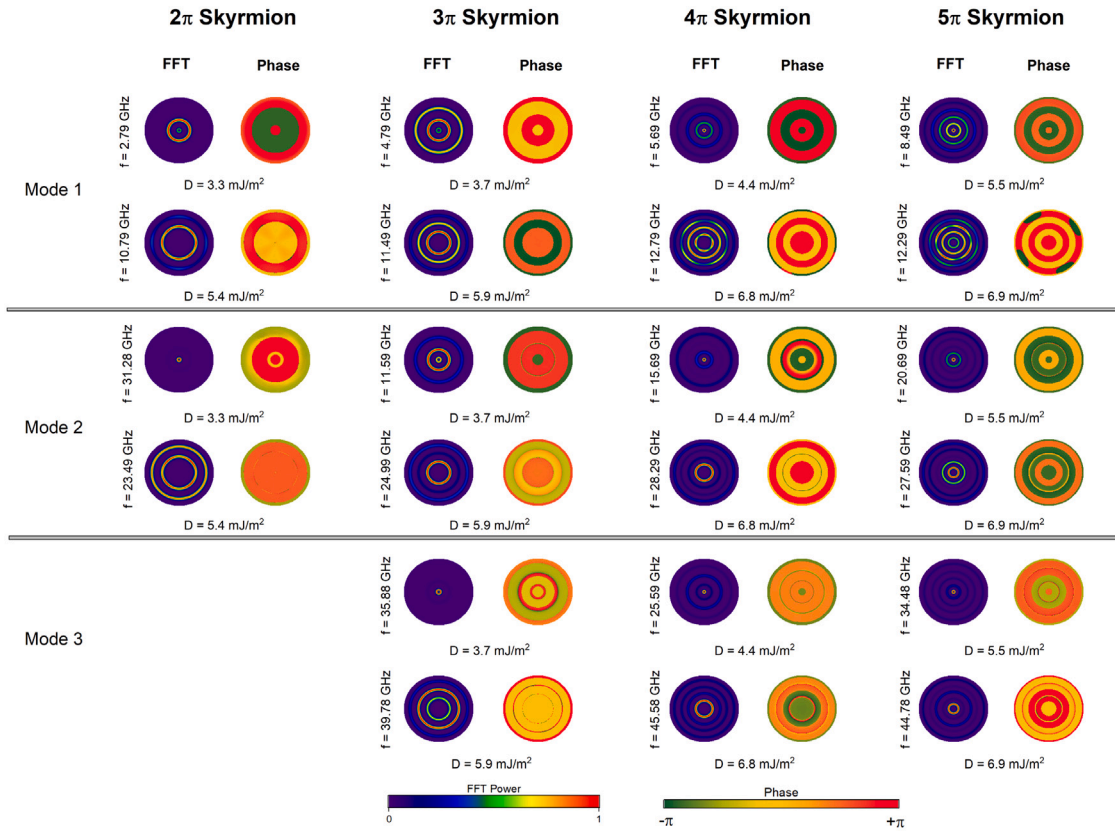


Fig. 4. Spatial distribution profiles of the Fourier power and the phases of the oscillations of the radial spin wave modes as a function of the Dzyaloshinskii–Moriya exchange constant  $D_{\text{int}}$ .

the small amplitude oscillations, and as  $D_{\text{int}}$  increases, the diameters of both rings approach the edges of the disk. However, the outer ring increases its oscillation amplitude up to  $D_{\text{int}} = 3.8 \text{ mJ/m}^2$  and then its intensity decreases with the increase of  $D_{\text{int}}$ , whereas the inner ring increases its intensity with the increase of  $D_{\text{int}}$ .

For mode 2, initially the maximum amplitude oscillation is distributed in only one ring around the center of the disk and its diameter increases with the increase of  $D_{\text{int}}$  up to approximately  $D_{\text{int}} = 3.9 \text{ mJ/m}^2$ . For higher values of  $D_{\text{int}}$ , an additional outer ring of small oscillations emerges and both diameters and their amplitudes increase with the increase of  $D_{\text{int}}$ .

### $3\pi$ skyrmion:

For mode 1, the oscillation amplitudes of the radial spin wave modes are distributed in three rings (outer, middle and inner) around the center of the disk. Initially ( $D_{\text{int}} = 3.3 \text{ mJ/m}^2$ ) the middle ring has a higher intensity than the other two rings. The diameter and the intensity of the inner ring increase with the increase of  $D_{\text{int}}$ . The diameter of the middle ring also increases with the increase of  $D_{\text{int}}$  and keeps its intensity constant up to approximately  $D_{\text{int}} = 4.9 \text{ mJ/m}^2$ . For higher values of  $D_{\text{int}}$ , its intensity decreases gradually. On the other hand, the outer ring decreases its intensity monotonically with the increase of  $D_{\text{int}}$ .

For mode 2, the oscillation amplitudes of the radial spin wave modes are also distributed in three rings (outer, middle and inner) around the center of the disk. Initially ( $D_{\text{int}} = 3.3 \text{ mJ/m}^2$ ) the middle ring has a higher intensity than the other two rings. The inner ring increases its intensity and diameter with the increase of  $D_{\text{int}}$ ; the same occurs with the middle ring. However, this ring vanishes for approximately  $D_{\text{int}} = 4.6 \text{ mJ/m}^2$ , but it emerges again, for approximately  $D_{\text{int}} = 5.8 \text{ mJ/m}^2$ , with a lower intensity, increasing its diameter with the increase of  $D_{\text{int}}$ . The outer ring increases its diameter and intensity with the increase of  $D_{\text{int}}$  up to  $D_{\text{int}} = 4.0 \text{ mJ/m}^2$ . For higher values of  $D_{\text{int}}$ , its diameter keeps increasing, but its intensity decreases.

For mode 3, there is only one ring around the center of the nanodisk. Its diameter increases with the increase of  $D_{\text{int}}$ . On the other hand, its intensity remains constant up to  $D_{\text{int}} = 5.2 \text{ mJ/m}^2$ , and then its intensity decreases with the increase of  $D_{\text{int}}$ . When  $D_{\text{int}} = 4.4 \text{ mJ/m}^2$ , another ring emerges of small oscillation amplitude with a larger diameter than that of the first ring. The diameter of this ring increases with the increase of  $D_{\text{int}}$ , whereas its intensity increases for a maximum value of  $D_{\text{int}} = 5.3 \text{ mJ/m}^2$  and then it remains almost constant. For  $D_{\text{int}} = 5.3 \text{ mJ/m}^2$  an outer ring emerges and it has a lower intensity, and this remains almost constant with the increase of  $D_{\text{int}}$ , whereas its diameter increases with  $D_{\text{int}}$ .

### $4\pi$ skyrmion:

For mode 1, the oscillation amplitudes of the radial spin wave modes are distributed in four rings ( $D_{\text{int}} = 4.4 \text{ mJ/m}^2$ ) around the center of the disk. The larger the diameter, the lower the amplitude of the spin waves, and their diameters increase with the increase of  $D_{\text{int}}$ . The inner ring has initially a higher intensity when compared with the other rings, but for the next value of  $D_{\text{int}}$  ( $D_{\text{int}} = 4.5 \text{ mJ/m}^2$ ), its intensity decreases and its value remains almost constant with the increase of  $D_{\text{int}}$ . On the other hand, the intensity of the other rings increase their values for  $D_{\text{int}} = 4.5 \text{ mJ/m}^2$  and they remain also constant with the increase of  $D_{\text{int}}$ .

For mode 2, the oscillation amplitudes are distributed in three rings around the center of the disk. The intensity of each one of them is lower the further away they are from the center. The intensity of the inner ring is kept almost constant with the increases of  $D_{\text{int}}$ , whereas its diameter increases. The other two rings increase their diameters with the increase of  $D_{\text{int}}$ , but their intensities decrease.

For mode 3, the oscillation amplitudes are distributed initially in two rings around the center ( $D_{\text{int}} = 3.4 \text{ mJ/m}^2$ ). The inner ring has a higher intensity than the other ring. Its intensity remains almost constant, but its diameter increases with the increase of  $D_{\text{int}}$ . For  $D_{\text{int}} = 3.5 \text{ mJ/m}^2$ , there appears a middle ring and its diameter and intensity increase up to  $D_{\text{int}} = 4.7 \text{ mJ/m}^2$  and then the intensity vanishes for higher values of  $D_{\text{int}}$ . The second ring increases its diameter with the increase of  $D_{\text{int}}$ , whereas its intensity decreases gradually, vanishing for approximately  $D_{\text{int}} = 6.1 \text{ mJ/m}^2$ . From approximately  $D_{\text{int}} = 6.6 \text{ mJ/m}^2$ , there appear three extra rings around the center, with lower intensity and their diameters remain constant with the increase of  $D_{\text{int}}$ .

### $5\pi$ skyrmion:

For mode 1, the oscillation amplitudes are distributed in five rings around the center of the disk. Their diameters increase with the increase of  $D_{\text{int}}$ , whereas their intensities remain almost constant.

For mode 2, the oscillation amplitudes are also distributed in five rings around the center of the disk. The inner and middle ring have higher intensity in comparison with the outer ring. The diameter and the intensity of the outer ring remain almost constant with the increase of  $D_{\text{int}}$ , whereas the inner and middle ring increase their diameter, keeping constant their intensities with the increase of  $D_{\text{int}}$ .

For mode 3, the oscillation amplitudes are distributed in rings around the center of the disk. Their diameters increase with the increase of  $D_{\text{int}}$ , but their intensities remain almost constant.

In Fig. 4 it is also shown the spatial profile of the phase of the radial spin wave modes. In this figure, we can see that the presence of the Dzyaloshinskii–Moriya interaction leads to a nonuniform phase, due to the nonreciprocity induced by that interaction, resulting in phase jumps along the radial direction. Each ring of spin wave oscillates in its respective phase according to its position in the nanodisk. For example, in mode 1 of the  $2\pi$  skyrmion, the ring formed by the maximum amplitude of the spin wave oscillates in phase and the ring formed by the lower amplitudes also oscillates in phase.

The power spectra for azimuthal spin wave modes of the  $k\pi$  skyrmions ( $k = 2, 3, 4$  and  $5$ ), as a function of  $D_{\text{int}}$ , are shown in Fig. 5. Similarly to the case of the radial spin wave modes, the values of the frequencies of the azimuthal spin wave, in a general way, increase with the increase of  $D_{\text{int}}$ . The values of those frequencies are shown in the Supplementary Material for the minimum and maximum values of  $D_{\text{int}}$  where it is possible to obtain the  $k\pi$  skyrmion. In this stage, in order to determine the direction of rotation of these modes,  $m = +1$  for counterclockwise (CCW) and  $m = -1$  for clockwise (CW) rotation, an in-plane alternating magnetic field,  $B_x = B_0 \sin(2\pi ft)$ , was used, where  $f$  is the frequency of the spin wave mode. The rotation direction is also shown in the Supplementary Material.

In Fig. 6 are shown the spatial profiles of the FFT power and the phase of the azimuthal spin wave modes for a  $2\pi$  skyrmion. For these modes, the maximum oscillation amplitudes are located in rings around the center of the nanodisk and their diameters increase with the increase of  $D_{\text{int}}$ . The spatial profiles of the phases of these modes show that the values of the phases change continuously according to their positions in the nanodisk, resulting in spin waves oscillating with different phases. The spatial profile of the FFT power and the phase for the other  $k\pi$  skyrmions are shown in the Supplementary Material.

As shown in Figs. 3 and 5, the values of the frequencies of the radial and azimuthal modes are strongly influenced by the presence of the DMI interaction. This is due to the fact that it induces a nonreciprocity, which leads to having different wavevectors along specific directions (e.g., in the radial and azimuthal directions). In the case of radial modes, usually considered standing spin waves, the nonreciprocity leads to the existence of waves that are not exactly standing waves. In the case of azimuthal modes, the nonreciprocity is responsible for the frequency splitting of the modes, leading to a counterclockwise and a clockwise rotation modes [36].

We have also studied the evolution of the magnetization and the topological charge under an applied planar magnetic field, for  $k\pi$  skyrmions. In this stage, a  $k\pi$  skyrmion (state I in Fig. 7(a-c)) is stabilized in the nanodisk as initial magnetic configuration. After that, an in-plane magnetic field  $B_x$  along the  $x$ -axis was applied, from 0 up to maximum value of 1.8 T, in 100 mT steps. In each step, the relaxed magnetic state was stored and used as initial state for the next value of magnetic field. The maximum value of the magnetic field employed in this study was not sufficient to saturate the magnetization in the plane.

The magnetic hysteresis curves obtained are shown in Fig. 7(a-c). These loops are different from those obtained with perpendicular magnetic fields (see for example Refs. [37–39]). In that case, the core of the skyrmion expands and shrinks, increasing or decreasing its diameter depending on whether the magnetic field is parallel or antiparallel to the core magnetization. Additionally, there occurs switching of the magnetization, evidenced by sharp jumps in the hysteresis loops [38]. However, in our case the behavior of the magnetization is different (see



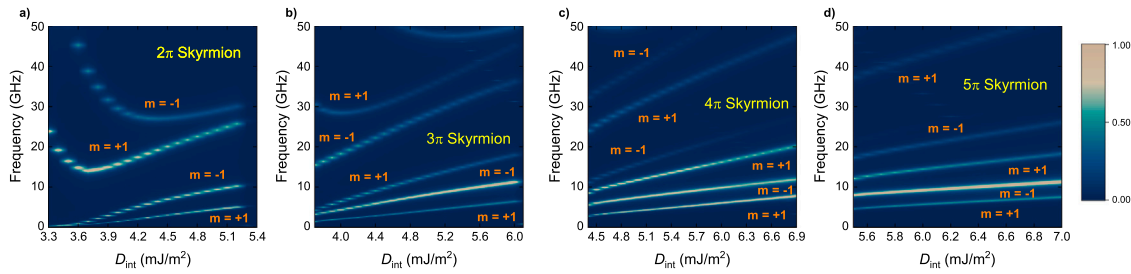


Fig. 5. Power spectra of the  $k\pi$  skyrmions as a function of the Dzyaloshinskii–Moriya exchange constant  $D_{int}$  for azimuthal modes, (a) for a  $2\pi$  skyrmion, (b) for a  $3\pi$  skyrmion, (c) for a  $4\pi$  skyrmion and (d) for a  $5\pi$  skyrmion.

## 2π Skyrmion

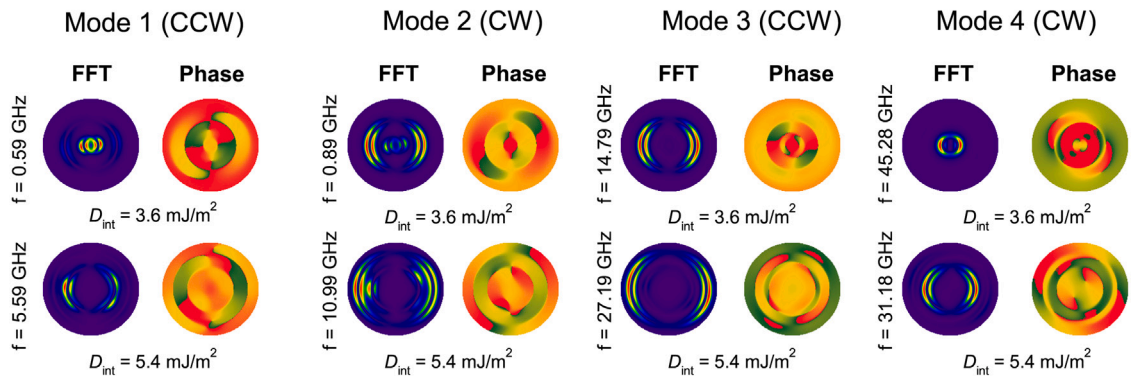


Fig. 6. Spatial distribution profiles of the Fourier power and the phases of the oscillations of the azimuthal spin wave modes as a function of the Dzyaloshinskii–Moriya exchange constant  $D_{int}$ , for a  $2\pi$  skyrmion.

Fig. 7(a–c) and no switching processes are evident. All the hysteresis loops obtained show small jumps in the evolution of the magnetization, corresponding to changes in the magnetic configuration in the nanodisk. The shape of the core is elongated along the direction of the applied magnetic field (state II in Fig. 7(a)) and its size along this direction increases with the increase of the magnetic field up to the point where the skyrmion is annihilated and a structure with a strange shape appears (state III in Fig. 7(a) and Fig. 7(c) and state II in Fig. 7(b)).

The magnetic hysteresis loops allowed the identification of two types of transitions between skyrmions. In the first type a  $k\pi$  skyrmion switches to a  $(k+1)\pi$  skyrmion and in the second type a  $k\pi$  skyrmion collapses to a  $(k-1)\pi$  skyrmion.

In Fig. 7(a–c) are shown the hysteresis loops where the first type of transition appears. For example, in Fig. 7(a) it is shown the hysteresis loop for a  $2\pi$  skyrmion ( $D_{int} = 4.5 \text{ mJ/m}^2$ ) as initial magnetic configuration (state I). Initially, the magnetic moments tend to align with the magnetic field elongating the  $2\pi$  skyrmion (state II). For a value of  $B_x = 0.79 \text{ T}$ , the elongated  $2\pi$  skyrmion is annihilated and a new distorted magnetic configuration is created (state III), which is manifest by a jump in the magnetization curve. Distorted magnetic configurations stay in the nanodisk up to the maximum value of the magnetic field (state IV), even when the magnetic field is decreased and it is reversed to negative values (state V). In this part of the magnetic cycle where the fields are negative, a  $3\pi$  skyrmion is created, for a field of  $B_x = -0.61 \text{ T}$  (state VI). This change in the magnetization is accompanied by another jump in the curve of magnetization. This new skyrmion is annihilated at  $B_x = -0.85 \text{ T}$  and a strange magnetic configuration appears again (small jump in the magnetization curve) and for the negative maximum value of the magnetic field this strange configuration collapses into another (state VII). The  $3\pi$  skyrmion appears again for a positive value  $B_x = 0.61 \text{ T}$ .

The dependence of the topological charge ( $Q$ ) on the intensity of magnetic field is also shown in this figure. The values of  $Q$  broadly agree with the theoretical values defined for the  $k\pi$  skyrmions. The jumps in the magnetic hysteresis loop are accompanied by jumps in the values of the topological charge. Similar behaviors are shown in Fig. 7(b–c) for  $3\pi$  and  $4\pi$  skyrmion as initial magnetic configuration.<sup>4</sup>

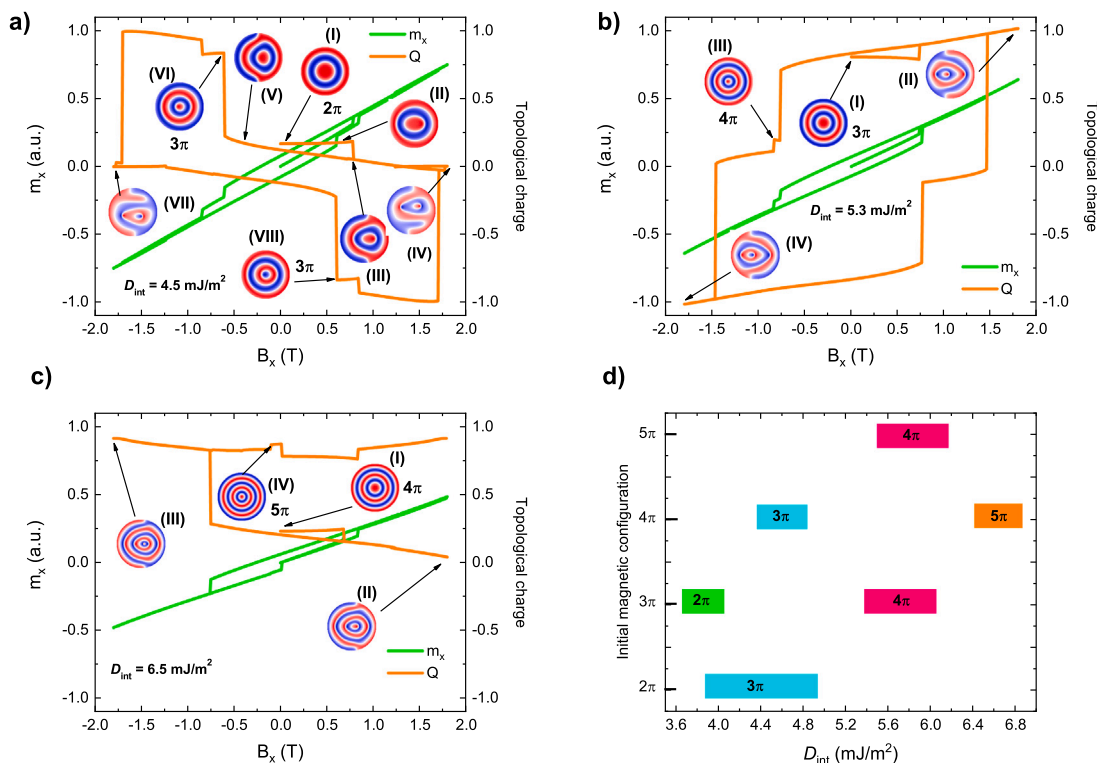
We repeated the same process of analysis of  $m_x$  vs.  $B$ , and  $Q$  vs.  $B$  curves, for the whole range of values of  $D_{int}$  used in this work. A simplified phase diagram is shown in Fig. 7(d). In this figure we can observe the regions where a transition from a  $k\pi$  skyrmion to a  $(k+1)\pi$  skyrmion is possible. The diagram also shows regions where it was observed the opposite type of transition, from  $(k+1)\pi$  to a  $k\pi$  skyrmion. In all cases, the  $(k+1)\pi$  skyrmion is always created when the magnetic fields are negative or when these are increasing from the most negative value, thus it is important to scan almost the complete hysteresis loop. When a  $5\pi$  skyrmion is used as initial magnetic configuration, it was not possible to obtain a  $6\pi$  skyrmion. This may be due to two reasons: (a) a  $6\pi$  skyrmion is not a metastable, or stable state in the nanodisk, or (b) the use of the hysteresis loop may not be suitable for this purpose.

The hysteresis loops for the second type of transitions, i.e., from  $k\pi$  to  $(k-1)\pi$  skyrmions, can be seen in the Supplementary Material. Those curves show behavior similar to that obtained for the first type of transition.

## 3. Conclusion

In this work, we have shown that a  $k\pi$  skyrmion can be stabilized in a ferromagnetic nanodisk tuning parameters such as the

<sup>4</sup> Transitions from a  $\pi$  skyrmion to  $2\pi$  skyrmion and vice versa are well studied, for example, see Refs. [2,12,15,17].



**Fig. 7.** Hysteresis loops (green curve) and evolution of the topological charge (orange curve) as a function of the planar magnetic field intensity, (a) for a  $2\pi$  skyrmion, (b) for a  $3\pi$  skyrmion, (c) for a  $4\pi$  skyrmion and (d) phase diagram showing the range of values of  $D_{int}$  where the corresponding transitions from initial values to final values of  $k\pi$  are observed. Note that the diagram contains transitions from  $k\pi$  to  $(k+1)\pi$  skyrmions, as well as transitions from  $(k+1)\pi$  to  $k\pi$  skyrmions.

value of the uniaxial perpendicular anisotropy constant  $K_z$  and the Dzyaloshinskii–Moriya interaction constant  $D_{int}$ .

Our results show that it is even possible to obtain a  $4\pi$  skyrmion or  $5\pi$  skyrmion as ground state of the nanodisk. A phase diagram with these results was presented.

We have also shown that the values of the frequencies of the power spectrum and the spatial distributions of the FFT power, for radial and azimuthal spin wave modes, are dependent on the value of  $D_{int}$  and the maximum oscillation tends to move towards the edges of the nanodisk when  $D_{int}$  increases. These oscillations may be in phase or out of phase.

Our results also show that the switching between a  $k\pi$  and a  $(k+1)\pi$  skyrmion and vice versa, with a cyclical applied planar magnetic field is possible. This method can be easily implemented experimentally and it is relevant for applications in Spintronics, where a  $k\pi$  and a  $k'\pi$  skyrmions would represent “0” or “1”.

#### CRediT authorship contribution statement

**H. Vigo-Cotrina:** Methodology, Software, Writing – original draft. **D.L. Monteiro:** Software. **J.P.V. Urruchua:** Software. **A.P. Guimarães:** Writing – review & editing, Supervision.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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#### Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jmmm.2022.169665>.

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