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ABSTRACT

Magnetic vortices have potential applications in the field of spintronics and medicine and studying their magnetic interactions is crucial for future applications. This work introduces a new method based on obtaining the gradient magnetic vortex echo (GMVE) using micromagnetic simulations following a magnetic resonance imaging protocol. The results show that it is possible to characterize the magnetic interaction of arrays of nanodisks, having equal diameter and vortex configuration, as a function of disk separation. This characterization was performed by creating an inhomogeneity in the system through the application of a magnetic field gradient perpendicular to the plane of the nanodisk array. The inhomogeneity allows refocusing the magnetization in a time-controlled way by inverting the sign of the gradient and obtaining the characteristic transverse relaxation time T_2^* from the GMVE that contains the information on the magnetic interaction.

I. INTRODUCTION

The study of magnetic vortices in nanostructures is of great interest due to potential spintronic[1–5] and biomedical applications.[6–10] The magnetic vortices are characterized by two parameters: circulation c and polarization p. Vortex magnetic moments can be oriented clockwise or counterclockwise. The circulation is related to the magnetic moment orientation in the plane of the array, with $c = +1$ for counterclockwise orientation and $c = -1$ for clockwise. The magnetization at the central point of the vortex is perpendicular to the plane of the nanodisks and can point up or down. That defines the second vortex property, its polarity "p," where $p = +1$ points in the positive z-direction and $p = -1$ in the negative z-direction.[11–13]

The core of the magnetic vortices performs a spiral-shaped movement called a gyrotropic movement, which is carried out with a characteristic frequency (gyrotropic frequency). This frequency depends on the intrinsic parameters of the material and the aspect ratio $\beta = L/R$.[14–17] However, it is possible to control the value of the gyrotropic frequency using external agents such as electric currents and magnetic fields.[18–20] Gyrotropic motion is counterclockwise (for $p = +1$) and clockwise (for $p = -1$).

The interaction between nanostructures with vortex configurations has been studied using micromagnetic simulations and analytic models.[21–25] The magnetic interaction varies with the inverse of the separation "d" between nanostructures following a d^{-n} law, where n is approximately 4.[26,27]

The intensity of the magnetic interaction is modified by the action of perpendicular magnetic fields on the magnetic vortices.[28] To study these interactions, one needs to consider the individual interaction of each vortex with its neighbors, a task that is complex and computationally expensive; therefore, it is necessary to search for alternative methods.

Previous work showed a new phenomenon called magnetic vortex echo (MVE),[27,29] which is similar to the spin echo (SE) obtained in nuclear magnetic resonance.[30,31] Both MVE and SE arise due to the recovery of the total magnetization of the system, previously lost by different types of mechanisms.[27,29–31]

A magnetic vortex echo (MVE) can be obtained using an array of nanodisks.[27,29] In those works, the inhomogeneity in the system arises from considering a Gaussian distribution of diameters (therefore, a Gaussian distribution of gyrotropic frequencies), and the process of refocusing of the phases is mediated by switching the magnetic vortex core using a perpendicular Gaussian magnetic pulse. However, due to the inhomogeneity arising from the distribution of nanodisk diameters, it is impossible to control the process of refocusing in real time.

This work aims to show that, following the steps used in magnetic resonance imaging (MRI) protocols, it is possible to obtain an MVE by controlling the inhomogeneity through the application of perpendicular magnetic fields without switching the vortex core.

II. RESULTS AND DISCUSSION

The micromagnetic simulations were performed using the open-source code MuMax3.[32] We used a square array of 10×10 Permalloy nanodisks (see Fig. 1) with 300 nm diameter and thickness L = 20 nm. The mesh was discretized in $5 \times 5 \times \mathrm{L\,nm^3}$ size cells, and the typical parameters of Permalloy used were:[27,29,33] saturation magnetization $M_s = 8.6 \times 10^5$ A/m, exchange stiffness $A = 1.3 \times 10^{-11}$ J/m and damping constant $\alpha = 0.005$. The distance between the nanodisks was varied from d = 450 nm to ∞, i.e., in the latter geometry, the simulations were made for each disk individually.

All nanodisks in the array have a magnetic vortex configuration in the ground state, with p = +1 and c = +1 and for the dimensions of nanodisks considered, the gyrotropic frequency has a value of approximately 574 MHz.

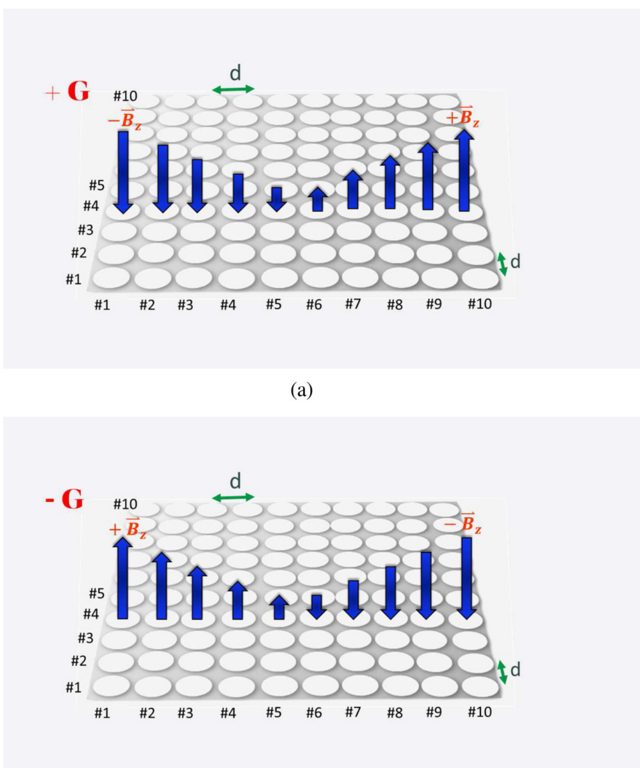

FIG. 1. Schematic representation of (a) the +G applied gradient along the x-axis and (b) the −G applied gradient along the x-axis.

First, we followed the method used by Garcia *et al.*[27] where, in order to induce the dynamics of the magnetization, an in-plane magnetic field was applied along the +x direction to displace all the vortex cores from the equilibrium positions (center of the nanodisks). In this stage, we have used a damping constant $\alpha = 1$ for fast convergence. After that, the magnetic field was turned off and $\alpha = 0.005$ (a realistic value) was used. This procedure allows the vortex core to perform the gyrotropic motion in counterclockwise rotation until t = τ, when the polarities of the vortex cores were switched from p = +1 to p = −1, using a Gaussian magnetic pulse $B_z = B_0 \exp(-0.5(t - t_0)^2/h^2)$, where $B_0 = -300$ mT, h = 200 ps, and $t_0 = \tau$. The vortex cores now perform the gyrotropic motion clockwise until they reach the equilibrium. The total x-component of the magnetization was monitored during the entire process, but a magnetic vortex echo was not observed.

To obtain the magnetic vortex echo phenomenon in our system, we have followed the method used in magnetic resonance imaging:[30] in this method, a bipolar gradient is applied, i.e., a gradient +G [see Fig. 1(a)] was applied until t = τ, and then the gradient was switched to −G [see Fig. 1(b)]. Additionally, there is an equivalent method, where a gradient +G is applied, and at time t = τ, the polarity is switched from +1 to −1.[30] We have also tried this last method, and similar results were obtained (not shown here).

Before applying the method used in MRI, we have obtained the temporal evolution of the total x-component of the magnetization of the system for $B_z = 0$ and $B_z \neq 0$ as follows: the simulation starts with the application of an in-plane static magnetic field of 20 mT along the +x direction, thus increasing the x-component of the magnetization and displacing all the vortex cores away from the equilibrium position (center of the nanodisks) along the +y direction, using a higher damping $\alpha = 1$ for faster convergence. After that, the in-plane magnetic field was turned off, using a realistic damping $\alpha = 0.005$, allowing the system to relax.

If no external agent (e.g., perpendicular magnetic field B_z) is applied during the gyrotropic motion, the temporal evolution of the total magnetization of the system undergoes a single free-induction decay, as shown in the inset of Fig. 2. Since all nanodisks have the same gyrotropic frequency, all vortex cores rotate in phase. The same behavior is obtained if a constant perpendicular magnetic field B_z is applied to all nanodisks during the gyrotropic motion because that field only increases or decreases the value of the gyrotropic frequency, depending on whether it is parallel or anti-parallel to the central magnetization of the vortices.[18,20]

Since all the nanodisks have the same dimensions, our system does not have a natural source of inhomogeneity. To introduce inhomogeneity, a perpendicular magnetic field gradient +G[34] along the x-direction was applied during the gyrotropic motion, i.e., the value of this field is different for each column of the square array of nanodisks. Thus, each column of nanodisks has a slightly different gyrotropic frequency, allowing for a phase shift between the vortex cores of each column (Fig. 1).

The values of the applied magnetic fields vary from $-B_z$ (1st column) to $+B_z$ (10th column) in $\triangle B_z$ steps. Thus, the vortex cores parallel to B_z move faster than the antiparallel cores.

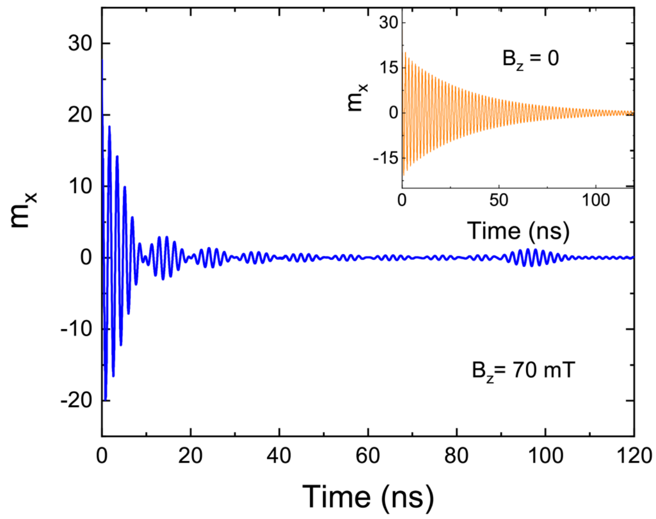

FIG. 2. Temporal evolution of the x-component of the reduced magnetization for all nanodisks in the presence of a perpendicular magnetic field. The inset shows the reduced magnetization for $B_z = 0$.

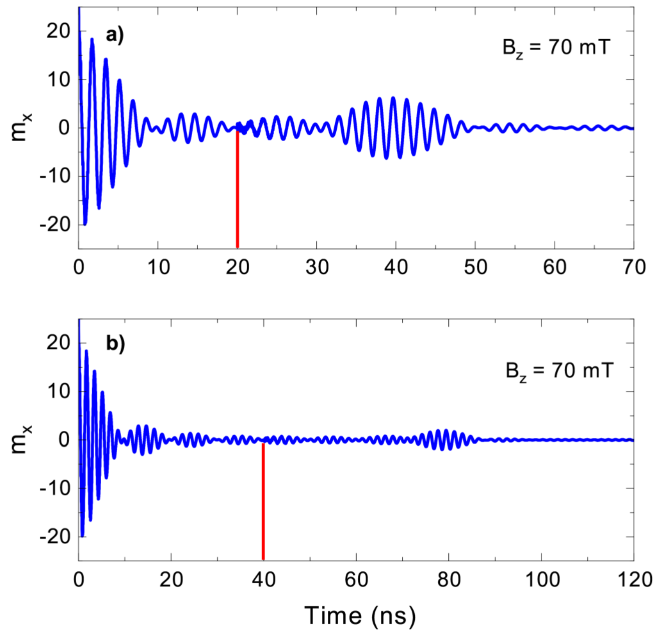

FIG. 3. Gradient magnetic vortex echo obtained for a square array of 100 nanodisks. The red line indicates the switching time τ in (a) at 20 ns and (b) at 40 ns.

If these magnetic fields are maintained until the vortex cores return to the center of the nanodisks, the temporal evolution of the total x-component of the magnetization of the array presents typical beating oscillations. This can be observed in Fig. 2.[35]

In order to recover the magnetization, we have followed the protocol used in MRI techniques. We have switched the gradient sign from +G to −G at time $t = \tau$. Hence, after time τ, the values of the magnetic fields decrease from $+B_z$ (1st column) to $-B_z$ (10th column) in $-\triangle B_z$ steps.

Due to the switching of G, the vortex cores that initially moved with a high gyrotropic frequency (p parallel to B_z) will now move with a lower gyrotropic frequency (p antiparallel to B_z). This allows refocusing the signal, canceling the phase shift induced by the first gradient +G. The recovery in the temporal evolution of the magnetization is the gradient magnetic vortex echo, and it occurs approximately at time $t = 2\tau$, analogous to gradient echo used in MRI.[36,37] This behavior can be seen in Figs. 3(a) and 3(b), where the switching of the signal of the gradient occurs at $\tau = 20$ ns and $\tau = 40$ ns, with concurrent maximum echo amplitude at $t = 40$ ns and $t = 80$ ns. Similar behavior was obtained for other values of B_z.[38]

Getting more than one echo (echo train) is possible if the gradient sign is inverted a second time (from −G to +G). These changes in the gradient sign can be made until the total signal tends to zero, as can be observed in Fig. 4(a).

It is important to emphasize that the gradient sign inversion ($-G$) only cancels the loss in magnetization from the first gradient application but does not cancel the loss due to the magnetic interaction between the disks. The exponential decay of the initial magnetization and the gradient magnetic vortex echoes is characterized by the transverse relaxation time T_2^* [see Fig. 4(a)], which takes into account the following contributions: the inhomogeneity in the frequencies of the nanodisks introduced by the application of the magnetic field gradient (Δf), the loss in magnetization due to the damping constant α (T_α) and the loss of magnetization due to the magnetic interaction between the disks (T_2'). Then, it is possible to write an expression for T_2^* as follows:[27,29]

$$\frac{1}{T_2^*} = \frac{1}{T_2'} + \frac{1}{2T_\alpha} + \Delta f. \quad (1)$$

We have calculated the value of T_2^* by determining the decay of the echo amplitudes for different values of τ at which the gradient G is inverted [see Fig. 4(a)] for different separations d. These results are shown in Fig. 4(b). For a better approximation, we use the signal modulus m of the two magnetization components ($m = \sqrt{m_x^2 + m_y^2}$) to obtain the value of T_2^*. A similar behavior was obtained for $B_z = 60$ mT and $B_z = 50$ mT (not shown here).

In Fig. 4(b), it can be seen that the values of T_2^* decrease with the inverse of the distance d. This occurs because the shorter the distance between the nanodisks, the faster will be the decay of the signal.

Studying the magnetic interaction between the nanodisks as a function of the separation distance d is sufficient to calculate the relaxation time T_2^*, as shown in previous works.[27,29] This dependence can be written as

$$T_2^* = A + B\left(\frac{1}{d}\right)^n. \quad (2)$$

The values of T_2^* obtained from fitting using Eq. (2) are shown in Fig. 4(b). We have obtained a value of $n = 4.59$ for $B_z = 70$ mT,

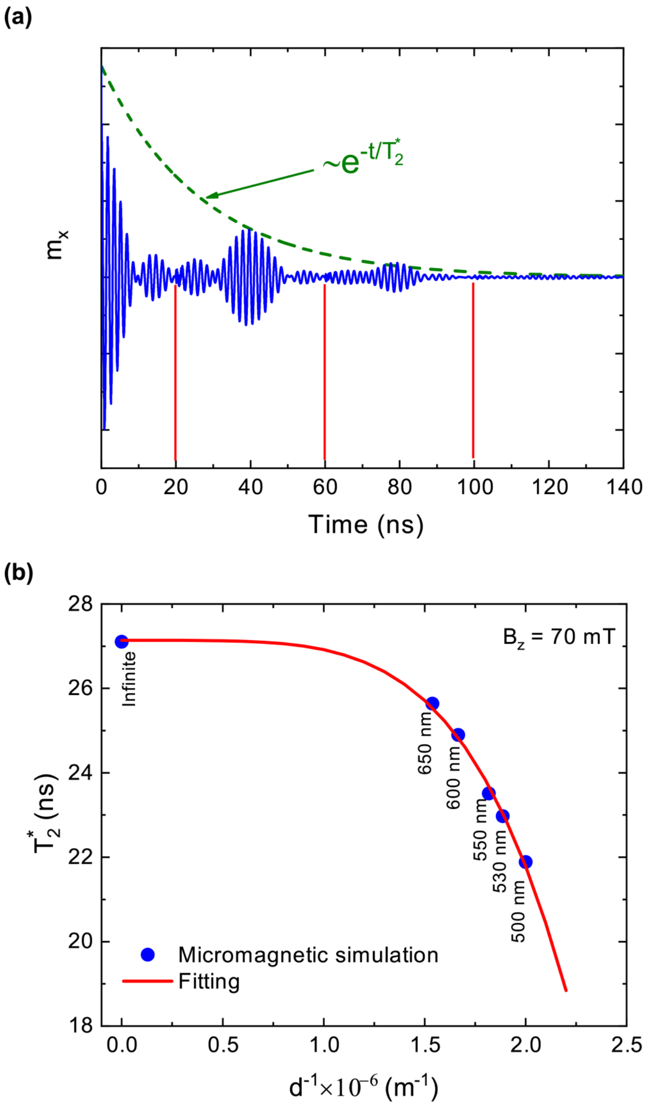

FIG. 4. (a) Time dependence of the x-component of the magnetization of the array of vortices, showing the presence of the gradient magnetic vortex echoes at t = 40, 80, and 120 ns. The dotted line is the curve of exponential decay of the echo amplitude vs separation time between pulses. (b) T_2^* values as a function of the separation distance between the nanodisks obtained by micromagnetic simulation (blue dots) and by fitting the values using Eq. (2) (red line).

while $B_z = 60$ mT and $B_z = 50$ mT yield values for n of 4.41 and 4.04, respectively. This is expected since the interactions between the disks decrease with increasing B_z;[20] therefore, at fields less than $B_z = 50$ mT, the interactions between the disks increase, leading to the standard value of $n \approx 4$, as reported in other works.[26,39]

For distances d less than 500 nm, the magnetic interaction is so strong that the disks tend to have the same gyrotropic frequency, canceling the inhomogeneity introduced by the gradient; therefore, the GMVE is not formed.

The values of the constants A and B from Eq. (2) depend on the intensity of B_z since the interactions between the disks depend on the value of the applied field (as mentioned above). For example, we obtained 2.713×10^{-8} s and -5.663×10^{-45} m^4s for A and B and for $B_z = 70$ mT; however, when $B_z = 50$ mT, the values for A and B changed to 2.699×10^{-8} s and -1.820×10^{-34} m^4s.

As already pointed out, the GMVE can be used to characterize an array of nanodisks. For example, considering d = inf, the first term of Eq. (1) is zero. The second term can be obtained without the need to use the GMVE ($T_\alpha \approx 3.43 \times 10^{-8}$ s). That is, it is sufficient to measure the decay time in the total magnetization of the system without the presence of the magnetic field gradient ($B_z = 0$). The term T_2^* is obtained as mentioned above (see Fig. 4) and has a value of $\approx 2.71 \times 10^{-8}$ s. Replacing these values in Eq. (1), we obtain a value of $\Delta f = 0.022$ GHz, which is quite close to the value of the characteristic frequency for the distribution induced by the magnetic field gradient, which is equal to $\Delta f = 0.020$ GHz (frequency difference between even and odd columns in the array).

III. CONCLUSIONS

In this work, using micromagnetic simulations that follow the protocol used in MRI, we obtained a gradient magnetic vortex echo (GMVE) in a square array of permalloy nanodisks with a magnetic vortex configuration. The GMVE arises due to the refocusing of the magnetization of the system.

We have used the GMVE to measure the magnetic interaction using the characteristic time T_2^* as a function of the distance d of the separation between the nanodisks.

The fits obtained for T_2^* show an inverse dependence on the power of d, in the form d^{-n}, with a value of n of approximately 4.22, a standard value also obtained in other magnetic interaction studies.

In order to characterize a magnetic system, the ferromagnetic resonance technique could be used to estimate the system's inhomogeneity by measuring the linewidth of the spectrum. In this type of measurement, all contributions to the decay process are considered at the same time (damping, interaction, among others), but we cannot identify each contribution individually.[27,29] However, using the GMVE, it is possible to calculate each contribution independently, as shown in the text.

Additionally, a potential application would be the next: it is often difficult to obtain a matrix with disks of the same diameter or thickness in manufacturing processes. However, following the MRI protocol, we can perform spatial encoding to identify disks with different diameters or thicknesses. Using field gradients encodes the rows by phase and the columns by frequency. After applying a Fourier transform to the magnetization signal, we can obtain the value of the gyrotropic frequency of each disk according to the position they occupy in the matrix and identify which disk has a frequency that differs from the frequency of the system.

One of the challenges in experimentally confirming these results is the shortest signal relaxation time (total magnetization), which is in the order of nanoseconds, making it difficult to measure the echo efficiently. Also, the impurities produced during their manufacturing process affect the gyrotropic frequency values or, in some cases, can suppress the gyrotropic motion.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

H. Vigo-Cotrina: Conceptualization (equal); Software (equal); Writing – original draft (equal). **S. Urcia-Romero:** Methodology (equal); Software (equal); Writing – original draft (equal). **A. P. Guimarães:** Conceptualization (equal); Supervision (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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[35]An increase in the amplitude of the beat pattern can be observed between t = 90 and t = 100 ns (which, in principle, is not expected). This increase should not be interpreted as a GMVE since, in reality, the following occurs: In general, it is assumed that the

gyrotropic motion is carried out at a constant frequency (gyrotropic frequency), whose value depends only on intrinsic parameters of the material and the thickness-to-radius aspect ratio (L/R). However, when the nucleus is very close to the center of the disk (approximately 4 nm), it oscillates much faster, leading to an increase in the value of the gyrotropic frequency in the last cycles of the motion.[40] Probably, this increase in frequencies leads to a slight increase in the amplitude of the beat pattern.

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[38] Also, we have used values of $B_z = 60$ mT and 50 mT. However, for lower values of 40 mT, the GEVM it is not obtained, due to the fact that the intensity of the magnetic field is not sufficient to create a frequency distribution in the nanodisks.

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