



## Research articles

# Writing and storing information in an array of magnetic vortex nanodisks using their azimuthal modes

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## ABSTRACT

The switching of a vortex core of a single disk in an array of a multilayer system is investigated by micro-magnetic simulation. We found that the perpendicular uniaxial anisotropy decreases the frequencies of the azimuthal mode in disks with magnetic vortex configuration. We obtained a phase diagram of magnetic field intensity vs. frequency of the azimuthal mode, as a function of the value of perpendicular uniaxial anisotropy. We demonstrated that rotating magnetic fields (CW and CCW) with frequency equal to azimuthal modes can be used to switch the vortex core of single disks in a disk array. This allows obtaining different memory states with a single array of nanodisks, and therefore writing information through the application of rotating fields.

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## 1. Introduction

The magnetic vortex configuration is characterized by an in-plane curling magnetization, and a core, where the magnetization points out of the plane. The curling direction defines the circulation  $C = +1$  (counterclockwise (CCW)) and  $C = -1$  (clockwise (CW)). The core has polarity  $p = +1$  when it points along the  $+z$  direction and  $p = -1$  in the  $-z$  direction [1].

A magnetic vortex core presents a translation mode of low frequency, in the sub-gigahertz range, known as gyrotropic motion [1,2] and two other modes of higher frequency ( $>1$  GHz): azimuthal and radial modes, which have their origin in magnetostatic interactions and thus are also dependent on the dimensions of the disk [3–5].

Depending on the ratio of the thickness to the radius of the disk ( $\beta = L/R$ ), the azimuthal modes have a splitting in the frequencies [6]. Their frequencies ( $\omega$ ) have CCW and CW senses of rotation [3].

Magnetic vortices have many potential applications in magnetic data storage devices [2,7–9]. For example, a vortex with  $p = +1$  can store bit 1, and a vortex with  $p = -1$  can store bit 0, or vice versa. In these applications, the issue of switching vortex cores is a topic of great interest, that has been studied for a long time [10–19].

Using rotating magnetic fields with a frequency equal to the gyrotropic frequency, it is possible to switch the vortex core polarity [18,20], but with the downside that this only happens when the

sense of rotation of the gyrotropic motion (which is determined by  $p$ ) coincides with the sense of rotation of the magnetic field [11,18,21,22]. Another proposal found in the literature is to use rotating magnetic fields with frequencies equal to the characteristic frequency of the azimuthal modes [14,17]. This method has the great advantage that magnetic rotating fields can be used with both directions of rotation (CCW and CW), and also involve shorter switching times [14,17].

Switching of vortex cores using azimuthal modes allows higher switching critical velocities (of the order of  $\sim 800$  m/s) [14], compared to those of the gyrotropic mode ( $\sim 330$  m/s) [20].

Nanodisks with a magnetic vortex configuration are generally produced by nanolithography in the form of arrays (matrices of nanodisks) on substrates that can influence their dynamic properties. For example, in a multilayer system, a perpendicular uniaxial anisotropy (PUA) can be induced due to the interface contribution, as has already been demonstrated by Garcia et al. [23]. This PUA influences the processes of switching of the vortex core [18].

An array of disks can be used to build an information storage device and/or build logic gate circuits [8,9]. In these arrays, the polarity has an important role, since it determines the type of logic gate to be obtained [8]. Consequently, it is necessary to search for mechanisms to control the polarity of a single disk in an array, without altering the polarity of the neighboring disks.

The goal of this work is to propose a novel method for controlling the selectively switching of one single vortex core in a matrix of nanodisk multilayer system, in order to obtain the desired combinations of bits in this matrix. For this purpose, we have used

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micromagnetic simulation. All simulations were made using the open source software Mumax3 [24], with discretization cell size of  $2 \times 2 \times L \text{ nm}^3$ , where  $L$  is the thickness of the disk. The material used is Permalloy (NiFe), with typical parameters [1,25,26]: saturation magnetization  $M_s = 8.6 \times 10^5 \text{ A/m}^2$ , exchange stiffness  $A = 1.3 \times 10^{-11} \text{ J/m}$ , and damping constant  $\alpha = 0.01$ . The perpendicular uniaxial anisotropy constant<sup>1</sup>  $K_z$  varied from 0 to 200  $\text{kJ/m}^3$ . For larger values of  $K_z$ , skyrmion type magnetic structures emerge [18,26].

## 2. Results and discussion

### 2.1. Isolated disk

We used disks with thickness  $L = 20 \text{ nm}$  and diameter  $D = 500 \text{ nm}$ . For these dimensions, the magnetic vortex configuration is stable [18]. We assumed that the vortex core is initially at the equilibrium position at the center of the disk, and has polarity  $p = +1$  and circulation  $C = +1$ .

In order to excite the azimuthal spin wave modes, we have applied an in-plane sinc pulse magnetic field<sup>2</sup>  $\mathbf{B}(t) = (B_0 \sin(x)/x, 0, 0)$ , with  $x = 2\pi f(t-t_0)$ , centered on  $t_0 = 1 \text{ ns}$ , where  $B_0 = 1 \text{ mT}$  is the magnetic field amplitude and  $f = 50 \text{ GHz}$  is the frequency. The frequencies of the modes are obtained by fast Fourier Transform (FFT) from the time evolution of the x-component of the magnetization. These frequencies are shown in Fig. 1. We repeat the same procedure for each value of  $K_z$ .

There are three frequencies for each value of  $K_z$ . The lowest value frequency corresponds to the gyrotropic mode ( $f_0 \approx 0.35 \text{ GHz}$ ) and the other two frequencies correspond to the  $m = -1$  (clockwise) and  $m = +1$  (counterclockwise) azimuthal modes, respectively [14,17].

The frequency of the gyrotropic mode remains practically constant with the increase<sup>3</sup> of  $K_z$ . The azimuthal frequencies decrease with increasing anisotropy, as shown in Fig. 1(a), because the influence of PUA modifies the configuration of the magnetic vortex [23,26]. It is important to note that the effect of PUA on the vortex configuration is totally different from that produced by a perpendicular magnetic field (PMF). Whereas PUA does not alter the gyrotropic mode, PMF does so in a manner proportional to the intensity of this field [17].

In Fig. 1(b) are shown the three frequencies ( $f = \omega/2\pi$ ) for each value of  $K_z$ .

In order to switch the vortex core, we have used an in-plane rotating magnetic field  $\mathbf{B}(t) = B_0 \cos(\omega t)\hat{x} + B_0 \sin(\omega t)\hat{y}$  ( $+\omega$  for CCW and  $-\omega$  for CW) bursts with duration of 24 periods, as suggested by Kammerer et al. [14]. After the magnetic field is turned off, we have monitored the micromagnetic simulation for an additional 1.5 ns with zero magnetic field, to observe possible switching that may occur due to delayed processes [29].

We start by exploring the switching vortex core using  $f = 0.35 \text{ GHz}$  (gyrotropic mode); we encountered a minimum magnetic field intensity  $B_0 = 1.2 \text{ mT}$ , to obtain the switching of vortex cores for the entire range of perpendicular uniaxial anisotropy constant  $K_z$  used in this work (see Section 1).

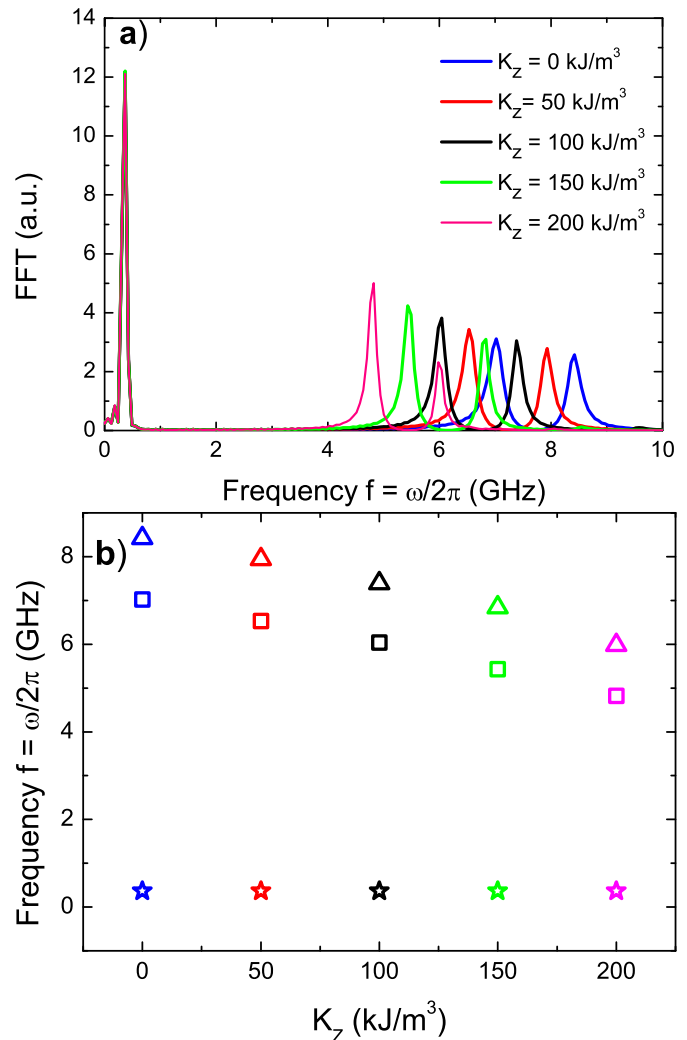
In Fig. 2 are shown the switching times<sup>4</sup> ( $t_{sw}$ ) as a function of  $B_0$ , for each value of  $K_z$  used in this work, using the gyrotropic mode. We found a decrease in the values of  $t_{sw}$  with the increase of  $B_0$ .

<sup>1</sup> These values of  $K_z$  can be obtained experimentally increasing the thickness of the disk as shown in Ref. [23,27] or increasing the thickness of the substrate [28].

<sup>2</sup> The sinc function has a Fourier Transform of constant intensity, up to a frequency  $f$ . This function has often been used in the literature to simulate FMR spectra of magnetic systems. This reveals the amplitude of the different modes.

<sup>3</sup> There is a variation of approximately 3% for the maximum value of  $K_z$ , but this change is negligible. This too was demonstrated by Fior et al. [18].

<sup>4</sup> See Supplementary material for details of how these values were obtained.



**Fig. 1.** (a) Values of the gyrotropic mode frequency and azimuthal spin wave frequencies for  $m = +1$  (counterclockwise) and  $m = -1$  (clockwise) obtained by a fast Fourier transform (FFT) from the time evolution of the x-component of the magnetization for each value of  $K_z$ , for  $p = +1$ . The largest peak represents the gyrotropic mode. (b) The same frequencies obtained previously but in other presentation. Triangles correspond to the  $m = +1$  azimuthal mode, squares to the  $m = -1$  azimuthal mode, and stars to the gyrotropic mode.

For  $K_z = 0 \text{ kJ/m}^3$  it is obtained a switching time ( $t_{sw}$ ) of approximately 15 ns (Fig. 2). This time is reduced by approximately 88% with the increase of  $B_0$  from 15 ns ( $B_0 = 1.2 \text{ mT}$ ) to 1.82 ns ( $B_0 = 7 \text{ mT}$ ). For larger intensity magnetic field, undesirable multiple switching events appear. The same behavior is observed for  $K_z \neq 0$ .

Although  $t_{sw}$  decreases with the increase of  $B_0$ , the critical velocity (approximately 329 m/s) that the vortex core reaches before switching, is the same for all values of  $B_0$ . This is known as the universal criterion of switching, as has already been demonstrated by Lee et al. [20]. However, this critical velocity decreases when  $K_z \neq 0$ , from 329 m/s ( $K_z = 0 \text{ kJ/m}^3$ ) to 200 m/s ( $K_z = 200 \text{ kJ/m}^3$ ), but is still independent on  $B_0$ . These values are consistent with those obtained by Fior et al. [18].

In order to obtain the magnetic field intensity to switch the vortex core using azimuthal modes, we have varied  $B_0$  from 1 mT to 6 mT for  $m = -1$  mode, and from 1 mT to 8 mT for the  $m = +1$  mode, in 0.2 mT steps for both  $m = +1$  (CCW) and  $m = -1$  (CW) modes. The switching phase diagram ( $B_0$  vs. frequency) is shown in Fig. 3. We have varied slightly the values of the frequencies shown in Fig. 1, as suggested in Ref. [17], in order to obtain lower values of the threshold magnetic field intensity.

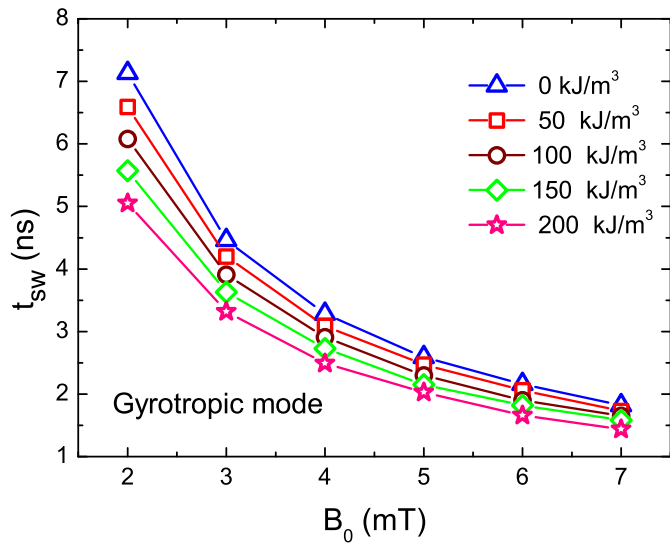


Fig. 2. Switching times through the gyrotropic mode as a function of magnetic field intensity ( $B_0$ ) for different values of  $K_z$ .

We encountered three regions for both modes  $m = +1$  and  $m = -1$ : 1) no switching, 2) single switching, and 3) multiple switching. We are interested in the region of single switching, with the purpose of having total control in the selectivity of the resulting polarity  $p$ .

In Fig. 3 we do not show the region for  $1 \text{ mT} < B_0 < 2 \text{ mT}$  for  $m = +1$  mode, and the region for  $1 \text{ mT} < B_0 < 3.6 \text{ mT}$  for  $m = -1$  mode, since there is no switching for all the values of  $K_z$  used in this work.

The threshold of the magnetic field intensity ( $B_0$ ) and the range of single switching of vortex core are different for each value of  $K_z$  (see Fig. 3), and for modes  $m = +1$  and  $m = -1$ . A wider range of  $B_0$  values is found for  $m = -1$  mode that result in a single switching (green squares), in comparison with  $m = +1$ . For  $m = +1$  mode, multiple switching events are dominant in the phase diagram, whereas for  $m = -1$ , single switching events are more frequent.

Multiple switching events appear because the applied magnetic field pumps enough energy to reverse repeatedly the vortex core between  $p = +1$  and  $p = -1$  [16].

For the  $m = +1$  mode, we used a magnetic field ( $B_0$ ) of up to 8 mT to obtain single switching for  $K_z = 200 \text{ kJ/m}^3$ . This is different for the  $m = -1$  mode, where a threshold of magnetic field  $B_0 = 3.6 \text{ mT}$  is necessary to obtain a single switching of vortex core.

All these differences between modes  $m = +1$  and  $m = -1$  are due to the fact that the modes act differently in the creation of a dip, which is the first step in the switching process [14,30]. Whereas the reversal of the  $m = -1$  mode begins with a single dip, in the  $m = +1$  mode, a double dip is formed [14].

Fig. 4 shows the switching times ( $t_{sw}$ ) for both modes, and for the case  $K_z = 0 \text{ kJ/m}^3$  and  $K_z = 100 \text{ kJ/m}^3$ . These times decrease with increasing intensity of the magnetic field for  $m = +1$  mode, however, for  $m = -1$ , it is observed that for some values of  $B_0$ ,  $t_{sw}$  does not have the same behavior<sup>5</sup>. This can also be attributed to nonlinear dynamics.

Similar behaviors were obtained by Kammerer et al. [14] for Permalloy disks, using  $K_z = 0 \text{ kJ/m}^3$ .

It is important to note that although the switching time can be reduced increasing the magnetic field intensity, and using the gyrotropic mode (Fig. 2), we obtain shorter switching times with lower field intensities using the azimuthal modes (Fig. 4). As mentioned earlier, we get  $t_{sw} = 1.82 \text{ ns}$  ( $B_0 = 7 \text{ mT}$ ) using the gyrotropic

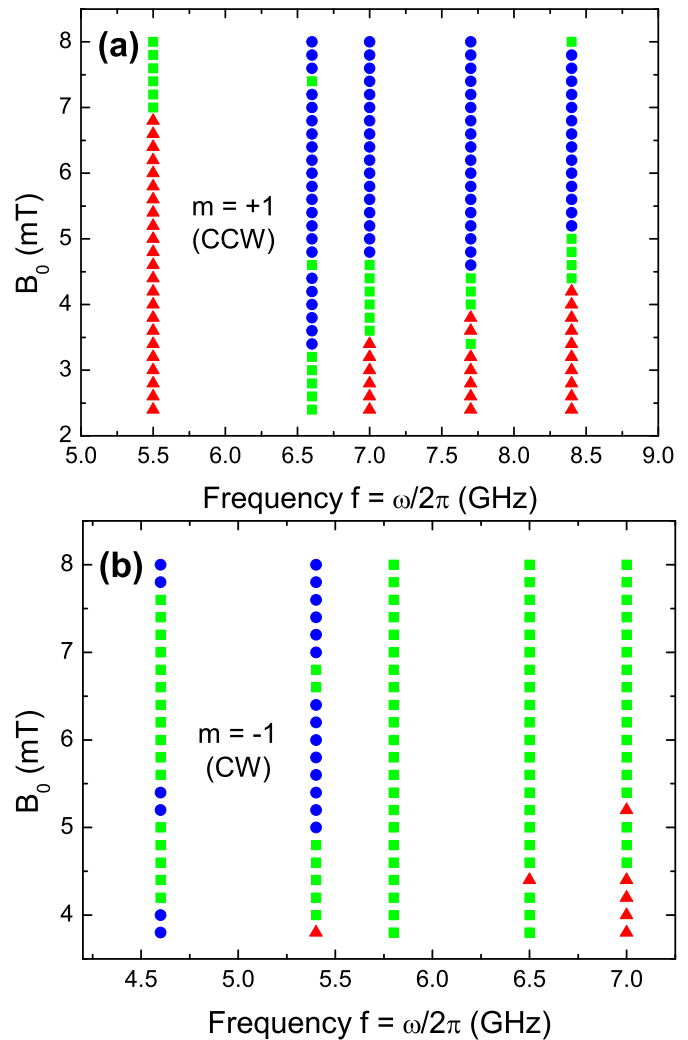


Fig. 3. Switching phase diagrams for (a)  $m = +1$  (CCW) and (b)  $m = -1$ . Red triangles indicate no switching, green squares indicate single switching and blue circles indicate multiple switching. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

mode for  $K_z = 0 \text{ kJ/m}^3$ , but using the azimuthal mode we get shorter times, of approximately  $t_{sw} = 1.29 \text{ ns}$  for the  $m = +1$  mode, and  $t_{sw} = 0.85 \text{ ns}$  for  $m = -1$  mode. Both values of  $t_{sw}$  were obtained with magnetic field intensity smaller than 7 mT (Fig. 4).

The nonlinear dynamics breaks the universal criterion for switching of vortex cores found for the gyrotropic mode. We have obtained different values for critical velocities. For example, for  $K_z = 0 \text{ kJ/m}^3$ , we found an average critical velocity<sup>6</sup> ( $v_{sw}$ ) for the entire region where there is single switching (Fig. 3) of approximately 816 m/s and 400 m/s, for modes  $m = +1$  and  $m = -1$ , respectively. These values are higher than those found for the gyrotropic mode, and similar to those found in Ref. [14].

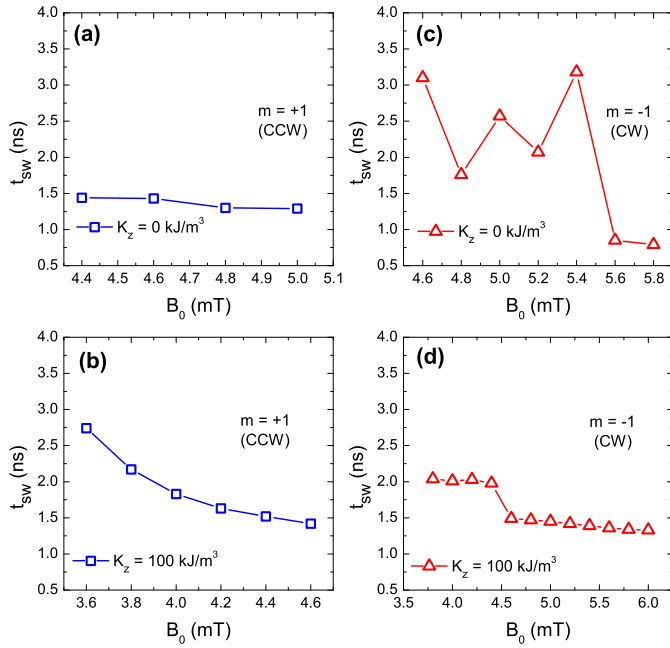
In Table 1 are shown the values of the average critical velocities for modes  $m = +1$  and  $m = -1$ .

All values of critical velocities shown in Table 1, using the azimuthal modes, are higher than the universal critical velocity found using the gyrotropic mode, of approximately 330 m/s. These higher velocities are responsible for the shorter switching times.

The average critical velocities do not show a monotonic behavior

<sup>6</sup> The average critical velocity was obtained by averaging the values of the critical velocities for each value of magnetic field and for each value of  $K_z$ .

<sup>5</sup> Supplementary material shows  $t_{sw}$  for all values of  $K_z$  used in this work.



**Fig. 4.** Switching times versus magnetic field intensity for (a, b)  $m = +1$  (CCW) and (c, d)  $m = -1$  (CW) rotating magnetic fields, for  $K_z = 0 \text{ kJ/m}^3$  and  $K_z = 100 \text{ kJ/m}^3$ .

**Table 1**

Average critical velocity for different values of  $K_z$  and modes  $m = +1$  and  $m = -1$ .

$K_z \text{ (kJ/m}^3\text{)}$	$v_{sw} \text{ (m/s)}$	$v_{sw} \text{ (m/s)}$
	$m = +1$	$m = -1$
0	816	400
50	913	380
100	686	381
150	806	491
200	601	375

with the increase of  $K_z$ ; they increase for  $K_z = 50 \text{ kJ/m}^3$  ( $m = +1$ ), then decrease for  $K_z = 100 \text{ kJ/m}^3$  and then increase again. This behavior contrasts with the case where PMF increases or decreases  $v_{sw}$  depending on whether the PMF is parallel or antiparallel to  $p$  [17].

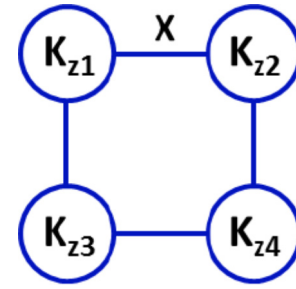
This difference is expected, since PUA does not modify the gyrotropic mode, whereas PMF does. Moreover, PUA and PMF act differently on the vortex core, leading to totally different behaviors in the switching processes, as has already been demonstrated by Fior et al. [18].

Next, we used the influence of PUA and rotating magnetic fields in order to obtain different final states in a  $2 \times 2$  matrix of disks.

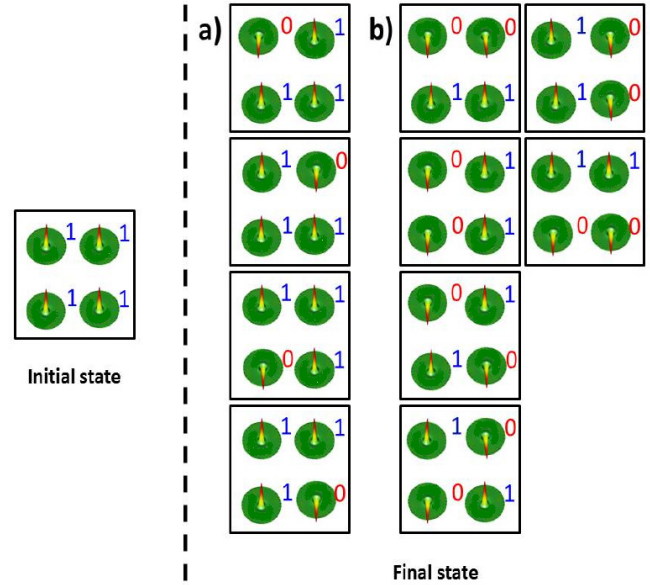
## 2.2. $2 \times 2$ matrix

We will now describe a matrix of four vortex disks, where we can write four bits of information using the rotating magnetic fields.

We used an array of four identical disks, as shown in Fig. 5, with thickness  $L = 20 \text{ nm}$ , diameter  $D = 500 \text{ nm}$  and separated by an edge to edge distance  $x = 500 \text{ nm}$ . Each disk has its own  $K_{zn}$ , with  $n = 1, 2, 3, 4$  and  $K_{z1} < K_{z2} < K_{z3} < K_{z4}$ . The initial state (Fig. 6) is one in which all disks have polarity and circulation  $p = C = +1$ . It is important to mention that the magnetostatic interaction between the disks can change the values of the intensity of magnetic fields for switching [13,31] from the values given in Fig. 3. However, this does not alter the principle that the magnetic field having frequency equal to either modes ( $m = +1$  or  $m = -1$ ) only reverses the vortex core of the disk to which these modes correspond.



**Fig. 5.** Array of disks with magnetic vortex configuration and different values of  $K_z$ .



**Fig. 6.** Initial configuration of the matrix of disks and the resulting configurations obtained after applying the rotating fields. Blue number 1 and red number 0 correspond to polarity  $p = +1$  and  $p = -1$ , respectively. All disks have  $C = +1$ .

In order to obtain different final states, as shown in Fig. 6, we used a global CCW rotating magnetic field  $\mathbf{B}(t) = B_0 \cos(\omega t)\hat{x} + B_0 \sin(\omega t)\hat{y}$  acting on the entire matrix, with duration of 24 periods. Additionally we have monitored the micromagnetic simulation for an additional 1.5 ns with zero magnetic field (as mentioned in subSection 2.1).

We used the following convention: blue digit 1 to indicate positive polarity  $p = +1$ , and red digit 0 to indicate negative polarity  $p = -1$  (see Fig. 6).

In order to obtain any of the final states that have a single 0 red digit (see column a in Fig. 6), we choose a frequency  $f$  equal to the azimuthal  $m = +1$  mode of the disk of interest to be switched. For example, to switch the vortex core of disk 1 ( $K_{z1} = 0 \text{ kJ/m}^3$ ), we used a frequency  $f = 8.4 \text{ GHz}$  (see Fig. 1) and  $B_0 = 4.6 \text{ mT}$ . The magnetic field will efficiently excite the disk 1, to which the frequency corresponds, switching the vortex core only in this disk (see videos in the Supplementary material). It is important to remark that this would be impossible using a frequency equal to that of the gyrotropic mode, because disks with higher  $K_z$  would switch before those of smaller values of  $K_z$  [18].

A final state with two red digits, or two zeros (see column b in Fig. 6) can be obtained in two steps using the azimuthal modes: first switching one of the disks, and next the second disk. This could be done in one step using the gyrotropic mode, depending on which disks the user wants the switching. For example, if one desires to switch only disks 3 and 4, the duration of applied magnetic field will have to be that necessary for the switching to occur



on disk 3, since the switching on disk 4 would have occurred [18], due to  $K_{z3} < K_{z4}$ , thus  $t_{sw3} < t_{sw4}$ . However, if one desired to switch disk 1 ( $K_{z1}$ ) and disk 4 ( $K_{z4}$ ), the application of magnetic field will switch the disks with intermediate values of  $K_z$ , such as  $K_{z2}$  and  $K_{z3}$ .

Final states with three red digits (see Fig. 3 in Supplementary material) can be obtained in three steps, following similar procedure, as in the case of two red digits.

Full switching of all disks is trivial and can be obtained using a frequency equal to that of the gyrotropic mode.<sup>7</sup>

Switching from negative polarity  $p = -1$  to positive polarity  $p = +1$  is also possible using the azimuthal modes, but changing the sense of rotation of these modes. For  $p = +1$ , we have  $m = +1$  (CCW) and  $m = -1$  (CW), and for  $p = -1$  we have  $m = +1$  (CW) and  $m = -1$  (CCW) [14].

### 3. Conclusion

In this work, we initially studied the influence of PUA on the azimuthal modes in individual disks with magnetic vortex configuration, using micromagnetic simulations. Our results show that azimuthal mode frequencies decrease with the increase of PUA, and modified the intensity of the magnetic field necessary to switching the vortex core.

Based on this initial study, we then demonstrated that the azimuthal modes can be used to selective switching the polarity in arrays of disks, therefore obtaining several different final state configurations using a single array of disks. This shows the great advantage of using the azimuthal modes for excitation, in comparison to the use of the gyrotropic mode.

This work also shows that when the intrinsic variable  $K_z$  is considered, the universality of the value of the critical velocity is broken, even for the gyrotropic mode.

Our proposal addresses a subject not very studied, the influence of the PUA in the magnetic vortex dynamics, allowing to write information in an array of disks through selective switching of vortex cores. This simple system can be expanded to larger arrays.

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### Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.jmmm.2018.03.064>.

### References

- [1] K.Y. Guslienko, Magnetic vortex state stability, reversal and dynamics in restricted geometries, *J. Nanosci. Nanotechnol.* 8 (2008) 2745–2760, <https://doi.org/10.1166/jnn.2008.003>.
- [2] A.P. Guimarães, *Principles of Nanomagnetism*, second ed., Springer, Cham, 2017.
- [3] K.Y. Guslienko, A.N. Slavin, V. Tiberkevich, S.-K. Kim, Dynamic origin of azimuthal modes splitting in vortex-state magnetic dots, *Phys. Rev. Lett.* 101 (2008) 247203, <https://doi.org/10.1103/PhysRevLett.101.247203>.
- [4] A.A. Awad, K.Y. Guslienko, J.F. Sierra, G.N. Kakazei, V. Metlushko, F.G. Aliev, Precise probing spin wave mode frequencies in the vortex state of circular magnetic dots, *Appl. Phys. Lett.* 96 (1) (2010) 012503, <https://doi.org/10.1063/1.3268453>.
- [5] M. Sproll, M. Noske, H. Bauer, M. Kammerer, A. Gangwar, G. Dieterle, M. Weigand, H. Stoll, G. Woltersdorf, C.H. Back, G. Schuetz, Low-amplitude magnetic vortex core reversal by non-linear interaction between azimuthal spin waves and the vortex gyromode, *Appl. Phys. Lett.* 104 (1) (2014) 012409, <https://doi.org/10.1063/1.4861779>.

- [6] J.P. Park, P.A. Crowell, Interactions of spin waves with a magnetic vortex, *Phys. Rev. Lett.* 95 (2005) 167201, <https://doi.org/10.1103/PhysRevLett.95.167201>.
- [7] S. Bohlens, B. Krüger, A. Drews, M. Bolte, G. Meier, D. Pfannkuche, Current controlled random-access memory based on magnetic vortex handedness, *Appl. Phys. Lett.* 93 (14) (2008) 142508, <https://doi.org/10.1063/1.2998584>.
- [8] H. Jung, Y.-S. Choi, K.-S. Lee, D.-S. Han, Y.-S. Yu, M.-Y. Im, P. Fischer, S.-K. Kim, Logic operations based on magnetic-vortex-state networks, *ACS Nano* 6 (5) (2012) 3712–3717, <https://doi.org/10.1021/nn3000143>, PMID: 22533663.
- [9] H. Vigo-Cotrino, A.P. Guimarães, Single array of magnetic vortex disks uses in-plane anisotropy to create different logic gates, *J. Magn. Magn. Mater.* 441 (2017) 14–20, <https://doi.org/10.1016/j.jmmm.2017.05.027>.
- [10] S.K. Kim, K.S. Lee, Y.S. Choi, Y.S. Yu, Low-power selective control of ultrafast vortex-core switching by circularly rotating magnetic fields: circular – rotational eigenmodes, *IEEE Trans. Magn.* 44 (11) (2008) 3071–3074, <https://doi.org/10.1109/TMAG.2008.2001539>.
- [11] M.-W. Yoo, K.-S. Lee, D.-E. Jeong, S.-K. Kim, Origin, criterion, and mechanism of vortex-core reversals in soft magnetic nanodisks under perpendicular bias fields, *Phys. Rev. B* 82 (2010) 174437, <https://doi.org/10.1103/PhysRevB.82.174437>.
- [12] Y.-S. Yu, H. Jung, K.-S. Lee, P. Fischer, S.-K. Kim, Memory-bit selection and recording by rotating fields in vortex-core cross-point architecture, *Appl. Phys. Lett.* 98 (5) (2011) 052507, <https://doi.org/10.1063/1.3551524>.
- [13] C.F. Adloff, M. Hänze, A. Vogel, M. Weigand, M. Martens, G. Meier, Self-organized state formation in magnonic vortex crystals, *Phys. Rev. B* 88 (2013) 224425, <https://doi.org/10.1103/PhysRevB.88.224425>.
- [14] M. Kammerer, M. Weigand, M. Curcic, M. Noske, M. Sproll, A. Vansteenkiste, B. Van Waeyenberge, H. Stoll, G. Woltersdorf, C.H. Back, G. Schuetz, Magnetic vortex core reversal by excitation of spin waves, *Nat. Commun.* 2 (279) (2011), <https://doi.org/10.1038/ncomms1277>.
- [15] M.-W. Yoo, J. Lee, S.-K. Kim, Radial-spin-wave-mode-assisted vortex-core magnetization reversals, *Appl. Phys. Lett.* 100 (17) (2012) 172413, <https://doi.org/10.1063/1.4705690>.
- [16] M. Kammerer, H. Stoll, M. Noske, M. Sproll, M. Weigand, C. Illg, G. Woltersdorf, M. Fähnle, C. Back, G. Schütz, Fast spin-wave-mediated magnetic vortex core reversal, *Phys. Rev. B* 86 (2012) 134426, <https://doi.org/10.1103/PhysRevB.86.134426>.
- [17] M.-W. Yoo, S.-K. Kim, Azimuthal-spin-wave-mode-driven vortex-core reversals, *J. Appl. Phys.* 117 (2) (2015) 023904, <https://doi.org/10.1063/1.4905689>.
- [18] G.B.M. Fior, E.R.P. Novais, J.P. Sinnecker, A.P. Guimarães, F. Garcia, Indirect switching of vortex polarity through magnetic dynamic coupling, *J. Appl. Phys.* 119 (2016) 093906, <https://doi.org/10.1063/1.4942534>.
- [19] X.M. Cheng, K.S. Buchanan, R. Divan, K.Y. Guslienko, D.J. Keavney, Nonlinear vortex dynamics and transient domains in ferromagnetic disks, *Phys. Rev. B* 79 (2009) 172411, <https://doi.org/10.1103/PhysRevB.79.172411>.
- [20] K.-S. Lee, S.-K. Kim, Y.-S. Yu, Y.-S. Choi, K.Y. Guslienko, H. Jung, P. Fischer, Universal criterion and phase diagram for switching a magnetic vortex core in soft magnetic nanodots, *Phys. Rev. Lett.* 101 (2008) 267206, <https://doi.org/10.1103/PhysRevLett.101.267206>.
- [21] M. Noske, A. Gangwar, H. Stoll, M. Kammerer, M. Sproll, G. Dieterle, M. Weigand, M. Fähnle, G. Woltersdorf, C.H. Back, G. Schütz, Unidirectional sub-100-ps magnetic vortex core reversal, *Phys. Rev. B* 90 (2014) 104415, <https://doi.org/10.1103/PhysRevB.90.104415>.
- [22] M. Curcic, B. Van Waeyenberge, A. Vansteenkiste, M. Weigand, V. Sackmann, H. Stoll, M. Fähnle, T. Tyliczszak, G. Woltersdorf, C.H. Back, G. Schütz, Polarization selective magnetic vortex dynamics and core reversal in rotating magnetic fields, *Phys. Rev. Lett.* 101 (2008) 197204, <https://doi.org/10.1103/PhysRevLett.101.197204>.
- [23] F. Garcia, H. Westfahl, J. Schoenmaker, E.J. Carvalho, A.D. Santos, M. Pojar, A.C. Seabra, R. Belkhou, A. Bendounan, E.R.P. Novais, A.P. Guimarães, Tailoring magnetic vortices in nanostructures, *Appl. Phys. Lett.* 97 (2) (2010) 022501, <https://doi.org/10.1063/1.3462305>.
- [24] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, B. Van Waeyenberge, The design and verification of Mumax3, *AIP Adv.* 4 (2014) 107133, <https://doi.org/10.1063/1.4899186>.
- [25] K.Y. Guslienko, B.A. Ivanov, V. Novosad, Y. Otani, H. Shima, K. Fukamichi, Eigenfrequencies of vortex state excitations in magnetic submicron-size disks, *J. Appl. Phys.* 91 (2002) 8037–8039, <https://doi.org/10.1063/1.1450816>.
- [26] E.R.P. Novais, S. Allende, D. Altbir, P. Landeros, F. Garcia, A.P. Guimarães, Effect of perpendicular uniaxial anisotropy on the annihilation fields of magnetic vortices, *J. Appl. Phys.* 114 (15) (2013), <https://doi.org/10.1063/1.4824803>.
- [27] J.O. Rantschler, P.J. Chen, A.S. Arrott, R.D. McMichael, W.F. Egelhoff Jr., B.B. Maranville, Surface anisotropy of permalloy in nm-thin multilayers, *J. Appl. Phys.* 97 (10) (2005) 10J113, <https://doi.org/10.1063/1.1853711>.
- [28] J.H. Kwon, J. Yoon, P. Deorani, J.M. Lee, J. Sinha, K.-J. Lee, M. Hayashi, H. Yang, Giant nonreciprocal emission of spin waves in Ta/Py bilayers, *Sci. Adv.* 2 (2016) 2375, <https://doi.org/10.1126/sciadv.1501892>.
- [29] M. Kammerer, M. Sproll, H. Stoll, M. Noske, M. Weigand, C. Illg, M. Fähnle, G. Schütz, Delayed magnetic vortex core reversal, *Appl. Phys. Lett.* 102 (1) (2013) 012404, <https://doi.org/10.1063/1.4773592>.
- [30] K.Y. Guslienko, K.-S. Lee, S.-K. Kim, Dynamic origin of vortex core switching in soft magnetic nanodots, *Phys. Rev. Lett.* 100 (2008) 027203, <https://doi.org/10.1103/PhysRevLett.100.027203>.
- [31] Y. Lu, Z. Zhang, Y. Liu, Magnetic interaction effect on the critical switching current in vortex arrays, *J. Appl. Phys.* 109 (10) (2011) 103906, <https://doi.org/10.1063/1.3590333>.

<sup>7</sup> The same methodology can be used for the case of having different disks forming nanopillars.