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Magnetic vortex echoes

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The dynamic properties of magnetic vortices have many potential applications in fast magnetic devices. Here we present a micromagnetic study of the motion of magnetic vortices in arrays of 100 nanodisks that have a normal distribution of diameters, as expected in real array systems, e.g., produced by nanolithography. The micromagnetic simulated experiments follow a protocol with an initial preparation and magnetic pulses that enable the control of the magnetic vortices initial positions and circular motion direction. The results show a new effect—the magnetic vortex echo (MVE) that arises from the refocusing of the overall array magnetization. We show, by using arrays with different interdisk separations, that MVE affords a means of characterizing them as regards the homogeneity and intensity of the interaction between its elements, properties that are relevant for device applications. We also show that a simple analytical model, analogous to the one that describes the spin echo in magnetic resonance, can be used to explain most features of the simulated magnetic vortex echo. [http://dx.doi.org/10.1063/1.4768446]

I. INTRODUCTION

Many magnetic nanoobjects have as ground state a magnetic vortex configuration, i.e., a pattern of magnetization tangential to concentric circles with a singularity at the center, where the magnetization points out of plane, the vortex core.[1–3] When a core is excited, i.e., displaced from its equilibrium, it performs a spiral-like motion back to the original position, with a well defined eigenfrequency of several hundred MHz.[4] Because of these dynamic properties, the vortices have many potential applications.[1,4–8] Usually, in these applications, one desires high speed dynamics and high densities; therefore, the vortices should be organized in as compact as possible arrays, and the optimization of the performance of the device requires an adequate physical characterization of their dynamic properties. The magnetic vortices have two main features: one is the sense of magnetization curling, i.e., its circulation, which can be counterclockwise (c = 1) or clockwise (c = −1); and, the second one, the core polarity (p), being p = +1(−1) for upward (downward) magnetization direction of the vortex core. The vortex core translation eigenfrequency (usually called gyrotropic frequency) is closely related to the geometry of the nanoobject and, e.g., for a thin nanodisk, is given by $\omega_G \approx (20/9)\gamma M_s \beta$ (M_s is the saturation magnetization, γ is the Gilbert gyromagnetic ratio, and $\beta = h/R$ is the aspect ratio).[2] The sense of the gyrotropic core motion (or the sign of the gyrotropic frequency) is determined by the core polarity and, for an upward (downward) core magnetization, p = +1 (−1), the core will precess in the counter-clockwise (clockwise) direction. Therefore, by controlling the vortex polarity, it is possible to control the sense of gyrotropic vortex core motion.

Until recently most studies neglected any dipolar coupling between nanoobjetcs with vortices, since they present a magnetic flux closure in the relaxed form. However, an out-of-equilibrium core generates sufficient magnetostatic energy to dynamically couple neighbor vortices, as demonstrated in some very recent studies.[8–16] Particularly interesting is the fact that it is possible to transfer energy, with negligible loss, between two neighbor vortices by stimulated gyrotropic motion.[9] This dynamic coupling is strongly dependent on the distance *d* between the centers of the vortices. This has been shown by Vogel and co-workers,[9] using ferromagnetic resonance (FMR), who obtained for a 4×300 array a dependence of the form d^{-n}, with $n = 6$. The same was found by Sugimoto *et al.*[8] using a pair of disks excited with rf current. On the other hand, Jung *et al.*,[11] studying a pair of nanodisks with time-resolved X-ray spectroscopy, found $n = 3.91 \pm 0.07$. Likewise Sukhostavets *et al.*,[12] also for a pair of disks, in this case studied by micromagnetic simulation, obtained $n = 3.2$ and 3.6 for the x and y interaction terms, respectively.

Most works deal with idealized systems containing one, two or no more than few array elements, and effects such as magnetic vortex coupling, inhomogeneities, magnetic stability in large arrays of nanostructures, among others, have been neglected so far. The question of how to characterize the dynamic properties of large area arrays of magnetic vortices has thus very important implications.

In the present work we are proposing a new phenomenon, the magnetic vortex echo (MVE), and have developed an analytical model that describes its main features. This analytical description is analogous to that used for the spin echo observed in nuclear magnetic resonance (NMR), essential in applications such as magnetic resonance imaging (MRI).[17]

We present a micromagnetic study of the motion of magnetic vortices in arrays of 100 nanodisks that have a normal distribution of diameters, as expected in real array systems, e.g., produced by nanolithography. The results show the magnetic vortex echo effect arising from the collective magnetic vortex cores motion which leads to refocusing of the overall array magnetization, as shown in Fig. 1. Using

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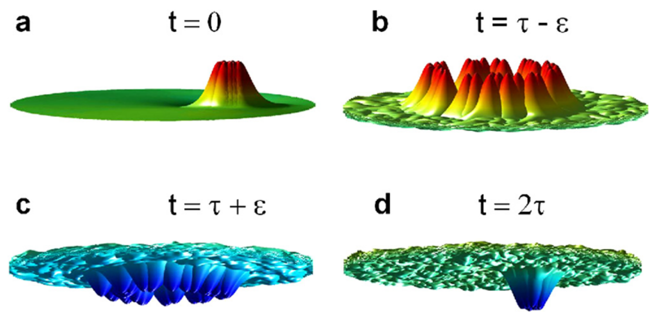

FIG. 1. Formation of magnetic vortex echoes: superposition of the individual simulated disks of the 10×10 matrix at different instants. (a) Top view of the disk at $t=0$, (b) top view at $t=\tau-\epsilon$ (before the inverting magnetic pulse), (c) bottom view at $t=\tau+\epsilon$ (after the inverting magnetic pulse), and (d) bottom view at $t=2\tau$ (at the moment of the vortices refocalization). All disks initially with same circulation $c=+1$ and polarity $p=+1$. (enhanced online) [URL: http://dx.doi.org/10.1063/1.4768446.1].

large arrays with different interdisk separations, MVE affords a means of characterizing large arrays as regards the homogeneity and intensity of the interaction between the array elements, properties that are relevant for device applications.

II. THE MODEL

The formulation of the model begins considering an infinite array of magnetic nanoelements with distance d between their centers. Let us now consider that their vortex cores perform gyrotropic motion, after being excited by the action of some external perturbation, e.g., an in-plane magnetic field applied along the y direction, which has displaced all cores along the x axis, increasing the overall M_y magnetization. As in a real vortex array, we assume that the disks do not have exactly the same gyrotropic frequencies, arising, for instance, from their size distribution.

To derive the time dependence of the array magnetization we will assume first that the gyrotropic frequencies vary continuously and have a Gaussian distribution $P(\omega)$ with standard deviation $\Delta\omega$. Second, we will also assume that the circulation and polarity are initially the same for every vortex: $c=+1$ and $p=+1$. This is not an issue, as will be clear in Sec. III; however, this configuration can be easily achieved by proper procedures (Antos *et al.*[18] and the references therein).

After the vortices are displaced at $t=0$, they will relax toward the equilibrium position in a spiral-like gyrotropic motion, with different frequencies ω, generating an oscillatory behavior of both in-plane magnetization components ($M_x(t)$ and $M_y(t)$). After a given elapsed time, since we are considering a distribution of gyrotropic frequencies, the cores will be completely out of phase, and as consequence, the overall array magnetization will be reduced and eventually will be damped to zero. Using the approach employed in the description of magnetic resonance (e.g., see (Refs. 19 and 20)), one can derive the array y component of the magnetization,

$$M_y(t) = M_y(0)\int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\frac{(\omega-\omega_0)^2}{\Delta\omega^2}}}{\Delta\omega\sqrt{2\pi}}\cos(\omega t)d\omega, \qquad (1)$$

an integral that is the Fourier transform of the gyrotropic frequency distribution $P(\omega)$.[21] One is now able to express $M_y(t)$ as a function of an important relaxation time $T_2^* = 1/\Delta\omega$,

$$M_y(t) = M_y(0)e^{-\frac{1}{2}\frac{t^2}{T_2^{*2}}}\cos(\omega_0 t). \qquad (2)$$

The same reasoning can be applied to $M_x(t)$. This result shows that the total magnetization tends to zero, as the different contributions of both $M_y(t)$ and $M_x(t)$ get gradually out of phase. This damping, with a characteristic time T_2^*, is analogous to the free induction decay (FID) in NMR.

After an elapsed time t, the angle rotated by each vortex core will be ωt; if at $t=\tau$ we invert the polarities of the vortices in the array, e.g., using an appropriate magnetic pulse, the motion of the cores will change direction (i.e., $\omega \to -\omega$). Therefore, for $t>\tau$ one obtains

$$M_y(t-\tau) = M_y(0)\int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\frac{(\omega-\omega_0)^2}{\Delta\omega^2}}}{\Delta\omega\sqrt{2\pi}}\cos[\omega(\tau-t)]d\omega. \qquad (3)$$

The y component of the magnetization becomes

$$M_y(t) = M_y(0)e^{-\frac{1}{2}\frac{(t-2\tau)^2}{T_2^{*2}}}\cos(\omega_0 t). \qquad (4)$$

Equation (4) means that $M_y(t)$ (and $M_x(t)$) increases for $\tau < t < 2\tau$, reaching a maximum at a time $t=2\tau$: this maximum is the magnetic vortex echo, analogous to the spin echo observed in magnetic resonance (Fig. 1). In the case of the NMR spin echo, the maximum is due to the refocusing of the in-plane components of the nuclear magnetization.

Up to now we have only considered a frequency distribution arising from geometric inhomogeneities.[22,25] In a real vortex array other irreversible processes should also be considered, and the decrease of $M_y(t)$ and $M_x(t)$ components will also be affected by these additional processes that we define to be characterized by a relaxation time T_2. Considering this, $M_y(t)$ will be

$$M_y(t) = M_y(0)e^{-\frac{1}{2}\frac{(t-2\tau)^2}{T_2^{*2}}}e^{-\frac{t-\tau}{T_2}}\cos(\omega_0 t). \qquad (5)$$

T_2 is a relaxation time analogous to the spin transverse relaxation time (or spin-spin relaxation time) T_2 in magnetic resonance: now $1/T_2^* = \Delta\omega + 1/T_2$. T_2 can be measured by determining the decay of the echo amplitude for different values of the interval τ. The processes contributing to T_2 are: (a) the interaction between the nanoelements, which, in the first approximation, amounts to random magnetic fields that will increase or decrease ω of a given element, producing a frequency spread of width $\Delta\omega' = 1/T_2'$ and (b) the loss in magnetization (of rate $1/T_\alpha$) arising from the energy dissipation related to the Gilbert damping constant α that appears in the Landau-Lifshitz-Gilbert[1] equation. Identifying T_α to the NMR longitudinal relaxation time T_1, one has[19] $1/T_2 = 1/T_2'$ $+1/2T_\alpha$. Therefore the relaxation rate $1/T_2^*$ is given by

$$\frac{1}{T_2^*} = \Delta\omega + \frac{1}{T_2} = \Delta\omega + \frac{1}{T_2'} + \frac{1}{2T_\alpha}. \qquad (6)$$

The vortex echo maximum at $t = 2\tau$, from Eq. (5), is $M_y(2\tau) \propto \exp(-\tau/T_2)$; one should note that the maximum magnetization recovered at a time 2τ decreases exponentially with T_2, i.e., this maximum is only affected by the homogeneous part of the total decay rate given by Eq. (6). In other words, the vortex echo cancels the loss in $M_y(t)$ due to the inhomogeneity $\Delta\omega$, but it does not cancel the decrease in $M_y(t)$ due to the interaction between the nanoelements (the homogeneous relaxation term $1/T_2'$), or due to the energy dissipation (term $1/2T_\alpha$).

Note also that if one attempted to estimate the inhomogeneity of an array of nanoelements using another method, e.g., measuring the linewidth of a FMR spectrum, one would have the contribution of this inhomogeneity together with the other terms that appear in Eq. (6), arising from interaction between the elements and from the damping. On the other hand, measuring the vortex echo it would be possible to separate the intrinsic inhomogeneity from these contributions, since T_2 can be measured separately, independently of the term $\Delta\omega$.

III. MICROMAGNETIC SIMULATION

In order to confirm the validity of the MVE model, we have performed micromagnetic simulations of an assembly of 100 magnetic nanodisks employing the OOMMF code.[23] The simulated system was a square array of 10×10 Permalloy disks, thickness 20 nm, with distance d from center to center. This distance was varied from $d = 350$ nm up to ∞, in which case the simulations were made on disks one at a time, adding the individual magnetic moments $\mu_i(t)$. To account for the inhomogeneities expected in a real vortex array, we have introduced a Gaussian distribution of diameters, centered on 250 nm with standard deviation $\sigma = 10$ nm and $\sigma = 20$ nm; $\sigma = 10$ nm corresponds to $\Delta\omega \approx 1.5 \times 10^8\ \text{s}^{-1}$. The disks were placed at random on a square lattice. We have also used different values of α.

The simulation initial state was prepared by applying a perpendicular magnetic field pulse of $B_z = +300$ mT to set all disks to the polarity $p = +1$, followed by an in-plane field of 25 mT along the y direction in order to displace all the vortex cores from the equilibrium positions. The system was then allowed to precess freely until $t = \tau$, when the vortex polarities were inverted by the action of a Gaussian magnetic pulse of amplitude $B_z = -300$ mT, with width 100 ps.

Simulations were performed either with random circulation or with $c = +1$; the result is that the value of c is irrelevant, as we can verify by comparing Figs. 2(a) and 2(b). For disks having different circulations ($c = \pm 1$), the cores will be displaced in opposite directions, but the MVE will be the same, since all the magnetizations will point along the same direction. On the other hand, in a configuration where the polarity of the disks is initially random (i.e., $p = \pm 1$) the $p = -1$ disks would not invert their polarities under the influence of the negative B_z field pulse at $t = \tau$, therefore they would not contribute to the echoes, and the echo amplitude would be reduced. However, since the preparation of the system involves an initial positive B_z pulse, all disks will have initially the same polarity ($p = +1$), as assumed in Sec. II.

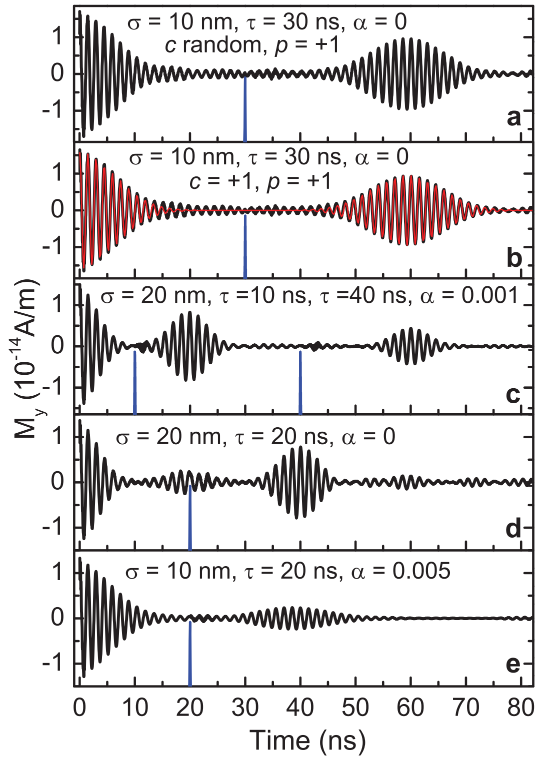

FIG. 2. Magnetic vortex echoes: simulations (black line) for 100 nanodisks, with $d = \infty$ (a) $\sigma = 10$ nm, $\tau = 30$ ns, $\alpha = 0$, $p = +1$, and random c; (b) $\sigma = 10$ nm, $\tau = 30$ ns, $\alpha = 0$ (in red, fit using Eq. (2) plus Eq. (5)); (c) $\sigma = 20$ nm, $\tau = 10$ ns, and $\tau = 40$ ns (two pulses), and $\alpha = 0.001$; (d) $\sigma = 20$ nm, $\tau = 20$ ns, $\alpha = 0$; (e) $\sigma = 10$ nm, $\tau = 20$ ns, $\alpha = 0.005$. The inversion pulses ($B_z = -300$ mT) are also shown (in blue). Disks in (b) to (e) initially with same circulation $c = +1$ and polarity $p = +1$.

We have chosen to present the simulations performed preparing all disks with same circulation ($c = +1$) and polarity ($p = +1$), without loss of generality.

As expected from the model, the array simulated overall in-plane magnetization is markedly damped as a result of the defocusing from the initial state, showing a clear FID with a characteristic time T_2^*. Moreover, the micromagnetic simulations have also confirmed the occurrence of the echoes at the expected times ($t = 2\tau$). For different values of σ, the T_2^* time, and consequently the duration of the FID and the width of the echo are modified (Figs. 2(b) and 2(d)); increasing α results in a faster decay of the echo intensity (Figs. 2(b) and 2(e)). We have also obtained multiple echoes, by exciting the system with two pulses (Fig. 2(c)).[24]

Figure 3 shows the dependence of $1/T_2$ on α for $\sigma = 10$ nm; essentially the same result is obtained for $\sigma = 20$ nm, since T_2 does not depend on $\Delta\omega$ (Eq. (6)). Taking a linear approximation, $1/T_2 \approx A\alpha$, and since for $d = \infty$ there is no interaction between the disks, $1/T_2 = 1/2T_\alpha$, and therefore

$$\frac{1}{T_\alpha} = 2A\alpha. \tag{7}$$

From the least squares fit (Fig. 3), $A = 1.6 \times 10^{10}\ \text{s}^{-1}$. This relation can be used to determine experimentally α, measuring T_2 with vortex echoes, for an array of well-separated disks. Note that we can only obtain directly the T_α and $\Delta\omega$ contributions to T_2^* using the echoes.

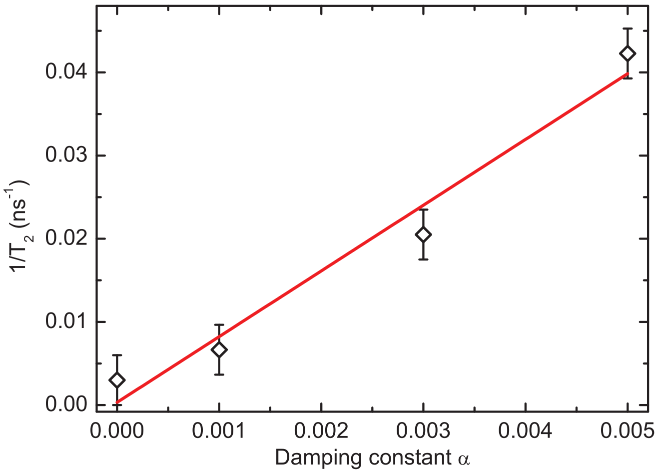

FIG. 3. Variation of $1/T_2$ obtained by fitting the curves of echo intensity versus τ to $M_0 \exp(-\tau/T_2)$, as a function of α, for $D = 250$ nm, $\sigma = 10$ nm, $d = \infty$; the continuous line is a linear fit.

Regarding the problem of determination of the interaction between nanoelements, from our micromagnetic simulations we could describe the dependence of the contribution to $1/T_2^*$ as a function of the distance d between the nanodisks as

$$T_2^* = B + Cd^{-n}. \tag{8}$$

Using Eq. (8) we found, from the best fit (Fig. 4), $n = 4.1 \pm 0.4$, in good agreement with Jung *et al.*[11] and reasonable agreement with Sukhostavets *et al.*[12]

In Fig. 5 we show the results of the simulations with $\sigma = 10$ nm and $\alpha = 0.001$. Assuming $n = 4$, a reasonable linear fit can be obtained with $B = (6.5 \pm 0.1) \times 10^{-9}$ s and $C = -(4.2 \pm 0.3) \times 10^{-26}$ m^{4}s. From $T_2^*(d = \infty)$ and using Eq. (7), we get $\Delta\omega \approx (1.53 \pm 0.1) \times 10^8$ s^{-1}, as expected from our initial choice of the Gaussian distribution of diameters ($\Delta\omega \approx 1.5 \times 10^8$ s^{-1}).

Combining Eqs. (6), (7), and (8), one can obtain the interaction term $1/T_2'$. The computation of $1/T_2'$ required the determination of the other individual contributions to $1/T_2^*$, which was done through the simulation of vortex echoes. To derive simply the dependence of the interaction on d it is sufficient to measure $1/T_2^*$ as a function of d, since the interaction term $1/T_2'$ is the only contribution that is dependent on d; this does not need the use of the echoes, only requiring the determination of the relaxation rate $1/T_2^*$.

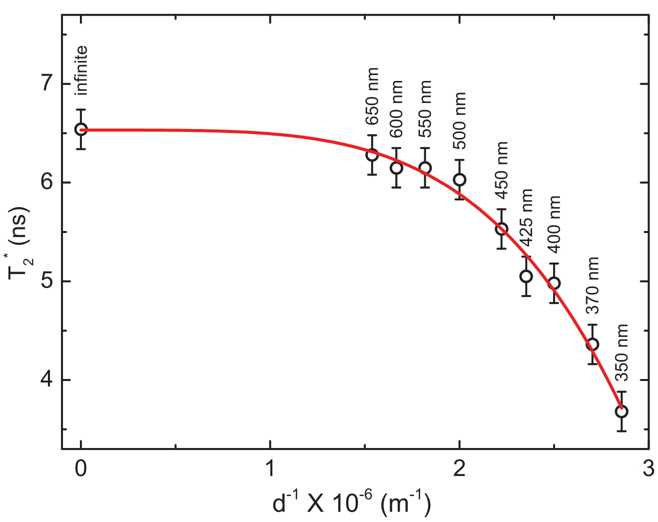

FIG. 4. Variation of T_2^* versus d^{-1} for an array of 10×10 nanodisks with a distribution of diameters centered on $D = 250$ nm ($\sigma = 10$ nm), $\alpha = 0.001$ and separation d; the continuous line is the best fit to Eq. (8).

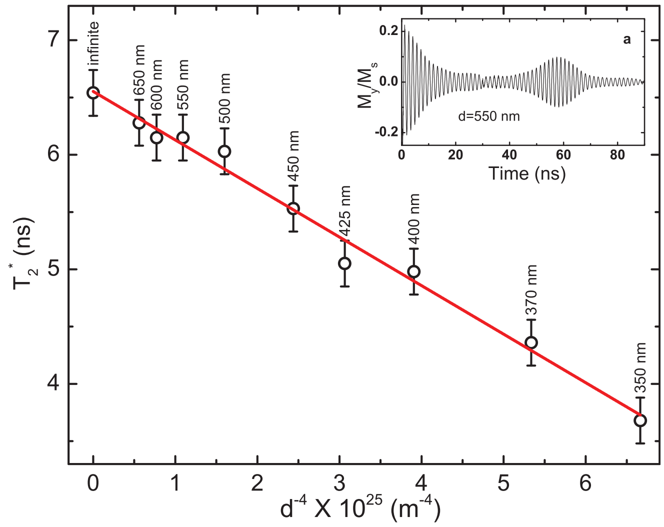

FIG. 5. Variation of T_2^* versus d^{-4} for an array of 10×10 nanodisks with a distribution of diameters centered on $D = 250$ nm ($\sigma = 10$ nm), $\alpha = 0.001$ and separation d; the continuous line is a linear fit. Inset (a) shows an echo simulation for $d = 550$ nm, $\tau = 30$ ns, $\alpha = 0.001$.

IV. CONCLUSIONS

Micromagnetic simulated experiments in large nanodisk arrays reveal a new effect—the magnetic vortex echo—that arises from the refocusing of the overall array magnetization. We have shown the MVE potential as a characterization technique, since it is a direct way of obtaining important parameters such as T_2, related to the interaction between the nanoelements with vortex ground states, and the Gilbert damping constant α; it therefore can be used to determine α in these systems. Applications of the MVE include the measurement of the inhomogeneity, such as the distribution of dimensions, aspect ratios, perpendicular magnetic fields, and so on, in a planar array of nanoelements with vortices; it may be used to study arrays of nanowires or nanopillars containing thin layers of magnetic material. These properties cannot be obtained directly, for example, from the linewidth of FMR absorption. In an actual MVE experiment the sequence of external magnetic field pulses has to be repeated many times (as in NMR), and the echo signals added to improve the signal to noise ratio.

We also show that a simple analytical model, analogous to the one that describes the spin echo in magnetic resonance, can be used to explain most features of the MVE. This model has validated the micromagnetic simulations of the new phenomenon and confirmed the applicability of the MVE as a useful tool for the characterization of large arrays of magnetic nanoobjects with ground state magnetic vortex configuration.

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[21] T. Butz, *Fourier Transformation for Pedestrians* (Springer, Berlin, 2006).
[22] The sources of inhomogeneity are the spread in radii, in thickness, or the presence of defects. An external perpendicular field H adds a contribution to $\omega, \omega = \omega_G + \omega_H$, with $\omega_H = \omega_0\, p(H/H_s)$, where p is the polarity and H_s the field that saturates the nanodisk magnetization.[25] A distribution ΔH is another source of the spread $\Delta\omega$.
[23] See http://math.nist.gov/oommf/ for information on the OOMMF micromagnetic simulation program.
[24] These echoes, however, are not equivalent to the stimulated echoes observed in NMR with two 90° pulses.[17]
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