Spin dynamics under continuous fields: noise spectroscopy

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Outline

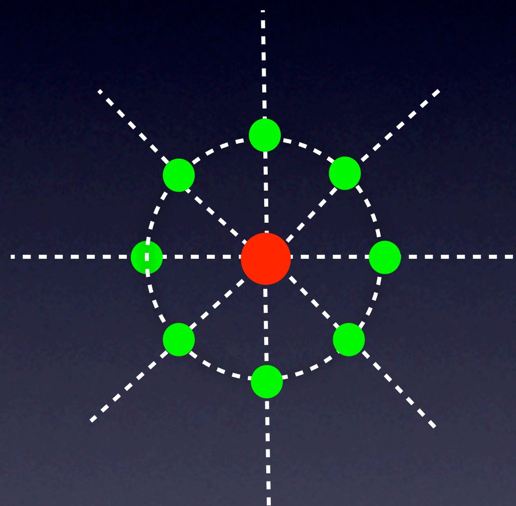
- Noise spectroscopy basics
- Continuous driving field approach
- Conclusions

Part I: Qubit noise spectroscopy with a continuous driving field

Knowing the characteristics of the noise is key to designing optimal controls and quantum error correction protocols for a given system

$$H_{SE} = S_z \sum_m \lambda_m I_{z,m}$$

$$H_E = \sum_{i<j} h_{ij}$$



pure dephasing

Consider the semi-classical approximation

$$H_{sc}(t) = f(t) S_z \quad \text{, where}$$

$$f(t) = Tr_E\{e^{-iH_E t}\Delta e^{+iH_E t}\rho_E\} \quad , \quad \Delta = \sum_m \lambda_m I_{z,m}$$

We treat $f(t)$ as a random variable with $\langle f(t) \rangle = 0$

characterized by a noise power spectral density $S(\omega)$

Apply an arbitrary sequence of pi pulses:

$\langle S_x(t) \rangle = \langle S_x(0) \rangle e^{-\chi(T)}$ where $\chi(T) = \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} d\omega S(\omega) \left| F(\omega, T) \right|^2$

$F(\omega, T)$ is the 'filter function' of the sequence

(Das Sarma 2008, Suter 2011, Biercuk 2011, etc)

One can think of this as a matrix equation, e.g. vectorized data from a real experiment:

$$\vec{\chi} = \mathbf{F} \cdot \vec{S}$$

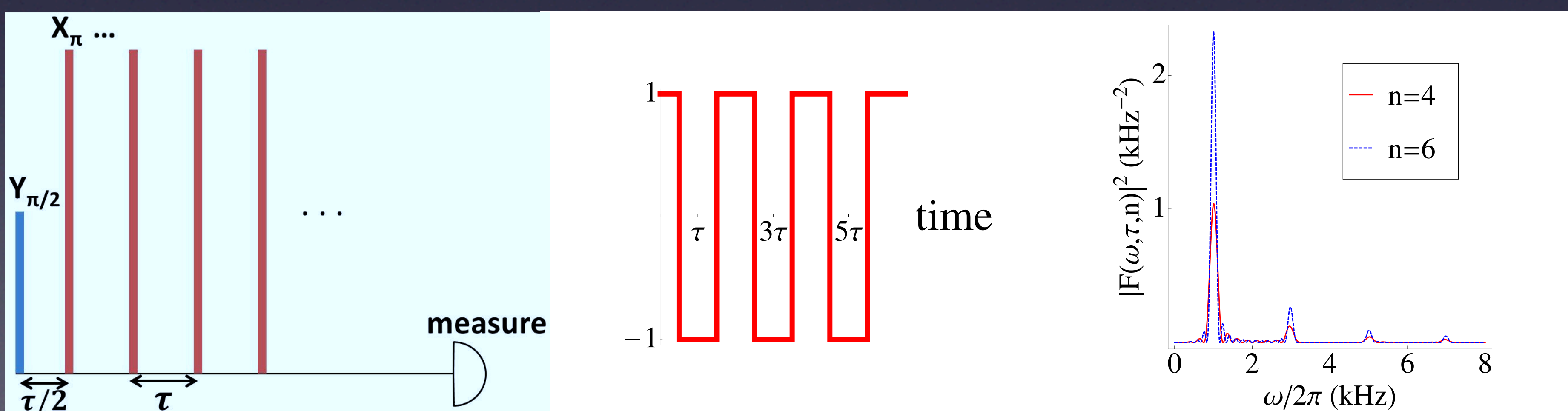
Determining $S(\omega)$ is therefore equivalent to inverting $\mathbf{F}$:

$$\vec{S} = \mathbf{F}^{-1} \cdot \vec{\chi}$$

However, $\mathbf{F}$ is singular since $F(\omega,0) = 0$

A dataset $\vec{\chi}$ does not in general correspond to a unique $\vec{S}$!

Solution: a sufficient number of decoupling cycles produces a filter function = sum of delta-like functions



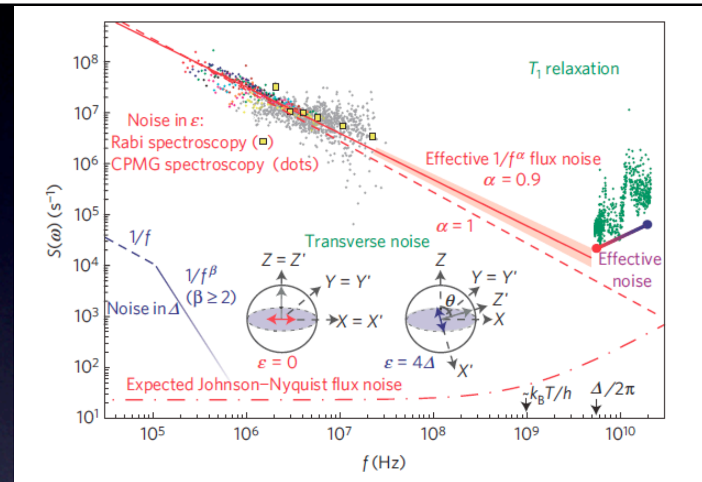
fundamental only
(neglecting harmonics):

Bylander et al 2011,
Meriles et al 2010, etc

nature physics
ARTICLES
PUBLISHED ONLINE: 8 MAY 2011 | DOI: 10.1038/NPHYS1994

Noise spectroscopy through dynamical decoupling with a superconducting flux qubit

Jonas Bylander[1]*, Simon Gustavsson[1], Fei Yan[2], Fumiki Yoshihara[3], Khalil Harrabi[3†], George Fitch[4], David G. Cory[2,5,6], Yasunobu Nakamura[3,7], Jaw-Shen Tsai[3,7] and William D. Oliver[1,4]



including harmonics:

Alvarez and Suter, 2011

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Measuring the Spectrum of Colored Noise by Dynamical Decoupling

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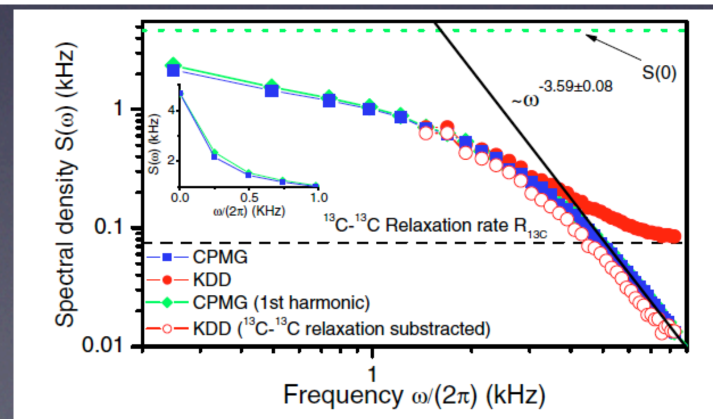
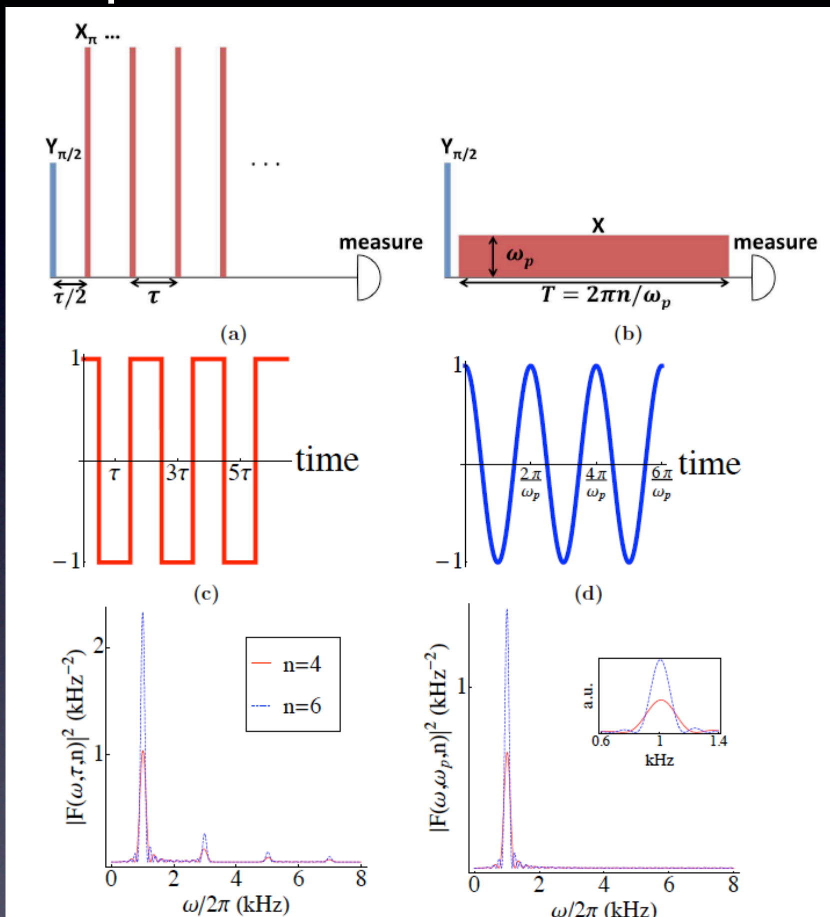
(Received 12 June 2011; published 30 November 2011)



What about continuous decoupling (à la spin-locking)?

pulsed CW



possible advantages:

- limited RF power

- simpler analysis (?)

- better robustness to control error (?)

$$\tilde{H}(t) = e^{-i\omega_{rf} t S_x}\left(f(t)S_z\right)e^{+i\omega_{rf} t S_x} = f(t)\left[\cos(\omega_{rf} t)S_z + \sin(\omega_{rf} t)S_y\right]$$

With the fictitious filter function, one would obtain:

$$\chi(\omega_{rf},T) \approx \frac{\sqrt{\pi}}{2\sqrt{2}} S(\omega_{rf})T \quad \text{when} \quad n = \frac{\omega_{rf}T}{2\pi} >> 1 \quad \text{(\# Rabi cycles)}$$

However this is only a guide for our intuition, not a correct calculation....

Rigorous methods:

1) Generalized Bloch equations (Geva, Kosloff, Skinner 1995)

2) Average Hamiltonian formalism (D. Park, JB 2013)

On the relaxation of a two-level system driven by a strong electromagnetic field

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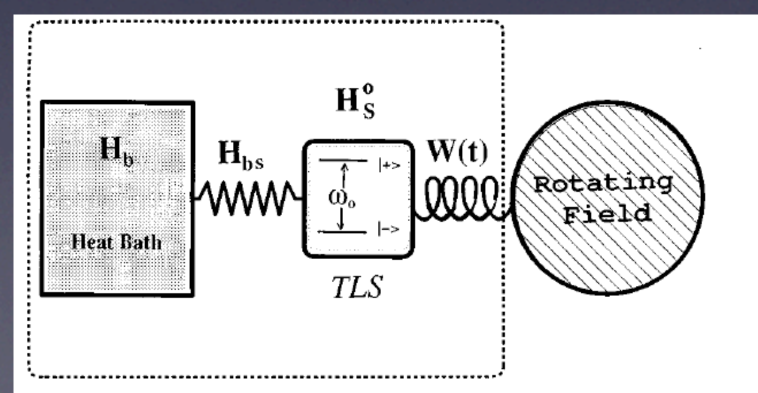


GBE assumptions:

1) system weakly coupled to bath, initial state is a product state

2) $T_{1,2}^{sys} >> \{T_{corr}^{bath}, 2\pi/\omega_{rf}\}$ (Markov approximation, many cycles)

3) bath-induced coherent dynamics are negligible

Relaxation tensor $\hat{\Gamma}(\omega_{rf}, \Delta\omega)$ is a function of the field amplitude, frequency

In our case, pure dephasing, $\rho_0 = S_x$, on-resonance field along $\vec{x}$, $kT >> \hbar\omega_{rf}$,

$$\frac{dS_x}{dt} = -\frac{1}{2}\tilde{C}_{\Delta\Delta}(\omega_{rf})S_x$$

$$C_{\Delta\Delta}(\tau) = Tr_E\{\Delta(\tau)\Delta(0)\rho_E\}$$

bath correlation function

We find that $S(\omega_{rf}) = \frac{1}{\sqrt{2\pi}} \tilde{C}_{\Delta\Delta}(\omega_{rf})$, hence the GBE predicts

a signal decay:

$$\langle S_x(t) \rangle = \langle S_x(0) \rangle \exp\left(-\sqrt{\frac{\pi}{2}} S(\omega_{rf}) t \right)$$

Twice the rate of the fictitious filter function result (but same form).

The GBE gives a result, but it's not very intuitive.

What does it say about conditions for reliable extraction of $S(\omega)$?

Can it be verified by an independent method?

Average Hamiltonian theory

$$\tilde{H}(t) = f(t)\left[\cos(\omega_{rf}t)S_z + \sin(\omega_{rf}t)S_y\right]$$

$$\tilde{U}(t) = \mathcal{T}\left(e^{-i\int_0^T \tilde{H}(t)dt} \right)$$, $\mathcal{T}$ is the Dyson time-ordering operator

Utilize the Magnus expansion:

$$\tilde{U}(t) = e^{-iT\sum_{k=0}^{\infty}\bar{H}_k} \quad , \quad \bar{H}_0 = \frac{1}{T}\int_0^T \tilde{H}(t)dt$$, ...etc

Consider a particular realization of the noise function, sampled for time *T* and its Fourier transform:

$$f_{j,T}(t) = \begin{cases} f_j(t), & 0 \le t \le T \\ 0, & otherwise \end{cases} \qquad F_{j,T}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_{j,T}(t)e^{-i\omega t}dt$$

D. Park, JB (2013) http://lanl.arxiv.org/abs/1308.6310

Rewrite the zeroth-order average Hamiltonian as

$$\bar{H}_{0,j}T = \frac{1}{\sqrt{2\pi}} \int_0^{T=2\pi n/\omega_{rf}} F_{j,T}(\omega)\left[\left(\alpha_1 + i\alpha_3\right)S_z + \left(\alpha_2 + i\alpha_4\right)S_y\right]d\omega$$

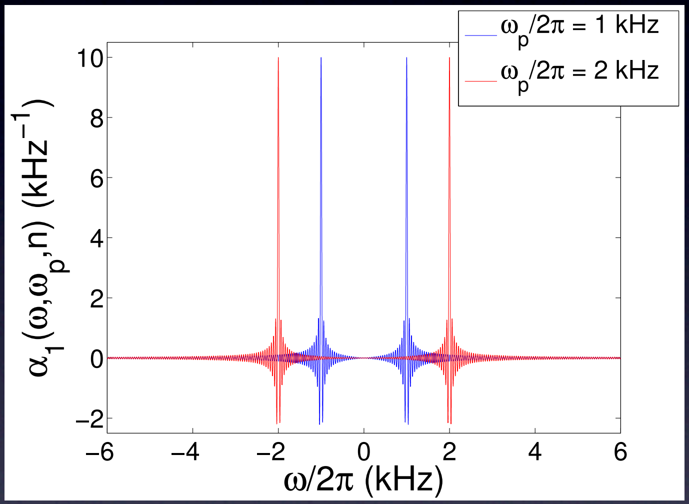
where

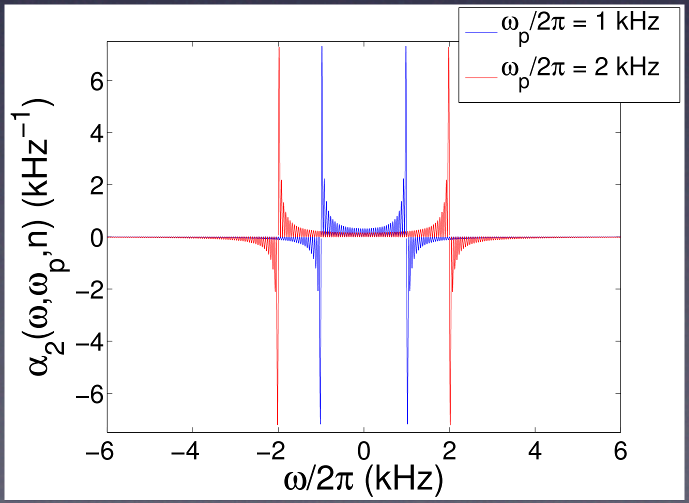
$$\alpha_1(\omega,\omega_{rf},n) = \int_0^{2\pi n/\omega_{rf}} \cos(\omega t)\cos(\omega_{rf} t)dt$$

$$\alpha_2(\omega,\omega_{rf},n) = \int_0^{2\pi n/\omega_{rf}} \cos(\omega t)\sin(\omega_{rf} t)dt$$

$$\alpha_3(\omega,\omega_{rf},n) = \int_0^{2\pi n/\omega_{rf}} \sin(\omega t)\cos(\omega_{rf} t)dt$$

$$\alpha_4(\omega,\omega_{rf},n) = \int_0^{2\pi n/\omega_{rf}} \sin(\omega t)\sin(\omega_{rf} t)dt$$





when $n >> 1$

$$\bar{H}_{0,j} T \approx \sqrt{\frac{\pi}{2}} \left(\mathrm{Re}(F_{j,T}(\omega_{rf})) S_z - \mathrm{Im}(F_{j,T}(\omega_{rf})) S_y \right)$$

this produces a signal

$$\frac{\langle S_{x,j}(T) \rangle}{\langle S_x(0) \rangle} = \cos\left(\sqrt{\frac{\pi}{2}} \mathrm{sgn}(\mathrm{Re}(F_{j,T}(\omega_{rf}))) \left| F_{j,T}(\omega_{rf}) \right| \right)$$

ensemble averaging over noise realizations gives:

$$\frac{\langle S_x(T) \rangle}{\langle S_x(0) \rangle} = \exp\left(-\left\langle \left| F_{j,T}(\omega_{rf}) \right|^2 \right\rangle_j \pi / 4 \right) = \exp\left(-\sqrt{\frac{\pi}{2}} S(\omega_{rf}) T \right)$$

The zeroth-order AHT result agrees with the GBE!

Here, we get an intuition for how good the approximation is based on the shape of the α_k functions (a bit like the filter function analysis)

We can go a step further and consider higher order terms:

$$\bar{H}_1 T = \frac{-i}{2}\int_0^T dt_1 \int_0^{t_1} dt_2 [\tilde{H}(t_1), \tilde{H}(t_2)]$$

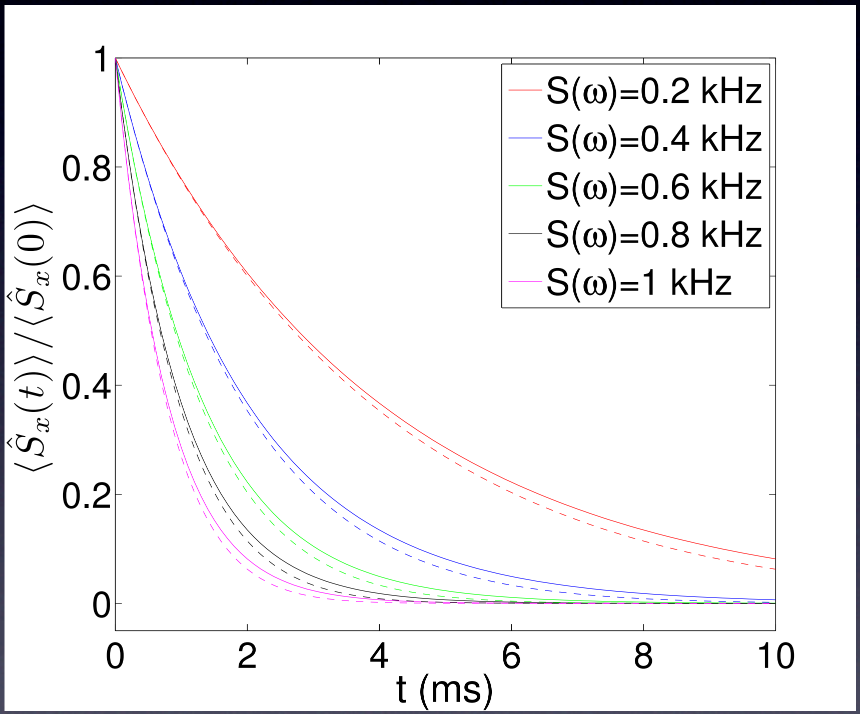
$$= \frac{S_x}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{j,T}(\omega_1) F_{j,T}(\omega_2) \xi(\omega_1, \omega_2, T) d\omega_1 d\omega_2 \approx 0$$

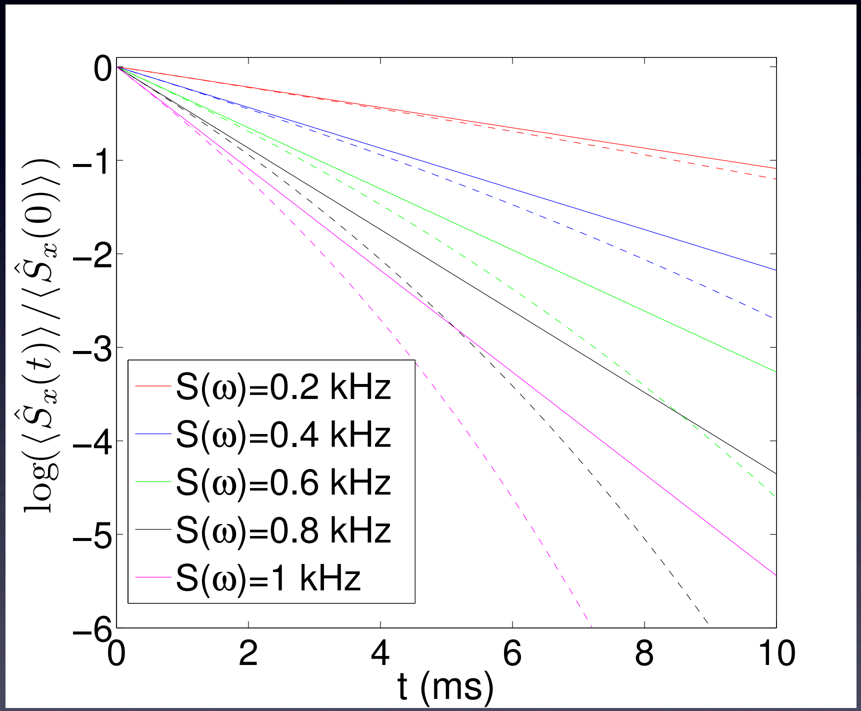
The second-order term is more complicated, we numerically estimate it and get a final result:

$$\frac{\langle S_x(T)\rangle}{\langle S_x(0)\rangle} \approx \exp\left(-\sqrt{\frac{\pi}{2}} S(\omega_{rf})T - 0.056\left(S(\omega_{rf})T\right)^2 - 0.005\left(S(\omega_{rf})T\right)^3\right)$$

signal decay, GBE / zeroth-order AHT ———————

signal decay, 2nd-order AHT - - - - - - - - - - - - - - -





Combining the conditions $S(\omega_{rf})T \sim 1$ and $n >> 1$ implies

$$\frac{\omega_{rf}}{2\pi} >> S(\omega_{rf})$$

and since

$$\left\langle \left| f(t) \right|^2 \right\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega)\, d\omega$$

we can say that reliable noise spectroscopy is guaranteed if

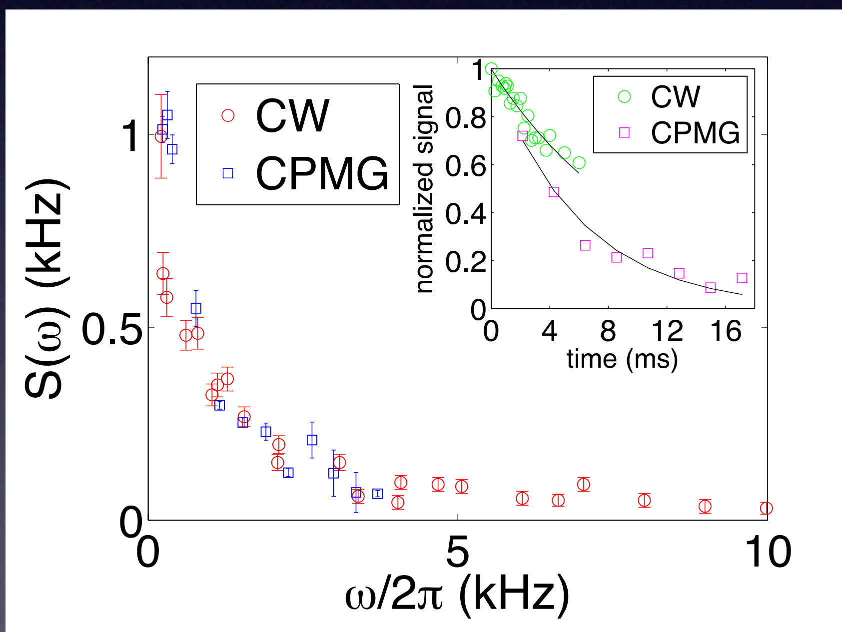
$$\left\langle \left| f(t) \right|^2 \right\rangle << \frac{\omega_{rf}^{\min}}{T\sqrt{2\pi}}$$

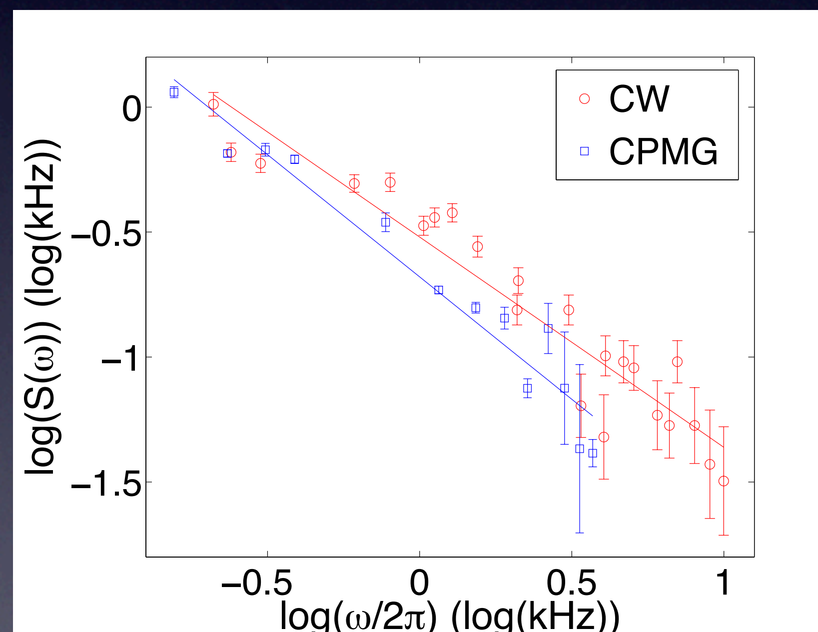
Experiment: solid-state NMR (malonic acid crystal)

qubit: carboxylic ^{13}C, natural abundance

environment: ^{1}H spins (weak decoupling on)

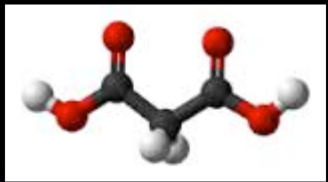
$$0.15 kHz \le \frac{\omega_{rf}}{2\pi} \le 10 kHz$$





$$CPMG \rightarrow S(\omega) \propto \omega^{-0.9\pm0.1} \quad CW \rightarrow S(\omega) \propto \omega^{-0.8\pm0.1}$$

Experiment: solid-state NMR (malonic acid crystal)



- system is only roughly described by the semiclassical Hamiltonian $H_{sc}(t) = f(t)S_z$

- 'bath' is certainly not Markovian!

Conclusions

- CW noise spectroscopy is equivalent to the pulsed method (practical advantages for CW method, perhaps)

- 0th-order AHT equivalent to the GBE result

- 2nd-order AHT correction adds Gaussian decay component

- Criteria can be shown for reliable CW noise spectroscopy

Future work:
can we generalize AHT method to noise along multiple axes?
non-Markovian bath?

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