Chapter 6

Aspects of black hole entropy

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ASPECTS OF BLACK HOLE ENTROPY

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Abstract

In 1985, 'tHooft presented a statistical derivation of the Bekenstein-Hawking entropy of a black hole (essentially interpreted as entanglement entropy), using a crude "brick-wall" cutoff to control divergences. This derivation seems to encounter difficulties which have led many (including 'tHooft himself) to question its soundness. This contribution offers a review and reappraisal of this and some other recent attempts to derive and interpret the Bekenstein-Hawking entropy.

6.1 Introduction

The Bekenstein-Hawking relation

$$S_{BH} = \frac{1}{4}A/\ell_{P\ell}^2 \tag{5.1}$$

was inferred by Hawking [1] from the formal resemblances between thermodynamics and classical black hole mechanics [2], combined with his 1974 discovery that a characteristic (Hawking) temperature

$$T_H = \hbar\kappa/2\pi \tag{5.2}$$

must be assigned to black holes in the quantum domain. This gave precise form and a firm foundation to Bekenstein's earlier heuristic arguments [3] for a proportionality between black hole entropy and area.

Much work and discussion have since been devoted to the problem of understanding the Bekenstein-Hawking entropy in a more direct and fundamental way. Here, I shall report briefly on work (done in collaboration with Shinji Mukohyama [4] at the Yukawa Institute, and with Frans Pretorius and Dan Vollick [5] at the University of Victoria, which I believe casts some useful sidelights on this issue.

In 1977 Gibbons and Hawking [6] gave a statistical derivation of (1), using analytic continuation to the Euclidean sector and imposing a Matsubara period T_H^{-1} on Euclidean time—i.e., they considered a black hole in thermal equilibrium with its own radiation at the Hawking temperature (Hartle-Hawking state). In this approach, S_{BH} appears already at zero-loop order, as a contribution to the partition function from the boundary terms accompanying the Einstein-Hilbert action. This affords no clue to the dynamical origins of S_{BH}; rather, it seems to imply that the entropy is in some sense topological in origin.

Interesting and suggestive, but still short of completely satisfying, are interpretations of S_{BH} which refer to the hole's past or future history or ensembles of histories; for example, "S_{BH} is the logarithm of the number of ways the hole could have been made" [7], or those which attempt to relate S_{BH} to thermal entropy of the evaporation products [8].

Ideally, it ought to be possible to regard S_{BH} as a *state function*, defined at each moment of time in terms of the dynamical degrees of freedom existing at that moment. One also wants to understand how it comes to have the simple universal form (1), independent of the hole's internal structure and all details of the microphysics.

Perhaps the most promising view is that S_{BH} is entanglement entropy, associated with modes and correlations hidden from outside observers by the horizon [9]. (If the black hole originates from a pure state, there is perfect correlation between internal and external modes, and the entanglement entropy can be found by tracing over either set of modes.) Remarkably, this yields an entropy proportional to area. Naively, the coefficient of proportionality is infinite, but reduces to the right order of magnitude when one allows for quantum fluctuations, which will prevent events closer to the horizon than about a Planck length $\ell_{P\ell}$ from being seen on the outside.

A 1985 calculation by 'tHooft [10] seems to be based, at least implicitly, on the idea of entanglement. This treats the statistical thermodynamics of quantum fields in the Hartle-Hawking state, propagating on a fixed (Schwarzschild) black hole background of mass M. Divergences are controlled by a "brick wall," a reflecting spherical surface, a little outside the gravitational radius. 'tHooft found—in addition to the expected volume-proportional terms describing hot fields inside a nearly flat, large cavity whose outer wall is at the Hawking temperature—additional wall terms proportional to the area. These latter terms diverge as α^{-2}, where α is the proper altitude of the wall above the gravitational radius. For a specific choice of α (which depends on the number of fields, etc., but is generally of order $\ell_{P\ell}$), one is able to recover the Bekenstein-Hawking result (1).

However, this calculation appears to run into contradictions which have led many [11], including 'tHooft himself to question its validity.

(i) Does 'tHooft's one-loop entropy, due to thermal excitations, have to be *added* to the Gibbons-Hawking zero-loop geometrical entropy? Each by itself already accounts for the full value of S_{BH}.

(ii) The quantum fields were supposed to be in the Hartle-Hawking state. Accordingly, their stress-energies should be bounded (of order M^{-4} in Planck units) near the gravitational radius, and negligible for astrophysical masses M. Yet, 'tHooft's calculation assigns enormous (Planck-level) energy densities to them near the wall.

(iii) The integrated field energy yields a wall contribution

$$\Delta M = \frac{3}{8}M \tag{5.3}$$

to the gravitational mass. This suggests that the gravitational back-reaction is substantial and the underlying assumption of a fixed geometrical background inconsistent.

However, these difficulties have a simple resolution [4]. In the next two sections I shall review the basic elements of the brick wall model, and then show that inconsistencies disappear once the ground state of this model is correctly identified. (It is really the Boulware state, not the Hartle-Hawking state.) In the following sections, I shall sketch a thermodynamical armchair experiment [5], involving the quasi static reversible compression of a massive thin shell to its gravitational radius, which yields the Bekenstein-Hawking entropy without resort to ad hoc cutoffs. Finally, I shall comment briefly on the implications of these results for the meaning of S_{BH}, and for the exceptional case of extremal black holes.

6.2 Hartle-Hawking and Boulware States

Let us begin by recalling the essential properties of the quantum states that will enter our discussion.

In any static spacetime with Killing time t, the Boulware state is the one annulled by the annihilation operators associated with "Killing modes" (positive-frequency in t). In the spacetime of a stationary eternal black hole, the Hartle-Hawking state is correspondingly associated with "Kruskal modes" (positive-frequency in Kruskal co-ordinates) [12]. This state appears empty of "particles" to free-falling observers at the horizon, and its stress-energy is bounded there (not quite zero, because of polarization effects).

To illustrate these definitions, consider a (1+1)-dimensional geometry with metric

$$ds^2 = \frac{dr^2}{f(r)} - f(r)\,dt^2, \tag{5.4}$$

and write $\kappa(r) = \frac{1}{2}f'(r)$ for the red shifted gravitational force, i.e., the upward acceleration $a(r)$ of a static test-particle reduced by the redshift factor $f^{\frac{1}{2}}(r)$. At a horizon $r = r_0$ we have $f(r_0) = 0$ and the surface gravity is $\kappa_0 = \kappa(r_0)$.

For quantum fields propagating on this background, there is an effective stress-energy T_{ab}, completely specified by an energy density $\rho(r) = -T_t^t$ and pressure $P(r) = T_r^r$ in a stationary, flux-free state (thus excluding the Unruh state). Their radial dependence is determined by the conservation law and the trace anomaly, which for a massless scalar field is

$$T_a^a = \frac{\hbar}{24\pi}R, \tag{5.5}$$

with $R = -f''(r)$ for the geometry (4). Integrating the conservation law gives

$$f(r)P(r) = -\frac{\hbar}{24\pi}(\kappa^2(r) + \text{const.}). \tag{5.6}$$

Different choices of the constant of integration correspond to different boundary conditions, i.e., different quantum states.

For the Hartle-Hawking state, P and ρ are required to stay bounded at the horizon $r = r_0$, giving

$$P_{HH} = \frac{\hbar}{24\pi}\frac{\kappa_0^2 - \kappa^2(r)}{f(r)}, \qquad \rho_{HH} = P_{HH} + \frac{\hbar}{24\pi}f''(r). \tag{5.7}$$

At large radii (setting $f(r) = 1$),

$$P_{HH} = \rho_{HH} = \frac{\pi}{6\hbar}T_H^2, \qquad T_H = \hbar\kappa_0/2\pi, \tag{5.8}$$

which corresponds to one-dimensional scalar radiation at the Hawking temperature T_H.

For the Boulware state, the boundary condition is $P = \rho = 0$ when $r = \infty$. The integration constant in (6) must vanish, and we find

$$P_B = -\frac{\hbar}{24\pi}\frac{\kappa^2(r)}{f(r)}, \qquad \rho_B = P_B + \frac{\hbar}{24\pi}f''(r). \tag{5.9}$$

This is the zero-temperature ground state for the space outside a static star. Stationary observers perceive it as empty of "particles," but vacuum polarization at small radii where curvature is appreciable induces non vanishing (negative) stress-energies. If a horizon were present, P_B and ρ_B would diverge there to $-\infty$.

The difference between the stress-energies (7) and (9) has the exactly thermal form

$$\Delta P = \Delta\rho = \frac{\pi}{6\hbar} T^2(r) \tag{5.10}$$

where, in accordance with Tolman's law

$$T\sqrt{-g_{00}} = \text{const.} \tag{5.11}$$

for thermal equilibrium in a static field, $T(r) = T_H/\sqrt{f(r)}$ is the local temperature in the Hartle-Hawking state.

In summary, the Hartle-Hawking state is thermally excited above the zero-temperature Boulware ground state to a local temperature $T(r)$ which grows without bound near the horizon. It is nonetheless the Hartle-Hawking state which best conforms to what a gravitational theorist would call a vacuum at the horizon. (Throughout I shall reserve the word "vacuum" for a condition of zero stress-energy; in a curved space there is no corresponding quantum state in general.)

This relationship between the two states was proved above for $(1+1)$-dimensions, but remains at least qualitatively valid generally, with obvious changes arising from the dimensionality. In particular, the $(3+1)$-dimensional analogue of (10),

$$3\Delta P \approx \Delta\rho \approx \frac{\pi^2}{30\hbar^3} T^4(r) \tag{5.12}$$

holds to a very good approximation, both far from the hole and very near the horizon. Deviations occur in the intermediate region [13], but they remain bounded and will be irrelevant to our considerations.

6.3 Brick Wall Model

I can now proceed to sketch the bare bones of 'tHooft's brick wall model [10, 4].

We consider the thermodynamics of hot quantum fields propagating outside a spherical star with a perfectly reflecting surface and radius r_1 a little larger than its gravitational radius r_0. To keep the total field energy bounded, suppose the system enclosed in a spherical container of radius $L \gg r_1$.

For the space outside the star, assume a metric of the form

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\Omega^2 - f(r)dt^2. \tag{5.13}$$

This is general enough to include the Schwarzschild, Reissner-Nordström and de Sitter geometries (or any superposition of these) as special cases.

Into this space we introduce a collection of quantum fields, raised to some temperature T_∞ at large distances, and in thermal equilibrium. The local temperature is then

$$T(r) = T_\infty f^{-\frac{1}{2}} \tag{5.14}$$

and becomes very large when $r \to r_1 = r_0 + \Delta r$. We shall presently identify T_∞ with the Hawking temperature T_H of the horizon that would appear if $r_1 \to r_0$.

Characteristic wavelengths of this radiation are small compared to other relevant scales (curvature, size of container) in the regions of interest to us here. For instance, very near the star's surface,

$$\lambda \sim \hbar/T = f^{\frac{1}{2}}\hbar/T_\infty \ll r_0. \tag{5.15}$$

Elsewhere in the large container, at large distances from the star,

$$f \approx 1, \qquad \lambda \sim \hbar/T_\infty \sim r_0 \ll L. \tag{5.16}$$

Therefore a particle description should be a good approximation to the statistical thermodynamics of the fields. (Equivalently, one can arrive at this conclusion by considering the WKB solution of the wave equation [10, 4].)

The extensive thermodynamical parameters then each receive two principal contributions for large L and small $\Delta r = r_1 - r_0$:

(a) A volume term, proportional to $\frac{4}{3}\pi L^3$, representing the entropy and mass-energy of a homogenous quantum gas in a flat space (since $f \approx 1$ almost everywhere in the container if $L/r_0 \to \infty$) at a uniform temperature T_∞. This is the conventionally expected result, and there is no need to consider it in detail.

(b) Of more interest is the contribution of gas near the inner wall $r = r_1$, which we now proceed to study further. We shall find that it is proportional to the wall area and at the same time diverging like $(\Delta r)^{-1}$ when $\Delta r \to 0$.

Because of the high local temperatures T near the wall for small Δr, we may use the ultra relativistic formulae

$$\rho = \frac{3\mathcal{N}}{\pi^2}T^4, \qquad s = \frac{4\mathcal{N}}{\pi^2}T^3 \tag{5.17}$$

for the energy and entropy densities of this gas. The numerical factor $\mathcal{N}$ takes care of helicities, the number of particle species and the factor $\frac{7}{8}$ which differentiates fermionic from bosonic contributions.

The total entropy of the thermal excitation is given by the integral

$$S = \int_{r_1}^{L} s(r) 4\pi r^2 dr / \sqrt{f}, \tag{5.18}$$

where we have taken account of the proper volume element for the metric (13). On the other hand the integral for their gravitational mass does not contain $\sqrt{f}$, as is well-known:

$$\Delta M_{\text{therm}} = \int_{r_1}^{L} \rho(r) 4\pi r^2 \, dr. \tag{5.19}$$

Substituting (17) and (14) into (18) gives for the wall contribution to the thermal entropy,

$$S_{\text{wall}} = \frac{4\mathcal{N}}{\pi^2} 4\pi r_1^2 T_\infty^3 \int_{r_1}^{r_1+\delta} \frac{dr}{f^2(r)}, \tag{5.20}$$

where δ is an arbitrary small length subject to $\Delta r \ll \delta \ll r_1$. It will be instructive to re-express this result in terms of the altitude

$$\alpha = \int_{r_0}^{r_1} f^{-\frac{1}{2}} \, dr \tag{5.21}$$

of the inner wall above the horizon $r = r_0$ of the analytically extended exterior geometry (13). (Really, of course, the physical space contains no horizon, since (13) is valid only for $r > r_1$.)

For a non-extremal horizon we can write $f(r) \approx 2\kappa_0(r - r_0)$ in (21), obtaining

$$\Delta r = \frac{1}{2} \kappa_0 \alpha^2, \tag{5.22}$$

and (20) can be written

$$S_{\text{wall}} = \frac{\mathcal{N}}{90\pi\alpha^2} \left(\frac{T_\infty}{\kappa_0/2\pi} \right)^3 \frac{1}{4} A \tag{5.23}$$

in Planck units, where $A = 4\pi r_1^2$ is the wall area.

From (19) and (17) we find similarly that thermal excitations near the wall contribute

$$\Delta M_{\text{therm, wall}} = \frac{\mathcal{N}}{480\pi\alpha^2} \left(\frac{T_\infty}{\kappa_0/2\pi} \right)^3 A T_\infty \tag{5.24}$$

to the gravitational mass of the system.

Following 'tHooft [10, 4], let us introduce a crude cutoff to allow for quantum-gravity fluctuations by adjusting α so that we obtain the Bekenstein-Hawking entropy from (23):

$$S_{\rm wall} = S_{BH} \quad \text{when} \quad T_\infty = T_H, \tag{5.25}$$

where S_{BH} and T_H were introduced in (1) and (2); notice that they are purely *geometrical* quantities, determined by the metric (13). From (23) and (25) (momentarily restoring conventional units)

$$\alpha = \ell_{Pl}\sqrt{\mathcal{N}/90\pi}, \tag{5.26}$$

so that the cutoff α is indeed of the order of the Planck length. It is significant and crucial that α turns out to be *universal*, independent of the mass and other characteristics of the system, depending only on the number of physical fields in nature.

This universality allows a clean separation between geometrical and thermodynamical variables in the free energy

$$F_{\rm wall} = -\frac{1}{16}\left(\frac{T_\infty}{T_H}\right)^3 A T_\infty, \tag{5.27}$$

so that the entropy can be derived from it either via the Gibbs relation $S = -\partial F_{\rm wall}/\partial T_\infty$ holding the geometrical variables fixed ("off-shell," i.e., breaking the equality $T_\infty = T_H$), or via the Gibbs-Duhem relation $F = \Delta M - T_\infty S$ (equivalent to $S = -\mathrm{Tr}(\rho \ln\rho)$). Thus, there is no need in this formulation to maintain a distinction between "thermodynamical" and "statistical" entropy [14].

The wall's thermal mass-energy is given "on-shell" ($T_\infty = T_H$) by

$$\Delta M_{\rm therm,wall} = \frac{3}{16} A T_H, \tag{5.28}$$

which reduces to 'tHooft's result (3) in the special case where (13) is the Schwarzschild metric.

6.4 The Brick Wall Model: Inconsistent?

As mentioned in the Introduction, 'tHooft's model seems to encounter serious difficulties [11]. The result (28) suggests that it is inconsistent to neglect the back-reaction of the thermal excitations on the background geometry (13); their energy density, given by (17), becomes huge near the wall, a behaviour quite unlike the Hartle-Hawking stress-energy; and it is unclear whether the thermal entropy (23) needs to be supplemented by the Gibbons-Hawking geometrical contribution.

All these problems have a remarkably simple resolution. The key remark is that *the brick wall model correctly interpreted does not represent a black hole.* It represents the exterior of a starlike object with a perfectly reflecting surface, compressed to nearly (but not quite) its gravitational radius. As noted in Sec. 2, the ground state for such an object is not the Hartle-Hawking state but the Boulware state, corresponding to zero outside temperature and with a quite different behaviour near the gravitational radius. It has a negative energy density, growing to Planck levels near the wall. Thus, the thermal energy density ρ given by (17) is not the only source of the wall's mass; it has to be supplemented by the ground-state energy.

As in (12), we have for the total stress energy (ground state + thermal excitations) near the wall,

$$(T_\mu^\nu)_B + (T_\mu^\nu)_{\text{therm}} = (T_\mu^\nu)_{HH}, \tag{5.29}$$

i.e., effectively the Hartle-Hawking stress-energy, which is bounded and small for large masses. The total gravitational mass of the inner wall is accordingly negligible. We are entirely justified in neglecting back-reaction.

Further, since there is no horizon in this model—it is replaced by the brick wall, an inner boundary with the quite different topology $S^1 \times S^2$ in the Euclidean sector (a horizon would be a regular point)—the Gibbons-Hawking "instanton" does not contribute. All of the entropy derives from the thermal contribution (23).

It thus emerges that we have two mutually exclusive and (in the Bohr sense) complementary ways of understanding S_{BH}. In the brick wall model (which is not a black hole but is externally indistinguishable from one), S_{BH} appears as entropy of thermal excitations above a zero-temperature ground state. In a real black hole, thermal energies near the horizon are negligible and S_{BH} has a purely geometrical origin. The deeper implications of this duality would be worth exploring [4].

6.5 Operational approach

Even if its self-consistency is no longer in question, the brick wall model still labours under the handicap that the wall altitude has to be adjusted by hand to reproduce S_{BH} with the correct coefficient.

I shall now outline an approach that gives S_{BH} without cutoffs or ad hoc adjustments. In thermodynamics, the entropy of any state can be found by devising an idealized reversible process which arrives at that state, starting from a state of known entropy; we then employ the first law to compute the change of entropy. Here, we consider the reversible quasi-static contraction of a massive thin spherical shell toward its gravitational radius [5]. (In the final stages of contraction, the shell violates the dominant energy condition and develops other "unphysical" features. This is irrelevant, since the shell is nothing more than an idealized working substance designed to reversibly reach the final black hole state, which is independent of its mode of formation.)

The ground state for the space outside the shell is the Boulware state, whose stress-energy diverges to large negative values in the limit. To neutralize the resulting back-reaction, let us fill the exterior with thermal radiation to produce a "topped-up" Boulware state (TUB) whose local temperature equals the acceleration temperature $T = \hbar a(R)/2\pi$ at the shell's radius R. To maintain thermal equilibrium (and hence applicability of the first law), the shell itself must be raised to the same temperature T. This gives it a definite equation of state

$$T = T(M, R), \tag{5.30}$$

whose specific form can be found once the exterior spherical geometry is specified.

This equation of state relates an intensive variable to two extensive variables, the shell's area $A = 4\pi R^2$ and proper mass $M = \sigma A$ as measured by a local observer. Since the shell is uniform and we require it to be a thermodynamical system, it must be possible to rewrite (30) in a purely intensive form

$$T = T(\sigma, n). \tag{5.31}$$

How we interpret the second intensive parameter $n = N/4\pi R^2$ is quite immaterial; for convenience I shall refer to it as "particle density." Unlike (30), there is a certain amount of freedom in the functional form (31) (i.e., our choice of n), but it is strongly constrained by the requirement of compatibility with both (30) and the Gibbs-Duhem relations [5]. One possible choice, n^* ("canonical equation of state," distinguished by an asterisk) makes

$$\mu^* n^* = \sigma \quad \Longrightarrow \quad \mu^* N^* = M \tag{5.32}$$

where μ is the chemical potential associated with N.

The shell's entropy at any stage of the slow contraction is given by

$$TS = M + PA - \mu N, \tag{5.33}$$

where P is surface pressure. In the limit of approach to the gravitational radius r_0, P and T both diverge in the non-extremal case, while M remains bounded. The key result is [5]

$$\lim_{R\to r_0} (P/T) = \frac{1}{4}\,\hbar^{-1} \qquad (\kappa_0 \neq 0). \tag{5.34}$$

This result is general and independent of thermodynamics—it follows solely from the condition of mechanical equilibrium.

From (32)–(34) it now follows easily that for a shell made of canonical material,

$$\lim_{R\to r_0} S = \frac{1}{4}\,A/\hbar \qquad (\kappa_0 \neq 0), \tag{5.35}$$

which is the Bekenstein-Hawking result.

Of course, one would expect the entropy of the final black hole to be independent of the material we choose for the shell, and, indeed, the limit (35) is very robust. The term PA in (33) dominates M in this limit, so that we recover (35) also for non canonical material, provided only that

$$\lim_{T\to\infty} \frac{1}{T}\,\frac{\mu N}{M} = 0. \tag{5.36}$$

(Note that *some* restriction is obviously required, since the black hole limit represents a singular state for the shell ($P \to \infty$), and there would otherwise be nothing to prevent the equation of state (31) from becoming "singularly eccentric" in this limit.)

This suggests an operational definition of the Bekenstein-Hawking entropy as the maximum thermodynamical entropy that could be stored in the material that forms the black hole when this material is gathered into a thin shell near the horizon. This idealized process bears no resemblance to actual black hole formation, but is not too dissimilar from the time-reverse of the evaporation process.

It is helpful to look at a concrete example. For a spherical shell of proper mass M and charge Q, the exterior metric (13) is Reissner-Nordström, with

$$f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{R^2}, \tag{5.37}$$

and the explicit formulas are

$$m = M - \frac{1}{2}\,\frac{M^2 - Q^2}{R}, \qquad P = \frac{1}{16\pi}\,\frac{M^2 - Q^2}{R^2(R - M)}, \tag{5.38}$$

$$T = \frac{\hbar}{2\pi}\,\frac{1}{2}\,\frac{f'(R)}{\sqrt{f(R)}}. \tag{5.39}$$

It is then easy to check that (35) follows in the non-extremal case ($Q^2 < m^2$).

This situation is quite different for an extremally charged shell ($Q^2 = m^2 \Longrightarrow Q^2 = M^2$), because P is no longer dominant in the limit $R \to r_0$ but actually vanishes, while T now remains finite. From (32) and (33) we now find $S = 0$ for a shell made of canonical material, so that a black hole formed out of this material would satisfy the third law in its strongest (Planck) form. However, it is clear from (34) that the limiting entropy now depends sensitively on the choice of shell material.

Thus it would appear that an extremal black hole differs from a generic one in that its entropy is not a thermodynamical state function, i.e., not independent of its mode of formation and past history. This may account for a current cleavage of opinion on this issue. Arguments based on black hole instanton topology and black hole pair creation suggest that the entropy of extremal black holes is zero. On the other hand, the uncannily successful indirect derivations of S_{BH} by counting states of strings on D-branes recover the traditional value $\frac{1}{4}A/\hbar$ for extremal black holes.

The shell model attributes the universality of the Bekenstein-Hawking formula for non-extremal black holes essentially to the circumstance that material equations of state tend to a simple universal form at high temperatures. This suggests that one should consider the difference between the extremal and non-extremal cases, not as a difference between zero and nonzero redshifted (i.e., Hawking) temperatures but rather as a difference between the finite and infinite temperatures measured by *local* observers in the two cases.

6.6 Concluding Remarks

It is widely thought that the entropy contributed by thermal excitations or entanglement is a one-loop correction to the zero-loop (or "classical") Gibbons-Hawking contribution. The viewpoint suggested here is (at least superficially) quite different. One may consider these two entropy sources—(a) brick wall, no horizon, strong thermal excitations near the wall, Boulware ground state; and (b) black hole, horizon, weak (Hartle-Hawking) stress-energy near the horizon, Hartle-Hawking ground state—as equivalent but mutually exclusive (complementary in the sense of Bohr) descriptions of what is externally virtually the same physical situation. The near-vacuum experienced by free-falling observers near the horizon is eccentrically but defensibly explainable, in terms of description (a), as a delicate cancellation between a large thermal energy and an equally large and negative ground-state energy—just as the Minkowski vacuum is explainable to a uniformly accelerated observer as a thermal excitation above his negative-energy (Rindler) ground state. (This corresponds to setting $f(r) = r$ in (9).) The artificiality of such a description is underlined by the fact that this delicate balance must extend to fluctuations: fluctuations of the Boulware ground state would have to be exactly correlated with the enormous thermal fluctuations near a horizon to reproduce the relatively small fluctuations of the Hartle-Hawking state.

The brick wall model (as well as numerous other attempts to derive S_{BH} statistically by focusing on the neighbourhood of the horizon) presents us with a feature which is logically possible but strange and counterintuitive from a gravitational theorist's point of view. Although the wall is insubstantial (just like a horizon)—i.e., space there is practically a vacuum and the local curvature low—it is nevertheless the repository of all of the Bekenstein-Hawking entropy in the model.

Frolov and Novikov [9] and others have argued that this is just what may be expected of black hole entropy in the entanglement picture. Entanglement will arise from virtual pair-creation in which one partner is "invisible" and the other "visible" (although only temporarily—nearly all get reflected off the external potential barrier). Thus, on this picture, the entanglement entropy arises almost entirely from the strong correlations between nearby field variables on the two sides of the partition, an effect already present in flat space [9].

This in turn suggests that S_{BH} is (in the literal sense) a *superficial* property, that is should be considered as an *effective entropy* of a black hole, in the same sense that $6000K$ is an effective temperature for the sun. As far as their interactions with the environment are concerned, both objects are indistinguishable from shells (of the same size and mass) whose entropy or temperature have the effective values. This point of view provides a rationale for horizon-oriented derivations of S_{BH}, but at the same time offers little encouragement for the hope that such efforts will lead to insights into the deeper properties of black holes.

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