Chapter 2

Physics and astrophysics of black holes and physics of time machines

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Physics and Astrophysics of Black Holes and Physics of Time Machines

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Abstract. This review focuses on recent achievements in ones of the most important branches of the General Relativity: physics of Time Machines and Physics and Astrophysics of Black holes.

2.1 Introduction

The discussion in this review is organized as follows.

In Section 2 we discuss the physics of wormholes and time machines. Section 3 is devoted to the analysis of the membrane paradigm in the black hole physics. According to this paradigm the event horizon of a black hole looks for an external observer (outside the black hole) and behaves as a physical membrane with definite mechanical, electrical and thermodynamical properties. In Section 4 we discuss astrophysics of black holes and in final Section 5 we analyze the problem of tidal interaction of stars with a supermassive black hole.

For systematic discussion of the problems see the books: Thorne *et al.* (1986), Novikov and Frolov (1989), and Frolov and Novikov (1998).

2.2 Wormholes and time machines

2.2.1 Nontrivial topology of spacetime

An interesting feature of the General Relativity is possible existence of spacetimes with non-trivial topological structure. Wormhole solutions described by Wheeler (1962) are well known examples of spaces with non-trivial topology. The simplest example of such a spacetime is an Einstein-Rosen bridge. The spacetime of an eternal black hole can be considered as the evolution of the Einstein-Rosen bridge. To the future of the moment of time symmetry the throat shrinks to zero size and the singularity arises. No causal signal can propagate through the throat from one asymptotically flat region (R') to another one (R''). This property is directly connected with non-trivial causal structure of spacetime in the presence of a black hole.

In the general case the wormhole consists of a throat (corridor) connecting two holes in asymptotically flat space.

In the same manner as in the case of the eternal black hole in the absence of matter the throat of the wormhole pinches off so quickly that it cannot be traversed even by light. It is a generic property of spacetimes with a non-simply connected Cauchy surface. According to the theorem proved by Gannon (1975): *Any asymptotically flat spacetime with a non-simply connected Cauchy surface has singular time evolution if it satisfies the weak energy condition.* Moreover such singularities arise so quickly that no information carrying signal can propagate through a wormhole to the asymptotic region before creation the singularity. One can formulate a *topological censorship* as the statement that no observer remaining outside the region with strong gravitational field has time to probe the topology of spacetime (Friedman,Schleich, and Witt (1993)).

For more accurate formulation we shall use the energy condition which is weaker than weak energy condition. The *null energy condition* (NEC) is the requirement that $T_{\alpha\beta}l^{\alpha}l^{\beta} \geq 0$, for all null vectors l^{α}. It is implied by each of the other common positive energy conditions: the weak energy condition, the strong energy condition, and the dominant energy condition. A spacetime is said to satisfy the *averaged null energy condition* (ANEC) if the integral of $T_{\alpha\beta}l^{\alpha}l^{\beta}$ is non-negative along every inextendible null geodesic with affine parameter λ and tangent vector l^{α}:

$$\int T_{\alpha\beta}l^{\alpha}l^{\beta}d\lambda \geq 0. \tag{5.1}$$

This condition is evidently weaker then the NEC.

Consider an asymptotically flat spacetime and denote by $gamma_0$ a timelike curve with past end point at $\mathcal{J}^-$ and future end point in $\mathcal{J}^+$ that lies in a simply connected neighborhood of $\mathcal{J}^- \cup \mathcal{J}^+$. The theorem proved by Friedman,Schleich, and Witt (1993) reads: *If an asymptotically flat, globally hyperbolic spacetime satisfies the averaged null energy condition , then every causal curve from $\mathcal{J}^-$ to $\mathcal{J}^+$ can be continuously deformed to $gamma_0$.* If we assume that a spacetime is not simply connected then a causal curve beginning at $\mathcal{J}^-$ and passing through a wormhole cannot reach $\mathcal{J}^+$.

In order to prevent shrinking of a wormhole and to make it traversable one needs to fill up its throat with matter violating ANEC. It follows from the above theorem. This property of traversable wormholes can be easily shown directly. For any traversable wormhole a two-sphere surrounding one mouth , as seen through the wormhole from the other mouth, is an outer trapped surface. This implies (since there is no event horizon) that the stress-energy tensor of such matter $T_{\mu\nu}$ must violate ANEC (1). On the possibility of existence of matter violating weak energy condition see e.g. Thorne (1993), Visser (1995), and Flanagan and Wald (1996).

The spacetime with a traversable wormhole (if it only exists) can be transformed into a spacetime with closed timelike curves. Namely it was shown by Morris, Thorne, and Yurtsever (1988) and Novikov (1989) that closed timelike curves may arise as a result of the relative motion of the wormhole's mouths. This principal possibility to create closed timelike curves in a spacetime where they originally were absent attracted new interest to the wormhole-like solutions. From more general point of view this result indicates on deep relations between topological and causal structure of spacetime.

The main features of such spacetimes can be illustrated by considering a simple model proposed by Morris, Thorne, and Yurtsever (1988). The simplest a wormhole is obtained by removing two balls of equal radius from Euclidean space and identifying their surfaces. The surfaces then become the wormhole's mouths. In the process of identification of surfaces their extrinsic curvature jumps. It means that if such a spacetime is solution of the Einstein equations there must be a delta-like distribution of stress-energy at the junction between the two mouths, which violate NEC. In this particular model it is evident, because any bundle of radially traveling null geodesics that passes through the wormhole is converging as it enters and diverging as it leaves, and therefore gets de-focused by the wormhole. One can make the matter distribution smooth by connecting mouths by a handle of finite length instead of gluing them together. In what follows we always assume that the length of the handle is small enough.

One can construct wormhole spacetimes, so that wormhole's mouths are moving along arbitrary chosen world lines, by removing world tubes along those lines and identifying their surfaces with each other. The junction conditions requires that the intrinsic geometries of tubes' surfaces must be the same. This may require a distortion of spacetime geometry near the mouths if they are accelerated. But the distortion can be made arbitrary small by taking the value of (acceleration)$\times$(mouth radius) to be small enough (Morris, Thorne, and Yurtsever (1988), Friedman et al (1990)). Since the mouth's intrinsic geometries are the same, the proper time interval between two identified events on the mouths must be the same as seen through either mouth.

In order to accelerate a mouth one must apply a force to it. It can be done for example when the mouths are charged (i.e. when there exist flux of electric field through the mouth). In this case it is sufficient to apply electric field to one of the mouths. The mouth becomes moving in the external space. It is possible to show that during this motion the position of the other mouth in the external space remains unchanged.

One can choose the clocks located near both mouths to be synchronized in the external space before the beginning of the motion. As the result of the motion the synchronization in the external space is lost. The moving clock (measuring proper time near a first mouth) shows less time than the clock which remains at rest ("twin paradox"). Denote by $\Delta\tau$ this time difference. If $\Delta\tau > L/c$, where L is the distance between the final positions of the mouths, a *closed timelike curve* becomes possible. The region formed by events through which a closed timelike curve passes is restricted from the past by the *future chronology horizon*.

Consider a spacetime with a wormhole in case when the mouths are rotating one with respect another in the external space (Novikov (1989)). Closed timelike curves in this model arise for the same reason as in the above model with an accelerated mouth. As seen in the external space there is a dilation of proper time on the moving mouth relative to the static one, but as seen through the wormhole there is no such time dilation.

There is even simpler model when the mouths are moving with constant velocity one with respect the other and closed timelike curves also arise (Morris, Thorne, and Yurtsever (1988). In this case closed timelike curves are confined to bounded non-chronal region of spacetime: the region that begins at the future chronology horizon and ends at the past chronology horizon

The formation of non-chronal regions containing closed timelike curves is a generic property of spacetimes with wormholes. Generic relative motions of a wormhole's mouths will always produce closed timelike curves (Morris, Thorne, and Yurtsever (1988), as will the gravitational redshift when wormhole's mouths are placed in a generic gravitational field (Frolov and Novikov (1990)).

In the paper Frolov and Novikov (1990) it was argued that the interaction of a wormhole with classical matter generically generates a non-potential component of the gravitational field. That is why a locally static wormhole is generically unstable with respect to the processes which transform it into a 'time machine'. The relative motion of a wormholes mouths also generates a time gap for the clocks' synchronization. One can interpret the above results in the following way. There exist internal relation between topological and causal properties of a spacetime. The existence of closed timelike curve is a generic property of multiply connected locally static spacetimes.

2.2.2 Chronology Horizons

In this section we describe some general properties of spacetimes with closed timelike curves (for more details see (Friedman et al (1990), Hawking (1992), Thorne (1993)).

Solutions of Einstein equations which allow closed timelike curves have been known for a long time. The earliest example of such a spacetime is a solution obtained by Van Stockum (1937), which describes an infinitely long cylinder of rigidly and rapidly rotating dust. Another well known example is Gö del (1949) solution representing a stationary homogeneous universe with nonzero cosmological constant, filled with rotating dust. Closed timelike curves also present in the interior of the eternal Kerr black hole in the vicinity of ring singularity.

A spacetime whose closed timelike curves are not eternal can be divided into *chronal regions* without closed timelike curves, and *non-chronal regions* that contain closed timelike curves everywhere. The boundaries between the chronal and non-chronal regions are called *chronology horizons.* Chronal regions end and non-chronal region begins at *future chronology horizon.* Non-chronal regions end and chronal region begins at *past chronology horizon.* A future chronology horizon is a special type of future Cauchy horizon, and as such it is subject to all the properties of such horizons. In particular, it is generated by null geodesics that have no past endpoints but can leave the horizon when followed into the future. If the generators, monitored into past, enter one or more compact regions of spacetime and never thereafter leave them, the future chronology horizon is said to be *compactly generated.* In a wormhole model with closed timelike curves the future chronology horizon is compactly generated. The inner horizon of a Kerr-Newman solution is an example of a Cauchy horizon that is not compactly generated. The compactly generated chronology horizon cannot form in a spacetime developed from a spacelike non-compact surface without boundary if the null energy condition holds (Hawking (1992)).

The past-directed generators of the compactly generated future chronology horizon have no past end points. They will enter and remain a compact region

C. Hawking (1992) showed that there exist a nonempty set E of generators, each of which remains in the compact set C in the future direction, as well as in the past direction. The sets E generically contain at least one closed null geodesic. More exactly, Hawking (1992) proved that: (i) if E contains such a closed null geodesic, small variations of metric preserve this property; (ii) if E does not contain closed null geodesic, in geometries obtained by small variation of the metric such curves exist. The generators traced into the past either wander ergodically around C or they asymptote to one or more smoothly closed geodesics. In the latter case followed forward in time they are seen to originate in fountains and spew out of them. That is why Thorne (1993) proposed to name such closed null geodesics *fountains*. The Hawking's result indicates that in the generic case, C will contain such fountains, and it is likely that generically almost all the horizon generators will emerge from them (Thorne (1993)).

2.2.3 Possible Obstacles for a 'Time Machine' creation

In order to create a 'time machine' by using a wormhole, one need to assume that there exist principle possibility to make them long living and traversable. It is impossible without ANEC violation. Moreover ANEC must be violated near the fountain of any compactly generated future chronology horizon (Hawking, 1992). It means that it is impossible to create a 'time machine' in a finite region of space time without violating ANEC.

It is unclear whether it is possible to provide such a violation in physically acceptable conditions. It has been shown that ANEC is satisfied for noninteracting quantized scalar and electromagnetic fields in flat spacetime (Klinkhammer (1992), Folacci (1993)),, and in generic, curved 1+1 dimensional spacetimes (Wald and Yurtsever (1991)). On the other hand, in 3+1 dimensional spacetime, both non-trivial topology (Klinkhammer (1992)) and curvature (Wald and Yurtsever (1991)) can induce ANEC violations. Moreover in the latter paper it was shown that there are generic classes of spacetimes where quantum effects may violate ANEC. Under these conditions it is impossible at least now to exclude possibility such a violation of ANEC which is required for the 'time machine' creation.

If there is no eternal traversable wormholes in the Universe in order to create a 'time machine' by the proposed mechanism one need at first to create a wormhole. In the absence of wormholes the space initially (at some spacelike surface S) was simply connected. If a wormhole is created, then a later spacelike surface S' has different topology. If the processes connected with a 'time machine' formation are restricted to a bounded in space region, then it is natural to assume that one can surround this region by a timelike cylinder T which intersect the spacelike surfaces S and S' in compact regions S_T and S'_T of different topology. In other words the topology change occurs in a spacetime region M_T bounded by S, S', and T. In the absence of singularities and impossibility for M_T to spread to infinity M_T is compact. Hawking (1992) proved that the change of spatial topology inside M_T is impossible until it contains closed timelike curved. This is the generalization of the well known theorem by Geroch (1967). This result means that that even creation of a wormhole cannot be possible without creation a 'time machine'. Whether it is forbidden by some fundamental physical laws remains unclear.

There is another danger: quantum instability of the compactly generated future chronology horizon. We will not discuss this problem here (see Frolov and Novikov (1998)).

2.2.4 Physics in the Presence of 'Time Machines'

Here we discuss whether and how the laws of physics can deal with closed timelike curves (CTCs).

Usually one worries that the laws of physics cannot deal reasonably with a time machine. The crucial problem here is the problem of causality. The existence of CTCs allows one to travel into the past. At first sight it inevitably leads to the possibility of changing the past, thereby producing causality violations. But it is not so.

In the works: Novikov (1983), Zeldovich and Novikov (1975), Novikov and Frolov (1989), Friedman *et al* 1990, Novikov (1992) the principle of self-consistency (PSC) was declared and discussed. The meaning of this principle is the following.

In the case of an open timelike curve any event X divides other events on this curve into two parts: future events and past events with respect to X. All past events can influence X, but future events cannot. On a CTC the choice of the event X divides other events on the curve into future events and past ones only locally. In this case events which locally are in the future with respect to X can influence the event X circularly around the CTC. There is no global division of events on the CTC into future and past. The future influences the present around the closed timelike line, with a local directed into the future part of a light cone at each event of the closed curve. Not only the future is the result of evolution of the past but the past is the result of the future also. All events in a spacetime with CTCs must be self-consistent. According to the principle of self-consistency all events on CTCs influence each other around the closed timelike line in a self-adjusted way.

More precise formulation of this principle is: the only solution to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent. The PSC by fiat forbids changing the past. All events happen only once, and cannot be changed.

In order to demonstrate how this principle works we shall consider the so-called "billiard ball problem", (Friedman et al (1990), Echeverria, Klinkhammer, and Thorne (1991), Mikheeva and Novikov (1993)) which is the following: a solid perfectly elastic ball moves relative to the mouths of the wormhole. Its speed is assumed to be small compared with speed of light, so it can be treated non-relativistically. The ball enters the wormhole through mouth B, appears from A on the past and continuing its motion, it can encounter and collide with itself.

At first glance there is a "paradox" in this problem)the so-called Polchinski paradox" (Polchinski (unpublished)). The initial position and velocity of the ball are chosen in such a way that the ball moves along the trajectory α_1, enters mouth B, and exits from mouth A before it entered into B. The ball continues its motion along the trajectory α_2[1]. The timing is just right for the ball to hit itself at the point Z, knocking its "younger" self along trajectory a_3 and thereby preventing itself from ever reaching mouth B. Such an evolution is not self-consistent and impossible. It is not the solution of the evolution equations.

The mistake (the reason of the "paradox") is obvious: when at the beginning of our discussion we continued the trajectory α_1 after point Z, we did not take into account the influence of the impact and considered the motion of this ball along the trajectory a_2 without taking into account this impact. This means that we did not take into account the influence of the future on the past.

In the paper Echeverria, Klinkhammer, and Thorne (1991) the authors demonstrated that for initial data which give self-inconsistent "solutions" there are also self-consistent solutions. The initial data (initial position and velocity of the ball) are the same as in the previous consideration. The part of trajectory a_1 before the collision with the "older" self coming from the future is the same. This "older" ball moves along trajectory β_1 which is a little different from the one a_2. The "older" ball on β_2 strikes itself on α_2 gently, deflecting itself into slightly altered trajectory β_1. This altered trajectory β_1 takes the ball into the mouth B at a slightly altered point compared to the point in the self-inconsistent consideration. The ball exits from the mouth A before it went into mouth B, and moves along the trajectory β_2 to the collision event. This solution is self-consistent.

The general logic of construction of the self-consistent solutions looks like the following. Let us forget for a moment that the "older" ball is from the future and, during its motion after appearance from the mouth A we will treat it as a usual ball independent from the "younger" one. Let us suppose that we know the moment t_2 place $\vec{x}_2$ of its appearance from the mouth A and its velocity $\vec{V}_2$ at t_2. We can treat them as "initial conditions" for the "older" ball. Now using the standard physical laws we can calculate the collision between the "younger" and "older" balls, change of the trajectory of the "younger" one and the place $\vec{x}_1$, velocity $\vec{V}_1$ and the moment t_1 of its arrival to the mouth B. Of course $\vec{x}_1$, $\vec{V}_1$ and t_1 are function of $\vec{x}_2$, $\vec{V}_2$ and t_2 (as well as of the initial data for the "younger" ball). Now we must remember that the "older" ball is the "younger" one returned back from the future and that $\vec{x}_2$, $\vec{V}_2$ and t_2 are not the "initial conditions" but the consequence of the motion of the "younger" ball. Thus, because we know the laws of passage through the time machine, we we can express $\vec{x}_2$, $\vec{V}_2$ and t_2 in the terms of $\vec{x}_1$, $\vec{V}_1$ and t_1. Finally we have the system of equations, allowing us to calculate all motions if the initial data were specified.

Next questions arise: the first about the existence of the solution to this system for any initial conditions and the second about the uniqueness of the solution.

The positive answer to the first question was obtain by Echeverria, Klinkhammer, and Thorne (1991) for an ideal elastic billiard ball and by Mikheeva and Novikov (1993) for inelastic one. The answer to the second question is not

[1]The trajectory α_2 is well defined if the trajectory α_1 is given (see Echeverria, Klinkhammer, and Thorne (1991)).

trivial. In the paper by Echeverria *et al* (1991) it has been demonstrated that there could be infinite number of self-consistent solutions in the general case. These solutions include a number wormhole traversals.

For some initial data there is only one solution. For example a ball initially at rest far from the wormhole has only one solution to its equations of motion: namely it remains forever at rest. There are arguments also that there is only a single solution for any ball with an with an initial speed small enough and an initial path of motion that, if extended forever, remains far from the wormhole. Echeverria, Klinkhammer, and Thorne (1991) tried to find initial data for which there is not self-consistent solutions at all, but none were found.

What does the multiplicity of the solutions mean? Does it have any physical reason or it means that the laws of physics cannot deal with the time machine in a reasonable way?

Once again the answer is not trivial. The point is that Physics is quantum mechanical at heart, not classical. If one considers the classical problem as a limiting case of the quantum one, then the Cauchy problem turns out to be well posed in the formalism of quantum mechanics (Klinkhammer and Thorne (unpublished)). In classical (not quantum mechanical) physics more complicated interacting system than the billiard ball have been investigated (see, for example, Novikov 1992, Lossev and Novikov 1992, Novikov 1993). It was though, at first, that for some initial data there might be no self-consistent evolution. But so far no clean example of such thing have been exhibited.

In the work by Carlini *et al.* (1995) it was demonstrated that for a billiard-ball problem the principle of self-consistency directly follows from the principle of least action, in which the initial and final positions of the ball are fixed. This result motivates the authors to formulate the conjecture that the "principle of self-consistency" is a consequence of the "principle of least action" in the general case for all physical phenomena, not only for the simple mechanical problem considered there. Carlini and Novikov (1996) extended the analysis to the case of point-like "billiard balls" moving with relativistic velocity. It was shown that for the case under consideration the only possible trajectories for which the action is extremal are those which are globally self-consistent. This gives additional support for the conjecture.

2.3 Physics of Black Holes.

2.3.1 Membrane paradigm in black hole physics

A black hole is region in space-time from which no signal can escape to an external observer. A black hole's boundary is a so called event horizon.

After the gravitational collapse of a celestial body and formation of a black hole the external gravitational field of it asymptotically approaches a standard equilibrium configuration known as the Kerr-Newman field and characterized by just three numbers: mass, angular momentum and charge.

Black holes reside in curved space. If a black hole has nonzero angular momentum then anything near a black hole will be dragged along by the vortex gravitational field. In this section I will consider a black hole without electric charge (Kerr black hole). The horizon's surface area can be written in terms of its mass M and angular momentum $J = aM$, where a is an angular momentum per unit mass ($c = 1$, $G = 1$):

$$A = 4\pi(r_H^2 + a^2), \tag{5.2}$$

$$r_H = M + \sqrt{M^2 - a^2}. \tag{5.3}$$

The rotational energy, or corresponding mass M_{rot}, of a Kerr black hole is the following

$$M_{rot} = M - \left[\frac{1}{2}M\left(M + \sqrt{M^2 - a^2}\right)\right]^{1/2}. \tag{5.4}$$

This rotational energy (energy of the vortex gravitational field) can be extracted (in principle) from a black hole.

The black hole is a clot of gravity, there is not any real matter on the horizon. In spite of this fact the horizon looks for an external observer (outside the black hole) and behaves as a physical membrane which is made from a two-dimensional viscous fluid with definite mechanical, electrical and thermodynamic properties.

This remarkable viewpoint is known as the membrane paradigm (see Thorne et al (1986) for a review). According to this paradigm the interaction of the horizon with the external universe is described in terms of familiar laws for the horizon fluid, e.g. the Navier-Stokes equation, Maxwell's equations, a tidal force equation, and the equations of thermodynamics. It is very important to emphasize that the membrane paradigm is not an approximation method or some analogy. It is an exact formalism which gives exactly the same results as the standard formalism of the General Relativity. Because the laws governing the horizon's behavior have familiar forms, they are powerful for understanding intuitively and computing quantitatively the interaction of black holes with complex environments.

In subsequent parts of this section we will consider some manifestations of the physical properties of the black hole's membrane, that resided in the three dimensional space.

2.3.2 Mechanical properties of the horizon's membrane

According to the membrane formalism, from the point of view of an external observer the hole's membrane has definite surface mass density and the surface pressure and viscosity.

The formula for the mass density is

$$\sum = -\frac{1}{8\pi}gth^H, \qquad \theta^H \equiv \frac{d(\Delta A)}{\Delta A dt}, \tag{5.5}$$

where θ^H is a fractional change of area of a surface element per unit time of an observer at infinity. One can see that for the case of a black hole in equilibrium (for example, a nonrotating (Schwarzschild) or a Kerr hole in the empty space) $\sum = 0$. The value of θ^H is always non-negative ,consequently $\sum_H = 0$ is always non-positive.

There is surface pressure p^H in the membrane. For a Schwarzschild hole it is:

$$p^H = \frac{1}{32\pi M} \approx \left(10^{42}\frac{dyne}{cm}\right)\left(\frac{M_\odot}{M}\right). \tag{5.6}$$

¿From the point of view of the membrane formalism the gravity of a black hole in equilibrium is produced by p^H.

The horizon's shear viscosity η^H and the horizon's bulk viscosity ζ^H are correspondingly:

$$\eta^H = \frac{1}{16\pi} \approx 10^{37}\frac{g}{sec}, \tag{5.7}$$

$$\zeta^H = -\frac{1}{16\pi} = -10^{37}\frac{g}{sec}. \tag{5.8}$$

Because the membrane paradigm regards a black hole as a two dimensional membrane with familiar mechanical properties it is rather easier to understand intuitively and compute quantitatively what happens with a black hole under some definite conditions. Let us consider a few examples.

If a black hole occurs as a result of gravitational collapse of an asymmetric celestial body (without rotation), then a nonspherical hole arises at the first moment. The hole's membrane is deformed and there is no balance between the surface pressure of the membrane and its gravity. So the membrane vibrates and radiates gravitational waves. The waves carry away the energy of the membrane deformation. This together with the membrane viscosity makes the horizon settle down into an absolutely spherical equilibrium shape.

Another example is a shape of the membrane of a rotating black hole. Centrifugal forces make a hole's membrane bulge out at its equator. The balance between the surface pressure, gravity and centrifugal forces determines the shape of the horizon's membrane.

Let us consider one very unusual property of the horizon's membrane. We emphasized above that the differential equations which describe the interaction of the horizon with the external universe are familiar physical laws (e.g. the Navier-Stokes equation and so on). But the solutions of the equations are determined also by the boundary conditions. In the case of standard physics the boundary conditions must be imposed at some initial moment or in the infinite past. That is not so for the hole's horizon! The point is that the horizon is a boundary between light-speed signals that can and those that cannot ever escape to spatial infinity. But this fact depends on the processes in the future, not in the past.

Whether a signal can escape depends on the region of spacetime to the future of the signal's source. It means that the motion of the horizon at any moment of time depends not on what has happened to the horizon in the past but what will happen to the horizon in the future.

This property can be illustrated by the problem of a free fall of a thin spherical shell of a matter of mass ΔM into a Schwarzschild hole with mass M. The spacetime geometry is that of Schwarzschild both interior to the shell and outside it. In the interior the Schwarzschild mass is M and outside it is $M + \Delta M$. Now the light-speed signals with world lines at $r = 2M$ cannot be the boundary of the non-escape region because these signals and outgoing signals just outside $r = 2M$ will get caught and pulled into the hole by the added gravity of the shell when in the future the shell passes through them. The real boundary is generated by light-speed signals world lines of which are just outside of the surface $r = 2M$. In the past, long before the shell arrives at the horizon this surface practically coincides with $r = 2M$. Then, as the shell nears it the surface (which is the real boundary, meaning the real horizon) starts to expand. This is because the world lines of its generators go farther an farther from $r = 2M$. This is their property in the Schwarzschild spacetime, and it does not depend on the approaching shell. When the shell finally passes through it, the added shell's gravity starts influence the motions of the generators of the surface, the horizon suddenly stops expanding and freezes at $r = 2(M + \Delta M)$. These behaviors of the horizon are dictated by the properties of propagation of the light-speed signals which generate the horizon and which have the property to propagate at $r = 2(M + \Delta M)$ **after** crossing with the shell. Thus, this behavior of the horizon **before** crossing with the shell (its expansion) depends on the events in the future (the crossing with the shell).

One refers to this dependence of future events as the "theological" nature of the horizon (see Thorne et al 1986). I would like to emphasize that these behaviors looks as if the hole's membrane lives in time which flows in the opposite direction: from the future into the past. Indeed in this case the change of the size of the horizon looks very natural and causal. If we accept this point of view, we should consider the extraction of the shell from the hole, and just after this extraction of the shell from the membrane at $r = 2(M + \Delta M)$, the horizon starts contracting and settles down to $r = 2M$. We will see in Section 3 that this unusual property, namely, "feeling" information from the infinite future of the external observer, is a characteristic property not only of the horizon but also of the interior of a black hole.

2.3.3 Black-Hole Electrodynamics

A black hole horizon behaves as an electrically conducting sphere. To understand this let us ask what could be the external manifestation of the electric conductivity of a body in a flat spacetime. The simplest manifestation is the following. If one brings a positive electric charge close to a metal sphere then free electrons on the sphere's metal surface will be displaced with respect to the ions by the Coulomb electric forces. It polarizes the sphere. As a result, the electric field lines form a characteristic configuration in the space around the sphere. Now if one moves the charge parallel to the surface of the sphere from one position to another one, the characteristic configuration of the electric lines comes to a new place with some delay. This delay is determined by the resistivity of the sphere's metal surface. It turns out that if one brings a charge close to a non-spinning black hole, there is a similarity between the picture of the field lines in the vicinity of the black hole and the analogous picture in the vicinity of a metal sphere in a flat spacetime. Now the curvature of spacetime distorts the field line rather than displacement of real charges on the horizon. Nevertheless, it looks like the field of the charge polarizes the horizon.

If one moves the charge parallel to the hole's horizon to another position, then the configuration of the electric lines will settle down at the new place with some delay. Now it is determined by the finite time of propagation of electromagnetic signals. Nevertheless one can interpret it as an effective resistivity of the horizon.

In general one can say that a horizon's membrane behaves as a metal sphere with a surface resistivity equal to $R_H = 4\pi \approx 377$ohms.

The membrane paradigm gives insight into possible behaviors of rotating black holes in interaction with magnetized plasma. We will draw an analogy with a dynamo. In its rotor the motion of wire coils in a magnetic field produces an electromotive force compelling the charges to flow through the conductor. A black hole is also a special dynamo of great size. If a spinning black hole is immersed in an external magnetic field, a powerful electric field will also develop in its vicinity. The magnetic field is created by the interstellar gas flowing into a black hole. The magnetic field lines will tend to rotate along with the spinning black hole. The motion of any magnetic field generates an electric field. In the case of a rapidly rotating, magnetized black hole, the electric field generated near its edges can produce an enormous voltage difference between the poles of the hole and its equatorial region:

$$\Delta V \approx (10^{20}\text{volts}) \left(\frac{a}{M}\right) \left(\frac{M}{10^9 M_\odot}\right) \left(\frac{B}{10^4 G}\right), \tag{5.9}$$

where B is the magnetic field in the vicinity of the black hole. It is as though the spinning black hole was a huge battery. The electric field is responsible for accelerating the charged particles of the plasma and causing them to move along the magnetic lines of force. The total power output is

$$P \approx \left(10^{45}\frac{\text{erg}}{\text{sec}}\right) \left(\frac{a}{M}\right) \left(\frac{M}{10^9 M_\odot}\right) \left(\frac{B}{10^4 G}\right), \tag{5.10}$$

Probably this mechanism is the main "engine" of the active galactic nuclei.

2.3.4 Thermodynamics of black holes

¿From many aspects of the thermodynamics of black holes, I will discuss the problem of the black hole's thermal quantum radiation and the related problem of the thermal atmosphere of a black hole.

S.Hawking (1974) claimed that a black hole should emit thermal radiation with temperature

$$T_H = \frac{\hbar}{8\pi k} M^{-1} \approx (10^{-7} K) \left(\frac{M_\odot}{M}\right). \tag{5.11}$$

How, in simple physical terms, could one understand that a black hole behaves like an ordinary body with temperature T_H. A key insight into thermal emission from a hole come from theoretical discoveries in the mid-1970s (see Unruh, 1976). The crucial point is the existence of the event horizon for some definite classes of observers. For example, an accelerated observer in an empty spacetime has a horizon. This observer cannot receive information from the region beyond the horizon. The virtual particles' vacuum fluctuation waves are not confined solely to the region above the horizon; part of each fluctuation wave is beyond the horizon and part is within the region which the observer can see. According to quantum mechanics this principle lack of information about vacuum fluctuation waves leads to the conclusion (for an accelerated observer) that they are real waves. As a result, this observer is bathed in a perfect bath of thermal radiation with temperature $T = \hbar a/(2\pi k)$, where a is the observer's acceleration. Since a static observer just above a Schwarzschild horizon can be viewed as analogous to an accelerated observer in flat spacetime with acceleration $a = c^2/z$, where z is the distance from the horizon, such an observer should feel himself bathed in thermal radiation with local temperature $T = \hbar/(2\pi kz)$. This thermal radiation forms a thermal atmosphere of the hole. The radiation, climbing up through the hole's gravitational field, would be redshifted by a factor $\left(1 - \frac{2M}{r}\right)^{1/2}$. It will emerge with temperature T_H. Most of the photons and other particles fly upward a short distance and are then pulled back down by the hole's enormous gravity. A few of the particles manage to escape the hole's gravitational grip and evaporate into space. These particles form the Hawking radiation.

Note that a free falling observer does not feel this thermal atmosphere. He "sees" only vacuum fluctuations to consist of pairs of virtual particles.

The process of the Hawking quantum evaporation is very slow. The total lifetime is proportional to the cube of M. For a 20 solar mass black hole it is 10^{70}years.

In principle, the interactions of a black hole with the external Universe can change the process of extraction of the thermal energy from a black hole atmosphere drastically (see the review in referenced books).

2.4 Astrophysics of black holes

Do black holes exist in the Universe or are they only an abstract concept of the human mind? In principle, a black hole could be built artificially. However, this meets such grandiose technical difficulties that it looks impossible, at least in the immediate future. In fact, the artificial building of a black hole looks even more problematic than an artificial creation of a star. Thus we have to conclude that the physics of black holes, as well as the physics of stars, is the physics of celestial bodies. Stars definitely exist, but what may one say about the existence of astrophysical black holes?

Modern astrophysics deals with three types of black holes in the Universe:
1) *stellar black holes*, that is black holes of stellar masses, that were born when massive stars died;
2) *supermassive black holes* with masses up to $10^9 M_\odot$ and greater at centers of galaxies ($M_\odot = 2 \times 10^{33}$g is the mass of the Sun).

These two types of black holes have been discovered. The third possible type of astrophysical black holes - primordial black holes will be discussed in subsection 4.5. Our main attention in Section 4 is focused on the possible observational manifestation of black holes.

2.4.1 The origin of stellar black holes

"When all the thermo-nuclear sources of energy are exhausted a sufficiently heavy star will collapse" – this is the first phrase of the abstract of a remarkable paper by Oppenheimer and Snyder (1939). Every statement of this paper accords with ideas that remain valid today (including the terminology). The authors conclude the abstract by the following sentence: "... an external observer sees the star shrinking to its gravitational radius." This is the modern prediction of the formation of black holes when massive stars die.

How heavy should a star be to turn into a black hole? The answer is not simple. A star that is not massive enough ends up either as a white dwarf or a neutron star. There are upper limits on the masses of both these types of celestial bodies. For white dwarfs it is the *Chandrasekhar limit*, which is about $(1.2 - 1.4) \times M_{\odot}$. For neutron stars it is the *Oppenheimer-Volkoff limit*. The exact value of this limit depends on the equation of state at matter density higher than the density of nuclear matter $\rho_0 = 2.8 \times 10^{14}\mathrm{g\ cm}^{-3}$. The modern theory gives for the maximal mass of a non-rotating neutron star the estimate $(2-3) \times M_{\odot}$. Rotation can increase maximal mass of a non-rotating neutron star only slightly up to 25%. Thus one can believe that the upper mass limit for neutron stars should not be greater than $M_0 \approx 3M_{\odot}$. If a star at the very end of its evolution has mass greater than M_0 it must turn into a black hole. However this does not mean that all normal stars (on the "main sequence" of the Herzsprung-Russell diagram, with masses $M > M_0$ are black hole progenitors. The point is that the final stages of evolution of massive stars are poorly understood. Steady mass loss, catastrophic mass ejection and even disruption at supernovae explosions are possible. These processes can considerably reduce mass of a star at the end of its evolution. Thus the initial mass of black hole progenitors could be essentially greater than M_0.

There are different estimates for the minimal mass M_* of a progenitor star that still forms a black hole. Uncertainty is $M_* \approx (10 - 40)M_{\odot}$ and even more. Note that the evolution of stars in close binary systems differs from the evolution of sole stars because of mass transfer from one star to another. The conclusions about masses of black hole progenitors in this case could be essentially different.

One can try to estimate how many black holes have been created by stellar collapse in our Galaxy during its existence. The estimates give the number of the order 10^9.

2.4.2 Disk accretion onto black holes

For the purpose of finding and investigating black holes, two specific cases of accretion are of a particular importance: accretion in binary systems and accretion onto supermassive black holes that probably reside at the centers of galaxies. In both cases the accreting gas has big specific angular momentum.

As a result the gas elements circle around the black hole in Keplerian orbits, forming a disk or a torus around it. Viscosity plays a crucial role for the accretion. It removes angular momentum from each gas element, permitting it to gradually spiral inward toward the black hole. At the same time the viscosity heats the gas, causing it to radiate. Probable sources of viscosity are turbulence in the gas disk and random magnetic fields. Unfortunately, we are not near to a good physical understanding of the effective viscosity. Large-scale magnetic fields can also play an important role in the physics of accretion.

The properties of the accreting disk are determined by the rate of gas accretion. An important measure of any accretion luminosity of a black hole is provided by the Eddington critical luminosity

$$L_E = 4\pi G M_h \mu m_p c/\sigma_T = \left(1.3 \times 10^{38} \frac{\text{erg}}{\text{s}}\right) \mu \left(\frac{M_h}{M_{odot}}\right). \tag{5.12}$$

Here M_h is the mass of a black hole, μ is the molecular weight of electron, m_p is the rest mass of the proton, and σ_T is the Thomson cross section. It is the luminosity at which the radiation pressure just balances the gravitational force of the mass M_h for a fully ionized plasma.

A useful measure of the accretion rate $\dot{M}$ is the so-called "critical accretion rate":

$$\dot{M}_E = L_E c^{-2}, \tag{5.13}$$

where L_E is given by equation (12). We shall use the dimensionless expression $\dot{m} \equiv \dot{M}/\dot{M}_E$. The first models of the disk accretion were rather simple. They focused on the case of moderate rate of accretion $\dot{m} < 1$. Subsequently theories for $\dot{m} \sim 1$ and $\dot{m} \gg 1$ were developed. They take into account complex processes in radiative plasmas and various types of instabilities.

The source of luminosity for disk accretion is the gravitational energy that is released when gas elements in the disk spiral down. Most of the gravitational energy is released, generating most of the luminosity, from the inner parts of the disk. According to the theory for these simplest models the total luminosity of the disk is the following

$$L \approx \begin{cases} \left(3 \times 10^{36} \frac{\text{erg}}{\text{s}}\right) \left(\frac{\dot{M}}{10^{-9} M_\odot/\text{yr}}\right), & \text{nonrotating hole}, \\ \left(3 \times 10^{37} \frac{\text{erg}}{\text{s}}\right) \left(\frac{\dot{M}}{10^{-9} M_\odot/\text{yr}}\right), & \text{maximally rotating hole}. \end{cases} \tag{5.14}$$

The accretion rate $\dot{M}$ is an arbitrary external parameter, which is determined by the source of gas (for example, by the flux of gas from the upper atmosphere of the companion star in a binary system). We normalized $\dot{M}$ by the value $\dot{M}_0 = 10^{-9} M_\odot/\text{yr}$ because this is probably the typical rate at which a normal star is dumping gas onto a companion black hole. In this model the accretion gas is assumed to be relatively cool, with its temperature much less than the virial temperature corresponding to the potential energy in the gravitational field. As estimates show, a *geometrically thin disk* (with heights

$h \ll r$) might be formed under these conditions. This is the so-called *standard disk model*. In this model the electron and ion temperatures are equal, and the disk is effectively optically thick. The temperature of the gas in the inner parts of the disk reaches $T \approx 10^7 - 10^8$K. In this region electron scattering opacity modifies the emitted spectrum so that it is no longer the blackbody spectrum. Instead, the total spectrum of the disk radiation is a power law $F \sim \omega^{1/3}$ with an exponential cut off at high frequencies. The innermost regions of such "standard" disks are probably unstable. The thin accretion disk model is unable to explain the hard spectra observed in accretion flows around black holes in many observable cases.

A few types of hot accretion flow models have been proposed. Among them a model with a hot corona above a standard thin accretion disk. In another model the ions in the inner region are hot $T_i \approx 10^{11}$K but the electrons are considerably cooler $T_e \approx 10^9$K. This inner disk is thicker than in the "standard" model and produces most of the X-ray emission. The models with hot ions and cooler electrons are optically thin.

Further development of the theory of disk accretion led to more sophisticated models. It have been demonstrated that when the luminosity reaches the critical one (corresponding to $\dot{m} \equiv \dot{M}/\dot{M}_g$ of the order of unity) radiation pressure in the inner parts of the disk dominates the gas pressure and the disk is thermally and viscously unstable. For especially big $\dot{m} > 80$ the essential part of the energy of the plasma is lost by advection into the black hole horizon because the radiation is trapped in the accretion gas and is unable to escape. This process stabilizes the gas flow against perturbations. Advection can also be important for smaller $\dot{m}$. For high mass accretion rates the height of the accretion disk becomes comparable to its radius. In modern models the radial pressure gradients and the motion of gas elements along radius are taken into account. In the innermost parts of the disk and down to the black hole the flow of gas supersonic.

Recently, a new class of optically thin hot disk solutions has been discovered. In this model the most of the viscously dissipated energy is advected with the accreting gas, with only a small fraction of the energy being radiated. It is because the gas density is so low that the radiative efficiency is very poor. These models were named advection-dominated. They have been applied successfully to a few concrete celestial objects.

In conclusion we note that in some models of disk accretion electron-positron pair production can be important. We believe that new models involving recent achievements of plasma physics will play a key role in the modern astrophysics of black holes.

2.4.3 Evidence for Black Holes in Stellar Binary Systems

Probably the best evidence that black holes exist comes from studies of X-ray binaries. The arguments are as follows:

1. The X-ray emitting object in a binary system is very compact, and therefore cannot be an ordinary star. Thus it is either a neutron star or a black hole. This argument comes mainly from analysis of the features of emitted X-rays.

2. Analysis of the observational data allows one to determine the orbital motion in the binary system makes it possible to obtain the mass of the compact object. The data on the observed velocity of the optical companion star is of the most importance. Note that the Newtonian theory is always sufficient for the analysis. The technique of weighing stars in binaries is well known in astronomy. If the mass of the compact component is greater than the maximal possible mass of neutron stars $M_0 \approx 3M_\odot$ (see Section 4.1), then it is a black hole.

It is worth noting that this evidence is somewhat indirect because it does not confront us with the specific relativistic effects that occur near black holes and which are peculiar to black holes alone. However, it is the best that modern astronomy has proposed so far. In spite of these circumstances, we believe that the logic of the arguments is reliable enough.

According to the generally accepted interpretation, we have the necessary observational confirmation only for a few systems at the present time. For these systems, we have strong reasons to believe that the compact X-ray emitting companions are black holes. Some characteristics of these leading black hole candidates are summarized in Table 1 [according to Cherepashchuk (1996)].

The most plausible masses of compact objects in these systems are considerably larger than $M_0 \approx 3M_\odot$. The strongest candidate are those which have a *dynamical lower limit* of the mass of the compact object (or so-called *mass function* $f(M)$) greater than $3M_\odot$. ¿From this point of view the strongest candidates are GS 2033+338 (f(M)=$6.5M_\odot$), GS 2000+25 (f(M)=$5M_\odot$), and XN Oph 1977 (f(M)=$4M_\odot$).

The total number of systems that are frequently mentioned as possible candidates for black holes of stellar mass is about 20. All seriously discussed candidates a re X-ray sources in binary systems. Some of them are persistent, other are transient. Begelman and Rees (1996) summarize the present status as follows: "There is also overwhelming evidence for black holes in our own galaxy, formed when ordinary massive stars die, each weighting a few times as much as the Sun". Most of experts now agree with this unambiguous conclusion.

During the more than 25 years since the discovery of the first black hole candidate Cyg X-1 only a few new candidates have been added. This is in contrast to the rapid increase of the number of identified neutron stars. At present many hundreds of neutron stars have been identified in the Galaxy. About 100 of them are in binary systems. One might conclude that black holes in binary systems are exceedingly rare objects. This is not necessarily true, however. The small number of identified black hole candidates may as well be related to the specific conditions which are necessary for their observable manifestation.

According to estimations the evolutionary stage when a black hole binary continuously radiates X-ray may last only 10^4 years. We can thus detect it only during this short period. In effect, the population of black-hole binaries may be much larger than what we can presently observe. Such systems may be as common as neutron star binaries.

Table 2.1: Black-hole candidates in binary systems [Cherepashchuk (1996)].

System	Spectral type of the optical companion	Orbital period (days)	Mass of the compact companion (in $M_\odot$)	Mass of the optical companion (in$M_\odot$)	X-ray luminosity (erg/sec)
Cyg X-1 (V 1357 Cyg)	O9.7Iab	5.6	7-18	20-30	$\sim 8 \times 10^{37}$
LMC X-3	B(3-6)II-III	1.7	7-11	3-6	$\sim 4 \times 10^{38}$
LMC X-1	O(7-9)III	4.2	4-10	18-25	$\sim 2 \times 10^{38}$
A0620-00 (V616 Mon)	K(5-7)V	0.3	5-17	~ 0.7	$\leq 10^{38}$
GS 2023+338 (V 404 Cyg)	K0IV	6.5	10-15	0.5-1.0	$\geq 6 \times 10^{38}$
GRS 1121-68 (XN Mus 1991)	K(3-5)V	0.4	9-16	0.7-0.8	$\leq 10^{38}$
GS 2000+25 (QZ Vul)	K(3-7)V	0.3	5.3-8.2	~ 0.7	$\leq 10^{38}$
GRO J0422+32 (XN Per 1992= =V518 Per)	M(0-4)V	0.2	2.5-5.0	~ 0.4	$\leq 10^{38}$
GRO J1655-40 (XN Sco 1994)	F5IV	2.6	4-6	~ 2.3	$\leq 10^{38}$
XN Oph 1977	K3	0.7	5-7	~ 0.8	$\leq 10^{38}$

2.4.4 Supermassive Black Holes in Galactic Centers

Since the middle of this century astronomers have come across many violent or even catastrophic processes associated with galaxies. These processes are accompanied by powerful releases of energy and are fast not only by astronomic but also by earthly standards. They may last only a few days or even minutes. Most such processes occur in the central parts of galaxies, the galactic nuclei.

About one percent of all galactic nuclei eject radio-emitting plasma and gas clouds, and are themselves powerful sources of radiation in the radio, infrared, gamma, and especially, the "hard" (short wavelength) X-ray regions of the spectrum. The full luminosity of the nucleus is in some cases $L \approx 10^{47}$erg/s and millions of times the luminosity of the nuclei of more quite galaxies, such as ours. These objects were called *active galactic nuclei* (AGN). Practically all the energy of activity and of the giant jets released by galaxies originates from the centers of their nuclei.

Quasars form a special subclass of AGN. Their characteristic property is that their total energy release is hundreds of times greater than the combined radiation of all the stars in a large galaxy. At the same time the average liar dimensions of the radiating regions are small: a mere one-hundred-millionth of the linear size of a galaxy. Quasars are the most powerful energy sources registered in the Universe to date. What processes are responsible for the extraordinary outbursts of energy from AGN and quasars?

Learning about the nature of these objects involves measuring their sizes and masses. This is not easy at all. The central emitting regions of AGN and quasars are so small that telescope view reveals them just as point sources of light. Fortunately quite soon after the discovery of the quasar 3C 273 it was shown that its brightness changed. Sometimes it changes very rapidly, in less than a week. After this discovery, even faster variability (at timescale of a few hours or less) were detected in other galactic nuclei. From these variations one could estimate the dimensions of the central parts of the nuclei that are responsible for radiation. The conclusion was that these regions were not more than a few light-hours in diameter. That is, they are comparable to the solar system in size.

In spite of the rather small linear dimensions of quasars and many galactic nuclei, their masses turned out to be enormous. They were first estimated using formula (12). For quasistatic objects the luminosity cannot be essentially greater than L_E. A comparison of the observed luminosity with the expression (12) gives an estimate of the lower limit of the central mass. In some quasars this limit is $M \approx (1 - 10^2) \times 10^7 M_\odot$. These estimates are supported by data on the velocities within the galactic nuclei of stars, and gas clouds accelerated in the gravitational fields of the center of the nuclei. We will discuss this in the end of this section.

Great mass but small linear dimensions prompt the guess that there could be a black hole. This would account for all the extraordinary qualities of these objects. Now it is generally accepted that in AGN there are supermassive black holes with accretion gas (and maybe also dust) disks. One of the most important facts implied by observations, especially by means of radio telescopes, is the existence of directed jets from the nuclei of some active galaxies. For some of the objects there are evidence that radio components move away from the nucleus at ultrarelativistic velocities. The existence of an axis of ejection strongly suggests the presence of some stable compact gyroscope, probably a rotating black hole. In some cases one can observe evidence that there is also precession of this gyroscope. An essential role in the physics of processes in the centers of AGN is probably played by black-hole electrodynamics.

In the model of a supermassive black hole with an accretion disk for AGN one requires sources of fuel – gas or dust. The following sources have been discussed: gas from nearby galactic companion (the result of interaction between the host galaxy and the companion), interstellar gas of the host galaxy, disruption of stars by high velocity collisions in the vicinity of a black hole, disruption of stars by the tidal field of the black hole and some others.

Clearly, the processes taking place in quasars and other galactic nuclei are still a mystery in many respects. But the suggestion that we are witnessing the work of a supermassive black hole with an accretion disk seems rather plausible. Rees (1990) advocates a hypothesis that the massive black holes are not only in the active galactic nuclei but, also in the centers of "normal" galaxies (including nearby galaxies and our own Milky Way) (Rees 1990). They are quiescent because they now starved of fuel (gas). Observations show that galactic nuclei were more active in the past. Thus, "dead quasars" (massive black holes without fuel) should be common at the present epoch.

How can these black holes be detected? It has been pointed out that black holes produce cusp-like gravitational potentials and hence they should produce cuspy-like density distributions of the stars in the central regions of galaxies. Some authors have argued that the brightness profiles of the central regions of particular galaxies imply that they contain black holes. However the arguments based only on surface brightness profiles are inconclusive. The point is that a high central number density of stars in a core with small radius can be the consequence of dissipation, and a cusp-like profile can be the result of anisotropy of the velocity dispersion of stars. Thus these properties taken alone are not sufficient evidence for the presence of a black hole.

The reliable way to detect black holes in the galactic nuclei is analogous to the case of black holes in binaries. Namely, one must prove that there is a large dark mass in a small volume, and that it can be nothing other than a black hole. In order to obtain such a proof we can use arguments based on both stellar kinematics and surface photometry of the galactic nuclei.

If the distribution of the mass M and the luminosity L as functions of the radius are known we can consider the mass-to-light ratio M/L (in solar units) as a function of radius. This ratio is well known for different types of stellar populations. As a rule this ratio is between 1 and 10 for elliptical galaxies and globular clusters (old stellar population dominates there). If for some galaxy the ratio M/L is almost constant at rather large radii (and has a "normal" value between 1 and 10) but rises rapidly (toward values much larger than 10) as one approaches the galactic centre, then there is evidence for a central dark object (probably a black hole).

As an example consider galaxy NGC 3115 which is at a distance of 9.2Mpc from us [Kormendy and Richstone (1992)]. For this galaxy $M/L \approx 4$ and almost constant over a large range of radii $r > 4''$ (in angular units). This value is normal for a bulge of this type of galaxy. At radii $r < 2''$ the ratio

Table 2.2: Supermassive holes [Rees 1998]

	$M_h/M_\odot$	Method
M87	2×10^9	Stars+opt.disc
NGC 3115	10^9	Stars
NGC 4486 B	5×10^8	Stars
NGC 4594 (Sombrero)	5×10^8	Stars
NGC 3377	8×10^7	Stars
NGC 3379	5×10^7	Stars
NGC 4258	4×10^7	Masing H_2O disc
M31 (Andromeda)	3×10^7	Stars
M32	3×10^6	Stars
Galactic centre	2.5×10^6	Stars+3-D motions

M/L rises rapidly up to $M/L \approx 40$. If this is due to a central dark mass added to a stellar distribution with constant M/L, then $M_H = 10^{9.2\pm0.5} M_\odot$.

Is it possible to give another explanation of the large mass-to-light ratio in the central region of a galaxy? We cannot exclude the possibility that a galaxy contains a central compact cluster of dim stars. But it is unlikely. The central density of stars in the galaxy NGC 3115 is not peculiar. It is the same as in the centers of globular clusters. The direct observational data (spectra and colors) of this galaxy do not give any evidence of a dramatic population gradient near the center. Thus, the most plausible conclusion is that there is a central massive black hole.

Unfortunately, it is difficult to detect massive black holes in giant elliptical galaxies with active nuclei, where we are almost sure black holes must exist because we observe their active manifestation [Kormendy (1993)]. The reason for this is a fundamental difference between giant elliptical galaxies (the nuclei of some of them are among the most extreme examples of AGN), dwarf elliptical galaxies and spiral galaxies. Dwarf ellipticals rotate rapidly and star velocity dispersions are nearly isotropic. Giant elliptical galaxies do not rotate significantly and they have the anisotropic velocities. It is not so easy to model these dispersions. Furthermore, giant elliptical galaxies have large cores and shallow brightness profiles. Consequently, the projected spectra are dominated by light from large radii, where a black hole has no effect.

The technique described above has been used to search for black holes in galactic nuclei. Another possibility is to observe rotational velocities of gas in the vicinity of the galactic center. So far (the middle of 1998) black hole detections have been reported for the following galaxies: M32, M31, NGC 3377, NGC 4594, Milky Way, NGC3115, M87, and NGC 4258 [for review see Rees (1998). Some evidence for a supermassive black hole in NGC 4468B was reported by Kormendy et al. (1997).

Special investigations were performed in the case of the galaxy M87 [see Dressler (1989) for review of earlier works and Lauer *et al.* 1992]. This is a giant elliptical galaxy with active nucleus and a jet from the center. At present there is secure stellar-dynamical evidence for a black hole with mass $M \approx 3 \cdot 10^9 M_\odot$ in this galaxy. The Hubble Space Telescope has revealed a rotating disk of gas orbiting the central object in the galaxy [Ford *et al.* (1994), Harms *et al.* (1994)]. The estimated mass of the central object is $M = 2.4 \times 10^9 M_\odot$. The presence of a black hole in M87 is especially important for our understanding of the nature of the central regions of galaxies because in this case we observe also the activity of the "central engine".

Radio-astronomical observations of the nucleus of the galaxy NGC 4258 are of special interest [Miyoshi *et al* (1995)]. Using radio interferometry technique of observation of maser lines of molecules of water in gas clouds orbiting in the close vicinity of the nucleus, the observers obtained the angular resolution 100 times better that in the case of observations by the Hubble Space Telescope. The spectral resolution is 100 times better as well. According to the interpretation of the observations the center of NGC 4258 harbors a thin disk which was measured on scales of less than one light-year. The mass of the central object is $3.6 \times 10^7 M_\odot$. According to the opinion of Begelman and Rees (1996): "It represents truly overwhelming evidence for a black hole... NGC 4258 is the system for which it is hardest to envisage that the mass comprises anything but a single black hole".

In conclusion we list in Table 2 estimates of masses of black holes in the nuclei of some galaxies Rees (1998).

Progress in this field is very rapid and in the nearest future our knowledge about evidence of supermassive black holes in galactic nuclei will be more profound.

2.4.5 Primordial black holes

Modern astrophysics considers also the third possible type of black holes in the Universe - primordial black holes. These black holes might appear from inhomogeneities at the very beginning of the expansion of the Universe. Their masses can be arbitrary, but primordial black holes with $M \leq 10^{15}$g would have radiated away their mass by the Hawking quantum process in a time $t \leq 10^{10}$years (the age of the Universe) Only primordial black holes with mass $M > 10^{15}$g could exist in the contemporary Universe.

Searches for PBHs attempt to detect a diffuse photon (or another particle) background from a distribution of PBHs or to search directly for the final emission stage of individual black holes. Using the theoretical spectra of particles and radiation emitting by evaporating black holes of different masses, one can calculate the theoretical backgrounds of photons and other particles produced by a distribution of PBHs emitting over the lifetime of the Universe. The level of this background depends on the integrated density of PBHs with initial masses in the considered range.

A comparison of the theoretical estimates with the observational cosmic ray and γ-ray backgrounds place an upper limit on the integrated density of PBHs with initial masses in this range. According to estimates of MacGibbon and Carr (1991),, this limit corresponds to $\approx 10^{-6}$ of the integrated mass density of the visible matter in the Universe (matter in the visible galaxies).

The comparison of the theory with other observational data gives more weak limits (for review see Halzen *et al.* 1991, Coyne 1993).

The search for high energy gamma-ray bursts as direct manifestation of the final emission of the evaporating (exploding) individual PBHs has continued for more than 20 years. No positive evidence for existence of PBHs has been reported (see Cline and Hong (1992), (1994)).

A population of PBHs whose influence is small today may have been more important in the earlier epochs of the evolution of the Universe. Radiation from PBHs could perturb the usual picture of cosmological nucleosynthesis, distort the microwave background and produced too much entropy in relation to the matter density of the Universe. As we mentioned above, limits on the density of PBHs, now or at earlier times, can be used to provide information on the homogeneity and isotropy of the very early Universe, when they were formed. For review see Novikov *et al.* (1979), Carr *et al.* 1994.

The final state of the black hole evaporation is still unclear. There is a possibility that the endpoint of the black hole evaporation is a stable relic. The possible role of such relics in cosmology was first discussed by MacGibbon (1987), for review see Barrow *et al.* 1992.

2.5 Tidal interaction of star with a supermassive black hole

2.5.1 Introduction

The interaction of stars with a massive black hole probably plays an important role in physical processes in the central regions of QSOs and active galactic nuclei (AGNs), and even in global clusters and in the centers of normal g galaxies. (For review see Rees (1989a,b), Shlossman, Begelman and Frank (1990), Phinney (1989) and references therein).

Three dimensional numerical simulations are needed to describe the hydrodynamics and microphysics of processes during a close encounter of a star and a massive black hole. This approach is very time-consuming, even using supercomputers. There are also some approximate methods, namely, the linear theory for small deformations of a star, and the method based on the "affine stellar model". In latter method strong deformations are allowed for, but density contours inside a star are restricted to a homologous ellipsoidal shape.

Although the approximate methods are widely used, they do not allow one to address some key aspects of the problem. Among these are the hydrodynamics of tidal disruption, the distribution of the energy in the stellar debris, the dynamics of nonlinear oscillations in the case of a close encounter, stripping of the outer layers of a star by tidal forces, the physics of extremely close encounters, and many others. Also, it is unclear whether the affine model is reliable for describing numerous aspects of the close encounter problem. Thus, one needs a modern three-dimensional numerical approach to the problem.

We started the corresponding project five years ago. In this paper I give a brief review of our results in Khokhlov, Novikov and Pethick (1993a,b), Frolov, Khokhlov, Novikov and Pethick (1994), Diener, Frolov, Khokhlov, Novikov and Pethick (1997, 1998). As a rule I will not give references to the original papers. Corresponding references can be found in the papers mentioned above.

We will consider a star with mass M_* and radius R_* moving on a parabolic (or almost parabolic) orbit with a pericentric distance R_p around a black hole with mass M_h. This problem is important for the application to the discussion of the processes in the galactic centers. We assume that $M_h/M_* \gg 1$ and that $R_h/R_* \gg 1$. If the size of a black hole R_h is much smaller than R_p then $R_h/R_P \ll 1$, and Newtonian physics is valid. We start our consideration from this case and after that will consider relativistic tidal interaction of a star with a massive black hole.

We will not consider here our numerical methods. Their description see in the papers Khokhlov, Novikov and Pethick (1993a,b), Frolov, Khokhlov, Novikov and Pethick (1994), Diener, Frolov, Khokhlov, Novikov and Pethick (1997, 1998).

2.5.2 Newtonian encounters

Basic equation in the Newtonian approximation In this Section we assume that $R_h/R_P \ll 1$ and Newtonian physics is valid.

The star is described by the usual hydrodynamical equations for an inviscid fluid:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{U}), \tag{5.15}$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} = -\nabla \cdot (\rho \mathbf{U}\mathbf{U}) - \nabla P + \rho \mathbf{g}, \tag{5.16}$$

$$\frac{\partial E}{\partial t} = -\nabla \cdot [(E + P)\mathbf{U}] + \rho \mathbf{U} \cdot \mathbf{g}. \tag{5.17}$$

Here ρ, P and $\mathbf{U}$ are the density, pressure, and velocity of matter. In these first calculations we shall assume that the ratio of specific heats at constant pressure and constant volume, γ, is a constant, and therefore the energy density is given by $E = P/(\gamma - 1) + \rho \mathbf{U}^2/2$. The full acceleration includes contributions due to self-gravity and due to external forces

$$\mathbf{g} = -\nabla \Phi + \mathbf{g_e}, \tag{5.18}$$

where

$$\Phi(\mathbf{r}) = -\int \frac{G\rho(\mathbf{x})}{|\mathbf{r} - \mathbf{x}|} d^3\mathbf{x}. \tag{5.19}$$

We choose a nonrotating Cartesian coordinate system $OXYZ$, with its origin O located at the (moving) center of mass of the star. The X-axis lies in the orbital plane and is directed toward the pericenter of the orbit of the black hole relative to the star. The Z-axis is perpendicular to the orbital plane.

We assume that the orbit of the star is parabolic. In fact, since we intersted only in the part of the orbit which is close to the pericenter, our results should be applicable to very eccentric elliptical trajectories and to slightly hyperbolic trajectories as well.

The parabolic trajectory of the black hole $\mathbf{R}(t)$ is described in $OXYZ$ system by the equations

$$R(t)/R_p \equiv x = 1 + y^2, \qquad \Psi(t) = 2 \ \arctan \ y, \tag{5.20}$$

where $\mathbf{R}$ is the position of the black hole, Ψ is the angle between the direction to the pericenter and that to the black hole, and y is the real solution of

$$\frac{y^3}{3} + y = \left(\frac{GM_*}{2R_*^3}\right)^{1/2} \frac{t}{\eta}, \tag{5.21}$$

where

$$\eta = \left(\frac{M_*}{M_h}\right)^{1/2} \left(\frac{R_p}{R_*}\right)^{3/2} \tag{5.22}$$

characterizes the strength of the tidal interaction.

In our nonrotating coordinate system the external acceleration, $\mathbf{g_e}$, is reduced to the tidal acceleration, $\mathbf{g}_t$, of a black hole and is given by the gradient of the tidal potential:

$$\mathbf{g_e} \equiv \mathbf{g_t} = -\nabla\Phi_t. \tag{5.23}$$

Note that due to the inequality $R_* \ll R_p$ only the first term in the expansion of the tidal potential with respect to R_*/R_p is important and thus we take

$$\Phi_t(\mathbf{r}) = \frac{GM_h r^2}{R^3}\frac{1 - 3\cos^2\delta}{2} = \frac{GM_*}{R_*^3}\frac{r^2}{\eta^2 x^3}\frac{1 - 3\cos^2\delta}{2}, \tag{5.24}$$

where δ is the angle between $\mathbf{R}$ and $\mathbf{r}$. In polar coordinates r, θ, ϕ, the tidal potential (Eq.(10)) becomes

$$\Phi_t = \frac{GM_h}{R_*^3}\frac{3r^3}{4x^3\eta^2}\left\{\cos^2\theta - \sin^2\theta\cos[2(\phi - \Psi)] - \frac{1}{3}\right\}. \tag{5.25}$$

As an initial configuration we take a polytropic star of index $n = 1.5 - 3$. Thus we have three independent parameters η, n, and γ, which characterize our problem. If $\gamma > 1 + 1/n$, the entropy inside a star grows outward and the star is convectively stable. If $\gamma = 1 + 1/n$, the entropy is constant throughout the star and thus the star is neutrally stable with respect to convection. If $\gamma < 1 + 1/n$ the star is convectively unstable. The latter case is excluded from our consideration. In what follows, we use units in which $G = M_* = R_* = 1$ and present our results in nondimensional form. The time unit then corresponds approximately to the time for sound to cross the star. Time, density, energy, and angular momentum have to be multiplied by $(R_*^3/GM_*)^{1/2}$, M_*/R_*^3, GM_*^2/R_*, and $(GM_*^2R_*)^{1/2}$, respectively, to transform them to ordinary units.

Results for weak tidal encounters We have computed the tidal interaction of $n = 1.5, 2$ and 3 polytropic stars with $\gamma = 5/3$ for various values of η in the range from 01. to 3.5. In addition we have computed a few cases of $n = 2$, $\gamma = 1.5$ encounters. In this subsection we will discuss only cases in which the tidal interaction does not lead to disruption or stripping of the star (Khokhlov, Novikov and Pethick 1993a). Close encounters will be considered in the next subsection.

In the case of rather far encounters the result of tidal interaction can be roughly described as excitation of nonradial modes of oscillations. For closer encounters nonlinear effects are important. In our computations we observed very complicated and interesting motions of stellar matter.

The integral quantities important for various applications are the total energy E_{tot} and angular momentum L deposited in a star a during the encounter. During the encounter the variations of both the total energy and the angular momentum are nonmonotonic. The growth of E_{tot} and L occurs after passage of the pericenter when the lag between the tides and the direction to the black hole is maximum. Later on, the relative orientation of the tidal waves with respect to the black hole changes, and the tidal interaction gives rise to decreases in both the energy and the angular momentum. We note that the angular momentum acquired by a star does not correspond to its apparent rotation, which just reflects the propagation of tidally excited waves.

In the outer layers strong nonlinear hydrodynamical effects are observed. The apparent rotation of the outer layers is slower than of the inner parts of a star. This leads to the formation of interesting hole-like structures.

For encounters with $\eta \leq 3.5$, nonlinear effects lead to an increase of the energy transfer by a factor of 2 or more compared with the prediction of the linear theory.

Results for strong tidal encounters In this subsection we consider close encounters that lead to stripping or disruption of a star by tidal forces (Khokhlov, Novikov and Pethick, 1993b).

Let us consider an encounter of an $n = 2$, $\gamma = 3/5$ star with a black hole for $\eta = 1.5$.

Qualitatively the hydrodynamics of the interaction with the black hole is the same as for most distant encounters, but the deformations of the star are much greater. During the computations ≈15% of the mass flows out of the grid with roughly the parabolic velocity. This outflow occurs at the low-density edges of the tidal lobes. We expect this matter to become unbound and to be lost from the star. For the rest of the star, the hydrodynamical behavior is much the same as for the case of more distant encounters. After the encounter, the central density, $\tilde{\rho}$, averaged over several pulsations, drops to $\sim 0.55\rho_0$, where ρ_0 is the central density of the initial model. The numerical period of the model is $\prod \approx 4.5$, whereas the period of the radial f-mode, predicted by linear theory is 3.14. If one scales this period by the factor $(\rho_0/\tilde{\rho})^{1/2}$ to allow approximately for some of the nonlinear effects, one finds $\prod_{corr} = 4.23$, which is rather close to the period seen in the computations. In the previous subsection we found that for distant encounters where deformations are small, the numerical periods of the central density variations coincide with the period $l = 0$ fundamental mode calculated in linear theory. For close encounters these

Table 2.3: Dependence of η_{crit} on polytropic index n.

n	η_{crit}^{linear}	$\eta_{crit}^{numerical}$
3....................	0.4-0.5	0.5-0.6
2....................	0.7	1.1
1.5..................	1.0	1.5-1.9

periods are different, but even for the case $n = 2$, $\eta = 1.5$ considered here, the difference is only 6%. We conclude that the fundamental radial mode is excited here, just as it is in the case of more distant encounters.

The computations show that the energy transferred to the star would be $\Delta E \approx 0.15$,which is less than the binding energy of the initial stellar model, $|E_{tot}| = 0.5$. The final value of the angular momentum is $L \approx 0.1$.

The next encounter, for $n = 3$, $\gamma = 5/3$, and $\eta = 1$, was was computed on a moving grid to keep the expanding matter inside the computational domain. The hydrodynamical picture is qualitatively similar to that discussed above. In the outer layers strong nonlinear effects lead to stripping of material. Two lobes of outflowing material are now clearly seen on the expanding grid, but they contain only a few percent of the mass of the star.

After the encounter the total energy tends to a constant value. The amount of energy transferred to the star is $\Delta E \approx 0.16$, which is less than the total energy of the initial model $|E_{tot}| \approx 0.749$. The star is not totally disrupted in this case, and after the encounter the angular momentum transferred to the star is $L \approx 0.1$.

Results for $n = 3$, $\gamma = 5/3$, and $\eta = 0.5$ case show that stripping is much more pronounced than in the previous case. At the end of the computations the binding energy is positive and continues to increase. The angular momentum at the end of the computations is $L \approx 0.75$ and this increases too. The fact that the total energy becomes positive indicates that the star may be disrupted. However, computations show that the central condensed body survives. Moreover, the central density tends to a constant value. Both of these facts indicate that even though the energy transferred exceeds the binding energy of the star, the inner parts of the star may remain bound.

In Table 3 we give the extrapolated critical values of η, η_{crit}, at which the energy deposited is equal to the binding energy of the initial star. The estimates of η_{crit} from our calculations are systematically larger than those based on the linear theory, and the deviations tend to increase with decreasing polytropic index, n. Evans and Kochanek (1989) have computed the $\eta = 1$ encounter of the $n = 1.5$ polytrope and find that the star is totally disrupted. This agrees with our conclusion that for polytropic stars with $n = 1.5$ the critical value of η is greater than one.

Thus one can conclude that stripping of a star takes place for η larger than η_{crit} (see Rasio and Shapiro (1991)). For $\eta \leq \eta_{crit}$ a star may not be disrupted completely but the innermost parts of the star may form a bound configuration. In other words, part of the star may survive when the energy deposited exceeds the initial binding energy of the star.

2.5.3 Relativistic encounters

Tidal interaction of stars with a nonrotating black hole In this subsection we are interested in the case when relativistic effects of the black hole gravity are important. This is the case when the pericentric distance from a black hole R_p becomes comparable with the black hole radius, $r_g = 2GM_h/c^2$, $r_g/R_p \simeq 1$. In addition, we are interested in stellar encounters when the tidal acceleration at the pericenter is comparable with the acceleration due to self-gravity at the stellar surface, $GM_hR_*/R_p^3 \simeq GM_*/R_*^2$, where R_*, and M_* are the stellar radius and mass respectively. Combining these two conditions, we can express the mass of the black hole as a function of the radius and mass of the star

$$M_h \simeq 10^8 M_\odot \left(\frac{R_*}{R_\odot}\right)^{3/2} \left(\frac{M_*}{R_\odot}\right)^{-1/2}. \tag{5.26}$$

For this black hole mass both relativistic effects during tidal interaction are important, and tidal and self-gravity forces at the stellar surface are comparable. For stars with $M_* \simeq M_\odot$, $R_* \simeq R_\odot$ this formula gives $M_h \simeq 10^8 M_\odot$. For more compact stars with $M_* \simeq M_\odot$, relativistic tidal interaction of moderate strength would occur during encounters with black holes of smaller mass. Laguna et al. (1993) considered relativistic encounters of a polytropic stellar model with a massive black hole in the case when the tidal force substantially exceeds the acceleration due to self-gravity at the stellar surface (for their method see Laguna, Miller and Zurek, 1993).

In this subsection we consider the tidal interaction of a relativistic white dwarf model with a massive black hole Frolov, Khokhlov, Novikov and Pethick (1994). For white dwarfs (WDs), the stellar radius has a size $R_* = R_{wd} \simeq 2 \times 10^8 - 10^9$cm, and relativistic effects become important during a moderate tidal interaction when the black hole mass is $M_h \simeq 10^4 - 10^5 M_\odot$. A WD orbiting around a black hole can be source of gravitational radiation in the frequency range $\simeq 1\text{s}^{-1}$ that is important for future gravitational wave projects.

Following the approach of previous section, we consider here parabolic encounters of a carbon-oxygen WD of mass $0.6M_\odot$ with a $10^4 M_\odot$ black hole, analyze the hydrodynamics of the WD matter, study the deposition of energy and angular momentum in th WD and determine the critical conditions for tidal disruption. We want to understand how close the WD can approach a black hole without being destroyed.

About relativistic basic equation see [7]. We introduce the following notations: $r_g = 2GM_h/c^2$, r is the radial coordinate in the Schwarzschild metric, τ is the proper time of the star, $\widetilde{E}$ and $\widetilde{L}$ are dimensionless integrals of motion which are connected with the total energy E and angular momentum L by the relations

$$\widetilde{E} = E/M_*c^2, \qquad \widetilde{L} = L/M_*cr_\theta, \tag{5.27}$$

and M_* is the mass of the star. In what follows we consider parabolic motion for which $\widetilde{E} = 1$, so trajectories are characterized by one dimensionless constant $\widetilde{L}$. It is convenient to introduce also the following notation:

$$R_p = \widetilde{L}^2 r_\theta, \qquad t_p = \widetilde{L}^3 r_\theta / c, \qquad \mu = \widetilde{L}^{-1} = (r_g/R_p)^{1/2}, \tag{5.28}$$

$$\tau^* = \tau/t_p, \qquad x(\tau^*) = r(\tau)/R_p, \qquad t^* = t/t_p, \tag{5.29}$$

and

$$r_p = R_p x_p, \tag{5.30}$$

where

$$x_p = \frac{1}{2}(1 + \sqrt{1 - 4\mu^2}). \tag{5.31}$$

Relativistic effects influence the trajectory of a star and the tensor structure of the tidal forces. To clarify these effects, we compare the motion of a star with the parabolic velocity in relativistic and nonrelativistic theories. We hold fixed the same conserved quantity in the two problems (relativistic and nonrelativistic) - the angular momentum of the star, which unambiguously fixes the trajectory for the parabolic motion. The following four relativistic effects are important:

1. Relativistic shift of the pericenter distance,
2. Relativistic time delay,
3. Relativistic precession, and
4. Tensorial structure of tidal forces.

1. The difference Δx_p between x_p and its nonrelativistic limit, $x_{p,nr} = 1$ is

$$\Delta x_p = -\frac{1}{2}(1 + \sqrt{1 - 4\mu^2}). \tag{5.32}$$

For small μ this difference is $\Delta x_p \simeq -\mu^2$. In other words the radial coordinate of a relativistic pericenter is smaller than the nonrelativistic pericenter distance, R_p, by an amount $\Delta R_p = R_p \Delta x_p \simeq r_g$.

2. Let us denote nonrelativistic limit (that is when $c \to \infty$) τ^* by τ^*_{nr}. The difference

$$\Delta\tau^* \equiv \tau^* - \tau^*_{nr} \tag{5.33}$$

remains finite in the limit $r \to \infty$. In this limit,

$$\Delta\tau^*(r \to \infty) \equiv \tau^*_\infty = \frac{2}{3\sqrt{1 + 2\mu}}[(1 - \mu^2)K - 1(1 + 2\mu)E], \tag{5.34}$$

where $K = K(m)$ and $E = E(m)$ are the complete elliptic integrals of the first and second kind, respectively, and

$$m = \frac{4\mu}{1 + 2\mu}. \tag{5.35}$$

For small μ,

$$\Delta\tau^*(r\to\infty)\simeq\frac{3\pi}{8}\mu^4. \tag{5.36}$$

In other words, in the relativistic case it takes a longer time than in the nonrelativistic case for a body with the same angular momentum to pass near a black hole and return to the same initial radius. The time delay $\Delta T=2t_p\Delta\tau^*$ is

$$\Delta T\simeq\frac{3\pi}{4}\left(\frac{r_g}{R_p}\right)^{1/2}\frac{r_g}{c}. \tag{5.37}$$

3. By relativistic precession we mean the following effect. Let a nonrotating rigid body have three orthogonal axes rigidly attached to it. The orientation of these axes in space, after a test body passes near a black hole and goes away, will differ slightly from their initial orientations. The total precession angel is

$$\Delta\phi_{prec}=\frac{2}{\sqrt{1+2\mu}}K-\pi. \tag{5.38}$$

For small μ one has

$$\Delta\phi_{prec}\simeq\frac{3\pi}{4}\frac{r_g}{R_p}=\frac{3\pi}{4}\mu^2. \tag{5.39}$$

4. The tensorial structure of tidal forces is different in the relativistic and nonrelativistic cases (details see in Frolov, Khokhlov, Novikov and Pethick 1994). The extra term of the order $3\mu^2/x^2$ which enters the relativistic relations describes relativistic effects. These effects lead to an additional increase of the tidal acceleration in the $\tilde{1}-\tilde{2}$ plane, without changing the acceleration in the perpendicular $\tilde{3}$ direction. The maximum effect is reached for the limiting value of $\mu=1/2$, when at the pericenter the star is stretched in the radial direction with a strength 5/2 times larger, and compresses in the orthogonal direction with a strength 4 times larger than that in the nonrelativistic case.

Now we consider a WD of mass M_{wd} moving on a parabolic orbit around a black hole of mass M_h. As already mentioned above, the trajectory is uniquely defined by specifying the black hole mass M_h and angular momentum of the orbital motion L. The parameters η and μ are uniquely related to the black hole mass and orbital angular momentum:

$$\mu=\frac{2GM_*}{c}\frac{L}{M_h}, \tag{5.40}$$

$$\eta=\frac{1}{\sqrt{2GM_*^{5/3}R_*}}\frac{L}{M_h^{2/3}}. \tag{5.41}$$

Table 2.4: E_{tot} and ΔL values

N	μ	η	$\frac{L}{10^5 3\text{g cm}^2\text{s}^{-1}}$	$\frac{E_{tot}}{10^{49}\text{ergs}}$	$\frac{\Delta L}{10^{50}\text{g cm}^2\text{s}^{-1}}$
1.........	0.267	2.51	3.8	-0.67	3.2
2.........	0.283	2.14	3.6	3.2	9.7
3.........	0.300	1.76	3.4	6.4	14.6
4.........	0.316	1.50	3.2	7.6	15.7
5.........	0.000	1.50	3.2	3.5	9.7

The nonrelativistic limit correspond to $c \to \infty$. In this limit $\mu \to 0$ according to Eq.(40), while Eq.(41) corresponds to encounters with different L and M, but with a fixed strength of the interaction of the pericenter.

We describe both the hydrodynamics of the WD and its self-gravity in the framework of Newtonian physics. The WD is subjected to the action of the relativistic tidal acceleration field.

Computations begin on a stationary grid. When the expanding star reaches the boundary, the grid begins to expand uniformly, in order to keep most of the stellar matter inside the computational domain. The equation of state of the WD matter in tabular form takes the account the contributions from the ideal Fermi-Dirac gas of electrons and positrons with arbitrary degeneracy and degree of relativity, ions, and equilibrium Planck radiation.

The WD mass is taken to be $M_{wd} = 0.64 M_{\odot}$, its temperature to be 10^8K, and its composition to be equal amounts of C and O by mass. With our equation of state, the radius of the WD is $R_{wd} = 8.41 \times 10^8$cm, the central density is $\rho_c = 4.0 \times 10^6$g cm^{-3}, the central pressure $P_c = 2.2 \times 10^{23}$ ergs cm $^{-3}$ the gravitational energy is $E_g = -1.15 \times 10^{50}$ergs, the thermal energy is $E_t = 6.49 \times 10^{49}$ergs, and the total energy is $E_{tot} = -5.02 \times 10^{49}$ergs. With this central density, the WD is practically nonrelativistic and is close to an $n = 1.5$ polytrope. We start the integration when the radial coordinate of the WD, r, is $r = 2r_p$.

We computed five encounters of the WD with a black hole of mass $M_h = 8.94 \times 10^3 M_{\odot}$. The main parameters of the encounters are given in Table 4. The encounters N1 to N4 are relativistic, and are characterized by different orbital angular momenta L. The nonrelativistic encounter N5 was computed for comparison.

The encounter N1 during which the star partially survives, is nondisruptive. The encounters N2 to N4 are disruptive.

In Table 4 we give the total energy of the WD, E_{tot}, after the encounter and the deposited angular momentum ΔL.

In the nondisruptive case stripping of matter takes place. In the surface layers of the surviving core, complicated hydrodynamical phenomena are revealed. In both disruptive and nondisruptive encounter material flows put in the form of two thin S-shaped, supersonic jets. Our results provide realistic initial conditions for the subsequent investigation of the dynamic of the debris in the field of the black hole (Diener, Frolov, Khokhlov, Novikov and Pethick,

Table 2.5: The computed encounters

Model	α	L_z	Q	θ_p	M_*	R_*	η_r
1	1.0	3.33333	0.0	$(\pi/2)$	1.0	1.0	3.1517
2	1.0	3.15955	0.0	$(\pi/2)$	1.0	1.0	2.6539
3	1.0	3.00491	0.0	$(\pi/2)$	1.0	1.0	2.2566
4	1.0	2.85255	0.0	$(\pi/2)$	1.0	1.0	1.9051
5	1.0	2.72945	0.0	$(\pi/2)$	1.0	1.0	1.6486
6	-1.0	3.33333	0.0	$(\pi/2)$	1.0	1.0	2.8116
7	-1.0	3.15955	0.0	$(\pi/2)$	1.0	1.0	2.3038
8	1.0	0.0	11.1111	π	1.0	1.0	3.0014
9	1.0	0.0	9.98276	π	1.0	1.0	2.5018
10	1.0	0.0	11.1111	$(\pi/2)$	1.0	1.0	3.0014
11	1.0	0.0	9.98276	$(\pi/2)$	1.0	1.0	2.5018
12	0.5	3.34725	0.0	$(\pi/2)$	1.0	1.0	3.1224
13	0.5	2.14139	5.0	1.2	1.0	1.0	2.3891
14	1.0	2.5	0.0	$(\pi/2)$	2.0	1.5	2.4

1998).

We evaluated the critical conditions for the complete disruption of the white dwarf. The critical angular momentum of the orbital motion is estimated as $L_{crit} \simeq 3.7 \times 10^{53}$ g cm^2 s^{-1}. The corresponding critical η is $\eta_{crit} \simeq 2.0$. Comparison was done with the corresponding nonrelativistic encounters. The relativistic effects lead to more strong and complicated tidal interaction.

2.5.4 Tidal interaction of stars with a rotating black hole

The tidal interaction of $n = 1.5$ polytropic stars with a massive rotating black hole (BH) was studied numerically in Diener, Frolov, Khokhlov, Novikov and Pethick, 1997. The general relativistic tidal potential for the Kerr metric was used to evaluate tidal forces exerted on a star. The hydrodynamic response of a star to these forces was treated in the Newtonian approximation. In Diener, Frolov, Khokhlov, Novikov and Pethick, 1997 we have computed the energy, ΔE, and angular momentum, ΔL, transferred into a star, and the mass, ΔM, lost by the star during the interaction.

We have performed computations for encounters of different strength of the interaction, and for different values of the BH rotation parameter α. The parameters of computed encounters are listed in Table 5. Detail description of all used parameters see in Diener, Frolov, Khokhlov, Novikov and Pethick, 1997.

We choose the z-component of the orbital angular momentum, $\widetilde{L}_z$, Carter's fourth constant, $\widetilde{Q}$, and the polar angle at the pericenter, θ_p, to characterize the orbit. Next we choose M_* and R_* to characterize the initial star. The BH mass was selected in most cases to be $M_h = 1.0853 \times 10^7 M_\odot$. The only exception is model 14 in Table 3 for which $M_h = 2.7455 \times 10^7 M_\odot$. In

Table 2.6: The results of the numerical simulations

Model	$\Delta E/E_B$	$\Delta M_*/M_*$	$L/(GM_*^3R_*)^{1/2}$
1	0.027	0.0	0.0159
2	0.127	0.0035	0.0563
3	0.344	0.049	0.116
4	0.648	0.165	0.149
5	0.876	0.32	0.0993
6	0.110	0.0053	0.0562
7	0.475	0.1	0.127
8	0.057	0.0002	0.0294
9	0.238	0.023	0.0860
10	0.060	0.0005	0.0306
11	0.258	0.03	0.0943
12	0.034	0.0	0.0194
13	0.284	0.036	0.0993
14	0.300	0.035	0.0931

Frolov, Khokhlov, Novikov and Pethick, 1994 a relativistic parameter $\eta_r = \eta(x_p/x_p^{nr})^{3/2}$, was introduced to characterize the strength of the encounter. It takes into account the decrease of the radial coordinate at the pericenter due to relativistic effects in the case of a non-rotating BH. Here $\eta = (M_*/M_h)^{1/2}(R_p/R_*)^{3/2}$ characterizes the strength of the encounter in the Newtonian case and $x_p^{nr} = \tilde{L}_z^2 + \tilde{Q}$ is the pericentric distance for parabolic orbits in units of r_g in the non-relativistic case. In the general case of a rotating BH, the strength of the interaction depends on all parameters of the orbit, but the parameter η_r gives a rough idea of the strength of the interaction. With decreasing pericentric distance the strength of the interaction increases while η_r decreases. This parameter η is listed in Table 5 for the purpose of comparison.

Most of the computations were performed for an extremely rotating BH, $\alpha = 1$, and some for the BH with half the extremal rotation, $\alpha = 0.5$, (see column 2 of Table 5). Models 1 to 7 are encounters with the orbit lying in the equatorial plane of the BH. Encounters 1 to 5 were prograde, while 6 and 7 were retrograde. For the encounters 8 to 11, the initial angular momentum of the orbit was orthogonal to the angular momentum of the BH. These orbits are not planar. For encounters 8 and 9, the pericenter of the orbit is located at the axis of the BH rotation. For encounters 10 and 11 the pericenter is located in the equatorial plane of the BH. Encounter 12 is a prograde encounter with the orbit lying in the equatorial plane. For encounter 13, the pericenter is located neither in the equatorial plane nor on the axis of the BH rotation. Encounters 12 and 13 are for a rotating BH with half the extremal rotation. Encounter 14 is in the equatorial plane and prograde. In this case the stellar mass and radius are $M_* = 2M_\odot$ and $R_* = 1.5R_\odot$ respectively.

The energy and angular momentum transfer and the mass loss for the computed encounters are listed in Table 6.

The quantities ΔE, ΔL and ΔM depend on the stellar orbit, stellar structure, and the black hole's mass and angular momentum in a complicated way.

We show in Diener, Frolov, Khokhlov, Novikov and Pethick (1997) that the dependence can be factorized by introducing a single dimensionless parameter $\widehat{C}$ proportional to the integral of the square of the trace of the tidal tensor along the stellar trajectory. The energy and angular momentum transfer, and the mass loss as functions of $\widehat{C}$ are found in hydrodynamical simulations. Analytical approximations to $\Delta E(\widehat{C})$ and $\Delta M(\widehat{C})$ are constructed. The value of $\widehat{C}$ does not depend on the stellar structure. It is a universal function on the parameters of the orbit, and can tabulated once and for all. Tables of $\widehat{C}$ are presented in Diener, Frolov, Khokhlov, Novikov and Pethick (1997).

The results of Diener, Frolov, Khokhlov, Novikov and Pethick (1997) allow one to easily determine the outcome of tidal interaction for every possible combination of the input parameters. We find that the final energy of a star or a stellar remnant (if mass is lost) and its internal angular momentum as well depend most strongly on the angle between the initial orbital angular momentum and the angular momentum of the black hole.

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